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DESIGNING AN INTEGRATED MULTI-ECHELON, MULTI-PRODUCT AND MULTI-PERIOD SUPPLY CHAIN NETWORK WITH SEASONAL RAW MATERIALS

Abstract. *This paper deals with the problem of designing a supply chain network where availability of raw materials is limited and their price and quality vary over time due to their seasonal nature. A multi-echelon supply chain with multiple products and multiple time periods is considered and a Dynamic Mixed-Integer Linear Programming (DMILP) model is developed that integrates and optimizes various aspects of network design including supplier selection, location and size of production and distribution facilities, and quantity of flows along the supply chain, simultaneously. It is assumed that closing/ reopening production plants and distribution centers is possible during the planning horizon. The objective is to maximize total cash value within the supply chain under a constrained budget. A three-stage solution procedure is developed based on Genetic Algorithm (GA) and its efficiency is demonstrated through numerical experiments. The paper discusses specific considerations in optimizing supply chain networks with seasonal raw materials and highlights the benefits of the proposed modeling approach.*

Keywords*: Supply chain network design; Production-distribution network; Seasonal raw materials; Mathematical model; Genetic Algorithm (GA).*

JEL Classification: C02, C44, C61, C63, C65, M10

1. Introduction

The supply chain network design seeks to optimize strategic aspects of supply chain such as the number of supply chain facilities, their locations and capacities, and quantity of flows (Pishvaee et al., 2010). It is a complex decision-making problem since it needs to integrate various supply chain components such as procurement, production, distribution, inventory management, etc. (Sha and Che, 2006). The complexity increases when additional features that reflect characteristics of the real-world supply chains are incorporated into the problem.

In this paper, we focus on the specific case of a supply chain wherein seasonal raw materials such as agricultural and natural products are used in production processes. The availability of raw materials is limited and their price and quality vary over time. At the first look, it seems to be economical to produce and store a large quantity of products in the low-price high-quality period of the raw materials in order to satisfy demand in all future periods. However, taking capacity and budget constraints into account, makes it inefficient in practice. This implies that the seasonality of raw materials will affect decisions regarding the purchasing time and quantity as well as planning of production and distribution and, in turn, determining the optimal configuration of the supply chain.

We consider a multi-echelon supply chain with multiple products and multiple time periods. The available budget is limited and the facilities can be closed/ reopened during the planning horizon. A mathematical modeling approach is presented and a solution procedure is developed using Genetic algorithm (GA).

The remainder of the paper is organized as follows. Section 2 provides a brief review of related studies. Section 3 gives the problem description and Section 4 presents the mathematical modeling framework. Section 5 develops a solution procedure based on GA for the presented problem. In Section 6, the proposed modeling approach and the solution procedure are evaluated in a set of numerical experiments, and finally, the paper ends up with conclusions in Section 7.

2. Literature review

According to Thanh et al. (2008) and Hajipour and Pasandideh (2012), facility location and strategic planning of the supply chain has been the subject of many previous works. Herein, we review important studies that are more relevant to the focus of the current research.

Koksalan and Sural (1999) have considered both the location of new malt plants and the distribution of barley and malt with the objective of minimizing the present value of total costs. Melo et al. (2005) have developed a mathematical model that captures dynamic planning horizon, supply chain network configuration, external supply of materials, inventory for goods, distribution of commodities, availability of capital for investments, and storage limitations.

Pishvaee et al. (2010) have developed a bi-objective model for designing an integrated forward/ reverse logistics network that minimizes the total costs and maximizes the responsiveness of the network. Olivares-Benitez et al. (2012) have presented a bi-objective model for a two-echelon single-product supply chain design considering several transportation channels for each pair of facilities between echelons that introduce a cost-time tradeoff in the problem.

In more relevant studies, Martel (2005) has focused on designing an international production-distribution network for make-to-stock products. The author has discussed the manufacturing process and product structures, the logistics network structure, demand and service requirements, facility layouts and capacity options, product flows and inventory modeling, as well as financial flows modeling. Moreover, Thanh et al. (2008) have proposed a dynamic decision-making for designing a multi-echelon multi-commodity supply chain over a multi-period horizon and developed a Mixed-Integer Linear Programming (MILP) model that involves decisions about opening, closing, or enlargement of facilities, supplier selection, and quantity of flows along the supply chain.

In some other studies, the problem of supply chain network design has been investigated in an uncertain environment. Azaron et al. (2008) have developed a multi-objective stochastic programming model for supply chain design with uncertain supply chain costs. The model seeks to minimize the sum of current investment costs and the expected future processing, transportation, shortage and capacity expansion costs, the variance of the total cost, and the financial risk. Liang and Cheng (2009) have proposed a Fuzzy Multi-Objective Linear Programming (FMOLP) model for manufacturing/ distribution planning in a multi-product and multi-period supply chain under an uncertain environment. El-Sayed et al. (2010) have developed a stochastic programming model for designing a multi-period multi-echelon forward-reverse logistics network under risk. Also, Cardona-Valdez et al. (2011) have developed a stochastic model for designing a two-echelon production-distribution network that minimizes total cost and total service time.

The research on supply chain network design has been ever-increasing with the idea of reflecting real cases and/ or specific features of the problem (e.g. Babazadeh et al., 2012; Pishvaee and Razmi, 2012; Pishvaee et al., 2012, 2014; Pan and Nagi, 2013; Tabrizi and Razmi, 2013). However, little research has been conducted that addresses the effect of seasonality of raw materials on designing and optimizing the supply chain. The main contribution of this paper is to develop a comprehensive mathematical modeling framework that considers several practical aspects of designing a supply chain network with seasonal raw materials.

3. Problem description

Suppose a multi-product supply chain comprising suppliers, production plants, distribution centers, and customer zones. The problem is to select suppliers, determine location and capacity of plants and distribution centers, quantity of raw materials purchased from suppliers and shipped to plants, quantity of products shipped from plants to distribution centers, and the allocation of customer demands to distribution centers. The candidate suppliers as well as potential sites for the facilities and their available capacity levels have already been identified. A planning horizon of multiple years is considered each year is divided into several periods, i.e. the seasons. A limited quantity of all products can be purchased through outsourcing and shipped from distribution centers to customers.

The raw materials required for producing products are seasonal which is reflected in their varying price and availability as well as their production efficiency, i.e. the amount of raw material needed to produce a unit of product, during the planning horizon. The production efficiency will be the best at the highest-quality period. It is assumed that each product is made from only one type of raw material. Also, customer zones are fixed and their demand quantities are known. All customer demand should be satisfied, i.e. shortage is not allowed.

Each type of raw material and each product occupies a certain amount of capacity. Moreover, the facilities can be closed/ reopened during the planning horizon at a certain cost. The supply chain revenue comes from sales of products and each product has a residual value at the end of the planning horizon. The costs should be paid at the beginning of the period that they appear while the revenue sources are available at the end of the period. Available budget is limited and the objective is to maximize total cash value at the end of the planning horizon which incorporates total profit and the value of established facilities considering investment in establishing the facilities and depreciation of their buildings, equipments, machinery, etc. over time.

4. Mathematical model

4.1. Sets

I set of candidate suppliers ($i \in I$)

J set of potential sites for locating production plants ($j \in J$)

R set of potential sites for locating distribution centers ($r \in R$)

D set of customer zones ($d \in D$)

K set of products ($k \in K$)

T set of years in planning horizon ($t \in T$)

C set of available capacity levels for production plants ($c \in C$)

Z set of available capacity levels for distribution centers ($z \in Z$)

S set of time periods in each year ($s \in S$)

4.2. Parameters

B available budget at the beginning of planning horizon

fs_{jc}^{ts} investment cost of establishing a production plant with capacity level c at potential site j in period s of year t

FS_{rz}^{ts} investment cost of establishing a distribution center with capacity level z at potential site r in period s of year t

fc_{jc}^{ts} fixed cost of closing production plant j with capacity c in period s of year t

FC_{rz}^{ts} fixed cost of closing distribution center r with capacity level z in period s of year t

fo_{jc}^{ts} fixed cost of reopening plant j with capacity c in period s of year t

FO_{rz}^{ts} fixed cost of reopening distribution center r with capacity level z in period s of year t

VPC_{ijkc}^{ts} cost of purchasing a unit of raw material from supplier *i* and its processing for producing product *k* at production plant *j* with capacity *c* in period *s* of year *t*

$TC1_{jrk}^{ts}$ shipping cost per unit of product *k* from production plant *j* to distribution center *r* in period *s* of year *t*

$TC2_{rdk}^{ts}$ shipping cost per unit of product *k* from distribution center *r* to customer zone *d* in period *s* of year *t*

i_k inventory cost per unit of product *k* per period (as a percentage of price)

C_{rk}^{ts} purchasing price per unit of outsourced product *k* shipped to distribution center *r* in period *s* of year *t*

dc_{jc}^{ts} depreciation cost of facilities at plant *j* with capacity *c* in period *s* of year *t*

DC_{rz}^{ts} depreciation cost of facilities at distribution center *r* with capacity level *z* in period *s* of year *t*

P_{kd}^{t} selling price per unit of product *k* at customer zone *d* in year *t*

RV_{rk} residual value per unit of product *k* at the end of planning horizon

SC_{ki}^{t} capacity of supplier *i* for raw material of product *k* in year *t*

AC_{ik}^{ts} availability factor of raw material of product *k* at supplier *i* in period *s* of year *t*

P_k capacity consumption per unit of raw material for product *k*

E_k capacity consumption per unit of product *k*

PC_{jc}^{ts} available capacity at plant *j* with capacity level *c* in period *s* of year *t*

DC_{rz}^{ts} available capacity at distribution center *r* with capacity level *z* in period *s* of year *t*

e_{ijk}^{ts} quantity of product *k* produced at production plant *j* per unit of raw material purchased from supplier *i* in period *s* of year *t*

D_{kd}^{ts} demand for product *k* at customer zone *d* in period *s* of year *t*

O_k^{+} maximum level of outsourcing product *k*, as a percentage of total demand

O_k^{-} minimum level of outsourcing product *k*, as a percentage of total demand

4.3. Decision variables

XEF_{jc}^{ts} 1 if production plant with capacity level *c* is established at potential site *j* in period *s* of year *t*; 0 otherwise

x_{jc}^{ts} 1 if plant *j* with capacity *c* is in operation in period *s* of year *t*; 0 otherwise

x^{+}_{jtsc} 1 if production plant *j* with capacity level *c* is in operation in period *s* of year *t*, but is not in its previous period; 0 otherwise

x^{-}_{jtsc} 1 if production plant *j* with capacity level *c* is not in operation in period *s* of year *t*, but it is in its previous period; 0 otherwise

XED^{ts}_{rz} 1 if distribution center with capacity level *z* is established at potential site *r* in period *s* of year *t*; 0 otherwise

y^{ts}_{rz} 1 if distribution center *r* with capacity level *z* is in operation in period *s* of year *t*; 0 otherwise

y^{+}_{rtsz} 1 if distribution center *r* with capacity level *z* is in operation in period *s* of year *t*, but is not in its previous period; 0 otherwise

y^{-}_{rtsz} 1 if distribution center *r* with capacity level *z* is not in operation in period *s* of year *t*, but it is in its previous period; 0 otherwise

$X1^{ts}_{ijk}$ quantity of raw materials shipped from supplier *i* to production plant *j* for producing product *k* in period *s* of year *t*

$X2^{ts}_{jrk}$ quantity of product *k* shipped from production plant *j* to distribution center *r* in period *s* of year *t*

$X3^{ts}_{rk}$ quantity of product *k* purchased through outsourcing and shipped to distribution center *r* in period *s* of year *t*

$X4^{ts}_{rdk}$ quantity of product *k* shipped from distribution center *r* to customer zone *d* in period *s* of year *t*

4.4. Objective function

Maximize $Z = SB^{TS} + SSR^{TS} - SDP - \sum_{t=1}^{T}\sum_{s=1}^{S} SE^{ts} + \sum_{r=1}^{R}\sum_{k=1}^{K} DS^{TS}_{rk}.RV_{rk}$ (1)

As given in Eqs. (2) and (3), SB^{ts} is total cash value at the end of period *s* of year *t* and BA^{ts} is total cash value at the beginning of this period ($BA^{0} = B$).

$$SB^{ts} = BA^{ts} - TC^{ts} \quad (2)$$

$$BA^{ts} = SSR^{t,s-1} + SB^{t,s-1} \quad (3)$$

The total cost incurred at the end of period *s* of year *t* (TC^{ts}) comprises the costs of establishing production and distribution facilities (SE^{ts}), closing and reopening the facilities (SOC^{ts}), purchasing and processing raw materials (SVC^{ts}), purchasing products through outsourcing (SPP^{ts}), shipping products from plants to distribution centers and then to customers (STC^{ts}), and inventory of products (SIC^{ts}) in period *s* of year *t*. These are formulated in Eqs. (5) to (10).

$$TC^{ts} = SE^{t,s+1} + SOC^{ts} + SVC^{ts} + SPP^{ts} + STC^{ts} + SIC^{ts} \quad (4)$$

$$SE^{ts} = \sum_{j=1}^{N}\sum_{c=1}^{C} XEF_{jc}^{ts}.fs_{jc}^{ts} + \sum_{r=1}^{R}\sum_{z=1}^{Z} XED_{rz}^{ts}.FS_{rz}^{ts} \quad (5)$$

$$SOC^{ts} = \sum_{j=1}^{N}\sum_{c=1}^{C} x_{jtsc}^{+}.fo_{jc}^{ts} + \sum_{j=1}^{N}\sum_{c=1}^{C} x_{jtsc}^{-}.fc_{jc}^{ts} + \sum_{r=1}^{R}\sum_{z=1}^{Z} y_{rtsz}^{+}.FO_{rz}^{ts} + \sum_{r=1}^{R}\sum_{z=1}^{Z} y_{rtsz}^{-}.FC_{rz}^{ts} \quad (6)$$

$$SVC^{ts} = \sum_{k=1}^{K}\sum_{j=1}^{N}\sum_{i=1}^{I}\sum_{c=1}^{C} VPC_{ijkc}^{ts}.X1_{ijk}^{ts} \quad (7)$$

$$SPP^{ts} = \sum_{r=1}^{R}\sum_{k=1}^{K} X3_{rk}^{ts}.C_{rk}^{ts} \quad (8)$$

$$STC^{ts} = \sum_{k=1}^{K}\sum_{j=1}^{N}\sum_{r=1}^{R} X2_{jrk}^{ts}.TC1_{jrk}^{ts} + \sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{d=1}^{D} X4_{rdk}^{ts}.TC2_{rdk}^{ts} \quad (9)$$

$$SIC^{ts} = \sum_{r=1}^{R}\sum_{k=1}^{K} i_k.C_{rk}^{ts}.DS_{rk}^{ts} \quad (10)$$

SSR^{ts} is total revenue from sales and DS_{rk}^{ts} is the amount of inventory of product k in distribution center r at the end of period s of year t ($DS_{rk}^{0} = 0$). Moreover, SDP gives total depreciation cost of the facilities over the planning horizon.

$$DS_{rk}^{ts} = DS_{rk}^{t,s-1} + X3_{rk}^{ts} + \sum_{j=1}^{N} X2_{jrk}^{ts} - \sum_{d=1}^{D} X4_{rdk}^{ts} \quad (11)$$

$$SSR^{ts} = \sum_{k=1}^{K}\sum_{d=1}^{D}\sum_{r=1}^{R} P_{km}^{ts}.X4_{rdk}^{ts} \quad (12)$$

$$SDP = \sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{c=1}^{C} XEF_{jc}^{ts}.dc_{jc}^{ts}.((T-t).S+S-s) + \sum_{r=1}^{R}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{z=1}^{Z} XED_{rz}^{ts}.DC_{rz}^{ts}.((T-t).S+S-s) \quad (13)$$

4.5. Constraints

$$x_{jtsc}^{+} - x_{jtsc}^{-} = x_{jc}^{ts} - x_{jc}^{t,s-1} \qquad \forall s \in S;\ t \in T;\ j \in J;\ c \in C \quad (14)$$

$$y_{rtsz}^{+} - y_{rtsz}^{-} = y_{rz}^{ts} - y_{rz}^{t,s-1} \qquad \forall s \in S;\ t \in T;\ r \in R;\ z \in Z \quad (15)$$

$$\sum_{j=1}^{N}\sum_{s=1}^{S} X1_{ijk}^{ts} + \sum_{j=1}^{N} \left(X1_{ijk}^{ts} / AC_{ik}^{ts} \right) \le SC_{ik}^{t} \quad \forall i \in I;\ t \in T;\ k \in K \quad (16)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} X1_{ijk}^{ts}.P_k \leq PC_{jc}^{ts}.x_{jc}^{ts} \qquad \forall j \in J;\ t \in T;\ s \in S;\ c \in C \qquad (17)$$

$$\sum_{k=1}^{K} DS_{rk}^{ts}.E_k \leq DC_{rz}^{ts}.y_{rz}^{ts} \qquad \forall r \in R;\ t \in T;\ s \in S;\ z \in Z \qquad (18)$$

$$\sum_{i=1}^{M} X1_{ijk}^{ts}.e_{ijk}^{ts} = \sum_{r=1}^{R} X2_{jrk}^{ts} \qquad \forall j \in J;\ t \in T;\ s \in S;\ k \in K \qquad (19)$$

$$DS_{rk}^{t,s-1} + X3_{rk}^{ts} + \sum_{j=1}^{N} X2_{jrk}^{ts} = \sum_{d=1}^{D} X4_{rdk}^{ts} + DS_{rk}^{ts}$$

$$\forall r \in R;\ t \in T;\ s \in S;\ k \in K \qquad (20)$$

$$\sum_{r=1}^{R} X4_{rdk}^{ts} = D_{kd}^{ts} \qquad \forall s \in S;\ t \in T;\ k \in K;\ d \in D \qquad (21)$$

$$O_k^{+}.\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{d=1}^{D} D_{dk}^{ts} \geq \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R} X3_{rk}^{ts} \qquad \forall k \in K \qquad (22)$$

$$O_k^{-}.\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{d=1}^{D} D_{dk}^{ts} \leq \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R} X3_{rk}^{ts} \qquad \forall k \in K \qquad (23)$$

$$x_{jc}^{ts} \leq \sum_{t'=1}^{t} XEF_{jc}^{t's} \qquad \forall s \in S;\ t \in T;\ j \in J;\ c \in C \qquad (24)$$

$$y_{rz}^{ts} \leq \sum_{t'=1}^{t} XED_{rz}^{t's} \qquad \forall s \in S;\ t \in T;\ r \in R;\ z \in Z \qquad (25)$$

$$\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{c=1}^{C} XEF_{jc}^{ts} \leq 1 \qquad \forall j \in J \qquad (26)$$

$$\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{z=1}^{Z} XED_{rz}^{ts} \leq 1 \qquad \forall r \in R \qquad (27)$$

$$y_{rz}^{0} = 0 \qquad \forall r \in R;\ z \in Z \qquad (28)$$

$$x_{jc}^{0} = 0 \qquad \forall j \in J;\ c \in C \qquad (29)$$

$$DS_{rk}^{ts} \geq 0 \qquad \forall s \in S;\ t \in T;\ k \in K;\ r \in R \qquad (30)$$

$$SB^{ts} \geq 0 \qquad \forall s \in S;\ t \in T \qquad (31)$$

$$XEF_{jc}^{ts}, x_{jc}^{ts}, x_{jtsc}^{+}, x_{jtsc}^{-}, XED_{rz}^{ts}, y_{rz}^{ts}, y_{rtsz}^{+}, y_{rtsz}^{-}, X1_{ijk}^{ts}, X2_{jrk}^{ts}, X3_{rk}^{ts}\ X4_{rdk}^{ts} \in \{0,1\}$$

$$\forall s \in S;\ t \in T;\ j \in J;\ c \in C;\ i \in I;\ r \in R;\ z \in Z;\ d \in D;\ k \in K \qquad (32)$$

Constraints (14) and (15) imply that a facility is closed when it is in operation in previous period, but is not in current period. These constraints also represent that a facility is reopened when it is in operation in current period, but is not in previous period. Constraints (16) to (18) represent capacity constraints. Constraint (19) shows that quantity of products produced in a production plant is equivalent to the quantity of products shipped from the plant to distribution centers. Constraint (20) represents the inventory in a distribution center at the end of a time period. Constraint (21) guarantees that shortage is not allowed. Constraints (22) and (23) ensure that quantity of product purchased through outsourcing does not exceed the predetermined maximum and minimum levels.

Constraints (24) and (25) imply that unestablished facilities cannot be in operation. Moreover, constraints (26) and (27) guarantee that the facilities can be established only once during the planning horizon and constraints (28) and (29) represent that no facility is established before the planning horizon starts. In addition, constraints (30) and (31) enforce the non-negativity of cash value and inventory and constraint (32) requires that the decision variables are binary.

5. Solution algorithm

In order to solve the presented Dynamic MILP (DMILP) model, we divide it into three inter-related sub-problems. In this manner, the optimal solution of the model is built step by step and instead of a highly complex problem, we solve three problems with less complexity. Such an approach has been adopted in several previous studies (e.g. Farahani and Asgari, 2007; Pishvaee et al., 2011). Each sub-problem is solved using GA which has been widely applied in supply chain network design (e.g. Prakash et al., 2012).

5.1. Sub-problem (1): Determining the optimal number, location, and capacity of distribution centers and the allocation of customer demand

Step 1: converting the multi-product model into a volume-based model by obtaining demand of each customer zone in each period (DV_d^{ts}) using Eq. (33).

$$DV_d^{ts} = \sum_{k=1}^{K} D_{kd}^{ts}.E_k \tag{33}$$

Step 2: determining proportion of each product (U_{kd}^{ts}) in total products shipped from distribution center *r* to customer zone *d* ($X'4_{rd}^{ts}$) using Eq. (34).

$$U_{kd}^{ts} = {(D_{kd}^{ts}.E_k)} \Big/ {DV_d^{ts}} \tag{34}$$

Step 3: determining average cost of shipping one unit of product from each potential distribution center *r* to each customer zone *d* ($TC'2_{rd}^{ts}$) using Eq. (35).

$$TC'2_{rd}^{ts} = {\sum_{k=1}^{K}(D_{kd}^{ts}.TC2_{rdk}^{ts})} \Big/ {DV_d^{ts}} \tag{35}$$

Step 4: determining the optimal location and capacity of distribution centers (XED_{rz}^{ts}) and proportion of each customer's demand satisfied by each distribution center ($X'4_{rd}^{ts}$) such that Eq. (36) is minimized.

$$Z_1 = \sum_{r=1}^{R}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{z=1}^{Z} XED_{rz}^{ts}.DC_{rz}^{ts}.((T-t).S+S-s) + \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{d=1}^{D} TC'2_{rd}^{ts}.X'4_{rd}^{ts} \quad (36)$$

Step 5: determining the quantity of each product shipped from distribution centers to customer zones ($X4_{rdk}^{ts}$), total quantity of each product shipped from each distribution center ($D2_{rk}^{ts}$), proportion of demand satisfied by each distribution center in each year (F_{rk}^{t}), and price ($P1_{rk}^{ts}$) using Eqs. (37) to (40).

$$X4_{rdk}^{ts} = {X'4_{rd}^{ts}.U_{kd}^{ts}} \Big/ {E_k} \quad (37)$$

$$D2_{rk}^{ts} = \sum_{d=1}^{D} X4_{rdk}^{ts} \quad (38)$$

$$F_{rk}^{t} = {\sum_{s=1}^{S} D2_{rk}^{ts}} \Big/ {\sum_{r=1}^{R}\sum_{s=1}^{S} D2_{rk}^{ts}} \quad (39)$$

$$P1_{rk}^{ts} = {(\sum_{d=1}^{D}(P_{kd}^{t} - TC2_{rdk}^{ts}).X4_{rdk}^{ts})} \Big/ {D2_{rk}^{ts}} \quad (40)$$

Every solution is encoded as a chromosome containing four parts: (1) percentage of demand of customer *d* satisfied by distribution center *r* in period *s* of year *t* (${X4'^{ts}_{rd}} \big/ {DV_d^{ts}}$); (2) location of distribution centers and times of their establishing, closing, and reopening; (3) capacity of located distribution centers; and (4) total investment cost of establishing distribution centers, total depreciation cost of their facilities, total shipping cost, and the value of objective function Z_1. Z_1 is used as the fitness function and the roulette wheel algorithm is used for reproduction operation. Also, mutation is done independently on each gene based on an assigned probability. The algorithm stops if the objective value remains unchanged or the number of iterations exceeds a pre-determined value.

5.2. Sub-problem (2): Determining the optimal number, location, and capacity of production plants, proportion of total raw materials purchased from each supplier, and proportion of total products produced at each production plant

Step 1: converting the multi-year model into a single-year model in which capacity of each supplier for each type of raw material (TSC_{ki}) and demand of each customer zone for each product (D'_k) are equivalent to sum of capacity and sum of demand over all years, as given in Eqs. (41) and (42). Also, the cost of purchasing and processing a unit of raw material ($VPCA^s_{ijk}$), production efficiency of raw materials at each plant (me^s_{ijk}), and their availability factors (MAC^s_{ik}) are equivalent to their average values over the years, as defined in Eqs. (43) to (45).

$$TSC_{ki} = \sum_{t=1}^{T} SC^t_{ki} \tag{41}$$

$$D'_k = \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R} D2^{ts}_{rk} \tag{42}$$

$$VPCA^s_{ijk} = \sum_{t=1}^{T} VPC^{ts}_{ijk} \Big/ T \tag{43}$$

$$me^s_{ijk} = \sum_{t=1}^{T} e^{ts}_{ijk} \Big/ T \tag{44}$$

$$MAC^s_{ik} = \sum_{t=1}^{T} AC^{ts}_{jk} \Big/ T \tag{45}$$

Step 2: determining average shipping cost per unit of product from production plants to distribution centers in each period ($TC'1^s_{jrk}$) using Eq. (46).

$$TC'1^s_{jrk} = \sum_{t=1}^{T} TC'1^{ts}_{jrk} \Big/ T \tag{46}$$

Step 3: determining average investment cost of establishing a production plant with capacity level *c* in potential site *j* (fs_{jc}) using Eq. (47).

$$fs_{jc} = \sum_{t=1}^{T}\sum_{s=1}^{S} fs^{ts}_{jc} \Big/ (T.S) \tag{47}$$

Step 4: determining average rate of return on investment using Eq. (48).

$$i^* = S.\sum_{k=1}^{K} i_k \Big/ K \tag{48}$$

Step 5: determining the optimal location and capacity of plants (XEF^{ts}_{jc}), proportion of total raw materials purchased from supplier *i* for product *k* (a_{ik}), and proportion of total quantity of product *k* produced at plant *j* (b_{jk}) such that the costs of purchasing and processing raw materials, shipping products to distribution centers, and average investment in production facilities (Z_2) is minimized.

$$Z_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{s=1}^{S} X'1_{ijk}^{s}.(VPCA_{ijk}^{s} + me_{ijk}^{s}.\sum_{r=1}^{R} F_{rk}^{s}.TC'1_{jrk}^{s}) + \sum_{j=1}^{N}\sum_{c=1}^{C}(XEF_{jc}.fs_{jc}.i^{*}.T/2) \quad (49)$$

In applying GA, every solution to this sub-problem is represented by a chromosome consisting of five segments: (1) percentage of total raw material requirements in plant *j* purchased from supplier *i* in period *s*; (2) establishment of a plant at a potential site (1 if a plant is established, and 0 otherwise); (3) capacity of established plants; (4) proportion of total demand for product *k* produced at plant *j*; (5) total cost corresponding to the candidate solution (Z_2). Z_2 is used for evaluating the solutions. The crossover and mutation operators and the stoppage criteria are applied similar to the procedure described for the first sub-problem.

5.3. Sub-problem (3): Determining the optimal policy for establishing, closing, and reopening production plants and the level of outsourcing for each product

Step 1: applying the information obtained from previous sub-problems: (1) location and capacity of plants, (2) proportion of total demand for product *k* satisfied by plant *j* (a'_{jk}), (3) proportion of total raw material requirements for product *k* purchased from supplier *i* (a_{ik}^{s}) and applying it to all periods ($a_{ik}^{t'}$), (4) location and capacity of distribution centers, their closing/ reopening policy, and costs of establishing distribution centers in period $s+1$ of year t ($SE2^{ts}$), closing and reopening ($SOC2^{ts}$), and depreciation of the established centers ($DP2$), (5) cost of shipping products from distribution centers to customers ($STC2^{ts}$).

Step 2: determining the maximum level of outsourcing (O_k^{+}), and then, solving the sub-problem and obtaining the optimal value of Z_3 using Eq. (50).

Step 3: determining the minimum level of outsourcing (O_k^{-}), and then, solving the sub-problem and obtaining the optimal value of Z_3 using Eq. (50).

Step 4: assuming three random values for outsourcing level, then solving their corresponding sub-problems and obtaining the optimal values of Z_3.

Step 5: identifying the optimal interval based on the obtained values of Z_3, assuming a new set of three random values within this interval, and then solving the three sub-problems and obtaining the optimal values of Z_3 using Eq. (50).

Step 6: repeating Step 5 until the difference between two obtained values of Z_3, as defined in Eq. (50), is less than 5%. The parameter values obtained in this way characterize the optimal solution to the original problem.

$$Z_3 = BA^{t'+1} + SE + IV - DP \tag{50}$$

$$BA^{t'} = \sum_{k=1}^{K}\sum_{d=1}^{D}\sum_{r=1}^{R} P_{kd}^{t'-1}.X4_{rdk}^{t'-1} + BA^{t'-1} - TC^{t'} \tag{51}$$

$$SE = \sum_{t'=1}^{T'} SE^{t'} \tag{52}$$

$$IV = \sum_{r=1}^{R}\sum_{k=1}^{K} DS_{rk}^{T'}.RV_{rk} \tag{53}$$

$$DP = DP1 + DP2 \tag{54}$$

IV and DP are total residual value of inventory and total depreciation costs, respectively. The total cost $TC^{t'}$ in Eq. (51) is defined using Eq. (55).

$$TC^{t'} = SE^{t'} + SOC^{t'} + SVC^{t'} + SPP^{t'} + STC^{t'} + SIC^{t'} \tag{55}$$

$$SE^{ts} = SE1^{ts} + SE2^{ts} \tag{56}$$

$$SOC^{ts} = SOC1^{ts} + SOC2^{ts} \tag{57}$$

$$SVC^{ts} = \sum_{k=1}^{K}\sum_{j=1}^{N} VPC_{ijkc}^{ts}\left(\sum_{i=1}^{I} X1_{ijk}^{ts}\right) \tag{58}$$

$$SPP^{ts} = \sum_{r=1}^{R}\sum_{k=1}^{K} X3_{rk}^{ts}.C_{rk}^{ts} \tag{59}$$

$$STC^{ts} = STC1^{ts} + \sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{d=1}^{D} X4_{rdk}^{ts}.TC2_{rdk}^{ts} \tag{60}$$

$$SIC^{ts} = \sum_{r=1}^{R}\sum_{k=1}^{K} i_k.C_{rk}^{ts}.DS_{rk}^{ts} \tag{61}$$

The value of $STC1^{ts}$ in Eq. (60) is obtained using Eq. (62).

$$STC1^{ts} = \sum_{k=1}^{K}\sum_{j=1}^{N}\sum_{r=1}^{R} X2_{jrk}^{ts}.TC1_{jrk}^{ts} \tag{62}$$

The first segment of the chromosome structure for this sub-problem indicates times of establishing, closing, and reopening plants using binary values. It is used to determine the costs of establishing plants in period $s+1$ of year t ($SE1^{ts}$), closing and reopening the plants ($SOC1^{ts}$), and depreciation of established facilities ($DP1$), and to obtain available capacity at each plant in each period based on the capacities determined in sub-problem (2).

The second segment denotes demand for product *k* at plant *j* in period *s* of year *t* as defined in Eq. (63). T' in Eq. (64) is the counter of time periods and the values of $D''^{t'}_{jk}$ are obtained considering total demand for the product ($D''^{t'}_{k}$), allowable outsourcing quantity ($OUT^{t'}_{k}$), and proportion of total demand satisfied by plant *j* (a'_{jk}). $D''^{t'}_{k}$ and $OUT^{t'}_{k}$ are formulated in Eqs. (65) and (66).

$$D''^{t'}_{jk} = D''^{t'}_{k}.a'_{jk} \tag{63}$$

$$T' = T.S \tag{64}$$

$$D''^{t'}_{k} = D'_{k} - \sum_{t'=1}^{t'-1}\sum_{r=1}^{R} D2^{t'}_{rk} - \sum_{r=1}^{R} DS^{t'-1}_{rk} \tag{65}$$

$$OUT^{t'}_{k} = O_{k}.D'_{k} - \sum_{t'=1}^{t'}\sum_{r=1}^{R} x3^{t'}_{rk} \tag{66}$$

The value of $x3^{t'}_{rk}$ in Eq. (66) is obtained using Eq. (67) to (69).

$$x3^{t'}_{rk} = \begin{cases} D2^{t'}_{rk} - \sum_{j=1}^{N} x2^{t'}_{jrk} - DS^{t'-1}_{rk} & if \quad x3^{t'}_{rk} \geq 0 \\ 0 & otherwise \end{cases} \tag{67}$$

$$x2^{t'}_{jrk} = D''^{t'}_{jk}.F^{t}_{rk} \tag{68}$$

$$DS^{t'}_{rk} = \sum_{t'=1}^{t'}\sum_{j=1}^{N} x2^{t'}_{jrk} + \sum_{t'=1}^{t'} x3^{t'}_{rk} - \sum_{t'=1}^{t'} D2^{t'}_{rk} \tag{69}$$

The feasibility of $D''^{t'}_{jk}$ is checked considering operation plan of the facilities in the first segment and available capacity of the suppliers (SC^{ts}_{ki}) given in Eq. (70). Then, we determine $x1^{t'}_{ijk}$ based on the $D''^{t'}_{jk}$, $a^{t'}_{ik}$, and $e^{t'}_{ijk}$ using Eq. (71). The required budget for purchasing and processing raw materials in each period is obtained using Eq. (58) based on the $x1^{t'}_{ijk}$ values.

$$SC^{ts}_{ki} = (SC^{t}_{ki} - \sum_{j=1}^{N}\sum_{s=1}^{s-1} X1^{ts}_{ijk} \Big/ AC^{ts}_{ik}).AC^{ts}_{ik} \tag{70}$$

$$x1^{t'}_{ijk} = D''^{t'}_{jk} \Big/ e^{t'}_{ijk} .a^{t'}_{ik} \tag{71}$$

The $x1^{t'}_{ijk}$ values are used in determining quantity of products produced in each period which is used along with F^{t}_{rk} to calculate $x2^{t'}_{jrk}$ as given in Eq. (68). Based on the obtained $x2^{t'}_{jrk}$ values, shipping costs ($STC1^{ts}$), quantity of product inventory ($DS^{t'}_{rk}$), quantity of products purchased through outsourcing ($x3^{t'}_{rk}$), inventory costs (SIC^{ts}), and outsourcing costs (SPP^{ts}) are determined using Eqs.

(62), (69), (67), (61), and (59), respectively. The level of outsourcing each product (O_k), total costs of inventory, shipping, purchasing raw materials as well as outsourced products, and closing and reopening the facilities (FTC) as given in Eq. (72), total cost of establishing production and distribution facilities (SE), total residual value of product inventory (IV), and the value of objective function Z_3 constitute the next five segments of the solution structure, respectively.

$$FTC = \sum_{t'=1}^{T'} (SOC^{t'} + SVC^{t'} + SPP^{t'} + STC^{t'} + SIC^{t'}) \tag{72}$$

6. Numerical experiments

A set of test problems with different sizes is randomly generated and solved using LINGO 11.0. Results of these experiments are used to classify the problems into three size categories based on the maximum number of parameters (given in Table 1). The problems categorized in the large-sized group are those that their optimal solution cannot be obtained using exact solution procedure.

Table 1: Classification of the problem size based on maximum number of parameters

Problem size	Products	Time periods	Candidate suppliers	Potential prod. sites	Available capacity levels for prod. sites	Potential dist. sites	Available capacity levels for dist. sites	Customer zones
Small	3	12	3	3	3	3	3	5
Medium	10	20	5	6	5	6	5	10
Large	> 10	> 20	> 5	> 6	> 5	> 6	> 5	> 10

We randomly generate a set of small-sized problems and compare their solutions with the corresponding exact optimal values. Results indicate that the proposed algorithm works well, considering the average error of 2.379% (Table 2).

Table 2: Comparison of results for a set of small-sized test problems

Problem no.	Exact solution	Proposed GA	Error (%)
1	825434	813052	1.532
2	889401	869834	2.249
3	1134577	1124366	0.908
4	1300056	1257154	3.413
5	1325561	1321584	0.301
6	1490087	1434954	3.842
7	1663211	1614978	2.987
Average error (%)			2.379

We also compare computation times using six randomly-generated test problems, two of each size as given in Table 3. Table 4 shows that the proposed algorithm achieves a near-optimal solution in a relatively short time and the error percentage remains within a reasonable range even by increasing problem size.

Table 3: Characteristics of test problems used for comparison of computation times

Problem no.	No. of products	No. of time periods	No. of candidate suppliers	No. of potential prod. sites	No. of available capacity levels for prod. sites	No. of potential dist. sites	No. of available capacity levels for dist. sites	No. of customer zones
1	2	4	2	2	2	2	2	3
2	3	12	3	3	3	3	3	5
3	6	16	4	4	3	4	3	6
4	9	20	5	6	4	6	4	10
5	12	30	8	8	5	8	5	20
6	15	36	12	12	5	12	5	30

Table 4: Comparison of computation times for the test problems given in Table 3

Problem characteristics			Exact solution		Proposed GA		
Problem no.	**No. of parameters**	**No. of constraints**	**Objective value**	**Computation time (s)**	**Objective value**	**Computation time (s)**	**Error (%)**
1	311	211	1821301	12	1746628	4	4.275
2	1387	1179	8052115	16	7947437	4	1.317
3	7839	2916	28562280	345	27648287	14	3.306
4	28879	7675	9145400	23455	8980782	17	1.833
5	---	---	---	---	7984623	48	---
6	---	---	---	---	8964817	73	---

7. Conclusions

In this paper, an integrated mathematical model was developed for the problem of optimizing the supply chain configuration and product flow decisions where the raw materials required for producing products are seasonal in nature. The effect of seasonality was modeled in terms of time-varying cost of purchasing and processing the raw materials, production efficiency, and availability factor of raw materials. The production efficiency was considered to indicate ever-decreasing quality of raw materials. Also, the availability factor was used to set limitations on the availability of raw materials over the time.

The proposed model designs the network such that the total cash value within the supply chain is maximized, so that it provides a more realistic picture of the effect of network design decisions on economic performance of the supply chain. Also, a three-stage solution algorithm was developed based on GA and the results indicated that it can be used effectively and efficiently for solving large-sized problems where the exact optimal solution cannot be obtained.

The presented modeling approach can be applied in many industries, especially those using agricultural or natural products in their production processes. For future research, additional features that reflect characteristics of the real-world supply chains such as uncertainty in customer demand and supply chain costs, allowing backorder, and expansion / contraction of production/ distribution capacity during the planning horizon can be incorporated into the model.

REFERENCES

[1] **Azaron, A., Brown, K.N., Tarim, S.A. and Modarres, M. (2008),** ***A Multi-objective Stochastic Programming Approach for Supply Chain Design Considering Risk.*** *International Journal of Production Economics*, 116, 129-138;

[2] **Babazadeh, R., Razmi, J. and Ghodsi, R. (2012)** ***Supply Chain Network Design Problem for a New Market Opportunity in an Agile Manufacturing System*****.** *Journal of Industrial Engineering International*, 8, 1-8;

[3] **Cardona-Valdez, Y., Alvarez, A. and Ozdemir, D. (2011),** ***A Bi-Objective Supply Chain Design Problem with Uncertainty.*** *Transportation Research Part C: Emerging Technologies*, 19, 821-832;

[4] **El-Sayed, M., Afia, N. and El-Kharbotly, A. (2010),** ***A Stochastic Model for Forward-reverse Logistics Network Design under Risk.*** *Computers and Industrial Engineering*, 58, 423-431;

[5] **Farahani, R.Z. and Asgari, N. (2007),** ***Combination of MCDM and Covering Techniques in a Hierarchical Model for Facility Location: A Case Study.*** *European Journal of Operational Research*, 176, 1839-1858;

[6] **Hajipour, M.S.V. and Pasandideh, S.H.R. (2012),** ***Proposing an Adaptive Particle Swarm Optimization for a Novel Bi-objective Queuing Facility Location Model.*** *Economic Computation and Economic Cybernetics Studies and Research*; *ASE Publishing*, 46, 223-240;

[7] **Koksalan, M. and Sural, H. (1999),** ***Efes Beverage Group Makes Location and Distribution Decisions for its Malt Plants.*** *Interfaces*, 29, 89-103;

[8] **Liang, T.F. and Cheng, H.W. (2009),** ***Application of Fuzzy Sets to Manufacturing/ Distribution Planning Decisions with Multi-product and Multi-time Period in Supply Chains.*** *Expert Systems with Applications*, 36, 3367-3377;

[9] **Martel, A. (2005),** ***The Design of Production-distribution Networks: A Mathematical Programming Approach*****.** In: *Geunes, J. and Pardalos, P.M. (Eds.),* Supply chain optimization, Springer, 265-305;

[10] **Melo, M.T., Nickel, S. and Saldanha da Gama, F. (2005),** ***Dynamic Multi-commodity Capacitated Facility Location: A Mathematical Modeling Framework for Strategic Supply Chain Planning.*** *Computers and Operations Research*, 33, 181-208;

[11] **Olivares-Benitez, E., Gonzalez-Velarde, J.L. and Rios-Mercado, R.Z. (2012),** ***A Supply Chain Design Problem with Facility Location and Bi-objective Transportation Choices.*** *TOP*, 20, 729-753;

[12] **Pan, F. and Nagi, R. (2013),** ***Multi-echelon Supply Chain Network Design in Agile Manufacturing.*** *Omega*, 41, 969-983;

[13] **Pishvaee, M.S., Farahani, R.Z. and Dullaert, W. (2010),** ***A Memetic Algorithm for Bi-objective Integrated Forward/reverse Logistics Network Design.*** *Computers and Operations Research*, 37, 1100-1112;

[14] **Pishvaee, M.S., Rabbani, M. and Torabi, S.A. (2011),** ***A Robust Optimization Approach to Closed-loop Supply Chain Network Design under Uncertainty.*** *Applied Mathematical modeling*, 35, 637-649;

[15] **Pishvaee, M.S. and Razmi, J. (2012),** ***Environmental Supply Chain Network Design Using Multi-objective Fuzzy Mathematical Programming.*** *Applied Mathematical Modelling*, 36, 3433-3446;

[16] **Pishvaee, M.S., Razmi, J. and Torabi, S.A. (2012),** ***Robust Possibilistic Programming for Socially Responsible Supply Chain Network Design: A New Approach.*** *Fuzzy Sets and Systems*, 206, 1-20;

[17] **Pishvaee, M.S., Razmi, J. and Torabi, S.A. (2014),** ***An Accelerated Benders Decomposition Algorithm for Sustainable Supply Chain Network Design Under Uncertainty: A Case Study of Medical Needle and Syringe Supply Chain*****.** *Transportation Research Part E: Logistics and Transportation Review*, 67, 14-38;

[18] **Prakash, A., Chan, F.T.S., Liao, H. and Deshmukh, S.G. (2012),** ***Network Optimization in Supply Chain: A KBGA Approach.*** *Decision Support Systems*, 52, 528-538;

[19] **Sha, D.Y. and Che, Z.H. (2006),** ***Supply Chain Network Design: Partner Selection and Production/distribution Planning Using a Systematic Model.*** *Journal of the Operational Research Society*, 57, 52-62;

[20] **Tabrizi, B.H. and Razmi, J. (2013),** ***A Multi-Period Distribution Network Design Model under Demand Uncertainty.*** *Journal of Industrial Engineering International*, 9, 1-9;

[21] **Thanh, P.N., Bostel, N. and Peton, O. (2008),** ***A Dynamic Model for Facility Location in the Design of Complex Supply Chains.*** *International Journal of Production Economics*, 113, 678-693.