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HIGHER ORDER ADAPTIVE KALMAN FILTER FOR TIME VARYING ALPHA AND CROSS MARKET BETA ESTIMATION IN INDIAN MARKET

Abstract: *First order Adaptive Kalman Filter (AKF) were successful for market risk beta estimation to accommodate the adaptive parameters better in a time varying CAPM. This paper presents a new formulation of a noise covariance adaptation based second and third order AKF for joint estimation of alpha (risk-free), co-incidental and cross market risks (betas) components of market returns in a "two factor" CAPM. Investigations reveal that the higher order AKFs perform as good as Kalman filter in spite of flexibility in the time varying noise covariance.*

Key Words: *Adaptive Kalman Filter, Time Varying Alpha, Cross Market Beta Estimation, Higher Order Filtering, Indian Market.*

JEL Classification: C13, C32, C58, G13.

1. Introduction

This paper extends the first order Adaptive Kalman Filter (AKF) to second and third order (henceforth AKF2 and AKF3 respectively) for estimation of time varying parameters of CAPM. Though classical CAPM assumed that the driving parameters α (market independent part) and β (market sensitivity) are constant over time, sufficient evidences [1, 2, 5, 9] have been mentioned in the literature that the β parameter is time varying and α is constant through out the considered time interval. Again literature [1] shows that α may be assumed to be time varying but the rate of variations is low (slow) as compared to β variability. Since first order AKF filtering techniques are not competent enough to estimate both these time varying CAPM parameters jointly. But first order AKF may be extended to its second order successor which can estimate β variability while α could be presumed time varying.

For the development of AKF2, necessary second order model is already empirically characterized in [5] for second order Kalman Filter (KF2). [5] used a second order state-space model (combining random walk model of state evolution and market model for observation modelling) for CAPM parameters (α and β) estimation in Italian market. This paper presents the report on the investigations of estimating these two parameters by second order KF (KF2), where both the noise covariances (Q and R of process and observation models respectively) are assumed to be known a priori. [8] have shown that there are justifications to treat these noise covariances as time varying. [6] used a technique for estimating the unknown variance R by OLS residuals as an estimate of the error sequence rather than estimate as a constant variance for the observation equation. Then R was calculated using different time lengths, i.e., the variance at period t was based on the said residuals from the first to the t-th period. After that they look Q to be proportional to that of the observation equation and the factor (a constant) of proportionality was the ratio between the OLS estimate of the variance of the estimated β and the OLS residuals variance for the entire sample. [5] used maximum likelihood based techniques for estimation of the above mentioned considered model parameters. Present work, on the contrary, proposed to solve Q and R estimation problems using the techniques QAKF and RAKF (along with their corresponding modified methods) respectively introduced in [2].

A single β model may not be sufficient to explain the cross-section of returns while describing a large portion of the common variation in returns in spite of the CAPM's popularity. According to [9] "the empirical deficiencies of the CAPM are most commonly explained by missing risk factors or by a mis-approximation of the total wealth portfolio". These lead to multifactor pricing models as motivated by the Intertemporal Capital Asset Pricing Model (ICAPM) introduced by [10]. A two factor model (with two separate βs say β_1 and β_2) may be considered for explaining the portfolio returns. One of the two factors identifies the sensitivity (β_1) of the market where the instruments of the portfolio are traded and other one identifies a cross market sensitivity (β_2). Then a third order model may be constructed with states α, β_1 and β_2. Present work also extended the proposed AKF2 techniques to third order AKF (AKF3) and applied the techniques for estimation of time varying α, β_1 (as sensitivity of Nifty) and β_2 (as sensitivity of Sensex) of four Indian sectoral portfolios (appeared as indices of Bank, Midcap, Defty and Junior) of NSE.

Noise covariances adaptation based first order AKF methods have been characterized by [2] where either Q or R have been assumed to be and estimated during filtering. Q is adapted in first order QAKF (QAKF1) by a scaling method introduced by [3] and R is adapted in first order RAKF (RAKF1) by covariance matching principles introduced by [7]. This paper concentrates on characterizing second and third order QAKF (QAKF2 and QAKF3) along with second and third order RAKF (RAKF2 and RAKF3). R is assumed to be known a priori in the first case and Q is assumed to be known a priori in the last case. The QAKF2 and RAKF2 are characterized for α and β estimation with full synthetic dataset (where

r_{it} and r_{mt} both synthetic) and semi-synthetic dataset (where r_{it} is synthetic prepared with r_{mt} from market data). Empirical (where r_{it} and r_{mt} both from market data) characterization of QAKF2 and RAKF2 are also presented here. Moreover, QAKF3 and RAKF3 are only characterized for empirical α, β_1 and β_2 estimation with Indian data in this work.

The following sections of this paper are organized as follows. Section 2 presents the second and third order state space model used for the higher order filter characterization. Section 3 presents the proposed AKF2 and AKF3. Section 4 presents the results of the investigations with AKF2 and AKF3 for alpha and beta estimation. The paper is ended with a section on summarizing the conclusions and contributions.

2. Background Theory

2.1 Second Order State Space Model

The relation between the portfolio returns and the market index returns can be expressed by the standard market model given by: $r_{it} = \alpha_{it} + \beta_{it} r_{mt} + \varepsilon_{it}$ (1)

where r_{it} is the return for the portfolio *i*, r_{mt} is the return for the market index, α_{it} is the random variable that describe the slow varying component of the return for the portfolio i which is independent to the market return (instead its time independent assumption), ε_{it} is the random disturbance vector all at time *t* such that: $E(\varepsilon_{it})=0 \ \forall i, \ \forall t$; $E(\varepsilon_{it}\varepsilon_{jt}^T)=0 \ \forall i, \ \forall j \ \forall t, \ i \neq j$; $E(\varepsilon_{it}\varepsilon_{i\tau}^T)=0 \ \forall i, \ \forall t, \forall \tau$, $t \neq \tau$; $E(\varepsilon_{it} r_{mt}^T)=0 \ \forall i, \ \forall t, \ i \neq j$. Eq. (1) demonstrates that the return for the portfolio i (r_{it}), at time instant t, depends on the return for the market index r_{mt} on the same time. Moreover, the relation between these two variables is linear. Coefficient β shows how portfolio returns vary with the market returns and is used to measure the portfolio's systematic risk (or market risk).

[5, 9] assumed that α and β follows a random walk (RW) model and [12] developed parametric statistical tests to verify if α and β follows this process. The RW model of α and β dynamics can be expressed as $\alpha_{it} = \alpha_{it-1} + u_{it}$ and $\beta_{it} = \beta_{it-1} + \eta_{it}$. We assume that random variables ε_{it}, u_{it} and η_{it} are Gaussians i.e., $\varepsilon_{it} \sim N(0,R)$, $u_{it} \sim N(0,Q_1)$, $\eta_{it} \sim N(0,Q_2)$. Initial conditions are $\alpha_0 \sim N(\alpha_0, P_{011})$, $\beta_0 \sim N(\beta_0, P_{022})$. It is supportive to represent the RW model in the state space framework before proceeding.

Observation equation: $y(t) = C(t)x(t) + \varepsilon_t = C(t)x(t) + R_{root}\psi(t)$ (2)

Observation equation represents the market model with time-varying coefficients where matrix $C(t)$ has dimensions $T \times 2$ so that each row will represent the market realizations at certain point in time, has the following structure: $C(t) = [1 \;\; | \;\; r_{mt}]$ and the values of $C(t)$ is assumed to be known. The state vector $x(t)$ has dimensions 2×1 and represents the α and β coefficients at time t: $x(t) = [\alpha_{it} \;\; \beta_{it}]'$. The variance of ε_{it} is unknown, assumed finite and modelled by matrix $R_{root} = \sqrt{R}$.

State equation: $x(t) = Fx(t-1) + \xi_t = Fx(t-1) + Q_{root}\chi(t)$ (3)

The covariance matrix Q is assumed diagonal, finite and its elements are unknown. Matrix F is the identity matrix in the adopted model (RW) in this work. i.e., $F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ while vector ξ_t models the random part of the state vector: $\chi(t) = [u_{it} \;\; \eta_{it}]'$. The covariance matrix of the state noise has the following structure $Q_{root} = \begin{bmatrix} \sqrt{Q_1} & 0 \\ 0 & \sqrt{Q_2} \end{bmatrix}$ and $Q = Q_{root} Q_{root}^T$. Vector $\vartheta_2 = (R, Q_1, Q_2)$ consisting of three stochastic parameters must be estimated

2.2 Third Order State Space Model

A possibly multidimensional set of common factors are mapped to the returns of a portfolio in "factor pricing model" [9] which are widely employed by investment professionals to characterize risk of a portfolio and hence obtain return predictions. Multifactor pricing models are called for instead of simple CAPM to improve the prediction quality of a portfolio returns with respect to distributed multi-market risk factors. For understanding risk based and non-risk based more justifications (like data-snooping biases, the existence of market frictions, transaction costs and liquidity effects) to think beyond CAPM the reader can refer to [9]. The present modeling approach (two factors pricing model) employed in this section has two aspects: sectorial portfolios of equities are used as dependent variables instead of single stocks and time-varying factor sensitivities are modelled as individual stochastic processes. A "two factors" pricing model (having two market factors) can characterize the return (r_{it}) of a portfolio with respect to return (r_{m1t}) of a market where the assets of that portfolio are traded together with the return (r_{m2t}) of a parallel secondary or complementary market in which those assets are also traded and arbitrage is possible. Mathematically, these can be expressed as:

$r_{it} = \alpha_{it} + \beta_{1t} r_{m1t} + \beta_{2t} r_{m2t} + \varepsilon_{it}$ (4)

where β_{1t} and β_{2t} are the sensitivities of the primary and cross (or secondary) markets returns respectively with respect to the portfolio returns. All other assumptions corresponding to the eqn. 1 are assumed to be valid with the above eq. 4 and also same assumptions are considered for both the market returns while considering a single market effect in eqn. 1.

In this case α, β_1 and β_2 assumed to follow a random walk model given by $\alpha_{it} = \alpha_{it-1} + u_{it}$, $\beta_{1t} = \beta_{1t-1} + \eta_{1t}$ and $\beta_{2t} = \beta_{2t-1} + \eta_{2t}$. We assume that random variables ε_{it}, u_{it}, η_{1t} and η_{2t} are Gaussians i. e. $\varepsilon_{it} \sim N(0,R)$, $u_{it} \sim N(0,Q_1)$, $\eta_{1t} \sim N(0,Q_2)$ and $\eta_{2t} \sim N(0,Q_3)$. Initial conditions are $\alpha_0 \sim N(\alpha_0, P_{011})$, $\beta_{10} \sim N(\beta_{10}, P_{022})$, $\beta_{20} \sim N(\beta_{20}, P_{033})$. It is again supportive (like second order model) to represent the RW model in the state space framework before proceeding. To do so, the observation and state equation can be taken as same as that stated in eqn. 2 and 3 respectively. This observation equation represents the market model with time-varying coefficients where matrix $C(t)$ has dimensions $T \times 3$ so that each row will represent the observations at certain point in time, has the following structure: $C(t) = [1 \quad | \quad r_{m1t} \quad | \quad r_{m2t}]$ and is assumed to be known. The state vector $x(t)$ has dimensions 3×1 and represents the α and β coefficients at time t: $x(t) = [\alpha_{it} \quad \beta_{1t} \quad \beta_{2t}]'$. The variance of ε_{it} is unknown, assumed finite and modelled by matrix $R_{root} = \sqrt{R}$. The process noise covariance matrix Q is assumed diagonal and finite. In the model (RW) adopted in the present work, matrix F is the identity matrix given by $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. while vector ξ_t or $\chi(t)$ models the random part of the state vector expressed as: $\chi(t) = [u_{it} \quad \eta_{1t} \quad \eta_{2t}]'$ The covariance matrix of the state noise has the following structure $Q_{root} = \begin{bmatrix} \sqrt{Q_1} & 0 & 0 \\ 0 & \sqrt{Q_2} & 0 \\ 0 & 0 & \sqrt{Q_3} \end{bmatrix}$ and $Q = Q_{root} Q_{root}^T$. Vector $\vartheta_3 = (R, Q_1, Q_2, Q_3)$ consisting of four stochastic parameters must be estimated in this case.

3. Higher Order AKF

Second order KF (KF2) were tried out systematically for the present application. This was necessary to confirm that the assumed structure of the covariance matrices and the unity system matrix assumptions are adequate. It was also necessary to establish the kind of accuracy one can expect in idealized situations. The simplest of cases to be tried out was with known Q_1, Q_2 and R and using synthetic data generated using the model given by state-space equations 4 and 8 with Gaussian noise and other system descriptions where $r_{mt} = \mathbf{1}_{Tx1}$. Note that this is the simplest case and no adaptation was involved. It is observed that the performances of joint α and β estimates are good for some true known combination like R =1e-8, Q_1 =1e-6 and Q_2 =1e-7 (Case 1) with respect to RMSE. However, the performance of KF2 is not so good while some true known combination like R=1e-6, Q_1=1e-6 and Q_2 =1e-3 (Case 2) is choosen.

The next experiment was carried out with "semi-synthetic" data [4]. The semi-synthetic data was generated by using the daily observed market returns r_{mt} (Nifty of NSE) instead of $r_{mt} = \mathbf{1}_{Tx1}$. It is observed that the performance of KF2 with semi-synthetic data is better than Case 1 and Case 2 of full synthetic data for time varying joint α and β estimation as expected. Since we have realized a better performance of KF2, the next question is "can the performance be improved more?". If so, then at what extent this performance can be enhanced. Or in other words, is there any optimal choice of these known parameters combinations. We have found out that the possible optimal (better) combination of the said parameters are R =1e-8, Q_1=1e-6 and Q_2 =1e-7 (Case 3) as evident with the experiments with semi-synthetic data. This optimal combination is decided on the basis that the average RMSE became stable after the above combination while conducting Monte Carlo experiments. This can be claimed to be optimal combination if such stability is not due to the numerical limitations of the estimation software (Matlab in our case). It is also observed that the performance of KF2 with semi-synthetic data and parameters of Case 3 is much better than Case 2 for time varying α and β estimation with respect to average RMSE of 1000 MC runs.

3.1 AKF2 Methods and Simulations

3.1.1 QAKF2 and MQAKF2 Algorithms

The QAKF2 has been developed by extending the algorithm for second order KF with known Q and known R. QAKF2 assumed that the values of R is known to the filter but Q is unknown. The formula proposed by [3] for time varying Q estimation has been used in the following QAKF2 for time varying α and β (state) estimation here similar to the usage at first order QAKF.

Algorithm 3.1: QAKF2 for joint α and β estimation with known R

Step 1: Initialize $\hat{x}(t)$, $P(t)$ and Q and set m, $C(t)$, F, and known R $\forall$ t.

Step 2: Calculate optimal prediction of the state vector $\hat{x}(t \mid t-1) = F\hat{x}(t-1)$

Step 3: Calculate prediction covariance matrix

$$P_p(t) = FP(t)F^T + Q_{root}(t,\hat{\vartheta})Q_{root}{}^T(t,\hat{\vartheta})$$

Step 4: Calculate innovation $v(t) = y(t) - C(t)\hat{x}(t \mid t-1)$.

Step 5: Calculate filter gain $K(t) = P_p(t)C^T(t)\left(C(t)P_p(t)C^T(t) + R_{root}(\hat{\vartheta})R_{root}{}^T(\hat{\vartheta})\right)^{-1}$.

Step 6: Calculate filter covariance $P(t) = [I - K(t)C(t)]P_p(t)$

Step 7: Calculate estimate of the state vector
$\hat{x}(t) = \hat{x}(t \mid t-1) + K(t)\left(y(t) - C(t)\hat{x}(t \mid t-1)\right)$

Step 8: Check the value of t.

Step 9: If t> m then carry out step 9 otherwise repeat step 2 to 8 with
$Q_{root}(t+1) = Q_{root}(t)$.

Step 10: Update state noise covariance $Q_{root}(t+1) = Q_{root}(t)\sqrt{\lambda}$ where

$$\lambda = \frac{\frac{1}{m}\sum_{i=0}^{m-1} v(t-i)v^T(t-i) - R}{trace\{C(t)P_p(t)C^T(t)\}}.$$

Step 11: Repeat steps 2 to 10 for all epochs.

Both fully synthetic and semi-synthetic data were used first to characterize this QAKF2. It was noted that the diagonal elements of Q can be tracked adequately. As a few failure cases (λ negative) were encountered, it was decided to use Modified QAKF2 (MQAKF2), that is the modified form, presented below.

Algorithm 3.2: MQAKF2 for α and β estimation with known R

Step 1: Initialize $\hat{x}(t)$, $P(t)$ and Q and set m, $C(t)$, F, and known R $\forall$ t.

Step 2: Repeat step 2 to 9 of algorithm 3.1 (QAKF2).

Step 3: Update state noise covariance: $Q_{root}(t+1) = Q_{root}(t)\sqrt{\lambda}$ where

$$\lambda = \left|\frac{\frac{1}{m}\sum_{i=0}^{m-1} v(t-i)v^T(t-i) - R}{trace\{C(t)P_p(t)C^T(t)\}}\right|.$$

Step 4: Repeat steps 2 and 3 for all epochs.

It is observed that the performance of the MQAKF2 is comparable with KF2 though the values of Q_1 and Q_2 are unknown in his case. Q_1 and Q_2 estimation performance is also observed to be acceptable for this modified AKF algorithm with full synthetic and semi-synthetic dataset.

3.1.2 RAKF2 and MRAKF2 Algorithms

The following algorithm RAKF2 has been developed by extending the algorithm for second order KF where both Q and R are known. RAKF2 assumed that the value of Q is known to the filter but R is unknown. The formula proposed by [3, 11] for time varying Q estimation has been used for adaptive time varying α and β (states) estimation.

Algorithm 3.3: RAKF2 for α and β estimation with known Q

Step 1: Initialize $\hat{x}(t)$, $P(t)$ and R and set m, $C(t)$, F, and known Q $\forall$ t.

Step 2: Calculate optimal prediction of the state vector $\hat{x}(t \mid t-1) = F\hat{x}(t-1)$

Step 3: Calculate prediction covariance matrix $P_p(t) = FP(t)F^T + Q_{root}(\hat{\vartheta})Q_{root}^T(\hat{\vartheta})$

Step 4: Calculate innovation: $v(t) = y(t) - C(t)\hat{x}(t \mid t-1)$.

Step 5: Calculate gain: $K(t) = P_p(t)C^T(t)\left(C(t)P_p(t)C^T(t) + R_{root}(t,\hat{\vartheta})R_{root}^T(t,\hat{\vartheta})\right)^{-1}$

Step 5: Calculate covariance: $P(t) = [I - K(t)C(t)]P_p(t)$

Step 7: Calculate state estimate: $\hat{x}(t) = \hat{x}(t \mid t-1) + K(t)\left(y(t) - C(t)\hat{x}(t \mid t-1)\right)$

Step 8: Check the value of t.

Step 9: If t> m then carry out step 10 otherwise repeat step 2 to 8 with

$R(t+1) = R(t)$.

Step 10: Update observation noise covariance

$$R(t+1) = \tfrac{1}{m}\sum_{i=0}^{m-1} v(t-i)v^T(t-i) - C(t)P_p(t)C^T(t).$$

Step 11: Repeat steps 2 to 10 for all epochs.

The RAKF2 failed with synthetic data when the true value of the parameters are R=1e-8, Q_1=1e-6 and Q_2=1e-7. The reason behind this failure was investigated and found that it is due to negativity occurance of R. The above numerical experiments indicated that a much less noisy estimation of β is possible provided suitable (small) values of noise covariances are chosen or initialized. The algorithm MRAKF2 is developed by modifying the RAKF2 where the R calculation formula has been changed by considering its absolute value so that negativity occurrences do not take place.

Algorithm 3.4: MRAKF2 for α and β estimation with known Q

Step 1: Initialize $\hat{x}(t)$, $P(t)$ and R and set m, $C(t)$, F, and known Q $\forall$ t.

Step 2: Repeat step 2 to 9 of algorithm 3.3 (RAKF2).

Step 3: Update observation noise covariance:

$$R(t+1) = \left|\tfrac{1}{m}\sum_{i=0}^{m-1} v(t-i)v^T(t-i) - C(t)P_p(t)C^T(t)\right|$$

Step 4: Repeat steps 2 and 3 for all epochs.

It has been observed that the performance of MRAKF2 is comparable with that of KF with known R for time varying joint α and β estimation with full synthetic as well as semi-synthetic data. The performance of R estimation by MRAKF2 is also fairly acceptable.

3.2 AKF3 Methods and Simulations

3.2.1 QAKF3 and RAKF3 Algorithm

Algorithms KF2, QAKF2 and RAKF2 are extended to KF3, QAKF3 and RAKF3 respectively where dimension of state variable is increased from 2 to 3. However the filtering algorithms do not change except the said change (increase) in the dimension of x(t), Q, P, Pp from 2 to 3. QAKF3 and RAKF3 are modified to MQAKF3 and MRAKF3 respectively to take care of the negativity occurrences similar to the second order filtering cases.

4. Results of Empirical Investigations

4.1 Data Source and Preparations

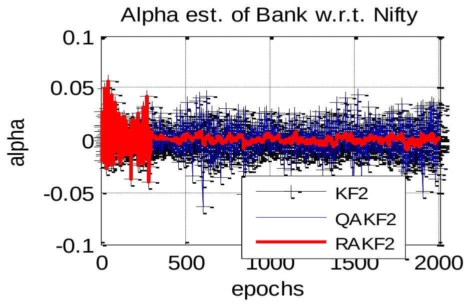
The stock indices data from National Stock Exchange (NSE) of India has been collected from NSE website. The daily closing data during 1st January, 2001 to 31st, December, 2008, total of 2003 days data, are considered for the study. The stock market indices are reasonably representative of a mixture of industry sectors and trading activity mostly revolves around the stocks comprising the indices. The sectoral indices, suitably designed portfolios of equities from specific sector, are considered as representatives of portfolios and gross index Nifty (S&P CNX NIFTY) data are fair representative of the diversified market together. The considered sectoral indices are Bank (BANK NIFTY), Midcap (CNX MIDCAP), Defty (S&P CNX DEFTY) and Junior (CNX NIFTY JUNIOR). The literature on formation and composition of the selected indices are available on NSE websites.

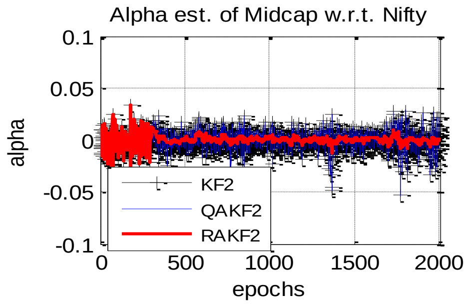
4.2 Results of AKF2

This section presents and analyses the results of the empirical investigations for α and β estimation carried out with data set from Indian market. Empirical time varying daily α and β estimates of four sectoral Indian indices portfolios (viz. Bank, Midcap, Defty and Junior) by KF2 and AKF2 (QAKF2 and RAKF2) proposed and modified here. The gross index Nifty (of NSE of India) daily returns are used as market returns during the considered period. Fig. 1 and fig. 2 present daily estimates of α and β of four sectoral Indian portfolios respectively for the considered time duration by KF2, MQAKF2 and MRAKF2. We can compare the said estimates from the graphs of these figures. Fig. 3 presents the time varying Q_1

and Q_2 estimates by MQAKF2 and fig. 4 presents the time varying R estimates by MRAKF2.

The assumed known or initial values of the parameters during applying the KF2, MQAKF and MRAKF are Q_1 =1e-6, Q_2 =1e-7 and R =1e-8. The initial values of α and β are 0.002 and 0.01 respectively where as initial values of P=[0.001 0; 0 0.01] in all filters. However chosen suitable value of m is 300 in both the AKF2.





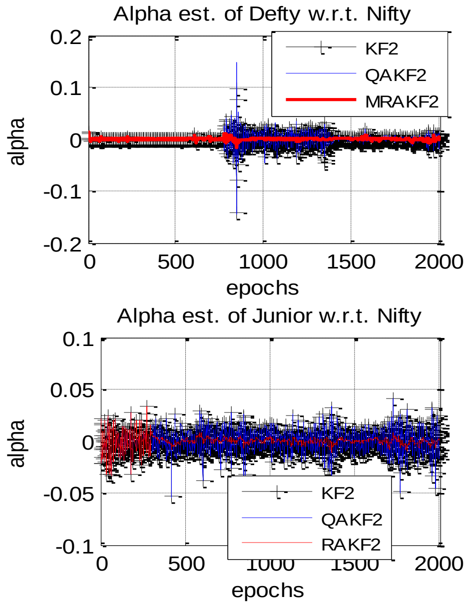
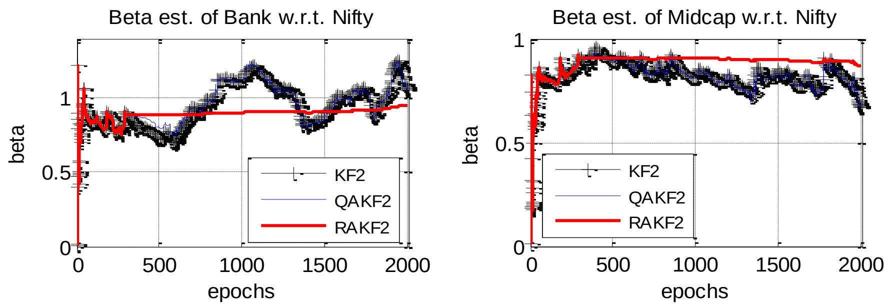


Figure 1: α estimates of the considered indices by KF2, MQAKF and MRAKF

It is observed (Fig. 1) that all three filters emerge daily α estimates comparable irrespective of all the four portfolios. It is also noted that the range of α estimates is least for Defty and greatest for the Bank among the considered portfolios. The range of α estimates is comparatively similar for Midcap and Junior but in between Bank and Defty. Comparing the estimates of these three filters, we have found that KF2 shows greater variability in the estimates than the estimates by MAKF2 variations. However, range of α estimates by MQAKF2 is revealed to be greater than that of MRAKF2. With the initializations and assumptions mentioned above, it is observed that negativity of R occurred in the case of Defty only and not in others portfolios and hence MRAKF2 called for. Above all, the assumption of constant α in CAPM can be challenged in view of the above results.



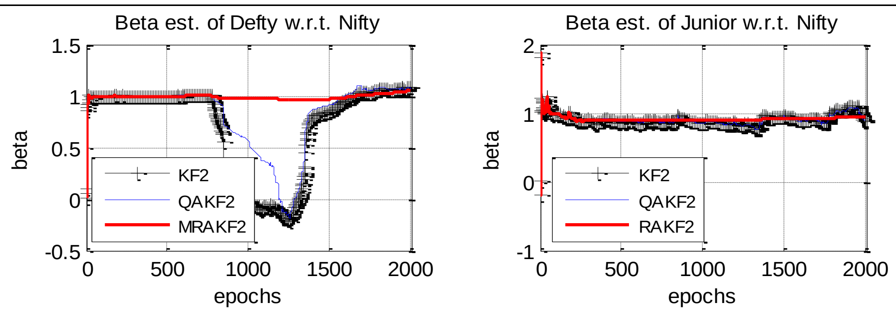


Figure 2: β estimates by KF2, MQAKF2 and MRAKF2

It has been observed (fig.2) that the daily β estimates by KF2 and MQAKF2 are close to each other for all the considered portfolios. Daily β estimates by MRAKF2 is comparable with that of KF2 and MQAKF2 for Junior only where as the nature of MRAKF2 β estimates are quite different than that emerged by KF2 and MQAKF2 for other three Indian portfolios. The variability of β estimates by MRAKF2 is very small compared to that provided by KF2 and MQAKF2. It is also noted that daily β estimates by KF2 and MQAKF2 are similar to that provided by first order KF and first order MQAKF.

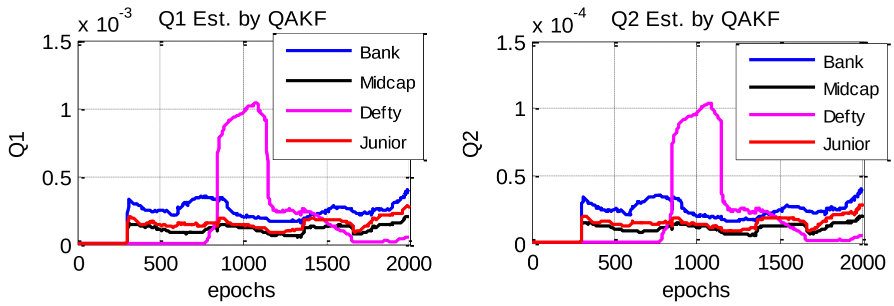


Figure 3: Time varying Q_1 and Q_2 estimates by MQAKF2 w.r.t. Nifty

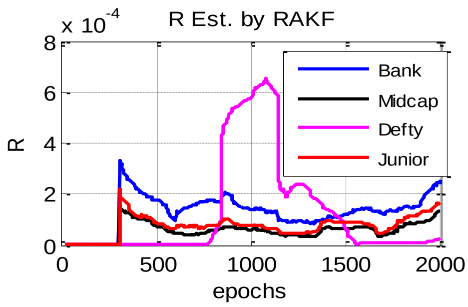


Figure 4: Time varying R estimates by MRAKF2 w.r.t. Nifty

It has been observed (fig 3 and 4) that the time varying Q_1, Q_2 and R estimates are analogous for Bank, Midcap and Junior but the estimates of these parameters for Defty is quite different from that of the other three portfolios. It is also interesting to note that the estimates of these parameters are quite close to each other for Midcap and Junior where as the estimates are different a little for Bank.

4.3 Results of AKF3

This section presents the empirical estimates of time varying α, β_1 and β_2 of the four sectoral portfolios. A two factor third order pricing model is appropriate to quantify the market risks of these portfolios since the assets in these portfolios are Indian equities and mostly traded in the NSE and BSE with arbitrage facilities. NSE should be the primary market for these portfolios since these portfolios are designed and maintained by NSE of India and hence Nifty returns are treated as primary market returns. Most of the assets under the considered portfolios are traded at BSE of India which is the oldest and having second market capitalization in India. It is also a matter of concern that many investors and traders take their risk decisions on the basis of the gross market index Sensex designed and maintained by BSE. Hence, Sensex returns have been considered as cross market returns in the above "two factor pricing model". This section of the paper aims at analyzing the time-varying impact of market risks quantified by the market sensitivities parameters β_1 and β_2 together with the market independent part of the portfolio returns identified by α of four Indian portfolios using KF3 and AKF3.

The fig. 5, 6 and 7 present daily estimates of α, β_1 and β_2 respectively for the said portfolios by KF3, MQAKF3 and MRAKF3. Fig. 8 presents the time varying Q_1. Fig. 9 presents time varying Q_2 and Q_3 estimates by MQAKF3. Fig. 10 presents the time varying R estimates by MRAKF2. The assumed known and initial values of the parameters during applying the above are Q_1=1e-6, Q_2=1e-7, Q_3=1e-7 and R=1e-8. The initial values of α, β_1 and β_2 are 0.002, 0.01 and 0.01 respectively whereas initial values of P=[0.001 0 0; 0 0.01 0; 0 0 0.01] in all filters. However chosen suitable value of m is 300 in both the AKF3.

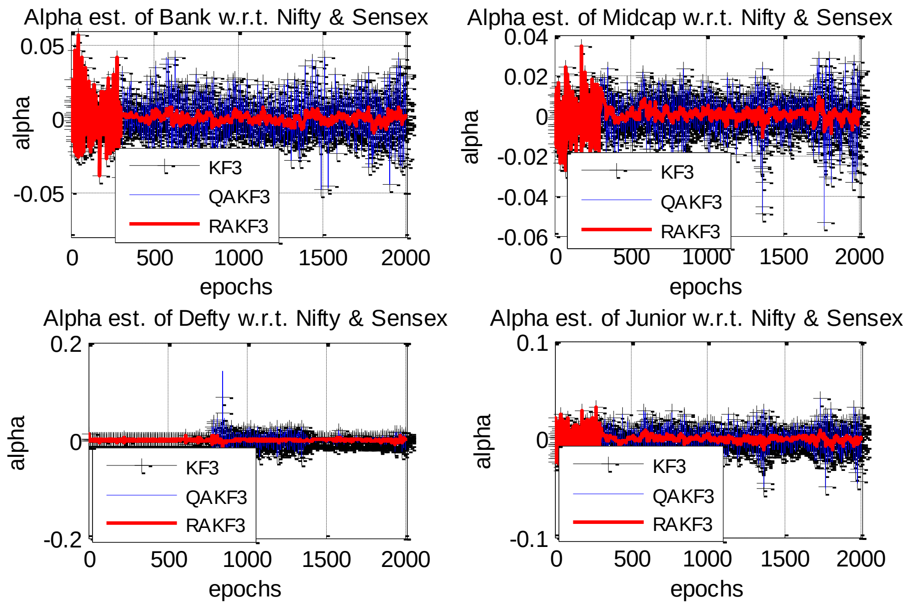


Figure 5: α estimates of the considered indices by KF3, MQAKF3 and MRAKF3

It is observed (fig. 5) that α estimates by second order and third order KF and AKF both are very akin to each others except for Midcap (though comparable).

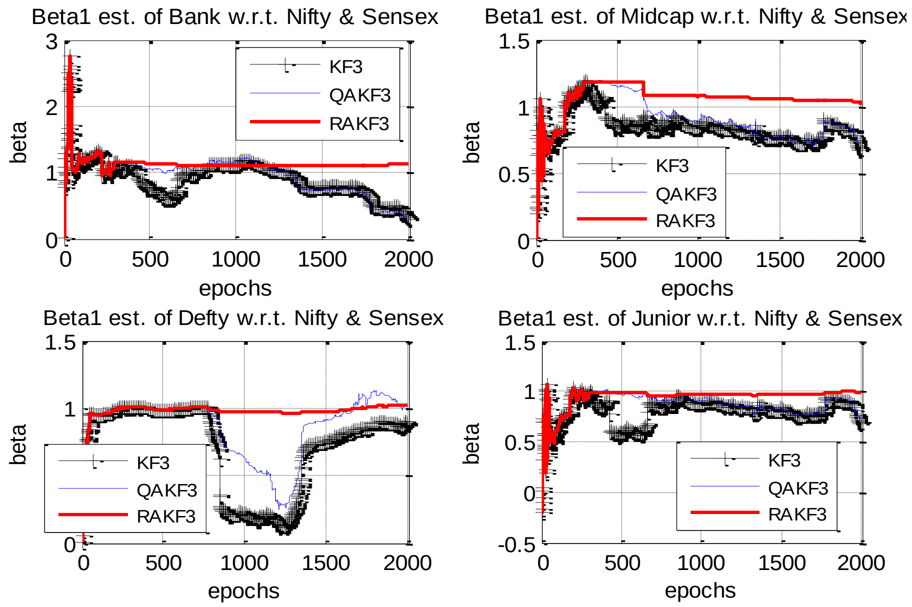


Figure 6: β_1 estimates of the considered indices by KF3, MQAKF3 and MRAKF3

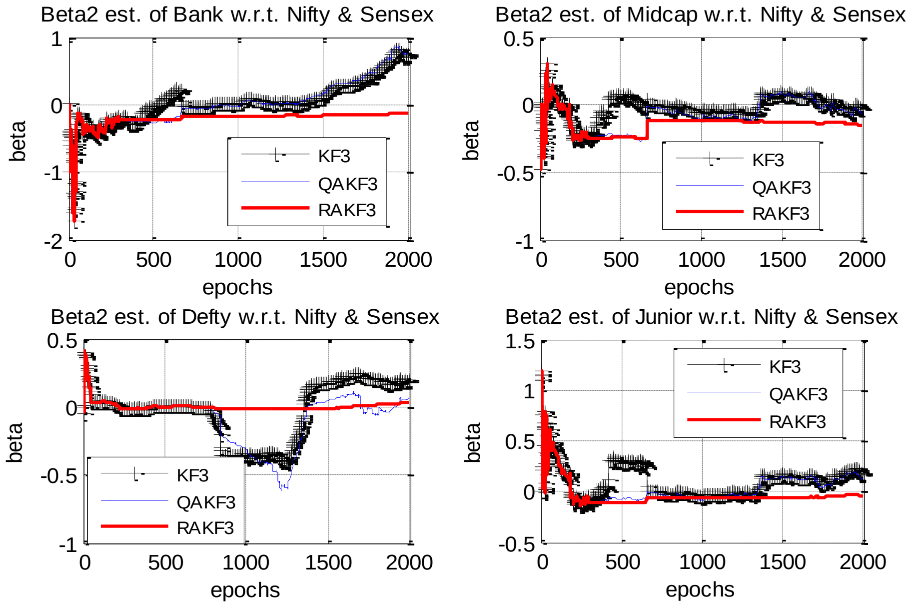


Figure 7: β_2 **estimates of the considered indices by KF3, MQAKF3, and MRAKF3**

It is observed that β_1 estimates by KF3 and AKF3 are comparable. But these β_1 estimates are different from β estimates by KF2 and AKF2 for all considered portfolios though both β and β_1 indicate Nifty sensitivity one in absence of cross market factor and other in presence of the same. The trends of β and β_1 estimates by second and third order filters are akin for all portfolios except for Bank. β_2 estimates by KF3 are comparable with β_2 estimates by AKF3 where either Q or R is unknown. The trends of β_1 and β_2 estimates are not alike for considered portfolios except minor likeness in trends for Defty. The trends of β_1 and β_2 estimates are revealed to be reverse for Junior portfolio by both KF3 and AKF3.

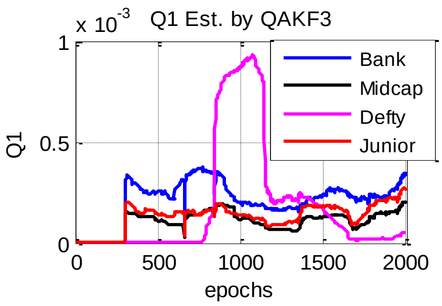


Figure 8: Q_1 **estimates by MQAKF3 w.r.t. Nifty & Sensex**

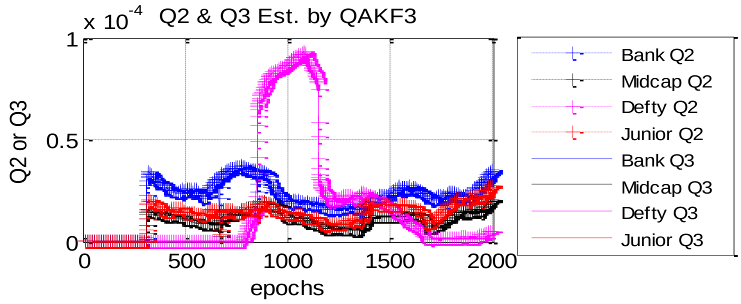


Figure 9: Q_1, Q_2 **and** Q_3 **estimates by MQAKF3 w.r.t. Nifty & Sensex**

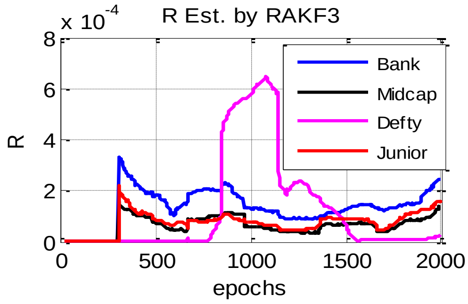


Figure 10: R estimates by MRAKF3 w.r.t. Nifty & Sensex

The trends of Q_1 estimates by AKF2 are analogous to that by AKF3 for all considered portfolios individually. Q_2 and Q_3 estimates by AKF3 are revelaed to be equal to one with another for all considered individual portfolios also. However trends of Q_2 estimates by AKF2 are similar to the trends of Q_2 or Q_3 estimates by AKF3 for all considered portfolios individually too. Since Q_1 and Q_2 estimates by AKF2 and AKF3 are different from one to another individually, it may be argued that the modeling uncertainties are different for the considered portfolios. On the contrary, Q_2 and Q_3 estimates by AKF3 being equal implies that the modeling uncertainties are same for both the market sensitivity parameters viz. β_1 and β_2.

The R estimates by AKF2 and AKF3 are equal individually for the considered portfolios as expected. However, R estimates ae different for all considered portfolios but trends are alike for all considered portfolios except for the Defty.

5. Conclusions

This paper explored the performance of second and third order AKF (with modifications to counter non-positive covariance) for estimation of CAPM parameters with Indian market data. Novelty of this work lies in applying second and third order AKFs. This work reconfirms the earlier findings that the CAPM parameters for Indian market should be considered as time varying. The novelty of the proposed higher order AKFs is that these can jointly estimate the market independent part, primary and cross market sensitivity (all time varying) of a portfolio returns. Moreover, this work used AKF2 and AKF3 for estimation of process and observation noise covariances. Applications of second and third order AKF techniques for such estimation are new in financial literatures.

The investigations reported here show that the estimated β's are comparable to earlier work in Indian market by first order KF and [5] in Italian market using KF2. Moreover, empirical β estimates by higher order AKFs do not suffer from "inadequate noise filtering" problem unlike first order AKFs. Even in case of RAKF2 and RAKF3, the time varying primary market sensitivity became smooth after some time iterations. These observation indicates that market sensitivity becomes constant (more or less) if market independent part α of portfolio return is assumed to be time varying (in case of higher order models) when observation noise covariance is not known beforehand.

Time varying Q_1, Q_2 and R estimates by suitable second order AKFs show that their trends are alike for all considered portfolios except Defty. Moreover, individual Q_1, Q_2 and R estimates of Midcap and Junior are very close to each other. It is also noted that MRAKF2 is used only in the estimation of α and β of Defty among the four considered portfolios because of negativity occurrences in R.

This work contributes to the knowledge corpus by exploring the empirical relevance of time-varying factor loadings in a multifactor pricing framework specially in Indian market. The qualititative comparison of the estimated of cross market factor sensitivities are presented which is at least novel in Indian market. Investigations may further be extended to understand the effect of other domestic and international factor sensitivities. Quantitative performance of the estimated parameters are evaluated through VaR backtesting, expected shortfall analysis and in-sample forecasting analysis, all of which confirmed that higher order filters perform at per with KF and AKFs.

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