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FUZZY MULTI OBJECTIVE PROJECT SCHEDULING UNDER INFLATIONARY CONDITIONS

Abstract. *Project scheduling is a major area of project management and planning. Due to the increasing prices of goods in most countries during the project and lack of information related to each parameters and variables, using fuzzy sets under inflationary condition can be efficient. In this paper, fuzzy project scheduling when project's revenues and costs are increased by different inflationary rate is considered. The duration of each project's activity is assumed as a fuzzy number. Two objectives of the problem are to minimize the project duration and to maximize the net present value of the project. The nonlinear multi-objective model will be solved by one of the multi-objective decision making methods named LP-Metric method and Frank-Wolf algorithm as one of the nonlinear programming procedure. The solving approach will be explained in a numerical example and the computational results will be reported.*

Keywords: *Fuzzy sets, Inflation, Project scheduling, Makespan, NPV.*

JEL Classification: C44, E31, E40

1. Introduction

Project scheduling is an important component of project management. The management of a project requires the scheduling of a set of activities through different methods based on the kind of parameters and variables. Critical Path Method (CPM) introduced by Kelley (1961) is one of the useful tools which can be applied when the duration of activities are deterministic. Due to the lack of information for parameter estimation, using deterministic methods were not applicable any more. Thus using stochastic variables or fuzzy number should be more reasonable. With respect to the above-mentioned reasons, Malcolm et al. (1959) used three estimation methods for modeling with stochastic variables based on beta distribution. The current contribution implies the concept of Project Evaluation and Review Technique (PERT) method in probabilistic area. Hence, some researchers like Ke and Liu (2005) were attracted to project scheduling with stochastic duration of activities. Tilson et al. (2009) considered project scheduling with stochastic activity duration to maximize expected net present value. Application of stochastic methods is dependent to have historical data about the information and characteristic of each project. In contrast, one of the features in any project is its uniqueness and implementation of stochastic procedures is not suitable any more. So scientists and researchers would use fuzzy set theory instead.

Considering the pioneering work of Zadeh (1965), researchers have started to reject the stochastic approach and recommended the use of fuzzy models (Demeulemeester and Herroelen, 2002). Nasution (1994) presented fuzzy CPM considering fuzzy float time of each activity in the specific level of α-cut and Chanas (1981) explained the fuzzy PERT method in this area as well. A rich literature review can be found in Herroelen and Leus (2005) and Verderame (2010) works. Past research has employed various objectives, for example minimization of the project duration, maximization of Net Present Value (NPV) of the project cash flows, maximization of the project resource utilization and minimization of the project total costs. A project scheduling problem in which the objective is to maximize NPV of the project cash flows is called Project Scheduling Problems with Discounted Cash Flows (PSPDCF). Russell (1970) introduced the problem of maximizing NPV in project scheduling problem. He proposed a successive approximation approach to solve the problem. Grinold (1972) added a project deadline to the model, formulated the problem as a linear programming problem, and proposed a method to solve it. Fathallahi and Najafi (2013) considered fully fuzzy project scheduling with discounted cash flows.

Since the prices of goods and services are continually increasing, the existing assumption that unchangeable revenues and costs respectively are the positive and negative cash flows for each project is not reasonable any more. Inflation is the term used in order to depict these changes. The inflation in the project scheduling problem was proposed by Jolayemi and Oluleye (1993). They developed a linear programming model for project scheduling problem in which the objective function is to minimize the project total cost and no payments are made for the project during its life cycle. Najafi et al. (2009) assumed project scheduling with discounted cash flows when the cash flows are increased by inflation rate and the objective function was to maximize the NPV of the project. According to the reasons of application of fuzzy sets and inflation condition, applying these two assumptions have not been worked yet. In this paper we consider project scheduling with discounted cash flows when the duration of each activities are considered as triangular fuzzy numbers and the positive and negative cash flows are increased by different rate of inflation. Two objective functions are considered to maximize the net present value and to minimize the makespan of the project. In the next section the model of fuzzy multi objective project scheduling with discounted cash flows will be formulated in which the cash flows are increased by inflation rate. In section three some basic arithmetic of fuzzy numbers and using of MODM techniques and application of nonlinear programming would be implemented. Finally a numerical example is proposed which can express comprehensive finding out.

2. Problem Formulation

Consider a project with *n* activities which the activities *1* and *n* are dummy activities. Suppose a project network as activity on node without any loops. Each activity *i* has a set of predecessor activities *p(i)*. We define C_i as the cash flows related to activity *i* which occurs at the end of activity *i*. It obtains two groups of cash flows: negative cash flow shown by C_i^- and positive cash flow shown by C_i^+ and the total cash flow for each activity as observed in equation (1) is evaluated by subtracting these cash flows. Also, α and *f* are the discount rate and the inflation rate, respectively. The positive cash flows are increased with the inflation rate f_1 and the negative cash flows are increased with the inflation ratef_2. Also,$\widetilde{d_\iota}$ and $\widetilde{F_\iota}$ are fuzzy activity duration and fuzzy finish time of activity *i*.

$$C_i = C_i^+ + C_i^- \tag{1}$$

The fuzzy multi objective model for project scheduling with discounted cash flows when the negative cash flows and positive cash flows are increased by inflation rate can be formulated as follows:

$$\max Z_1 = \sum_{i=1}^{n} C_i^+ e_i^{-(\alpha - f_1)\widetilde{F_\iota}} + C_i^- e_i^{-(\alpha - f_2)\widetilde{F_\iota}} \tag{2}$$

and

$$\min Z_2 = \tilde{F}_n \tag{3}$$

s.t

$\tilde{F}_j \leq \tilde{F}_i - \tilde{d}_i \qquad \forall\, j \in p(i), i = 1, \dots, n$ (4)

$\tilde{F}_i \geq 0 \qquad i = 1, \dots, n$ (5)

The first objective function is defined as maximizing the net present value of the project and the second objective function express minimizing the makespan of project which is equal to minimizing the finish time of the last project's activity. The constraint (4) demonstrates the precedence constraint which the activity predecessors should be done before it and the last constraint denote the domain of variables.

The first step for solving this fuzzy model is using the basic fuzzy operations in the next section, the definition of fuzzy numbers and the math operations would be described.

3- Preliminaries of Fuzzy Sets

One of the common forms of fuzzy numbers is triangular fuzzy numbers as shown in Fig. 1. Triangular fuzzy number is denoted by a triplet$\tilde{A} = (a^p, a^m, a^o)$, $(a^p < a^m < a^o)$. Wherea^p,a^m and a^o are the components of the number and when all of the components are positive then the fuzzy number is called positive. This definition is true for negative number, too. Each fuzzy number has a membership function which can be defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a^p}{a^m-a^p} & a^p \leq x \leq a^m \\ \frac{a^o-x}{a^o-a^m} & a^m \leq x \leq a^o \\ 0 & otherwise \end{cases} \qquad (6)$$

Where $\mu_{\tilde{A}}(x)$ is the degree of membership or the membership function value of x in fuzzy set $\mu_{\tilde{A}}(x)$ which continuously maps from R to closed interval [0,1].

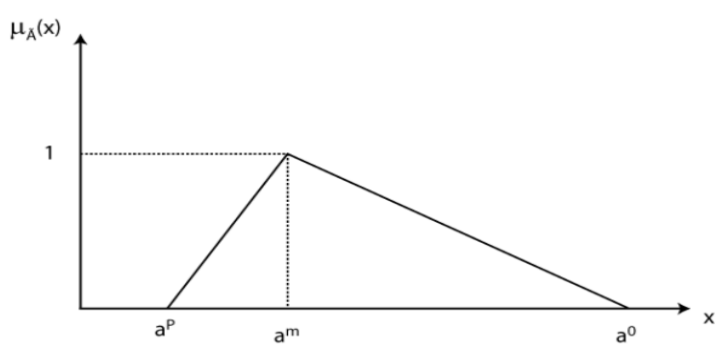

Figure 1:Triangular fuzzy number

Arithmetic Operations

Assume two triangular fuzzy numbers$\tilde{A} = ((a^p, a^m, a^o), \tilde{B} = (b^p, b^m, b^o)$, fuzzy arithmetic is as follows:

i. Additions

$$\tilde{A} + \tilde{B} = (a^p, a^m, a^o) + (b^p, b^m, b^o) = (a^p + b^p, a^m + b^m, a^o + b^o) \quad (7)$$

ii. Subtraction

$$\tilde{A} - \tilde{B} = (a^p - b^o, a^m - b^m, a^o - b^p) \quad (8)$$

iii. Multiplication

$$\tilde{A} * \tilde{B} = (\, a^p b^p, a^m b^m, a^o b^o) \quad (9)$$

If r is a constant parameter, its multiplication to a fuzzy number would be as bellow:

$$r * (a^p, a^m, a^o) = (ra^p, ra^m, ra^o) \qquad if\ r \geq 0\ , a \geq 0$$

$$r * (a^p, a^m, a^o) = (ra^o, ra^m, ra^p) \qquad if\ r < 0\ \ , a \geq 0 \quad (10)$$

$$r * (a^p, a^m, a^o) = (ra^p, ra^m, ra^o) \qquad if\ r \geq 0\ \ , a < 0$$

$$r * (a^p, a^m, a^o) = (ra^o, ra^m, ra^p) \qquad if\ r < 0\ \ , a < 0$$

iv. Division

$$\tilde{A}/\tilde{B} = (a^p/b^o, a^m/b^m, a^o/b^p) \quad (11)$$

4. Solving Approach

Assume that $\tilde{A}$ and $\tilde{B}$ are the feasible solutions of previous model. Also, $\tilde{Z}_{\tilde{A}}$ denotes the value of objective function $\tilde{Z}$ for solution $\tilde{A}$ and $R(\tilde{A})$ would be the value of ranking fuzzy number. By the method of ranking fuzzy numbers, they are transformed to the value which is suitable for comparison. Therefore, it can be concluded that if $\tilde{Z}_{\tilde{A}} \geq \tilde{Z}_{\tilde{B}}$ then the inequality $\tilde{Z}_{R(\tilde{A})} \geq \tilde{Z}_{R(\tilde{B})}$ will be held. So, all fuzzy decision variables in the objective function (2) may be replaced by their ranks ($\bar{x} = \frac{(a^p+2a^m+a^o)}{4}$) (Kaur and Kumar, 2012). In the other words, the previous model may be modified to a crisp model as follows:

$$\max Z_1 = \sum_{i=1}^{n} C_i^+ e_i^{-(\alpha-f_1)\widetilde{F_i}} + C_i^- e_i^{-(\alpha-f_2)\widetilde{F_i}} = \sum_{i=1}^{n} C_i^+ e_i^{-(\alpha-f_1)(F_i^p,F_i^m,F_i^o)} + C_i^- e_i^{-(\alpha-f_2)(F_i^p,F_i^m,F_i^o)} = \tag{12}$$

$$\sum_{i=1}^{n}(C_i^+ e_i^{-(\alpha-f_1)F_i^o}, C_i^+ e_i^{-(\alpha-f_1)F_i^m}, C_i^+ e_i^{-(\alpha-f_1)F_i^p}) + \sum_{i=1}^{n}(C_i^- e_i^{-(\alpha-f_1)F_i^p}, C_i^- e_i^{-(\alpha-f_1)F_i^m}, C_i^- e_i^{-(\alpha-f_1)F_i^o})$$

and

$$\min Z_2 = \tilde{F}_n = (F_n^p, F_n^m, F_n^o) \tag{13}$$

If each objective function is replaced by their ranking of fuzzy numbers, the objective functions can be transformed as bellow:

$$\text{Max } Z_1 = \sum_{i=1}^{n} \frac{1}{4}(\,(C_i^+ e_i^{-(\alpha-f_1)F_i^o} + 2\,C_i^+ e_i^{-(\alpha-f_1)F_i^m} + C_i^+ e_i^{-(\alpha-f_1)F_i^p}) + \sum_{i=1}^{n} \frac{1}{4}(\,(C_i^- e_i^{-(\alpha-f_1)F_i^p} + 2C_i^- e_i^{-(\alpha-f_1)F_i^m} + C_i^- e_i^{-(\alpha-f_1)F_i^o})) \tag{14}$$

and

$$\text{Min } Z_2 = \tilde{F}_n = \frac{1}{4}(F_n^p + 2F_n^m + F_n^o) \tag{15}$$

s.t.

$$\begin{cases} F_j^p \le F_i^p - d_i^p \\ F_j^m \le F_i^m - d_i^m \\ F_j^o \le F_i^o - d_i^o \end{cases} \quad \forall j \in p(i), i = 1,2,\dots,n \tag{16}$$

$$\begin{cases} F_i^p \ge 0 \\ F_i^m \ge 0 \\ F_i^o \ge 0 \end{cases} \quad i = 1,2,\dots,n \tag{17}$$

$$F_i^p < F_i^m < F_i^o \quad i = 1,\dots,n \tag{18}$$

The fuzzy model is transformed to a crisp model by using of fuzzy operations and mathematical fuzzy programming. The multi objective nonlinear model should be solved by the means of MODM techniques and nonlinear programming methods. Since the first objective function of the aforementioned model is not a convex model and the most of nonlinear programming techniques are based on the convexity of the model, we should do some changes in objective function (14) to transform the nonconvex model to a convex one or use the local optimal methods such as GA,SA,PSO. In the next section, we interpret some assumptions and make some changes to convert the nonconvex objective function to a convex one.

The exponential function $g(t) = Ke^{-\alpha t}$ prove be a convex function for all $K \geq 0$ by Fathallahi and Najafi(2013)and for all $K < 0$ the exponential function is concave if the coefficient of t is positive. By this way, the objective function (14) is divided into two groups of objective functions. One of them is convex and another one would be concave. The convex objective function contains the positive cash flows and the concave one contains the negative cash flows related to each activity. The objective function (14) can be transformed to a convex multi objective problem by using the following procedure:

1- Let $f_i(x)$ denote the convex terms of function (14).
2- Assign $g_i(x)$ to the concave ones.
3- Construct a new convex model as below:

$$\max \sum_{i \in Convex} f_i(x)$$

$$\min \sum_{i \in Concave} -g_i(x)$$

Since our original model consider two objective functions to optimize and the first objective function is divided into two objective functions to optimize, we encounter with three objective functions. Now, these objective functions are convex and we can implement most of the nonlinear methods for solving those functions. Before applying these techniques, we should use one of the MODM techniques.

4.1 LP-Metric method

There are different ways for solving multi objective models introduced as MODM techniques. One of the famous procedures used in this paper is LP-Metric method.

This method is based on minimizing each objective function from its best solution. Consider f_1, f_2, f_3as our objective functions which demonstrates respectively the convex part of NPV maximization, the concave part of NPV maximization and minimizing the project's makespan. The best solution for each objective function is shown byf_1^*, f_2^*, f_3^*. There are two parameters related to this method.Parameter *p* and λ express respectively the importance of derivation and the weight of each objective function. Because we have two objective functions in this paper and the first objective function is divided into two groups of objective function, the coefficient λ for *f*, *g* are supposed as 0.25 and for minimizing the makespan is supposed as 0.5. In addition, the value of *p* consider as 1.

$$Min\ z = \left(o.25\left(\frac{f_1^* - f_1}{f_1^*}\right)^1 + 0.25\left(\frac{f_2 - f_2^*}{f_2^*}\right)^1 + 0.5\left(\frac{f_3 - f_3^*}{f_3^*}\right)^1\right)^1 \quad (19)$$

St.

$$\begin{cases} F_j^p \leq F_i^p - d_i^p \\ F_j^m \leq F_i^m - d_i^m \\ F_j^o \leq F_i^o - d_i^o \end{cases} \quad \forall j \in p(i), i = 1,2,\dots,n \quad (20)$$

$$\begin{cases} F_i^p \geq 0 \\ F_i^m \geq 0 \\ F_i^o \geq 0 \end{cases} \quad i = 1,2,\dots,n \quad (21)$$

$$F_i^p < F_i^m < F_i^o \quad i = 1,\dots,n \quad (22)$$

The above-depicted model has a convex objective function. The constraint 20 demonstrates the precedence relations and constraint 21 denotes the domain of fuzzy variables. The last constraint is based on the reasonable relation of fuzzy numbers. For solving the nonlinear model, one of the NLP methods should be applied. One of the famous methods of solving nonlinear models is Frank-Wolf algorithm. In the next section this algorithm is briefly described.

4.2 Frank-Wolf algorithm

The Frank–Wolfe algorithm is a simple iterative first-order optimization algorithm for constrained convex optimization. In 1956, Frank and Wolfe developed an algorithm for solving quadratic programming problems with linear constraints. It is applicable to nonlinear programming with convex objective function. The Frank–Wolfe algorithm considers a linear approximation of the objective function as it is shown in equation (23). By computing the partial derivations for each variable and by putting one feasible solution to each of the partial derivation, the coefficient of each variable can be obtained. The model bellow is considered to solve iteratively. The linear programming for maximization objective function is demonstrated as follows and considers its solution as F_{Lp}^{k}in iteration k. The feasible solution is computed by the combination of the previous solutions in each iteration. Since computing the value of λ in the following statement:$F = \lambda F_{LP}^{k} + (1 - \lambda)F^{k-1}$maybe not available and in most of time of the time cannot be computed, the solution is obtained by the following combination:$F = 0.5F_{LP}^{k} + 0.5F^{k-1}$, and this algorithm which is based on Frank-Wolf algorithm will continue until two obtained solutions would be sufficiently near each other.

$$\max g(x) = \sum_{j=1}^{n} c_j F_j \tag{23}$$

$$st$$

$$AF_j \leq b \tag{24}$$

$$F_j \geq 0 \tag{25}$$

5. Numerical Examples

For more explanation, consider the following project with 8 activities as seen in Fig. 2. The duration and cash flow for each activity are presented in Table (1). For each cash flow consider the positive and negative cash flow. The interest rate is 0.03and the first activity and the last activity are supposed as dummy activities. The inflation rate for positive cash flows and negative number are considered 0.01 and 0.02, respectively.

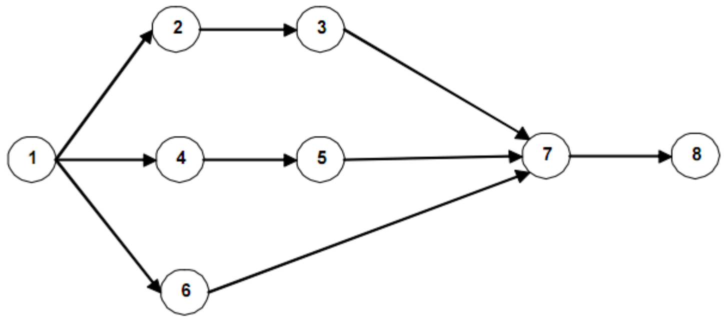

Figure 2: The project network of the example

The duration of each activity is assumed as a fuzzy number and the other parameters are considered crisp as well.

Table 1:Fuzzy activity duration and deterministic cash flows of activities

Activity(i)	Duration($\tilde{d}_i$)	Positive cash flow(C_i^+)	Negative cash flow(C_i^-)
1	(0,0,0)	0	0
2	(5,8,10)	120	-100
3	(3,5,8)	180	-80
4	(8,10,12)	130	-140
5	(3,4,5)	150	-100
6	(6,8,10)	90	-100
7	(1,2,3)	20	-100
8	(0,0,0)	0	0

The objective functions for this project are as follows:

$$
\begin{aligned}
maxZ_1 = 0.25(120e^{-0.02F_2^o} &+ 240\,e^{-0.02F_2^m} + 120e^{-0.02F_2^p} - 100e^{-0.01F_2^p} \\
&- 200e^{-0.01F_2^m} - 100e^{-0.01F_2^o} + 180e^{-0.02F_3^o} + 360\,e^{-0.02F_3^m} \\
&+ 180e^{-0.02F_3^p} - 80e^{-0.01F_3^p} - 160e^{-0.01F_3^m} - 80e^{-0.01F_3^o} \\
&+ 130e^{-0.02F_4^o} + 260\,e^{-0.02F_4^m} + 130e^{-0.02F_4^p} - 140e^{-0.01F_2^p} \\
&- 280e^{-0.01F_2^m} - 140e^{-0.01F_2^o} + 150e^{-0.02F_5^o} + 300\,e^{-0.02F_5^m}
\end{aligned}
$$

$$
\begin{aligned}
+150e^{-0.02F_5^p} - 100e^{-0.01F_5^p} &- 200e^{-0.01F_5^m} - 100e^{-0.01F_5^o} + 90e^{-0.02F_6^o} \\
&+ 180\,e^{-0.02F_6^m} + 90e^{-0.02F_6^p} - 100e^{-0.01F_6^p} - 200e^{-0.01F_6^m} \\
&- 100e^{-0.01F_6^o} + 20e^{-0.02F_7^o} + 40\,e^{-0.02F_7^m} + 20e^{-0.02F_7^p} \\
&- 100e^{-0.01F_7^p} - 200e^{-0.01F_7^m} - 100e^{-0.01F_7^o}
\end{aligned}
$$

$$minZ_2 = 0.25(F_8^p + F_8^m + F_8^o)$$

s.t

$$
\begin{cases} F_1^p \le F_2^p - 5 \\ F_1^m \le F_2^m - 8 \\ F_1^o \le F_2^o - 10 \end{cases}
\begin{cases} F_1^p \le F_6^p - 6 \\ F_1^m \le F_6^m - 8 \\ F_1^o \le F_6^o - 10 \end{cases}
\begin{cases} F_1^p \le F_4^p - 8 \\ F_1^m \le F_4^m - 10 \\ F_1^o \le F_4^o - 12 \end{cases}
\begin{cases} F_2^p \le F_3^p - 3 \\ F_2^m \le F_3^m - 5 \\ F_2^o \le F_3^o - 8 \end{cases}
$$

$$
\begin{cases} F_4^p \le F_5^p - 3 \\ F_4^m \le F_5^m - 4 \\ F_4^o \le F_5^o - 5 \end{cases}
\begin{cases} F_6^p \le F_7^p - 1 \\ F_6^m \le F_7^m - 2 \\ F_6^o \le F_7^o - 3 \end{cases}
\begin{cases} F_3^p \le F_7^p - 1 \\ F_3^m \le F_7^m - 2 \\ F_3^o \le F_7^o - 3 \end{cases}
\begin{cases} F_5^p \le F_7^p - 1 \\ F_5^m \le F_7^m - 2 \\ F_5^o \le F_7^o - 3 \end{cases}
\begin{cases} F_7^p \le F_8^p \\ F_7^m \le F_8^m \\ F_7^o \le F_8^o \end{cases}
$$

$$
\begin{cases} F_1^p \ge 0 \\ F_1^m \ge 0 \\ F_1^o \ge 0 \end{cases}
\begin{cases} F_2^p \ge 0 \\ F_2^m \ge 0 \\ F_2^o \ge 0 \end{cases}
\begin{cases} F_3^p \ge 0 \\ F_3^m \ge 0 \\ F_3^o \ge 0 \end{cases}
\begin{cases} F_4^p \ge 0 \\ F_4^m \ge 0 \\ F_4^o \ge 0 \end{cases}
\begin{cases} F_5^p \ge 0 \\ F_5^m \ge 0 \\ F_5^o \ge 0 \end{cases}
\begin{cases} F_6^p \ge 0 \\ F_6^m \ge 0 \\ F_6^o \ge 0 \end{cases}
\begin{cases} F_7^p \ge 0 \\ F_7^m \ge 0 \\ F_7^o \ge 0 \end{cases}
\begin{cases} F_8^p \ge 0 \\ F_8^m \ge 0 \\ F_8^o \ge 0 \end{cases}
$$

$$
\begin{cases}
F_1^p < F_1^m < F_1^o \\
F_2^p < F_2^m < F_2^o \\
F_3^p < F_3^m < F_3^o \\
F_4^p < F_4^m < F_4^o \\
F_5^p < F_5^m < F_5^o \\
F_6^p < F_6^m < F_6^o \\
F_7^p < F_7^m < F_7^o \\
F_8^p < F_8^m < F_8^o
\end{cases}
$$

The first objective function contains the convex functions and concave functions. By identifying the objective functions $f(x), g(x)$ which demonstrate the convex and concave functions and $h(x)$ as minimization of makespan, we consider $f_1 = f(x), f_2 = -g(x), f_3 = h(x)$. LP-Metric method can solve multi objective models and the optimal solution for each objective function should be found through it. The model can be written as below:

$$min = 0.25\left(\frac{2311 - f_1}{2311}\right) + 0.25\left(\frac{f_2 - 2146}{2146}\right) + 0.5\left(\frac{f_3 - 16.25}{16.25}\right)$$

And the constraints are as aforementioned. Frank-Wolf algorithm is applied for the above nonlinear convex objective function. The partial derivation for each parameter is computed as follows and the linear programming is solved. The number of iterations the algorithm converges is depicted in Figure 3. The difference between two solutions is considered lower than 0.1 and the final approximated solution is shown in Table 2. The vertical axis shows the amount of linear programming obtained by the new solution in each iteration and the horizontal axis denotes the number of iterations.

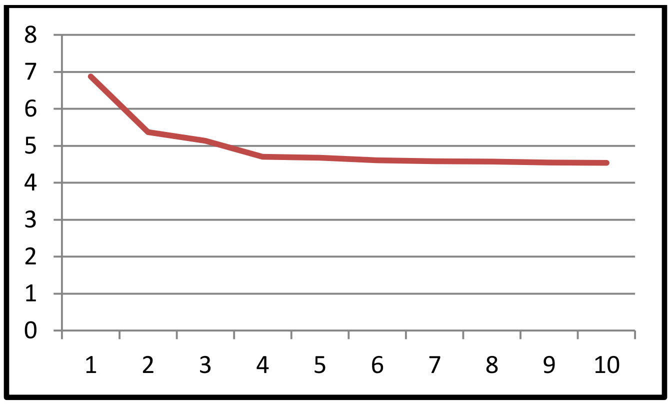

Figure 3: Frank-Wolf algorithm LP convergence

Table 2: Activities'finish time

Activity(i)	Finish Time ($\widetilde{F_i}$)
1	(0,0,0)
2	(5,8,10)
3	(14,14,18)
4	(8,10,12)
5	(11,14,17)
6	(0,0,0)
7	(16,16,21)
8	(16,16,21)

6. Conclusion

In this paper project scheduling under inflationary condition and uncertainty was considered. The duration of activities was considered as triangular fuzzy numbers. The positive cash flows for each activity were inflated by specific rate which was different with the negative cash flows inflation rate. The objectives were supposed as maximization of net present value of the project and minimization of the finish time of the project. The function of maximizing the net present value contains two parts: the convex part and concave part. These two parts are divided into two objective functions. For encountering with these three objective functions, the LP-metric method was used. The nonlinear convex objective function was solved by the method based on Frank-Wolf algorithm.

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