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GROUP DECISION MAKING PROCEDURE BASED ON TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS: MULTIMOORA METHODOLOGY

***Abstract**: This paper proposes a group multi-criteria decision making approach based on MULTIMOORA method and trapezoidal intuitionistic fuzzy numbers (ITFNs). Specifically, the definition of ITFN by Nehi and Maleki (2005) is followed. The proposed approach relies on the trapezoidal intuitionistic fuzzy power aggregation operators, which reduce the impact of biased assessments in the group decision making. An illustrative example is provided to demonstrate the operationality of the proposed methodology.*

***Keywords**: multi-criteria decision making, trapezoidal intuitionistic fuzzy number, MULTIMOORA.*

JEL Classification: C44, C61, D81

1. Introduction

Multi-criteria decision making (MCDM) aims at comparison of alternatives in terms of multiple conflicting criteria. As the real world decision making involves imprecision, uncertainty, ambiguity, and vagueness, modelling such phenomena requires appropriate techniques. In particular, it is important to represent and aggregate the decision variables with minimal loss of information. The fuzzy set theory (Zadeh, 1965) was introduced to model the uncertain phenomena and became the basis for further extensions. See Mardani et al. (2015) for a survey on development of the fuzzy MCDM methodology.

The traditional fuzzy sets allow to assign the elements thereof with the degrees of membership. Atanassov (1986) suggested intuitionistic fuzzy sets (IFSs) which can be considered as a generalization of the fuzzy sets. Specifically, an IFS allows to define not only the degree of membership, but also that of non-membership to a set. Subsequently, the degree of hesitancy can be taken into account. A particular case of IFS is intuitionistic fuzzy number (IFN), which can be ordinary, interval-valued, triangular, trapezoidal etc. IFNs have been employed in various instances of decision support systems (Wan, Dong, 2014), including MCDM (Wan et al., 2013; Zavadskas et al., 2014) and data envelopment analysis (Puri, Yadav, 2015).

Intuitionistic trapezoidal fuzzy numbers (ITFNs) allow for high flexibility in defining the membership and non-membership functions. The latter concept was offered by Nehi and Maleki (2005) and subsequently employed in many studies on MCDM. Ye (2011) proposed the expected value method for ITFNs. Ye (2012a, 2012b) proposed some similarity measures for ITFNs. Li and Chen (2015) and Wan and Dong (2015) developed MCDM techniques based upon ITFNs. Wan and Dong (2015) defined the power geometric operators for aggregation of ITFNs.

Among the existing MCDM techniques, the Multi-objective Optimization by Ratio Analysis (MOORA) is distinctive with vector normalization and both compensatory and non-compensatory aggregation. MOORA was introduced by Brauers and Zavadskas (2006), and updated with Full Multiplicative Form thus becoming known as MULTIMOORA (Brauers, Zavadskas, 2010). Brauers and Zavadskas (2011) also offered Dominance Theory to aggregate the ranks resulting from the three parts of MULTIMOORA.

The MULTIMOORA method has been updated to handle different types of information and applied for decision making in various fields (Baležentis, Baležentis, 2014). Deliktas and Ustun (2015) applied fuzzy MULTIMOORA to student selection, whereas Mishra et al. (2015) employed the latter technique to supplier selection. Liu et al. (2014, 2015) applied various extensions of MULTIMOORA to healthcare waste treatment planning. Zavadskas et al. (2015) extended MULTIMOORA into interval intuitionistic fuzzy environment.

Notably, Chen and Li (2014) proposed MULTIMOORA updated with ITFNs, as defined by Wang (2008). In this paper, we propose MULTIMOORA based upon ITFNs, as defined by Nehi and Maleki (2005). Therefore, different approaches towards representation of the fuzzy data can be taken by utilizing different techniques.

The paper proceeds as follows: Section 2 brings the preliminaries for intuitionistic trapezoidal fuzzy numbers. Section 3 presents the intuitionistic trapezoidal fuzzy power operators. Section 4 presents the proposed approach, MULTIMOORA-ITFN. Finally, a numerical example is given in Section 5.

2. Preliminaries for ITFNs

This section describes the preliminaries for intuitionistic fuzzy sets, fuzzy numbers, intuitionistic fuzzy numbers, and intuitionistic trapezoidal fuzzy numbers. This section is mainly based on Ye (2011).

Definition 1. Let X be a universe of discourse. Then an intuitionsitic fuzzy set A in X is defined by (Atanassov, 1986):

$$A = \left\{ \left\langle x, \mu_A(x), v_A(x) \right\rangle \middle| x \in X \right\}, \tag{1}$$

where $\mu_A(x): X \to [0,1]$ and $v_A(x): X \to [0,1]$ such that $0 \le \mu_A(x) + v_A(x) \le 1$. The functions $\mu_A(x)$ and $v_A(x)$ are, respectively, membership and non-membership functions. These two functions define the degrees of membership and non-membership of each element x in X to the intuitionistic fuzzy set A. Furthermore, for each A in X, one can estimate the intuitionistic index of the element x in the set A:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x),\ x \in X, \tag{2}$$

where $\pi_A(x)$ is a hesitancy degree of x to A with $0 \le \pi_A(x) \le 1, x \in X$.

Definition 2. Let A be a fuzzy number in the set of the real numbers, R, with its membership function (Dubois, Prade, 1978) defined as

$$\mu_A(x) = \begin{cases} 0, & x < a_1; \\ f_A(x), & a_1 \le x \le a_2; \\ 1, & a_2 \le x \le a_3; \\ g_A(x), & a_3 \le x \le a_4; \\ 0, & a_4 < x; \end{cases} \tag{3}$$

where $a_1, a_2, a_3, a_4 \in R$; $f_A : [a_1, a_2] \to [0,1]$ is a non-decreasing continuous function with $f_A(a_1) = 0$, $f_A(a_2) = 1$ called the left side of the fuzzy number A; and $g_A : [a_3, a_4] \to [0,1]$ is a non-increasing continuous function with $g_A(a_3) = 1$, $g_A(a_4) = 0$ called the right side of the fuzzy number A.

Definition 3. Let A be an intuitionistic fuzzy number (Grzegorzewski, 2003) in the set of the real numbers, R, with its membership function defined as

$$\mu_A(x)=\begin{cases}0, & x<a_1;\\ f_A(x), & a_1\le x\le a_2;\\ 1, & a_2\le x\le a_3;\\ g_A(x), & a_3\le x\le a_4;\\ 0, & a_4<x;\end{cases} \quad (4)$$

and its non-membership function defined as

$$v_A(x)=\begin{cases}1, & x<b_1;\\ h_A(x), & b_1\le x\le b_2;\\ 0, & b_2\le x\le b_3;\\ k_A(x), & b_3\le x\le b_4;\\ 1, & b_4<x;\end{cases} \quad (5)$$

where $0\le \mu_A(x)+v_A(x)\le 1$ and $a_1,a_2,a_3,a_4,b_1,b_2,b_3,b_4\in R$ such that $b_1\le a_1\le b_2\le a_2\le a_3\le b_3\le a_4\le b_4$; and the four functions $f_A,g_A,h_A,k_A:R\to[0,1]$ are called the side functions of a fuzzy number. The functions f_A and k_A are non-decreasing continuous functions, whereas functions h_A and g_A are non-increasing continuous functions.

Noteworthy, each intuitionistic fuzzy number $A=\{\langle x,\mu_A(x),v_A(x)\rangle | x\in X\}$ is a conjunction of two fuzzy numbers: A^+ with a membership function $\mu_{A^+}(x)=\mu_A(x)$ and A^- with a membership function $\mu_{A^-}(x)=1-v_A(x)$. Note that $SupA^+\subseteq SupA^-$ (Ye, 2011).

Definition 4. An intuitionistic trapezoidal fuzzy number A with parameters $b_1\le a_1\le b_2\le a_2\le a_3\le b_3\le a_4\le b_4$ is denoted as $A=\langle (a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4)\rangle$ in the set of real numbers R (Nehi and Maleki, 2005). Its membership and non-membership functions, therefore, can be given as

$$\mu_A(x) = \begin{cases} 0, & x < a_1; \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2; \\ 1, & a_2 \le x \le a_3; \\ \dfrac{x-a_4}{a_3-a_4}, & a_3 \le x \le a_4; \\ 0, & a_4 < x; \end{cases} \tag{6}$$

$$v_A(x) = \begin{cases} 1, & x < b_1; \\ \dfrac{x-b_2}{b_1-b_2} & b_1 \le x \le b_2; \\ 0, & b_2 \le x \le b_3; \\ \dfrac{x-b_3}{b_4-b_3}, & b_3 \le x \le b_4; \\ 1, & b_4 < x. \end{cases} \tag{7}$$

In case $b_2 = b_3$, and, hence, $a_2 = a_3$, a trapezoidal intuitionistic fuzzy number is reduced to a triangular intuitionistic fuzzy number.

Nehi and Maleki (2005) defined the following arithmetic operations on the intuitionistic trapezoidal fuzzy numbers. Let $A_1 = \langle (a_{11}, a_{12}, a_{13}, a_{14}), (b_{11}, b_{12}, b_{13}, b_{14}) \rangle$ and $A_2 = \langle (a_{21}, a_{22}, a_{23}, a_{24}), (b_{21}, b_{22}, b_{23}, b_{24}) \rangle$ be the two intuitionistic trapezoidal fuzzy numbers and *r* be a positive scalar number. Then,

$$A_1 + A_2 = \left\langle \begin{matrix} (a_{11}+a_{21}, a_{12}+a_{22}, a_{13}+a_{23}, a_{14}+a_{24}), \\ (b_{11}+b_{21}, b_{12}+b_{22}, b_{13}+b_{23}, b_{14}+b_{24}) \end{matrix} \right\rangle, \tag{8}$$

$$A_1 - A_2 = \left\langle \begin{matrix} (a_{11}-a_{24}, a_{12}-a_{23}, a_{13}-a_{22}, a_{14}-a_{21}), \\ (b_{11}-b_{24}, b_{12}-b_{23}, b_{13}-b_{22}, b_{14}-b_{21}) \end{matrix} \right\rangle, \tag{9}$$

$$A_1 \cdot A_2 = \left\langle \begin{matrix} (a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}, a_{14}a_{24}), \\ (b_{11}b_{21}, b_{12}b_{22}, b_{13}b_{23}, b_{14}b_{24}) \end{matrix} \right\rangle, \tag{10}$$

$$A_1 \div A_2 = \left\langle \begin{matrix} (a_{11}/a_{24}, a_{12}/a_{23}, a_{13}/a_{22}, a_{14}/a_{21}), \\ (b_{11}/b_{24}, b_{12}/b_{23}, b_{13}/b_{22}, b_{14}/b_{21}) \end{matrix} \right\rangle, \tag{11}$$

$$rA_1 = \langle (ra_{11}, ra_{12}, ra_{13}, ra_{14}), (rb_{11}, rb_{12}, rb_{13}, rb_{14}) \rangle . \tag{12}$$

Furthermore, the Euclidean distance between A_1 and A_2 can be computed as (Ye, 2012a):

$$d_E(A_1, A_2) = \sqrt{\frac{1}{8}\left(\sum_{p=1}^{4}(a_{1p} - a_{2p})^2 + \sum_{q=1}^{4}(b_{1q} - b_{2q})^2\right)}. \tag{13}$$

The distance measures can be employed to derive a similarity measures. In case $0 \le b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4 \le 1$, one can derive a similarity measure, viz. the support of A_1 by A_2, in the following way:

$$Supp(A_1, A_2) = 1 - d_E(A_1, A_2), \tag{14}$$

where $\mathrm{Supp}(A_1, A_2) = 1$ if and only if $A_1 = A_2$; otherwise we have $0 \le \mathrm{Supp}(A_1, A_2) \le 1$.

The comparison of the two intuitionistic trapezoidal fuzzy numbers, A_1 and A_2, can be carried out by computing their respective expected values (EV):

$$EV(A_1) = \frac{1}{8}\left(\sum_{p=1}^{4} a_{1p} + \sum_{q=1}^{4} b_{1q}\right), \tag{15}$$

with greater EV implying superiority of a certain intuitionistic trapezoidal fuzzy number.

3. Trapezoidal intuitionistic fuzzy power aggregation operators

Group decision making requires imputation of the expert weights. These weights can be obtained in the spirit of the ordered weighted average (OWA), which was introduced by Yager (1988). The method assigns lower significance for more biased ratings. Therefore, group decision making becomes more robust in the sense of expert rating aggregation.

The power average (PA) operator (Yager, 2001) accounts for the degree of discrepancy among the elements to be aggregated by involving the support measures into the computation. Wan (2013) offered power average operator, the weighted power average operator, the power ordered weighted average operator, and the power hybrid average operator of trapezoidal intuitionistic fuzzy numbers. This section presents a trapezoidal intuitionistic fuzzy power ordered weighted average (TIFPOWA) operator and a trapezoidal intuitionistic fuzzy power ordered weighted geometric (TIFPOWG) operator.

Let there be a collection of ITFNs,
$A_j = \langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}), (b_{j1}, b_{j2}, b_{j3}, b_{j4}) \rangle$, where $j = 1, 2, ..., n$; and the related

the ordered set of ITFNs,
$A_{(j)} = \langle (a_{(j)1}, a_{(j)2}, a_{(j)3}, a_{(j)4}), (b_{(j)1}, b_{(j)2}, b_{(j)3}, b_{(j)4}) \rangle$ with $A_{(j-1)} \geq A_{(j)}$ for $j = 2, 3, ..., n$. Then the TIFPOWA aggregates a set of ITFNs into a single ITFN in the following way:

$$TIFPOWA(A_1, A_2, ..., A_n) = \sum_{j=1}^{n} w_j A_{(j)}$$
$$= \left\langle \left(\sum_{j=1}^{n} w_j a_{(j)1}, \sum_{j=1}^{n} w_j a_{(j)2}, \sum_{j=1}^{n} w_j a_{(j)3}, \sum_{j=1}^{n} w_j a_{(j)4} \right), \left(\sum_{j=1}^{n} w_j b_{(j)1}, \sum_{j=1}^{n} w_j b_{(j)2}, \sum_{j=1}^{n} w_j b_{(j)3}, \sum_{j=1}^{n} w_j b_{(j)4} \right) \right\rangle \quad (16)$$

where $w_j = (w_1, w_2, ..., w_l, ..., w_n)$ is a set of weights such that

$$w_l = g\left(\frac{D_l}{TV}\right) - g\left(\frac{D_{l-1}}{TV}\right), \quad D_l = \sum_{j=1}^{l} V_{(j)}, \quad TV = \sum_{j=1}^{n} V_{(j)}, \quad V_{(j)} = 1 + T(A_{(j)}) \quad (17)$$

with $T(A_{(j)})$ being the support of the *j*-th largest ITFN by all the other ITFNs:

$$T(A_{(l)}) = \sum_{\substack{j=1 \\ j \neq l}}^{n} Sup\left(A_{(l)}, A_{(j)}\right), \quad (18)$$

where $Sup\left(A_{(l)}, A_{(j)}\right)$ is support of the *l*-th largest ITFN by the *j*-th largest ITFN, and $g : [0,1] \rightarrow [0,1]$ is a basic unit-interval monotonic (BUM) function. The following properties of BUM functions are valid: 1) $g(0) = 0$, 2) $g(1) = 1$, and 3) $g(x) \geq g(y)$, if $x > y$.

The TIFPOWG operator aggregates
$A_j = \langle (a_{j1}, a_{j2}, a_{j3}, a_{j4}), (b_{j1}, b_{j2}, b_{j3}, b_{j4}) \rangle$, $j = 1, 2, ..., n$, into a single ITFN as follows:

$$TIFPOWG(A_1, A_2, ..., A_n) = \prod_{j=1}^{n} (A_{(j)})^{w_j}$$
$$= \left\langle \left(\prod_{j=1}^{n} (a_{j1})^{w_j}, \prod_{j=1}^{n} (a_{j2})^{w_j}, \prod_{j=1}^{n} (a_{j3})^{w_j}, \prod_{j=1}^{n} (a_{j4})^{w_j} \right), \right.$$
$$\left. \left(\prod_{j=1}^{n} (b_{j1})^{w_j}, \prod_{j=1}^{n} (b_{j2})^{w_j}, \prod_{j=1}^{n} (b_{j3})^{w_j}, \prod_{j=1}^{n} (b_{j4})^{w_j} \right) \right\rangle \tag{19}$$

where $w_j = (w_1, w_2, ..., w_l, ..., w_n)$ is a set of weights satisfying Eqs. 17 and 18.

Both TIFPOWA and TIFPOWG can be employed to aggregate the ratings provided by different decision makers into a single decision matrix.

4. MULTIMOORA updated with intuitionistic trapezoidal fuzzy numbers

This section presents the MULTIMOORA method updated with ITFNs, MULTIMOOTA-ITFN. Initially, the expert ratings are aggregated into a single decision matrix and then the MULTIMOORA-ITFN is applied.

Step 1. Say there is a board of K experts. Accordingly, each of the experts establishes a decision making matrix, D^k, where $D^k = x_{ij}^k = \left\langle \left(a_1^{ij,k}, a_2^{ij,k}, a_3^{ij,k}, a_4^{ij,k} \right), \left(b_1^{ij,k}, b_2^{ij,k}, b_3^{ij,k}, b_4^{ij,k} \right) \right\rangle$, with $i = 1, 2, ..., m$ being the index of alternatives, $j = 1, 2, ..., n$ being that of criteria, and $k = 1, 2, \ldots, K$ that of experts. In case expert assessments do not satisfy the boundedness condition, $0 \le b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4 \le 1$, they should be normalized. Otherwise, the normalization can be carried out after aggregation. This paper focuses on linguistic reasoning and thus the second option.

Step 2. The TIFPOWG operator is employed to aggregate the expert decision matrices, A^k, into a single intuitionistic fuzzy decision matrix A with its elements $A = x_{ij} = \left\langle \left(a_1^{ij}, a_2^{ij}, a_3^{ij}, a_4^{ij,k} \right), \left(b_1^{ij}, b_2^{ij}, b_3^{ij}, b_4^{ij} \right) \right\rangle$, where

$$x_{ij} = TIFPOWG(x_{ij}^1, x_{ij}^2, ..., x_{ij}^K), \tag{20}$$

is the aggregated assessment of the *i*-th alternative against the *j*-th criterion. Eq. 15 can be employed to rank the ITFNs in case order of preference is unclear.

Step 3. In order to facilitate the MCDM, one needs to normalize the decision matrix and thus remove any differences amongst the criteria arising from their dimensions and types. Specifically, different dimensions are used for economic, environmental, or social indicators serving as respective criteria in decision making. As for the types of criteria, one can establish the two subsets of the criteria set, *J*, namely benefit criteria, *B*, and cost criteria, *C*. The former ones should be maximized, whereas the latter ones – minimized.

The modified vector normalization can therefore be employed to make IFNs comparable. Let v_j be the scaling factor:

$$v_j = \left[\frac{1}{8} \left(\sum_{p=1}^{4} (a_p^{ij})^2 + \sum_{q=1}^{4} (b_q^{ij})^2 \right) \right]^{-\frac{1}{2}}, \; j = 1, 2, ..., n. \tag{21}$$

By employing Eq. 21, one can now normalize the decision matrix. Specifically, we have:

$$x_{ij}^* = v_j x_{ij} = \left\langle \left(v_j a_1^{ij}, v_j a_2^{ij}, v_j a_3^{ij}, v_j a_4^{ij} \right), \left(v_j b_1^{ij}, v_j b_2^{ij}, v_j b_3^{ij}, v_j b_4^{ij} \right) \right\rangle, \forall j, \tag{22}$$

where $x_{ij}^* = \left\langle (\bar{a}_1^{ij}, \bar{a}_2^{ij}, \bar{a}_3^{ij}, \bar{a}_4^{ij}), (\bar{b}_1^{ij}, \bar{b}_2^{ij}, \bar{b}_3^{ij}, \bar{b}_4^{ij}) \right\rangle$ is a respective element of the normalized decision matrix. In addition, these values could be multiplied a respective weight, w_j, such that $\sum_{j=1}^{n} w_j = 1$. In case of fuzzy weights, their modal values should sum up to a unity.

Step 4 – The Trapezoidal Intuitionistic Fuzzy Ratio System. The normalized values are further aggregated in the spirit of Eqs. 8 and 9:

$$y_i = \sum_{j \in B} x_{ij}^* - \sum_{j \in C} x_{ij}^*, \; i = 1, 2, \ldots, m, \tag{23}$$

where y_i denoted the overall utility of the *i*-th alternative. Defuzzification is applied by the virtue of Eq. 15. Thereafter, the alternatives are ranked in descending order of $EV(y_i)$.

Step 5 – The Trapezoidal Intuitionistic Fuzzy Reference Point. The Maximal Utopian Reference Point (MURP), $r = r_j$, is defined as:

$$r_j = \begin{cases} \left\langle (1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0) \right\rangle, j \in B, \\ \left\langle (0, 0, 0, 0), (0, 0, 0, 0) \right\rangle, j \in C \end{cases} \tag{24}$$

Alternatively, one can employ Eq. 15 to identify the maximal (minimal) values of benefit (cost) criteria and thus construct a Maximal Objective Reference Point.

Then the Eq. 13 is employed to measure the distances between the alternative and the reference point (e. g. MURP) in terms of the Tchebycheff metric:

$$d_T(A_i,r)=\max_j d_E(x_{ij}^*,r_j)$$
$$=\max_j \sqrt{\frac{1}{8}\left(\sum_{p=1}^{4}(a_p^{ij}-r_p^j)^2+\sum_{q=1}^{4}(a_q^{ij}-r_q^j)^2\right)},\ i=1,2,\ldots,m, \quad (25)$$

The alternatives are ranked in ascending order of $d_T(A_i,r)$.

Step 6 – The Trapezoidal Intuitionistic Fuzzy Multiplicative Form. The overall utility of an alternative is determined by the multiplicative relationship:

$$U_i=\prod_{j\in B}x_{ij}\Big/\prod_{j\in C}x_{ij},\ i=1,2,\ldots,m, \quad (26)$$

where multiplication and division operators are those defined in Eqs. 10 and 11, respectively. The Eq. 15 is employed for defuzzification, with higher values of $EV(U_i)$ indicating higher preference of a certain alternative.

Step 7. The three ranks for each of alternatives obtained in Steps 4–6 are summarized by employing the Dominance theory (Brauers, Zavadskas, 2011).

5. Numerical example

Suppose that a telecommunication company intends to choose a manager for R&D department from four volunteers named A_1, A_2, A_3 and A_4 (adopted from Liu and Jin (2012)). The decision making committee assess the four concerned volunteers based on five attributes shown as follows:

(1) Proficiency in identifying research areas (C_1);
(2) Proficiency in administration (C_2);
(3) Personality (C_3);
(4) Past experience (C_4);
(5) Self-confidence (C_5).

The number of the committee members is three, with decision makers labeled as DM_1, DM_2 and DM_3, respectively. Each decision maker has presented his (her) evaluation information for four volunteers in Tables 2. The linguistic terms are same as those in Table 1.

Table 1. Linguistic variables and respective trapezoidal intuitionistic fuzzy numbers (Ye, 2012b)

Linguistic term	ITFN
Absolutely low (AL)	$\langle(0.001, 0.001, 0.001, 0.001), (0.001, 0.001, 0.001, 0.001)\rangle$
Low (L)	$\langle(0.001, 0.1, 0.2, 0.3), (0.001, 0.1, 0.2, 0.3)\rangle$
Fairly low (FL)	$\langle(0.1, 0.2, 0.3, 0.4), (0.001, 0.2, 0.3, 0.5)\rangle$
Medium (M)	$\langle(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)\rangle$
Fairly high (FH)	$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)\rangle$
High (H)	$\langle(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)\rangle$
Absolutely high (AH)	$\langle(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)\rangle$

Table 2. The ratings provided by the decision makers (DM_1–DM_3) to the candidates (A_1–A_4) in terms of the multiple criteria (C_1–C_5)

		C_1	C_2	C_3	C_4	C_5
DM_1	A_1	AH	AH	AH	AH	AH
	A_2	H	AH	AH	AH	M
	A_3	AH	FH	M	M	M
	A_4	M	M	M	FH	M
DM_2	A_1	AM	FH	M	M	AH
	A_2	M	AH	AH	AH	FH
	A_3	M	M	FH	AH	M
	A_4	AH	M	FH	M	M
DM_3	A_1	FH	M	M	AH	AH
	A_2	FH	FH	M	FH	M
	A_3	AH	AH	AH	AH	FH
	A_4	AH	AH	FH	AH	M

Step 1. Converting the linguistic terms into the ITFNs, we can get:

$$[D^1] = \begin{bmatrix} [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.7,0.8,0.9,1.0), & (0.7,0.8,0.9,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \end{bmatrix}$$

$$[D^2]=\begin{bmatrix}
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\
[(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)],
\end{bmatrix}$$

$$[D^3]=\begin{bmatrix} [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \\ [(0.5,0.6,0.7,0.8), & (0.4,0.6,0.7,0.9)], \\ [(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)], \end{bmatrix}$$

Step 2. In this example, we assume $g(x)=x$ and employ the TIFPOWG operator by the virtue of Eq. 20 to aggregate the expert decision matrices $D^k(k=1,2,3)$ into a single response matrix D. Indeed, one may choose $g(x)=x^k, k>0$ in order to obtain a different weight vector.

$$[D]=\begin{bmatrix}
[(0.77,0.83,0.88,0.92), & (0.71,0.83,0.88,0.96)], \\
[(0.49,0.60,0.70,0.80), & (0.41,0.60,0.70,0.87)], \\
[(0.63,0.70,0.77,0.82), & (0.54,0.70,0.77,0.87)], \\
[(0.63,0.70,0.77,0.82), & (0.54,0.70,0.77,0.87)], \\
[(0.51,0.61,0.69,0.77), & (0.41,0.61,0.69,0.85)], \\
[(0.77,0.83,0.88,0.92), & (0.71,0.83,0.88,0.96)], \\
[(0.51,0.61,0.69,0.77), & (0.41,0.61,0.69,0.85)], \\
[(0.41,0.52,0.61,0.69), & (0.31,0.52,0.61,0.77)], \\
[(0.41,0.52,0.61,0.69), & (0.31,0.52,0.61,0.77)], \\
[(0.63,0.70,0.77,0.82), & (0.54,0.70,0.77,0.87)], \\
[(0.51,0.61,0.69,0.77), & (0.41,0.61,0.69,0.85)], \\
[(0.43,0.53,0.63,0.73), & (0.2,0.53,0.63,0.83)], \\
[(0.63,0.70,0.77,0.82), & (0.54,0.70,0.77,0.87)], \\
[(0.77,0.83,0.88,0.92), & (0.71,0.83,0.88,0.96)], \\
[(0.63,0.70,0.77,0.82), & (0.54,0.70,0.77,0.87)], \\
[(0.51,0.61,0.69,0.77), & (0.41,0.61,0.69,0.85)], \\
[(1.0,1.0,1.0,1.0), & (1.0,1.0,1.0,1.0)], \\
[(0.35,0.45,0.56,0.66), & (0.35,0.45,0.56,0.76)], \\
[(0.35,0.45,0.56,0.66), & (0.35,0.45,0.56,0.76)], \\
[(0.3,0.4,0.5,0.6), & (0.2,0.4,0.5,0.7)],
\end{bmatrix}$$

Step 3. All of the criteria are benefit ones and expressed in the ITFNs, therefore we do not need to normalize them.

Step 4. The four candidates are ranked according to the Ratio System, cf. Eq. 23. Table 3 presents the results.

Table 3. The Ratio System

	y_i	$EV(y_i)$	Rank
A_1	[(2.32,2.66,2.95,2.51), (2.02,2.69,2.95,3.45)]	2.69	3
A_2	[(2.66,2.96,3.23,3.46), (2.37,2.96,3.23,3.66)]	3.07	1
A_3	[(2.28,2.62,2.92,3.18), (1.9,2.62,2.92,3.44)]	2.74	2
A_4	[(1.98,2.36,2.7,3.01), (1.58,2.36,2.7,3.32)]	2.50	4

Step 5. We define the Reference Point $r = [(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)]$ and thus rank the alternatives in terms of their distance from it (Eq. 13 and 25): those with smaller distances are attributed with higher ranks (Table 4).

Table 4. The Reference Point

	$d_T(A_i, r)$	Rank
A_1	0.85	4
A_2	0.66	1
A_3	0.73	2
A_4	0.73	2

Step 6. Eq. 27 is employed to obtain ranks for each of alternatives according to the Full Multiplicative Form (Table 5).

Table 5. The Full Multiplicative Form

	U_i	$EV(U_i)$	Rank
A_1	[(0.10,0.18,0.29,0.40), (0.05,0.18,0.29,0.55)]	0.25	2
A_2	[(0.18,0.29,0.42,0.56), (0.11,0.29,0.42,0.70)]	0.37	1
A_3	[(0.10,0.18,0.29,0.40), (0.05,0.18,0.29,0.55)]	0.25	2
A_4	[(0.06,0.14,0.23,0.35), (0.02,0.14,0.23,0.52)]	0.21	4

Step 7. Then, by using the Ratio System, the Reference Point and the Full Multiplicative Form to rank the candidates, we have the following results (Table 6). The Dominance theory (Brauers, Zavadskas, 2011) is employed to summarize the three different ranks provided by respective parts of MULTIMOORA. The last column in Table 6 presents the final ranking.

Table 6. Ranking of the candidates according to MULTIMOORA

	The Ratio System	The Reference Point	The Full Multiplicative Form	MULTIMOORA (Final rank)
A_1	3	4	2	3
A_2	1	1	1	1
A_3	2	2	2	2
A_4	4	2	4	4

According to the multi-criteria evaluation, the fourth candidate (A_2) should be recruited, whereas the second candidate (A_3) is the second-best option. At the other end of spectrum, candidates A_1 and A_4 are the last two.

6. Conclusion

The MULTIMOORA method has been extended with intuitionistic trapezoidal fuzzy numbers to handle the uncertainty in decision making. Furthermore, the trapezoidal intuitionistic fuzzy power aggregation operators have been employed to aggregate expert ratings and ensure that the biased ratings were attributed with lower significance. An illustrative example demonstrated the operationality of the proposed technique.

Further research should aim at allowing for higher degree of uncertainty in the modelling. For instance, application of the neutrosophic numbers would put fewer restrictions on the membership and non-membership functions. Interval intuitionistic trapezoidal fuzzy numbers would also allow to incorporate additional knowledge about uncertainty in the decision making process.

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