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# A CHOQUET CAPACITY AND INTEGRAL BASED METHOD TO IDENTIFY THE OVERALL IMPORTANCE OF ENGINEERING CHARACTERISTICS IN QUALITY FUNCTION DEPLOYMENT

***Abstract.*** *Deriving the overall importance of engineering characteristics from the information of the house of quality is one of the primary tasks in the quality function deployment. In this paper, we consider the information processing of house of quality as the multicriteria ordered sorting or classification problem, in which the multiple engineering characteristics are regarded as the multiple criteria, the partial evaluations of products on multiple engineering characteristics as the multiple inputs, and the customer perceptions of products in terms of ordered classes as the aggregate outputs. Given this consideration, we adopt the Choquet capacity to describe the importance degrees of engineering characteristics as well as the correlations among engineering characteristics, employ the Choquet integral to represent the grades of products on customer requirements, and construct the Choquet capacity and integral based methodology to derive the overall importance of engineering characteristics. The steps of the proposed methodology are discussed in detail and also illustrated with a digital camera design example.*

***Keywords:*** *Quality function deployment (QFD), House of quality (HOQ), Choquet capacity, Choquet integral, Shapley importance and interaction index.*

**JEL Classification: D81, O32**

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## 1. Introduction

Quality function deployment (QFD) is ‘‘an overall concept that provides a means of translating customer requirements into the appropriate technical requirements for each stage of product development and production” (Sullivan, 1986). Typically, a QFD system consists of four inter-linked phases (Zandi and Tavana, 2011): product planning, part deployment, process planning, and production planning. The output of one phase is employed in the next phase as an input (Liu and Wang, 2010). Generally, QFD utilizes four sets of matrices to translate customer requirements (CRs) into engineering characteristics (ECs), subsequently, into parts characteristics, process plans, and production requirements (Karsak, 2004). The set of matrices used in the product planning phase, usually called the house of quality (HOQ), contains information on CRs and ECs, relationship measures between CRs and ECs as well as correlation measures among ECs and customer perceptions compared to competitors (Chan and Wu, 2005). The HOQ is of fundamental and strategic importance in the QFD system, since the customer requirements for the product are identified, and incorporating the producing company’s competitive priorities, converted into appropriate ECs to achieving the desired customer satisfaction level (Chan and Wu, 2005). The HOQ is the engine that drives the entire QFD process (Cariaga et. al., 2007), considerable efforts must be committed to properly acquire ECs in order to keep the company successful (Li et. al., 2012). Following the results from the first phase of the QFD process, similar works are performed in the next three phases. In the product planning phase of the QFD, one of the key results of HOQ is the engineering priority which guides the design team in decision-making, resource allocation, and the subsequent QFD analysis (Chen et. al., 2006). Therefore, deriving the rankings of ECs from input variables is a crucial step towards successful QFD (Chen et. al., 2006).

Usually, in the HOQ, it is assumed that the relationship between the customer perceptions of CRs and the technical performance of the related ECs is linear, and the correlation between one EC and other ECs is also assumed to be linear. That is, these relationships and correlations can be modeled by the linear regression forms (Chen et. al., 2006). However, such linear assumptions are not appropriate in most realistic cases. In this paper, we propose a nonlinear model, the Choquet capacity and integral (Choquet, 1953) based model, to describe these relationships and correlations and then to derive the overall importance of ECs from the given information in HOQ. The main reasons for adopting such a model are that, firstly, the Choquet capacity can efficiently describe not only the importance of ECs but also the arbitrary kind of interaction, ranging from redundancy (negative interaction) to synergy (positive interaction), among the ECs. Secondly, in the HOQ, the customer perception is usually expressed and measured by predefined graded classes. For each CR, we can consider those ECs that affect the CR as the sources and the customer perception on this CR as the desired output. In this sense, the information processing of HOQ can be regarded as an ordered sorting (classification) problem in the multicriteria analysis theory. The Choquet integral with respect to capacity has been proved to be an appropriate tool to describe and solve the multicriteria ordered sorting (classification) problem (Grabisch, 1995).

The paper is organized as follows. After the introduction, Section 2 introduces some knowledge about the HOQ, the Choquet capacity and integral, and the TOMASO (Technique for Ordinal Multiattribute Sorting and Ordering) model. In Section 3, we discuss the Choquet integral based methodology for identifying the overall importance of ECs. A numerical example is given to illustrate the proposed methodology in Section 4. Finally, we conclude the paper in Section 5.

## 2. Preliminaries

### 2.1. The house of quality (HOQ)

In the product planning phase of QFD, the house of quality (HOQ) matrixes, as named by Hauser and Clausing (Hauser and Clausing, 1988), is used to translate qualitative customer requirements (CRs) into measurable engineering characteristics (ECs) and prioritize their importance (Liu, 2009). The typical structure of the HOQ, as shown in Fig. 1, consists of the following seven major components:

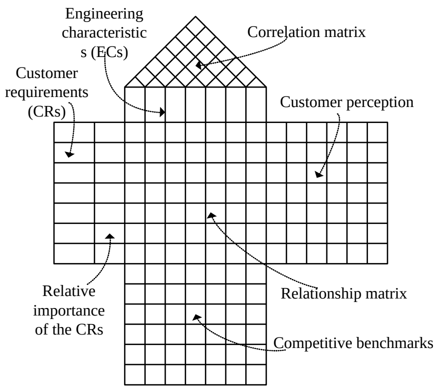


**Figure 1. The house of quality**

(1) Customer requirements (CRs). Customer requirements are requested functions or qualities that customers really desire (Liu, 2009). They are the most important input factors in QFD and are usually acquired through discussions with focus groups or individual interviews (Liu, 2009).

(2) Relative importance of the CRs. The CRs are weighted in order to express their relative importance, which can be obtained from some rating methods, such as customer survey, expert opinion, the analytic hierarchy process (AHP), the analytic network process (ANP), and so on (Lai et. al., 2008).

(3) Customer perception. The customer perception, or called customer competitive assessment, illustrates the company's and its competitors' performance in meeting these CRs. The customer perceptions is usually expressed and measured by graded classes (e.g., class 1: very poor; class 2: poor; class 3: neutral; class 4: good; class 5: very good.).

(4) Engineering characteristics (ECs). ECs describe the product in the language of the engineer, and referred to as the voice of the company. The CRs must be translated into relevant and measurable ECs that probably affect one or more CRs (Bottani and Rizzi, 2006).

(5) Relationship matrix. This is the core element of the HOQ. The relationships are expressed with graphic symbols that indicate how (the direction) and to what extent each EC meets each CR (Bottani and Rizzi, 2006). For example, these symbols can express four degrees: strong negative, negative, positive and strong positive. All these relationships form a matrix with the CRs as rows and the

ECs as columns (Chan and Wu, 2005). Absence of symbols means absence of relationships (Bottani and Rizzi, 2006).

(6) Correlation matrix. The correlation matrix, also called the roof matrix of the HOQ, indicates the inner dependences among the ECs, which can be obtained through engineering analysis and experience (Chan and Wu, 2005). A positive relationship indicates that two ECs can complement or improve each other, while a negative one suggests that trade offs are required (Bottani and Rizzi, 2006). Correlations are indicated with graphic symbols that express the degree of relation between ECs (Bottani and Rizzi, 2006). Symbols can be translated into four-value rating scale: strong negative, negative, positive, strong positive (Bottani and Rizzi, 2006). It is possible to have no correlations between ECs (Bottani and Rizzi, 2006).

(7) Competitive benchmarks. The competitive benchmark analysis, or called the technical competitive assessments, is to technically evaluate the performance of the company's product and its main competitors' similar products on each EC (Chan and Wu, 2005).

The main result of the HOQ is the overall importance of ECs (Chen et. al., 2006). Deriving the importance of ECs from input variables that obtained in the preceding steps is a crucial step in applying QFD, which provides important information to a design team to carry out resource allocation, design project planning and manpower planning (Chen et. al., 2006). In this paper, we propose a Choquet capacity and integral based model to derive the overall importance of ECs in HOQ.

### 2.2. The Choquet capacity and integral

Let $X$ be a set of objects of interest, $N = \{1,...,n\}$ be a set of $n$ points of view, and each object $x \in X$ be associated with a profile $x = (x_1,...,x_n) \in [0,1]^n$, where $x_i$ represents the partial score of $x$ on point of view $i$. Let $\mathcal{P}(N)$ be the power set of $N$, a capacity (Choquet, 1953) (or called nonadditive measure, fuzzy measure) on $N$ is a set function $\mu : \mathcal{P}(N) \to [0,1]$ such that $\mu$ is monotonic, $\mu(S) \le \mu(T)$ for $\forall S,T \subseteq N$ and $S \subseteq T$, and satisfies the boundary conditions: $\mu(\emptyset) = 0$, $\mu(N) = 1$. A set $S \subseteq N$ is said to be a carrier (or support) of the capacity $\mu$ if $\mu(T) = \mu(S \cap T)$ for $\forall T \subseteq N$. Thus, a capacity $\mu$ with the carrier $S \subseteq N$ is completely defined by the knowledge of the coefficients $\{\mu(T)\}_{T \subseteq S}$.

One of the equivalent representations of a capacity is its Möbius transformation (Grabisch, 1997) $m_\mu : \mathrm{P}(N) \to R$, $m_\mu(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} \mu(T)$ for $\forall S \subseteq N$. Accordingly, we can have $\mu(T) = \sum_{S \subseteq T} m_\mu(S)$ for $\forall T \subseteq N$. And the boundary and monotonicity conditions of a capacity can be represented, in terms of Möbius transformation, as (Grabisch, 1997; Wu et. al., 2015): $m_\mu(\varnothing) = 0$, $\sum_{T \subseteq N} m_\mu(T) = 1$, $\sum_{T \subseteq S, i \in T} m_\mu(T) \geq 0, \forall S \subseteq N, \forall i \in S$.

The Shapley importance index of a point of view $i \in N$ with respect to the capacity $\mu$ is defined by (Grabisch et. al., 2008)

$$I_\mu(\{i\}) = \sum_{T \subseteq N \setminus \{i\}} \frac{(n-|T|-1)!\,|T|!}{n!} [\mu(T \cup \{i\}) - \mu(T)] \tag{1}$$

The Shapley importance index of each point of view represents its overall importance, since the marginal contribution of the point of view with any presence ($T \subseteq N \setminus \{i\}$) has been comprehensively considered and integrated into the index.

The Shapley interaction index of any subset $S \subset N$ is defined as

$$I_\mu(S) = \sum_{T \subseteq N \setminus S} \frac{(n-|T|-|S|)!\,|T|!}{(n-|S|+1)!} \left( \sum_{C \subseteq S} (-1)^{|S \setminus C|} \mu(C \cup T) \right) \tag{2}$$

Similar to the Shapley importance index, the Shapley interaction index is also a comprehensive index of integrating the marginal interactions with all presences.

In terms of Möbius transformation, the Shapley importance and interaction index can be formulated as (Grabisch and Labreuche, 2008):

$$I_{m_\mu}(S) = \sum_{T \subseteq N \setminus S} \frac{1}{|T|+1} m_\mu(S \cup T) \tag{3}$$

In most situations, it is too difficult to take account of all the interaction between all the points of view. Grabisch (Grabisch, 1997) proposes the concept of $k$-additive capacity, which allows considering only the interactions among at most $k$ points of view. A capacity $\mu$ on $N$ is $k$-additive if its Möbius transformation satisfies $m_\mu(S) = 0$ for $S$ such that $|S| > k$ and there exist at least one subset $S$ such that $|S| = k$ and $m_\mu(S) \neq 0$. Since the broad applicability and ease of use, the 2-additive capacity has become one of the most popular ones in the field of the multicriteria analysis (Wu et. al., 2014).

When using a capacity to model the importance and interactions of the points of view, a suitable aggregation function is the Choquet integral (Grabisch et. al., 2008, Wu and Zhang, 2010). The discrete Choquet integral (Grabisch et. al., 2008, Wu et. al., 2013) of object $x \in X$, $x = (x_1, ..., x_n)$, with respect to a capacity $\mu$ on $N$ is defined by

$$C_\mu(x) := \sum_{i=1}^{n} x_{(i)}[\mu(A_{(i)}) - \mu(A_{(i+1)})] \tag{4}$$

where the parentheses used for indices represent a permutation on $N$ such that $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ and $A_{(i)}$ represents that subset $\{(i),...,(n)\}$, $A_{(n+1)} = \varnothing$. In terms of the Möbius transformation, the Choquet integral can be rewritten as

$$C_{m_\mu}(x) = \sum_{T \subseteq N} m_\mu(T) \min_{i \in T} x_i \tag{5}$$

The Choquet integral has some good aggregation properties, such as continuous, non-decreasing, and located between min and max (Meyer and Roubens, 2005).

### 2.3. The Choquet capacity and integral based ordered sorting model

As mentioned in the introduction, the information processing of HOQ can be regarded as the multicriteria ordered sorting problem. Hence, we introduce a Choquet capacity and integral based ordered sorting model, called the TOMASO (Technique for Ordinal Multiattribute Sorting and Ordering) (Marichal et. al., 2005).

The aim of ordered sorting model is to assign each object $x \in X$ into $l$-grade classes $\{Cl_t\}_{t=1}^{l}$. Let $P \subseteq X$ be the set of prototypes, the partial net score of object $x \in P$ for the point of view $i \in N$ is defined by

$$S_i(x) := \frac{S_i'(x) + (|P|-1)}{2(|P|-1)} \in [0,1] \tag{6}$$

where $S_i'(x)$ represents the number of times that $x$ better than any other alternative of $P$ minus the number of times that any other alternative of $P$ better than $x$ for the point of view $i$. Each object $x \in P$ can be represented by its net score vector $S(x) := (S_1(x),...,S_n(x)) \in [0,1]^n$. A dominance relation $D$ on $[0,1]^n$ can be defined as, for each $x, y \in [0,1]^n$, $xDy \Leftrightarrow S_i(x) \geq S_i(y)$ for all $i \in N$. Let $\{P_t\}_{t=1}^{l}$ be a partition of $P$, where $P_t := P \bigcap Cl_t$ for all $t \in \{1,...,l\}$. The set of non-dominating objects of $P_t$ is defined by $Nd_t := \{x \in P_t \mid \nexists x' \in P \setminus \{x\} : xDx'\}$ and the set of non-dominated objects of $P_t$ is defined by $ND_t := \{x \in P_t \mid \nexists x' \in P \setminus \{x\} : x'Dx\}$. Meyer and Roubens (Meyer and Roubens, 2005) suggested a quadratic program based capacity identification method:

$$\min \sum_{x \in \bigcup_{t=1}^{l}\{Nd_t \cup ND_t\}} [C_{m_\mu}(S(x)) - C_{m_\mu}(x)]^2$$
$$\text{subject to} \begin{cases} m_\mu(\varnothing) = 0, \sum_{T \subseteq N} m_\mu(T) = 1, \\ \sum_{T \subseteq S, i \in T} m_\mu(T) \geq 0, \forall S \subseteq N, \forall i \in S, \\ C_{m_\mu}(x) - C_{m_\mu}(x') \geq \varepsilon, \forall (x, x') \in Nd_t \times ND_{t-1}, t \in \{2, ..., l\}, \end{cases} \tag{7}$$

where $\varepsilon$ , $0 < \varepsilon < 1/l$ , is a given strictly positive threshold value for separating the classes $\{Cl_t\}_{t=1}^{l}$ .

The above program can be implemented with the Kappalab package for the GNU R statistical system (R Development, 2011), by executing the following command: "ls.sorting.capa.ident( $n$ , $k$ , $\mathbf{E}$ , $\mathbf{I}$ , $\varepsilon$ , A.Shapley.preorder, A.interaction.interval)", where the first argument '$n$' fixes the number of points of view, the second argument '$k$' sets the desired $k$ -additivity for the identified capacity, the third argument '$\mathbf{E}$' represents the matrix of the normalized partial evaluations of the $q$ objects, the fourth argument '$\mathbf{I}$' is the vector containing the indexes of the classes of the $q$ objects, the fifth argument '$\varepsilon$' is the threshold value for the classes, the sixth argument 'A.Shapley. preorder' is a matrix containing the constraints relative to the preorder of the Shapley values, and the last argument 'A.interaction.interval' is a matrix containing the constraints relative to the intervals of the Shapley interaction indices. That is, the inputs of the model TOMASO are the above seven type of information and its output is the optimal $k$ -additive capacity.

### 3. The Choquet capacity and integral based methodology

To facilitate the description of the methodology, the following assumptions and notation are used:

- There are $m$ identified CRs, denoted by $M = \{CR_1, ..., CR_m\}$, in the HOQ. Each $CR_i$ ( $i = 1, ..., m$ ) is associated with a relative weight $\lambda_i$ , $\sum_{i=1}^{m} \lambda_i = 1$ .
- The customer perceptions are expressed by the predefined graded classes $\{Cl\}_{t=1}^{l}$ . A threshold $\varepsilon$ , $0 < \varepsilon < 1/l$ , is given to separate those classes.
- $N = \{EC_1, ..., EC_n\}$ is the set of ECs, $N^i \subseteq N$ , ( $i = 1, ..., m$ ), consists of ECs that relevant to the $CR_i$ , the number of elements of set $N^i$ is denoted as $n^i$ .
- $\mathbf{R} = [r_{ij}]_{m \times n}$ is the relationship matrix, and $\mathbf{C} = [c_{jk}]_{n \times n}$ is the correlation matrix, where $j \neq k$ and $c_{jk} = c_{kj}$ . We assume that the relationships between CRs and ECs, as well as the correlations among ECs, are expressed in four

degrees: strong negative, negative, positive, strong positive.

- Let $P = \{p_1,..., p_q\}$ be the set of products to be evaluated, where $p_1$ is the company's product, and $p_2,..., p_q$ are the competitors' similar products. The technically evaluations of the performance of the $q$ products are expressed in the evaluation matrix $\mathbf{E} = [e_{rj}]_{q\times n}$, where $e_{rj}$ is the partial evaluation of product $p_r$ ( $r = 1,..., q.$ ) on $EC_j$ ( $j = 1,..., n.$ ).

We are now ready to give the steps of the Choquet capacity and integral based methodology of identifying the overall importance of ECs.

**Step 1:** Establish and normalize the products evaluation sub-matrix with respect to every CR.

We firstly extract the partial evaluations of ECs that relevant to every CR and constitute the evaluation sub-matrix with respect to every CR. Since the Choquet integral is non-decreasing with respect to every point of the view, there should be a positive or strong positive relationship between the every relevant EC and the customer perception on this CR. Hence, we can normalize the sub-matrix by the following procedure. For a given $CR_i \in M$ , if $EC_j \in N^i$ exerts a positive or strong positive (resp. negative or strong negative) effect on the $CR_i$, we can normalize the partial evaluation of the product $p_r \in P$ ( $r = 1,..., q.$ ) on $EC_j$ , $e_{rj}$ , by:

$$e^i_{rj} = \frac{e_{rj} - \min_r(e_{rj})}{\max_r(e_{rj}) - \min_r(e_{rj})} \quad (\text{resp. } e^i_{rj} = \frac{\max_r(e_{rj}) - e_{rj}}{\max_r(e_{rj}) - \min_r(e_{rj})}) \tag{8}$$

Accordingly, we should replace every negative or strong negative relationship in the relationship matrix by the positive or strong positive relatinoship, respectively. After Step 1, we will have $m$ normalized partial evaluation sub-matrixes, $\mathbf{E}^i = [e^i_{rj}]_{q\times n^i}$ , $i = 1,..., m$ , and the $m$ adjusted relationship matrix $\mathbf{R}' = [r'_{ij}]_{m\times n}$ .

**Step 2:** Construct the correlation sub-matrix with respect to every CR.

In view of the normalization and adjustment in the Step 1, we should accordingly adjust the correlation between the relevant ECs of every CR. For a given $CR_i$, the correlation $c_{jk}$ , $EC_j, EC_k \in N^i$ , is adjusted to be its opposite if only one of the two relationships, $r_{ij}$ or $r_{ik}$ , has been changed into its opposite in the Step 1. After this Step, we get $m$ adjusted correlation sub-matrixes, $\mathbf{C}^i = [c^i_{jk}]_{n^i\times n^i}$ .

**Step 3:** Represent the relationships between CRs and their relevant ECs by the Shapley importance index.

The new relationship matrix $\mathbf{R}' = [r'_{ij}]_{m\times n}$ only consists of two types of relationships: positive and strong positive. As mentioned before, we adopt the Shapely importance index to represent the contribution importance of the relevant

$EC_j \in N^i$ to the $CR_i$, denoted by $I^i(j)$. We can introduce a relative threshold $\eta > 0$ with respect to the Shapely importance indices to distinguish the degrees of positive and strong positive.

**Step 4:** Represent the correlations among ECs by the Shapley interaction index.

In the correlation matrix, there are four types of relationships between arbitrary two ECs: strong negative, negative, positive and strong positive. We represent correlation between $EC_j \in N^i$ and $EC_k \in N^i$ with respect to $CR_i$ by $I^i(j,k)$. Since the Shapley interaction index of arbitrary two points of view lies in interval $[-1,1]$, we can introduce a threshold $\xi > 0$ to distinguish the four types of correlations into subintervals $[-1,-\xi]$, $(-\xi,0]$, $[0,\xi)$ and $[\xi,1]$, respectively.

**Step 5:** Identify the optimal capacity with respect to each CR by using the TOMASO method.

As mentioned in Section 2.3, to executing the TOMASO model in the Kappalab package, we need to input the seven types of information. And these required information has already been determined by the four previous steps. So, we can establish the TOMASO model with respect to each $CR_i$, and solving it to obtain the optimal $k$ -additive capacity, usually a 2-additive capacity, $\mu^i$ on $N^i$.

**Step 6:** Generate the integrated capacity and derive the overall importance of every EC.

To unify the domain of capacity $\mu^i$ ( $i = 1,2,\ldots,m$ ) to the power set of $N$, $\mathrm{P}(N)$, we take $N^i$ as a carrier of $\mu^i$ and extend it by $\mu^i(T) = \mu^i(N^i \cap T)$, $\forall T \subseteq N$. Then we integrate the capacities $\mu^i$ on $N$ into the integrated capacity $\mu$ on $N$ according to the weights of ECs by $\mu(T) = \sum_{i=1}^{m} \lambda_i \mu^i(T)$, $\forall T \subseteq N$. The Shapley importance index of $EC_i$ with respect to $\mu$, $I_\mu(\{EC_i\})$, can be taken as its overall importance in quality function deployment.

From the above steps of the Choquet capacity and integral based method, we can see that the overall importance of each EC comprehensively reflects the its importance to every CR as well as its interaction with every relevant EC associated with every CR, since on the one hand, the capacity $\mu$ is an integrated capactiy of the optimal capacities with respect to all CRs, and on the other hand, the Shapley importance index is a comprehensive index by taking account of the marginal contributions with all presence.

## 4. An illustrative example

This section presents an illustration of the proposed methodology based on a digital camera design example (adapted from (Kwong et. al., 2007)). The HOQ was constructed to represent the information about the digital camera design problem (see Fig. 2).

- The HOQ has six CRs ($m = 6$) : $CR_1$: Photo quality, $CR_2$: Take distant image, $CR_3$: Low price, $CR_4$: Versatility, $CR_5$: Easy to operate, and $CR_6$: Portability.
- The customer perceptions are expressed by the predefined graded classes $\{Cl\}_{t=1}^{5}$ with a separate threshold $\varepsilon = 0.1$.
- These CRs are translated into corresponding six ECs ($n = 6$): $EC_1$: Max Resolution Support, $EC_2$: Optical Zoom, $EC_3$: Aperture Exposure Control, $EC_4$: LCD size, $EC_5$: Storage Media Support, and $EC_6$: Weight.

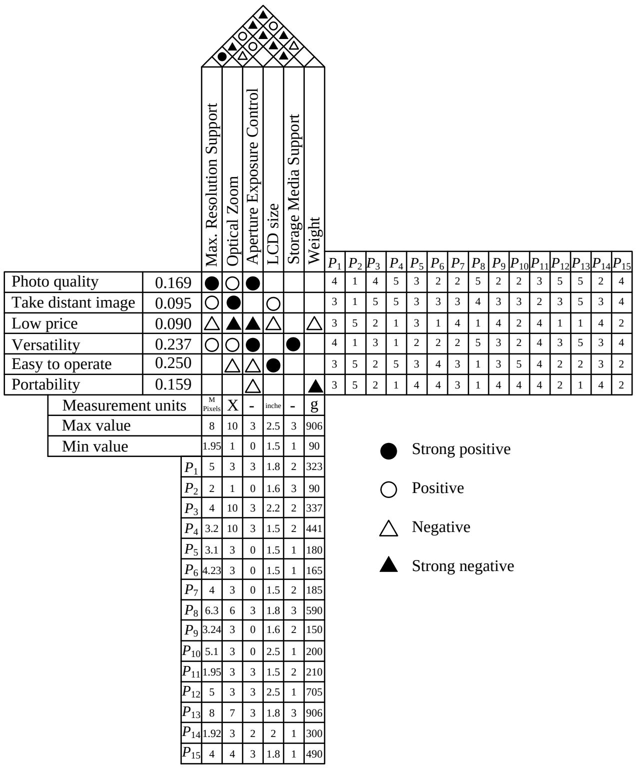


**Figure 2. The house of quality for digital camera design**

**Step 1:** Establish and normalize the products' evaluation sub-matrix with respect to every CR in sequence.

We take $CR_5$ (Easy to operate) as an example. Since $EC_2$ (Optical Zoom) exerts a negative effect on $CR_5$ (see Fig 2), $\max_r(e_{r2})=10$, $\min_r(e_{r2})=1$, and $EC_4$ (LCD size) exerts a strong positive effect on the $CR_5$, $\max_r(e_{r4})=2.5$, $\min_r(e_{r4})=1.5$, we can normalize respectively the partial evaluations $e_{12}$, $e_{14}$ of the product $p_1$ by

$$e_{12}^5=\frac{\max_r(e_{r2})-e_{12}}{\max_r(e_{r2})-\min_r(e_{r2})}=\frac{10-3}{10-1}=0.78 \tag{9}$$

$$e_{14}^5=\frac{e_{14}-\min_r(e_{r4})}{\max_r(e_{r4})-\min_r(e_{r4})}=\frac{1.8-1.5}{2.5-1.5}=0.3 \tag{10}$$

After getting the normalized evaluation matrix $\mathbf{E}^5=[e_{rj}^5]_{q\times 3}$, we accordingly change the negative relationships $r_{52}$ and $r_{53}$ into the positive relationship, see Fig. 3 (e). Finally, we can get the six normalized evaluation matrices $\mathbf{E}^i=[e_{rj}^i]_{p\times n^i}$, $i=1,...,6$, and the corresponding new relationships matrices, as shown in the Fig. 3. (a)~(f).

**Step 2:** Construct the correlation sub-matrix with respect to every CR.

Still considering $CR_5$, see Fig 3. (e), since both $r_{52}$ and $r_{53}$ have been changed to their opposites, the correlation between $EC_2$ and $EC_3$ will maintain its negative interaction. The positive correlation between $EC_3$ and $EC_4$ need to be changed into negative since only $r_{53}$ has been changed to its opposite in the Step 1. Similarly, we can adjust all the correlations, see Fig 3.

**Step 3:** Represent the relationship between CRs and their relevant ECs by the Shapley importance index.

In this example, the QFD team set the relative threshold of Shapely importance index as $\eta=0.1$. For $CR_4$, see Fig 3. (d), $EC_1$ and $EC_2$ exert positive efforts on $CR_4$, while $EC_3$ and $EC_5$ exert strong positive efforts on $CR_4$, we can construct the constraints of the four Shapley values, $I^4(1)$, $I^4(2)$, $I^4(3)$ and $I^4(5)$, as follows:

$$I^4(3)-I^4(1)\ge 0.1,\ I^4(3)-I^4(2)\ge 0.1,\ I^4(5)-I^4(1)\ge 0.1,\ I^4(5)-I^4(2)\ge 0.1. \tag{11}$$

**Step 4:** Represent the correlations among ECs by the Shapley interaction index.

The QFD team set the threshold of the type of correlations as $\xi=0.1$, then the Shapley interaction index of strong negative, negative, positive and strong positive correlations will lie in $[-1,-0.1]$, $(-0.1,0]$, $[0,0.1)$ and $[0.1,1]$, respectively. For $CR_4$, see Fig 3. (d), we will have the following constraints:

$$0.1\le I^4(1,2)\le 1,\ \ -1\le I^4(1,3)\le -0.1,\ \ -1\le I^4(1,5)\le -0.1,$$
$$-0.1\le I^4(2,3)\le 0,\ \ -1\le I^4(2,5)\le -0.1,\ \ -1\le I^4(3,5)\le -0.1. \tag{12}$$

**Step 5:** Identify the optimal capacity with respect to each CR by using the TOMASO method.

By using the Kappalab package (see the Appendix for the detail codes and annotations of the TOMOASO model of identifying the optimal 2-additvie capacity $\mu^4$ with respect to the $CR_4$), we can obtain the optimal capacities of the six CRs, denoted as $\mu^1$, $\mu^2$, $\mu^3$, $\mu^4$, $\mu^5$ and $\mu^6$, which are listed respectively in Tables 1~6, where the subscript is used as standing for the EC for convenience, e.g., the subset $\{1,2\}$ stands for the subset $\{EC_1, EC_2\}$.

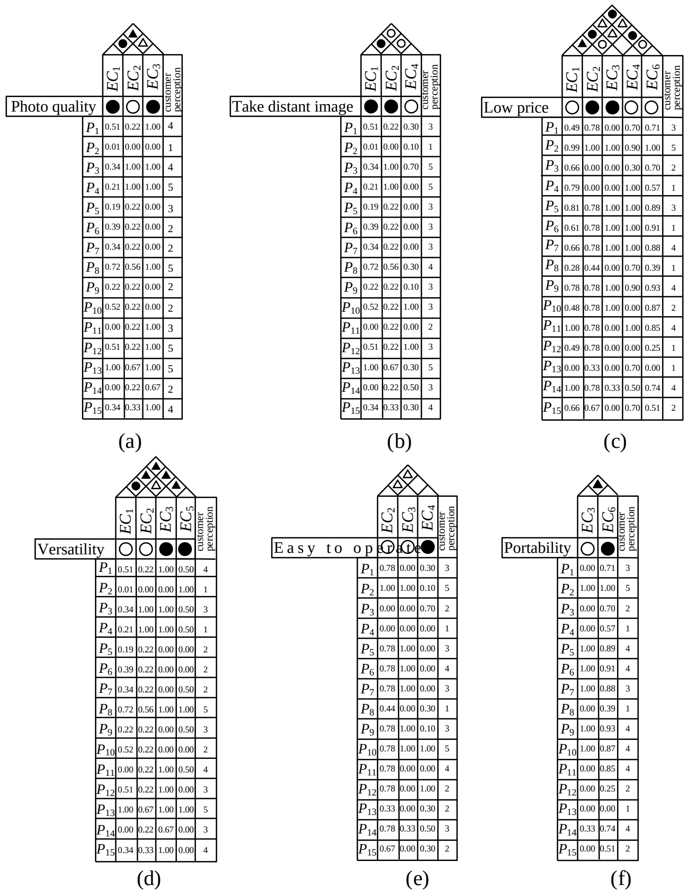


**Figure 3. The sub-HOQ with respect to each CR**

**Table 1. The capacity $\mu^1$ on $N^1 = \{EC_1, EC_2, EC_3\}$**

| $A$ | $\mu^1(A)$ | $A$ | $\mu^1(A)$ | $A$ | $\mu^1(A)$ | $A$ | $\mu^1(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {2} | 0.020 | {1,2} | 0.616 | {2,3} | 0.503 |
| {1} | 0.170 | {3} | 0.483 | {1,3} | 0.553 | {1,2,3} | 1.000 |

**Table 2. The capacity $\mu^2$ on $N^2 = \{EC_1, EC_2, EC_4\}$**

| $A$ | $\mu^2(A)$ | $A$ | $\mu^2(A)$ | $A$ | $\mu^2(A)$ | $A$ | $\mu^2(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {2} | 0.559 | {1,2} | 0.881 | {2,4} | 0.679 |
| {1} | 0.165 | {3} | 0.019 | {1,4} | 0.035 | {1,2,4} | 1.000 |

**Table 3. The capacity $\mu^3$ on $N^3 = \{EC_1, EC_2, EC_3, EC_4, EC_6\}$**

| $A$ | $\mu^3(A)$ | $A$ | $\mu^3(A)$ | $A$ | $\mu^3(A)$ | $A$ | $\mu^3(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {1,4} | 0.264 | {1,2,3} | 0.641 | {2,4,6} | 0.545 |
| {1} | 0.109 | {1,6} | 0.314 | {1,2,4} | 0.541 | {3,4,6} | 0.514 |
| {2} | 0.386 | {2,3} | 0.541 | {1,2,6} | 0.491 | {1,2,3,4} | 0.795 |
| {3} | 0.155 | {2,4} | 0.541 | {1,3,4} | 0.519 | {1,2,3,6} | 0.845 |
| {4} | 0.155 | {2,6} | 0.391 | {1,3,6} | 0.669 | {1,2,4,6} | 0.645 |
| {6} | 0.105 | {3,4} | 0.309 | {1,4,6} | 0.469 | {1,3,4,6} | 0.823 |
| {1,2} | 0.386 | {3,6} | 0.359 | {2,3,4} | 0.695 | {2,3,4,6} | 0.799 |
| {1,3} | 0.364 | {4,6} | 0.259 | {2,3,6} | 0.645 | {1,2,3,4,6} | 1.000 |

**Table 4. The capacity $\mu^4$ on $N^4 = \{EC_1, EC_2, EC_3, EC_5\}$**

| $A$ | $\mu^4(A)$ | $A$ | $\mu^4(A)$ | $A$ | $\mu^4(A)$ | $A$ | $\mu^4(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {5} | 0.400 | {2,3} | 0.699 | {1,2,5} | 0.700 |
| {1} | 0.200 | {1,2} | 0.500 | {2,5} | 0.500 | {1,3,5} | 0.899 |
| {2} | 0.200 | {1,3} | 0.699 | {3,5} | 0.899 | {2,3,5} | 0.899 |
| {3} | 0.599 | {1,5} | 0.500 | {1,2,3} | 0.899 | {1,2,3,5} | 1.000 |

**Table 5. The capacity $\mu^5$ on $N^5 = \{EC_2, EC_3, EC_5\}$**

| $A$ | $\mu^5(A)$ | $A$ | $\mu^5(A)$ | $A$ | $\mu^5(A)$ | $A$ | $\mu^5(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {3} | 0.346 | {2,3} | 0.648 | {3,5} | 0.798 |
| {2} | 0.402 | {5} | 0.452 | {2,5} | 0.754 | {2,3,5} | 1.000 |

**Table 6. The capacity $\mu^6$ on $N^6 = \{EC_3, EC_6\}$**

| $A$ | $\mu^6(A)$ | $A$ | $\mu^6(A)$ | $A$ | $\mu^6(A)$ | $A$ | $\mu^6(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {3} | 0.346 | {6} | 0.648 | {3,6} | 0.798 |

**Step 6:** Generate the integrated capacity and derive the overall importance of every EC.

We firstly unify each capacity $\mu^i$ into power set $\mathrm{P}(N)$. For example, for the capacity $u^1$ on $N^1 = \{EC_1, EC_2, EC_3\}$, the capacity values of the subsets $\{EC_1, EC_4\}$ and $\{EC_1, EC_2, EC_3, EC_4\}$ can be get by the following equations:

$u^1(\{EC_1, EC_4\}) = u^1(\{EC_1, EC_4\} \cap N^1) = u^1(\{EC_1\}) = 0.170$,

$$u^1(\{EC_1, EC_2, EC_3, EC_4\}) = u^1(\{EC_1, EC_2, EC_3, EC_4\} \cap N^1) = u^1(N^1) = 1.000. \quad (13)$$

Then we can integrate these capacities according to their EC's weight: for $\forall T \subseteq N$,

$$\mu(T) = \sum_{i=1}^{6} \lambda_i \mu^i(T) = 0.169 \times \mu^1(T) + 0.095 \times \mu^2(T) + 0.090 \times \mu^3(T) + 0.237 \times \mu^4(T) + 0.250 \times \mu^5(T) + 0.159 \times \mu^6(T). \quad (14)$$

The integrated capacity $\mu$ on $N$ is shown in the Table 7.

**Table 7. The integrated capacity $\mu$ on $N = \{EC_1, EC_2, EC_3, EC_4, EC_5, EC_6\}$**

| $A$ | $\mu(A)$ | $A$ | $\mu(A)$ | $A$ | $\mu(A)$ | $A$ | $\mu(A)$ |
|---|---|---|---|---|---|---|---|
| ∅ | 0.000 | {3,4} | 0.469 | {2,3,4} | 0.644 | {1,3,4,5} | 0.572 |
| {1} | 0.088 | {3,5} | 0.411 | {2,3,5} | 0.578 | {1,3,4,6} | 0.695 |
| {2} | 0.239 | {3,6} | 0.502 | {2,3,6} | 0.683 | {1,3,5,6} | 0.614 |
| {3} | 0.340 | {4,5} | 0.224 | {2,4,5} | 0.424 | {1,4,5,6} | 0.465 |
| {4} | 0.129 | {4,6} | 0.297 | {2,4,6} | 0.512 | {2,3,4,5} | 0.691 |
| {5} | 0.095 | {5,6} | 0.263 | {2,5,6} | 0.469 | {2,3,4,6} | 0.796 |
| {6} | 0.168 | {1,2,3} | 0.701 | {3,4,5} | 0.540 | {2,3,5,6} | 0.731 |
| {1,2} | 0.442 | {1,2,4} | 0.555 | {3,4,6} | 0.630 | {2,4,5,6} | 0.583 |
| {1,3} | 0.396 | {1,2,5} | 0.489 | {3,5,6} | 0.572 | {3,4,5,6} | 0.702 |
| {1,4} | 0.216 | {1,2,6} | 0.610 | {4,5,6} | 0.392 | {1,2,3,4,5} | 0.838 |
| {1,5} | 0.159 | {1,3,4} | 0.525 | {1,2,3,4} | 0.815 | {1,2,3,4,6} | 0.976 |
| {1,6} | 0.265 | {1,3,5} | 0.444 | {1,2,3,5} | 0.725 | {1,2,3,5,6} | 0.887 |
| {2,3} | 0.531 | {1,3,6} | 0.566 | {1,2,3,6} | 0.863 | {1,2,4,5,6} | 0.771 |
| {2,4} | 0.352 | {1,4,5} | 0.288 | {1,2,4,5} | 0.602 | {1,3,4,5,6} | 0.743 |
| {2,5} | 0.310 | {1,4,6} | 0.394 | {1,2,4,6} | 0.723 | {2,3,4,5,6} | 0.844 |
| {2,6} | 0.399 | {1,5,6} | 0.336 | {1,2,5,6} | 0.658 | {1,2,3,4,5,6} | 1.000 |

Finally, the overall importance of the six ECs, $I_\mu(EC_j)$ ($j = 1, 2, \ldots, 6$), can be obtained as: $I_\mu(EC_1) = 0.122$, $I_\mu(EC_2) = 0.248$, $I_\mu(EC_3) = 0.285$, $I_\mu(EC_4) =$, $0.121$, $I_\mu(EC_5) = 0.059$, and $I_\mu(EC_6) = 0.165$.

## 5. Conclusions

In this paper, we propose the Choquet capacity and integral based methodology to identify the overall importance of ECs from the information contained in HOQ. The feasibility of the proposed method was demonstrated by the digital camera design example. The main idea of this method is to consider the information processing of HOQ as a Choquet capacity and integral based multicriteria classification model, and to represent the overall importance of ECs and the correlations between them by the Shapley importance indices and Shapley interaction indices, respectively. The main advantage of this method is that the final overall importance of each EC comprehensively reflects its importance to every CR as well as its interaction with every relevant EC associated with every CR. Further research can focus on the sensitivity analysis of thresholds $\zeta, \eta$ to the final overall importance as well as the software of executing this method.

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**Appendix**

The code and annotations to identify the capacity $\mu^4$ on $N^4$ in Step 5 of the illustrative example in Section 4:

```
#create 15 R vectors representing the normalized partial evaluations of 15 productions
P1<-c(0.51,0.22,1.00,0.50)
# the symbol "<-" is the assignment operator, and "c" is R function for vector creation.
P2<-c(0.01,0.00,0.00,1.00)
…
P15<-c(0.34,0.33,1.00,0.00)
# concatenate into a 15 row matrix using the "rbind" matrix creation function.
E4<- rbind(P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12, P13, P14, P15,)
# the indexes of the classes of the 15 productions are encode into a 15 element R vector.
I4<- c(4,1,3,1,2,2,2,5,3,2,4,3,5,3,4)
# the threshold value to separate the classes.
ε <- 0.1
#the number of the relevant ECs.
n4<-4
# a 2-additive capacity.
k<-2
# the preorder constraint matrix of the Shapley importance values
Asp <- rbind(c(3,1,0.1), c(3,2, 0.1), c(4,1, 0.1), c(4,2, 0.1))
# e.g., the first row of the matrix Asp means ϕ⁴(3) − ϕ⁴(1) ≥ 0.1
# the interval constraint matrix constraint matrix of the Shapley interaction values
Aii <- rbind(c(1,2,0.1,1) , c(1,3,-1,-0.1) , c(1,4,-1,-0.1), c(2,3,-0.1,0), c(2,4,-1,-0.1), c(3,4,-1,-0.1))
# e.g., the first row of the matrix Aii means 0.1 ≤ ϕ⁴(12) ≤ 1 .
# model and solve the quadratic program.
lsc <- ls.sorting.capa.ident (n4, k, E4, I4, ε , A.Shapley.preorder=Asp, A.interaction.interval = Aii)
# the identified capacity.
zeta(lsc$solution)
```

*# the global evaluations of the 15 productions derived from the Choquet integral.*
lsc$glob.eval
*#the Shapley values of the relevant ECs with respect to the identified capacity.*
Shapley.value(lsc$solution)
*# the Shapley interaction indices of each pair of relevant ECs with respect to the identified capacity.*
interaction.indices(lsc$solution)

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