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FRACTAL MARKET HYPOTHESIS: THE EMERGENT FINANCIAL MARKETS CASE

Abstract*. The Efficient Market Hypothesis (EMH) is a long-stand frame in the analysis of the financial markets behaviour. Still, recent evidences point toward the limits of such as approach. Several alternative approaches have been proposed. Among them, Fractal Market Hypothesis (FMH) might provide interesting explanations for various types of market imperfections such as 'fat tail' effects, stochastic volatility and self-similarity. Based on this conceptual background, the aim of this study is twofold: 1) to directly address the issue of fractal dimension estimation by discussing some estimators which are more frequently used in literature and, respectively, 2) to provide empirical evidences for the potential fractal properties from nine important emergent markets. We find that emergent markets from Europe and Asia are closer to the 'non-persistence' status while Latin America markets exhibits more significant signs of local persistence. However, the current financial turmoil led to some changes in the time profile of the considered markets.*

Keywords: *Financial markets, Fractal Market Hypothesis, fractal dimension, emergent markets.*

JEL Classification: G14, G15

1. BACKGROUND AND RELATED LITERATURE

The Efficient Market Hypothesis (EMH), and especially its 'weak' form according to which current prices of financial assets are reflecting all historical market data such as past prices and trading volumes (Bodie et al., 2007), was for a long time a standard in financial market analysis.

However, various evidences from literature, points toward the fact that market returns are frequently displaying 'fat tail' effects, stochastic volatility and self-similarity (see Cutler et al., 1989, Guillaume et al. 1997, Cont, 2001, Anderson and Noss, 2013). Such evidences are casting reserves on the key assumptions underlying the EMH paradigm. Several alternative explanations about market dynamics have been proposed. Among them, is the *Fractal Market Hypothesis* (FMH)which was proposed by Peters (1990, 1994). This frame can be resumed as follows (see Panas and Ninni, 2010 and Anderson and Noss, 2013 for a more detailed discussion): (a) financial markets are composed by a mix of investors with distinctive trading horizons; (b) the newly arrived information impacts investors in a distinctive fashion, accordingly to their specific trading horizon (even if this information is received simultaneous by investors); (c) market stability is largely a matter of liquidity (the capability- the relative ease- of investors to engage in trades); (d) the financial assets' price dynamics reflects the outcomes of short-term technical analysis and long-term fundamental valuation trading decisions; (e) 'high-frequency traders' are able to pay an 'information premium' in order to collect and use the information incorporated in financial assets prices which is relevant for valuing these assets at a higher frequency than the longer-run traders; (f)if the degree of uncertainty related to macroeconomic conditions raises, the long-term investors may shift to short-run trades as they are less able to adopt trading decisions based on 'fundamentals'. Overall, from these hypotheses it emerged the view that "financial markets can be considered as embodying a 'special sort' of stability… The fractal structure seems to embody a certain 'tolerance to error' that guarantees the stability of the system" (Anderson and Noss, 2013:9).

A quite impressive body of literature explores the related issues of fractal and long-memory properties of financial markets. Lo (1991) develops a test for long-run memory which is robust to short-range dependence. Fang et al. (1994) study the fractional structure of markets directly based on fractional time series models developed by Geweke and Porter-Hudak (1983), Granger and Joyeux (1980) and Mandelbrot (1982). Panas and Ninni (2010) study the fractal properties of the London Metal Exchange (LME) return time series and conclude that the returns are displaying to some extent fractal properties. Mantegna and Stanley (1995) show that the scaling of the probability distribution of the Standard & Poor's 500 index can be described by a non-Gaussian process with dynamics that, for the central part of the distribution, correspond to that predicted for a Lévy stable process. Hall et al. (1989) tests the stable Paretian and mixture of normal distributions for daily closing future prices by applying a stability-under-addition test. Hall and Roy (1994) discuss the relationship between the fractal dimension and the fractal index of the covariance function in the wider context of non-Gaussian processes and consider the cases in which this relation holds. Blackledge (2008) explores in greater details the conceptual background to financial time series analysis and financial signal processing in terms of the Efficient Market Hypothesis and test the prediction of the conceptual frame on FTSE close-of-day data between 1980 and 2007. Anderson and Noss (2013) examine why and how the fractal properties of financial markets might arise and consider their implications for understanding the causes of financial (in) stability.

Still, only a limited number of studies are considering the case of emergent financial markets and are analysing if these markets possess, at least to a certain extent, a fractal structure. We argue that the structural, functional and institutional changes of this type of markets can lead to specific features in terms of market liquidity, asymmetric effects of information shocks, risk aversion of investors and their specific trading horizon. Such particularities might be translated in local persistence as well as in long-run memory of prices for financial assets traded on these markets. Indeed, this seems to be the picture which emerges from the existing literature. Panas (2001) estimates the corresponding fractal dimension by using stable distributions and exploring long memory through ARFIMA models in Athens Stock Exchange between January 1993 and May 1998. This study found for the considered market that: (1) there is a slowly decaying autocorrelation, (2) the presence of heteroskedasticity, (3) the distributions of return series are non-normal and (4) the return series are described by a long-memory fractional process. Oprean and Tănăsescu (2013) are studying the long range dependences for eight emergent European and BRIC markets. Wang et al. (2011) finds fractal characteristics for the Chinese stock market. Saleem (2014) finds evidence of long memory in all sectors of the Russian stock market. Mahalingam et al. (2012) provide evidences that the Indian Bombay Stock Exchange has a high degree of persistence. Pleşoianu et al. (2012) perform a multi-fractal analysis upon the intra-daily and the daily time series of BET index, BET-C index and ten stocks listed on the Bucharest Stock Exchange in order to assess the degree of informational efficiency of the Romanian stock market. The empirical results of the one-dimensional backward multi-fractal de-trended moving average (MFDMA) method provide support for the multi-fractal nature of this emerging market.

In such context, the aim of this study is twofold: 1) to directly address the issue of fractal dimension estimation by discussing some estimators which are more frequently used in literature and, respectively, 2) to provide empirical evidences for the potential fractal properties of some important emergent markets. The next section reviews the methodology and the basic statistics properties of the considered emerging markets. Section 3 reports and discusses the fractal dimension estimates while Section 4 concludes.

2. METHODOLOGY AND INTERNATIONAL DATA

2.1. FRACTAL DIMENSION

We follow the standard approach in defining the fractal dimension of a point set $X \subset \mathbb{0}^d$ to be the classical Hausdorff-Besicovitch dimension (Falconer, 1990; Gneiting and Schlather, 2004).Let $\varepsilon \subset [0,\infty)$. An ε - cover of X is a countable collection $\{B_i : i = 1,2,\ldots\}$ of 'balls' $X \subset \mathbf{O}^d$ of diameter $|B_i|$, less than or equal to ε that covers X .The δ -dimensional Hausdorff-Besicovitch content is defined by:

$$H^{\delta}(X) = \lim_{\varepsilon \to 0} \inf \left\{ \sum_{i=1}^{\infty} |B_i|^{\delta} \right\} \tag{1}$$

There exists a unique non-negative value D such that $H^{\delta}(X) = \infty$ if $\delta < D$ and $H^{\delta}(X) = 0$ if $\delta > D$. This is the Hausdorff-Besicovitch dimension, which under 'weak regularity' conditions coincides with the box-count dimension (see Gneiting et al., 2012):

$$D_{BC} = \lim_{\varepsilon \to 0} \frac{\log\left(N(\varepsilon)\right)}{\log\left(\frac{1}{\varepsilon}\right)} \tag{2}$$

Here $N(\varepsilon)$ denotes the smallest number of cubes of width ε which can cover X . It can be noticed that if $\{X_t : t \in \mathbf{O}^d\}$ is a standard Gaussian random functions with stationary incrementsand with $\alpha \in (0,2]$, its fractal index d and its fractal dimension D satisfy with a probability close to one (see Adler, 1981, Gneiting and Schlather, 2004, Gneiting et al., 2012) the following relation:

$$\mathbf{D} = d + 1 - \frac{\alpha}{2} \tag{3}$$

For a random series with no local trending or no local anti-correlations, it appears that $\mathbf{D} = 1.5$, while for a series with local persistence $\mathbf{D} < 1.5$ .Finally, if the series exhibit local anti-persistence, then $\mathbf{D} > 1.5$ (see for such interpretation, for instance, Krištoufek and Vosvrda, 2014).

There are several algorithms for the estimation of the fractal dimension (see, for discussions on their properties, Dubuc et al., 1989, Hall and Wood, 1993, Chan and Wood, 2000, 2004, Zhu and Stein,2002). We consider four of these which are among the most used in literature: the 'box-count' estimator, the Hall and Wood (1993) estimator, the robust Genton (1998) estimator and the wavelet estimator (Percival and Walden, 2000).

The box-count estimator

The standard box-count equals the slope in an ordinary least squares regression fitting the $N(\varepsilon)$ on $log(\varepsilon)$:

$$\widehat{D}_{BC} = -\frac{\left\{\sum_{k=0}^{K} (s_k - \bar{s}) \log\left(N(\varepsilon_k)\right)\right\}}{\left\{\sum_{k=0}^{K} (s_k - \bar{s})^2\right\}} \tag{4}$$

Here s_k stands for $\log(\varepsilon_k)$ and $\bar{s}$ is the average of s_0, s_1,…, s_k .However, several critics have been raised in literature in respect to this standard algorithm.

Liebovitch and Toth (1989) suggest an improvement by excluding the smallest scales ε_k for which $N(\varepsilon_k) > \frac{n}{5}$, as well as the two largest scales, from the regression fit. Gneiting et al. (2012) adopt this proposal and are arguing that the restrictions on the scales improve the statistical and computational efficiency of the estimator. Still, as they remark, one critical issue of this estimator is the fact that the information at very small scales is discarded.

The Hall and Wood estimator

Hall and Wood (1993) are proposing an alternative approach of the box-count that deals with information at the smallest observed scales.
Let:

$$\hat{A}\left(\frac{l}{n}\right) = \frac{l}{n}\sum_{i=1}^{\left[\frac{n}{l}\right]}\left|X_{\frac{il}{n}} - X_{\frac{(i-1)l}{\square}}\right| \qquad (5)$$

Here $\left[\frac{n}{l}\right]$ denotes the integer part of $\frac{n}{l}$. $\hat{A}\left(\frac{l}{n}\right)$ represents the absolute deviationof the series of length n within boxes of size l. At a scale $\varepsilon_i = \frac{l}{n}$, the estimator is based on an ordinary least squares regression fit of $\log\left(\hat{A}\left(\frac{l}{n}\right)\right)$ on $\log\left(\frac{l}{n}\right)$:

$$\hat{D}_{HW} = 2 - \frac{\left\{\sum_{l=1}^{L}(s_l - \bar{s})\log\left(\hat{A}\left(\frac{l}{n}\right)\right)\right\}}{\left\{\sum_{l=1}^{L}(s_l - \bar{s})^2\right\}} \qquad (6)$$

Here $L \geq 2$, $s_i = \log\left(\frac{l}{n}\right)$ and $\bar{s} = \frac{1}{L}\sum_{l=1}^{L} s_l$. Hall and Wood (1993) recommend the usage of $L = 2$ in order to minimize the potential biases. With this setting:

$$\hat{D}_{HW} = 2 - \frac{\log\left(\hat{A}\left(\frac{2}{n}\right)\right) - \log\left(\hat{A}\left(\frac{1}{n}\right)\right)}{\log(2)} \qquad (7)$$

The Genton estimator

Genton (1998) is a highly robust estimator based on the variogram, which is defined as:

$$\hat{V}_2\left(\frac{l}{n}\right) = \frac{1}{2(l-n)}\sum_{i=l}^{n}\left(X_{\frac{i}{n}} - X_{\frac{i-1}{n}}\right)^2 \qquad (8)$$

With this definition, the Genton estimator is:

$$\widehat{D}_G = 2 - \frac{\left\{\sum_{l=1}^{L}(s_l - \bar{s})\log\left(\widehat{V}_2\left(\frac{l}{n}\right)\right)\right\}}{2\left\{\sum_{l=1}^{L}(s_l - \bar{s})^2\right\}} \quad (9)$$

The notations are the same as for Hall and Wood estimator. If once again we consider $L = 2$, the estimator becomes:

$$\widehat{D}_G = 2 - \frac{\log\left(\widehat{V}_2\left(\frac{2}{n}\right)\right) - \log\left(\widehat{V}_2\left(\frac{1}{n}\right)\right)}{2\log(2)} \quad (10)$$

The wavelet estimator

The starting point in the construction of this estimator is represented by the maximal overlap discrete wavelet transform (MODWT). This transform involves the application of J_0 pairs of filters. More exactly, at each j -th level (with $j = 1, \ldots, J_0$), it implies to consider a wavelet (high-pass) filter $\tilde{h}_{j,l}$ yielding to a set of wavelet coefficients (where here L denotes the length of the wavelet filter) $\widetilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l}$ and, respectively, of a scaling (low-pass) filter $\tilde{g}_{j,l}$ yielding to a set of scaling coefficients $\widetilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l}$ (Percival and Walden, 2000). It can be noticed that the number of wavelet and scaling coefficients is the same as the number of sample observation at every level of the transform leading to an increase in computational complexity comparing with the standard discrete wavelet transform (DWT).

The underlying idea for this estimator is that at the j -th level the coefficients are associated with the scale $\tau_j = 2^{j-1}$, with corresponding frequencies band $\left(\frac{\pi}{2^j}, \frac{\pi}{2^{j-1}}\right]$. The average of these coefficients squared provides an estimator for the wavelet variance. For large τ_j, such variance varies approximately as τ_j^{α} (see for more details on this estimator and the corresponding frame Dubuc et al., 1989, Chan et al., 1995, Percival and Walden, 2000, Gneiting et al., 2012). The parameter α is the fractal index and can be used in order to estimate the fractal dimension.

We further involve these estimators for the analysis of local properties of nine important emergent capital markets. The implementation of the corresponding algorithms is performed via the R package fractal dim developed by Ševčíková et al. (2014). The MODWT uses the function modwt from the R package wavelets (Aldrich, 2013).

2.2. International Data

Data represents (logarithmic) daily close values for the indices of nine significant emerging markets. The data set was obtained by combining data from the Polish stock market portal Stooq (http://stooq.pl). The data is covering a time span between 14.09.1993-28.02.2014, including various events such as the Asian financial crisis, the DotCom bubble, the relatively stable path of growth between 2003 and 2007 as well as the recent financial turmoil period and its consequences. The gathered data was synchronized by removing periods without trading activity (weekends, night time and holidays) for the stock exchanges taken into account. One may argue that such procedure has the disadvantage that the observations which are separated in real time may become close neighbours in the synchronized time series. However, there are only a few observations comparing with the regular data and so the procedure does not lead to significant disturbances in data.

The main statistics of the (logarithm) close values are reported in Table 1.As per these statistics, the time series are mostly negatively skewed (with the notable exception of PX and SENSEX 30 indices) and more than half are platykurtic.

Table 1: Descriptive statistics for the considered indices

Market indices	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Shanghai Composite Index–China	3.224	3.220	3.785	2.531	0.215	-0.179	2.994
BUX Index – Hungary	4.007	4.044	4.479	3.047	0.350	-0.976	3.297
WIG20 – Poland	3.259	3.260	3.593	2.762	0.168	-0.353	2.679
PX Index – Czech Republic	2.883	2.896	3.287	2.500	0.208	0.153	1.698
SENSEX 30 Index – India	3.854	3.744	4.330	3.415	0.311	0.238	1.362
Hang Seng Index – Hong Kong	4.169	4.159	4.500	3.824	0.145	-0.036	1.887
KOSPI Index – South Korea	3.017	2.982	3.344	2.447	0.208	-0.236	2.245
Mexican Bolsa Index – Mexico	4.051	4.004	4.662	3.177	0.417	-0.083	1.625
Bovespa Index – Brazil	4.323	4.342	4.866	2.080	0.421	-0.784	4.114

Source: Author`s own calculations

3. RESULTS AND DISCUSSION

Table 2 reports the estimates of the corresponding fractal dimensions based on the mentioned estimators. These results indicate a diversified ecology of the markets. On average, all the included indices display fractal dimensions below 1.5 which indicates local persistent. In other words, they show strong positive autocorrelations for the analysed period. The highest deviation from the reference value of 1.5 corresponds to the Brazilian stock index Bovespa while the lowest one corresponds to the Polishstock index WIG20. One interesting feature is that there seems to be a regional component of short-run behaviour of markets with Asian markets being closer to no-local persistence status.

However, there are some differences between individual estimators. Usually, the Hall-Wood and wavelet estimators tend to exhibit higher levels while the box-count and Genton estimators points out toward stronger local persistence in data.

Table 2: Various estimators of fractal dimension: full sample (14.09.1993-28.02.2014)

	Box-count estimator	Hall–Wood estimator	Genton estimator	Wavelet estimator	Averages of all measures
	(1)	(2)	(3)	(4)	(5)
Shanghai Composite Index –China	1.369	1.433	1.380	1.500	1.438
BUX Index – Hungary	1.305	1.427	1.409	1.479	1.405
WIG20 –Poland	1.408	1.468	1.440	1.546	1.466
PX Index – Czech Republic	1.338	1.412	1.405	1.483	1.410
SENSEX 30 Index – India	1.423	1.457	1.398	1.501	1.445
Hang Seng Index –Hong Kong	1.411	1.466	1.380	1.539	1.449
KOSPI Index – South Korea	1.403	1.473	1.368	1.521	1.441
Mexican Bolsa Index – Mexico	1.347	1.456	1.360	1.517	1.420
Bovespa Index – Brazil	1.273	1.423	1.450	1.414	1.390

Source: Author`s own calculations

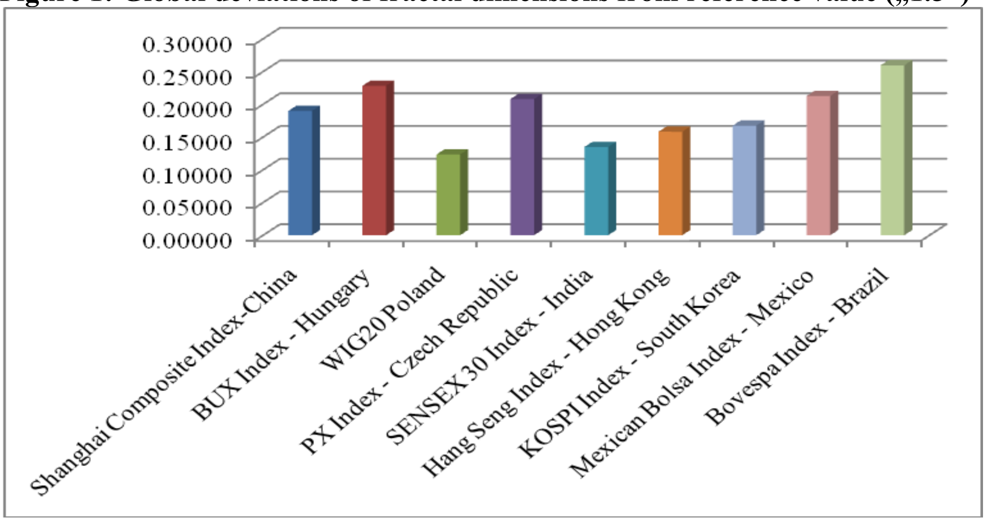
A possible alternative explanation of the findings might emerge if the fractal dimension is seen as a measure of market complexity. As this complexity will increase under the impact of better liquidity and the inclusion of more diversified and sophisticate financial instruments, the short-run traders will have more opportunities for arbitrage operations and hence less local persistence will be observed. On the other hand, if the markets are characterized by low liquidity and thin-trade, then the correlation between their evolution and macroeconomic fundamentals will be weakened. Also, their evolutionary path will have associated a higher degree of uncertainty. Hence, short-run trades and 'heard behaviour' will occur more frequently on such markets. Finally, a key role is played by regional and international capital flows as well as by the degree of financial integration for local markets.

Further, in order to provide an assessment of the fractal dimension deviations from the 'no persistence' reference value, we compute for each individual market j a global measure of such deviations which consider all four estimators i:

$$GD^j = \sqrt{\sum_{i=1}^{4}\left(\hat{D}_i - 1.5\right)^2} \tag{11}$$

The ranks of the markets accordingly to this measure are displayed in Figure 1. The same conclusions arise from this figure with Bovespa, BUX and Mexican Bolsa indices showing the largest global deviations while the smallest global deviations occur for WIG20, SENSEX 30 and Hang Seng indices.

Figure 1: Global deviations of fractal dimensions from reference value („1.5")



Source: Author`s own calculations

An interesting question which can be raise is the following: how did the current financial period affect the market behaviour? In order to search for empirical evidences, we re-estimate the fractal dimensions for a sub-sample of data between 02.01.2008 and 30.12.2010. The results are reported in Table 3.

Table 3: Various estimators of fractal dimension – 02.01.2008-30.12.2010 sample

	Box-count estimator	Hall-Wood estimator	Genton estimator	Wavelet estimator	Averages of all measures
	(1)	(2)	(3)	(4)	(5)
Shanghai Composite Index –China	1.330	1.459	1.374	1.465	1.407
BUX Index – Hungary	1.343	1.474	1.429	1.489	1.434
WIG20 – Poland	1.365	1.513	1.420	1.519	1.454
PX Index – Czech Republic	1.350	1.462	1.435	1.497	1.436
SENSEX 30 Index – India	1.350	1.452	1.454	1.487	1.436
Hang Seng Index – Hong Kong	1.385	1.497	1.456	1.553	1.473
KOSPI Index – South Korea	1.388	1.473	1.402	1.543	1.452
Mexican Bolsa Index – Mexico	1.360	1.460	1.384	1.539	1.436
Bovespa Index – Brazil	1.370	1.505	1.396	1.567	1.460

Source: Author`s own calculations

The largest decrease in the average fractal dimension during the crisis period was recorded for Shanghai Composite Index followed by WIG20 and SENSEX 30 while for other indices such as Bovespa or Mexican Bolsa the considered estimator suggests that there was in fact an increase in the fractal dimension comparing with the full sample. It can be argued that capital markets which are more integrated with the US are more sensitive to exogenous shocks located on these markets and thus the impact of the subprime crisis was broadly translated in an increase in their local persistence mechanisms. Once again, box-count and Genton methodologies are providing lower estimates comparing with other two methodologies. However, it can be noticed that, regardless what methodology is involved, all of them are highlighting the same type of changes at the level of individual indices.

What can be learned from these findings? At least several things might be stressed. Firstly, there empirical evidences support—for the included markets and during the analysed time span—the role played by dominant trading horizon: the shorter is this horizon, the more important are the trading decisions based on technical analysis, current status of market environment and levels and structures of market liquidity. Due to imperfect liquidity as well as to asymmetric impact exercised by new information, imperfect market mechanisms and institutions and significant trading risks, such short trading horizons are dominating emergent markets. This can be one possible explanation for the short-run persistence underlined by the values of fractal dimensions regardless of the considered methodology. Secondly, among emergent markets there can be quite substantial differences in terms of deviation from the 'non-persistence' case. Such differences might have endogenous sources in terms of particular market conditions but also might be induced by regional and international capital flows. For instance, as de Paula et al. (2013) note, recent inflows to markets in Latin America have been mostly determined by push (global) factors rather than by pull (domestic) factors. After a period of positive evolution, the recent financial and real turmoil has propagated in the area through mechanisms specific to emerging markets such as foreign capital withdraws perturbation of international trade flows and decline of exports. Thirdly, such information imperfection might be a consequence of markets' failure to attach a 'fair value' to the traded financial assets. As Anderson and Noss (2013:16) argue: "the concern is that more frequent and volatile valuations may risk leading to a reduction of the investment horizon of certain sorts of investor".

Of course, the considered methodological framework can be used in order to check for local persistence but cannot directly address the causes of such market imperfections. Thus, a more detailed analysis is required in order to deal with the involved market mechanisms and structures.

4. Conclusions

This paper assumes a twofold objective: 1) to discuss some methodological approaches that can provide empirical support for the Fractal Market Hypothesis and, respectively, 2) to apply these methodologies on some important emergent markets in order to stress some features specific to the fractal behaviour in their recent dynamics.

We find that emergent markets from Europe and Asia are closer to the 'non-persistence' status while markets from Latin America exhibit more significant signs of local persistence. We also find that recent financial turmoil contributed to the reduction of the markets' capacity to deal with information shocks.

Even if the fractal dimension estimators are not able per se to indicate the causes of markets information inefficiency, their usage can point out toward the importance of ensuring financial stability, and hence, can provide a piece of evidence in the discussion about regulatory measures aiming to support such stability.

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REFERENCES

[1] **Adler, R. J. (1981),** ***The Geometry of Random Fields***; *Wiley*, New York, 1981;

[2] **Aldrich, E. (2013),** ***Wavelets: A Package of Functions for Computing Wavelet Filters, Wavelet Transforms and Multiresolution Analyses (2013);*** R package version 0.3-0 (2013-12-17);

[3] **Anderson, N. and Noss, J. (2013),** ***The Fractal Market Hypothesis and its Implications for the Stability of Financial Markets***; Bank of England Financial Stability Paper No. 23 – August 2013;

[4] **Blackledge, J.M. (2008),** ***Application of the Fractal Market Hypothesis for Macroeconomic Time Series Analysis***; ISAST Transactions on Electronics and Signal Processing 2, 89-110;

[5] **Bodie, Z., Kane, A. and Marcus, A.J. (2007),** ***Essentials of Investments*, 6th edition**; *McGraw-Hill / Irwin*;

[6] **Chan, G., Hall, P. and Poskitt, D. S. (1995),** ***Periodogram-based Estimators of Fractal Properties;*** Annals of Statistics 23, 1684–1711;

[7] **Chan, G. and Wood, A. T. A. (2000),** ***Increment-Based Estimators of Fractal Dimension for Two Dimensional Surface Data***; Statistica Sinica 10, 343–376;

[8] **Chan, G. and Wood, A. T. A. (2004),** ***Estimation of Fractal Dimension for a Class of Non-Gaussian Stationary Processes and Fields***; Annals of Statistics 32, 1222–1260;

[9] **Cont, R. (2001),** ***Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues;*** Quantitative Finance, 1 (2001), 1–14;

[10] **Cutler, D., Poterba, J.and Summers, L. (1989),** ***What Moves Stock Prices?;*** Journal of Portfolio Management, 4–12;

[11] **De Paula, L. F., Ferrari-Filho, F. and Gomes, A.M. (2013),** ***Capital Flows, International Imbalances and Economic Policies in Latin America***; In Arestis, P., Sawyer, M. (Eds.), Economic Policies, Governance and the New Economics, Palgrave Macmillan, October, 209–248;

[12] **Dubuc, B., Quiniou, J. F., Roques-Carmes, C., Tricot, C. and Zucker, S. W. (1989),** ***Evaluating the Fractal Dimension of Profiles***; Physical Review A, 39 (1989), 1500-1512;

[13] **Falconer, K. (1990),** ***Fractal Geometry, Mathematical Foundations and Applications****; Wiley*, Chichester, 1990;

[14] **Fang, H., Lai, K.S. and Lai, M. (1994),** ***Fractal Structure in Currency Futures Price Dynamics***; Journal of Futures Markets 14, 169-181;

[15] **Genton, M. G. (1998),** ***Highly Robust Variogram Estimation***; Mathematical Geology 30, 213–221;

[16] **Geweke, J. and Porter-Hudak, S. (1983),** ***The Estimation and Application of Long Memory Time Series Models****;* Journal of Time Series Analysis 4, 221-238;

[17] **Gneiting, T. and Schlather, M. (2004),** ***Stochastic Models that Separate Fractal Dimension and the Hurst Effect****;* SIAM Review 46 (2004), 269–282;

[18] **Gneiting, T., Ševčíková, H. and Percival, D.B. (2012),** ***Estimators of Fractal Dimension: Assessing the Roughness of Time Series and Spatial Data****;* Statistical Science, Volume 27, Number 2 (2012), 247-277;

[19] **Granger, C. W. J. and Joyeux, R. (1980),** ***An Introduction to Long-Memory Time Series Models and Fractional Differencing***; Journal of Time Series Analysis, 1, l5-39;

[20] **Guillaume, D., Dacorogna, M., Davé, R., Müller, U., Olsen R. and Pictet, O. (1997),** ***From the Bird's Eye View to the Microscope: A Survey of New Stylized Facts of the Intraday Foreign Exchange Markets***; Finance and Stochastics, 1 (1997), 95–131;

[21] **Hall, J.A., Brorsen, W. and Irwin, S.H. (1989),** ***The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normal Hypotheses;*** Journal of Financial and Quantitative Analysis 24, 105-116;

[22] **Hall, P. and Roy, R. (1994), On the relationship between fractal dimension and fractal index for stationary stochastic processes**; Annals of Applied Probability 4 (1994), 241–253;

[23] **Hall, P. and Wood, A. (1993),** ***On the Performance of Box-Counting Estimators of Fractal Dimension***; Biometrika 80 (1993), 246–252;

[24] **Krištoufek, L. and Vosvrda, M. (2014),** ***Measuring Capital Market Efficiency: Long-Term Memory, Fractal Dimension and Approximate Entropy****;* The European Physical Journal B. Condensed Matter and Complex Systems 87: 162;

[25] **Liebovitch, L. S. and Toth, T. (1989),** ***A Fast Algorithm to Determine Fractal Dimensions by Box Counting****;* Physics Letters A 141 (1989), 386–390;

[26] **Lo, A.W. (1991),** ***Long-Term Memory in Stock Market Prices***; Econometrica 59, 1279-1313;

[27] **Mahalingam, G., Murugesan, S. and Jayapal, G. (2012),** ***Persistence and Long Range Dependence in Indian Stock Market Returns;*** International Journal of Management and Business Studies, Volume 2, Issue 4, October-December 2012, 72-77;

[28] **Mandelbrot, B. (1982),** ***The Fractal Geometry of Nature***; Freeman, New York;

[29] **Mantegna, R.N. and Stanley, H.E. (1995),** ***Scaling Behaviour in the Dynamics of an Economic Index;*** Nature 376, 46-49;

[30] **Oprean, C. and Tănăsescu, C. (2013),** ***Applications of Chaos and Fractal Theory on Emerging Capital Markets***; International Journal of Academic Research in Business and Social Sciences November 2013, Vol. 3, No. 11, 633-653.

[31] **Panas, E. (2001),** ***Estimating Fractal Dimension Using Stable Distributions and Exploring Long Memory through ARFIMA Models in Athens Stock Exchange***; Applied Financial Economics, 2001, 11, 395-402;

[32] **Panas, E. and Ninni, V. (2010),** ***The Distribution of London Metal Exchange Prices: A Test of the Fractal Market Hypothesis***; European Research Studies, Volume XIII, Issue (2), 2010, 193-201;

[33] **Percival, D. B. and Walden, A. T. (2000),** ***Wavelet Methods for Time Series Analysis;*** Cambridge University Press, Cambridge, New York (2000);

[34] **Peters, E. (1990),** ***Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility***; *Wiley*, New York;

[35] **Peters, E. (1994), Fractal Market Analysis:** ***Applying Chaos Theory to Investment and Economics;*** Wiley, New York;

[36] **Pleşoianu, A., Todea, A. and Căpuşan, R. (2012),** ***The Informational Efficiency of the Romanian Stock Market: Evidence from Fractal Analysis;*** International Conference Emerging Markets Queries in Finance and Business, Petru Maior University of Tîrgu-Mures, ROMANIA, October 24th - 27th, 2012 Procedia Economics and Finance, Volume 3, 2012, 111–118;

[37] **Saleem, K. (2014),** ***Modeling Long Memory in the Russian Stock Market: Evidence from Major Sectorial Indices***; The Journal of Applied Business Research, March/April 2014, Volume 30, Number 2, 567-574;

[38] **Ševčíková, H., Percival, D.B. and Gneiting, T. (2014),** ***Fractaldim: Estimation of Fractal Dimensions;*** R package version 0.8-4 (2014-02-24);

[39] **Wang, Y., Wang, J., Guo, Y., Wu, X. and Wang, J. (2011),** ***FMH and Its Application in Chinese Stock Market;*** Advanced Research on Computer Education, Simulation and Modeling Communications in Computer and Information Science Volume 176, 2011, 349-355;

[40] **Zhu, Z. and Stein, M.L. (2002),** ***Parameter Estimation for Fractional Brownian Surfaces;*** Statistica Sinica 12, 863–883.