ON THE POSSIBILITY TO USE A MODEL OF OPTIMISATION TO ACHIEVE SUSTAINABILITY

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Rezumat

Paradigma optimalității în dezvoltarea economică sau socială şi-a demonstrat caracterul inadecvat şi, în consecință, trebuie înlocuită c paradigma sustenabilității. În acest context, studiul are drept scop propunerea unor proceduri şi instrumente prin intermediul cărora să se obțină un portofoliu sustenabil de investiții financiare (deşi este, încă, folosit un model de tip Lagrange). Principala valoarea adăugată de natură conceptuală a studiului este aceea a mărcii de sustenabilitate a țintei de investiție financiară. În acest cadru, după obținerea soluției sustenabile a modelului, sunt realizate trei analize calitative, cu scopul de a obține semnificația modelului propus: a) impactul variației bugetului pentru un portofoliu sustenabil de investiții financiare; b) impactul variației costului de oportunitate al portofoliului sustenabil de investiții financiare; c) impactul variației mărcii de sustenabilitate.

Abstract

The optimality paradigm of the economic or social development proved to be inappropriate and, consequently, must be replaced by the sustainability paradigm. In such a context, the paper is aimed at to propose some procedures and instruments to reach a sustainable portofolio of financial investment (although a Lagrange model is still used). The main conceptual added value of the paper is the sustainability label of the financial investment target. In this

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framework, after the derivation of the sustainable solution, three qualitative analyses are performed, in order to get the significance of the proposed model: a) the impact of the variation of the budget for the sustainable portfolio of the financial investments; b) the impact of the variation of the opportunity cost of the investment portofolio; c) the impact of the variation of the sustainability label.

Keywords: sustainability, financial portfolio, Lagrange multiplier, indifference curve, opportunity cost.

Jel classification: G11, O16, P34.

1. General model

We will approach the possibility to form a sustainable portfolio of financial placements (actually, construction of a sustainable portfolio of financial assets).

- Notations:
  - x_i: „amount" of the sustainable financial asset „i" included in the „optimal" sustainable portfolio
  - t_i: opportunity cost of the sustainable destination of financial placement „i" within the sustainable portfolio of financial placement[1]
  - m_i: sustainability mark of the sustainable direction of financial placement „i"
  - B: budget (exogenous) allocated to construct the sustainable portfolio of financial assets
  - PS: sustainable portfolio of financial assets
  - M: sustainability mark of the sustainable portfolio of financial assets[2]
  - λ: Lagrange multiplier of the sustainability of the portfolio of financial assets[3]

[1] *This cost has the significance of a transaction cost.*
[2] *Which, according to the convention agreed above, will be maximised.*

- Formal equations:

  - Restriction of the optimising model: $B = \sum_{i=1}^{6} x_i \cdot t_i$

  - Target function[4]: $PS = M(x_1, x_2, x_3, x_4, x_5, x_6) = \frac{\prod_{i=1}^{6} (m_i \cdot x_i)}{\sum_{i=1}^{6} (m_i \cdot x_i)}$

  - The optimisation function (Lagrange) will have the following analytical form:

$$L = M(m_1 \cdot x_1, m_2 \cdot x_2, m_3 \cdot x_3, m_4 \cdot x_4, m_5 \cdot x_5, m_6 \cdot x_6) + \lambda \cdot \left[B - \sum_{i=1}^{6} (x_i \cdot t_i) \right] \rightarrow \max$$

- Economic significance of the Lagrange multiplier:

  - Conditions of maximisation: $\frac{\partial L}{\partial x_i} = 0$ for $(\forall) i \in \{1,6\}$, thus:

$$m_i \cdot \mu_i = \lambda \cdot t_i, \ (\forall) i \in \{1,6\}$$

where $\mu_i = \frac{\partial M}{\partial x_i}$, meaning it is the marginal mark of sustainability of the portfolio of financial assets.

  - Since, it is obviously that for the given t_i, $dB = \sum_{i=1}^{6} (dx_i \cdot t_i)$, it results that a variation of the mark of sustainability of the portfolio of financial assets, dPS will be written as:

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[3] The Lagrange multiplier shows with how many units will the value of the sustainability mark of the portfolio of financial assets modify when the budget allocated to construct the sustainable portfolio of financial assets changes with one unit.

[4] It may be easily demonstrated that such a function is decreasing and convex (its Hessian matrix is defined positively). These mathematical characteristics describe the qualitative hypotheses regarding a hyperarea of indifference.

$$dM = \sum_{i=1}^{6} (m_i \cdot \mu_i \cdot dx_i)$$

- As $m_i \cdot \mu_i = \lambda \cdot t_i$, we obtain:

$$dM = \sum_{i=1}^{6} (m_i \cdot \mu_i \cdot dx_i) = \lambda \cdot \sum_{i=1}^{6} (dx_i \cdot t_i) = \lambda \cdot dB$$

Hence the significance of λ: $\lambda = \frac{dM}{dB}$

- Optimal solution of the model:
  - It is given by the tangency point between the hyperarea of the budget (a concave hyperarea) and the hyperarea of the sustainable portfolio of financial assets (a convex hyperarea)
  - The system of equations that leads to the optimal solution is

$$\begin{cases} dM = 0 \\ B = \sum_{i=1}^{6} x_i \cdot t_i \end{cases}$$

which is:

$$\begin{cases} \sum_{i=1}^{6} m_i \cdot \mu_i \cdot dx_i = 0 \\ B = \sum_{i=1}^{6} x_i \cdot t_i \end{cases}$$

if we differentiate completely the second equation (B is given therefore it can be assimilated to a constant), we obtain:

$\sum_{i=1}^{6} (t_i \cdot dx_i) = 0$, hence: $\frac{dx_j}{dx_i} = -\frac{p_i}{p_j}$ for $(\forall) i, j \in \{1,6\}$, with $i \neq j$

On the other hand, from the equation of the hyperarea of indifference it results:

$\frac{dx_j}{dx_i} = -\frac{m_i \cdot \mu_i}{m_j \cdot \mu_j}$, for $(\forall) i, j \in \{1,6\}$, with $i \neq j$

Combining the two results (actually eliminating between the two results the expression $\frac{dx_j}{dx_i}$), we obtain the mathematical condition of the sustainable portfolio of financial assets which achieves the optimal decision under the given budgetary conditions:

$\frac{m_i \cdot \mu_i}{m_j \cdot \mu_j} = \frac{t_i}{t_j}$, hence: $\frac{\mu_i}{\mu_j} = \frac{\frac{t_i}{t_j}}{\frac{m_i}{m_j}} = \frac{\tilde{t}_{i/j}}{\tilde{m}_{i/j}}$, with the following notations:

$\tilde{t}_{i/j}$: relative cost of financing through sustainable sources of financing „i", and „j", respectively

$\tilde{m}_{i/j}$: relative mark of sustainability of the sustainable sources of financing „i", and „j", respectively

Therefore, will we obtain a sustainable portfolio of financial assets (optimal in terms of the budget allocated for the input costs of the sustainable destinations of financial placement within the portfolio of financial assets) when the following mathematical condition will be accomplished for all the specific sustainable destinations of financial placement: *the ratio of the marginal mark of sustainability of the portfolio of financial assets to any two sustainable destinations of financial placement is equal with the ratio of the relative price of opportunity to the relative mark of sustainability of the specific sustainable destinations of financial placement*.

If we note $m_i \cdot \mu_i = \overline{\mu}_i$, where $\overline{\mu}_i$, which we call adjusted marginal mark of sustainability, and we consider that it represents the marginal mark of sustainability of the sustainable portfolio of financial placement related to the sustainable destination of financial placement „i", corrected by multiplication with the mark of sustainability of the sustainable destination of financial placement „i", than we can make the following statements:

a. The ratio of the two adjusted marginal marks of sustainability of the portfolio of financial assets (marginal marks of sustainability determined in relation to two any sustainable destinations of financial placement) is the very marginal rate of substitution between the two sustainable destinations of financial placement, so that the mark of sustainability of the sustainable portfolio of financial assets doesn't change (in other words, so that the decision of formation of the sustainable portfolio of financial assets remains on the hyperarea of sustainability);

b. Although the marks of sustainability afferent to each sustainable destination of financial placement were considered to be constant, they can change due to the modification of the financial structure of the market; the model can therefore be complicated further, particularly for the cases of prognosis, considering that the marks of sustainability of the sustainable destinations of financial placement are, in turn, variable. The same reasoning can be done for the opportunity costs of the sustainable destinations of financial placement, susceptible to join the formation of the sustainable portfolio of financial assets.

2. A qualitative evaluation

To enable us observing the variable character of the budget, of the opportunity costs of the sustainable destinations of financial placement within the sustainable portfolio of financial assets, and of the mark of sustainability of the sustainable destinations of financial placement, we will make an analysis for two given destinations of financial placement, „i", and „j", with the opportunity costs „t_i", and „t_j", the marks of sustainability „m_i", and „m_j" and with the budget B.

a) Variation of the budget (B) allocated for the construction of the sustainable portfolio of financial assets

We will suppose that budget B is variable and the opportunity costs of the sustainable destinations of financial placement and the marks of sustainability of these destinations of financial placement remain constant.

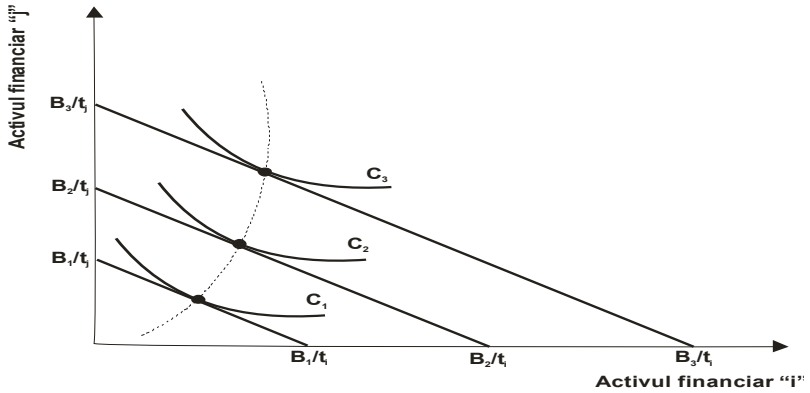
The equation of the initial budget will be: $B = x_i \cdot t_i + x_j \cdot t_j$, hence: $x_j = -\frac{t_i}{t_j} \cdot x_i + \frac{B}{t_j}$. As t_i and t_j are constant, the graphic representation of the budget will be a straight line with negative slope (with the size t_i/t_j) and with the free term B/t_j.

The curve of indifference of the mark of sustainability of the sustainable portfolio of financial assets will be decreasing and convex.

The optimum of the sustainable portfolio of financial assets will be obtained in all points of tangency between the straight line of the variable budget and the different curves of indifference (parallel between them[5]). Joining all these points of optimum we will obtain the path of the sustainable portfolio of financial assets for different values of the budget allocated for the construction of the sustainable portfolio of financial assets, as shown in *Figure 1*:

Figure 1

Path of the sustainable portfolio of financial placement for a variable budget



[5] *As it can be easily demonstrated, two curves of indifference can never intersect. As we are in the Euclidian space, it results that any two curves of indifference are parallel between them (we should note, nevertheless, the excessively simplifying character of this hypothesis, because the non-linear dynamics of the marks of sustainability in relation to the parameters of the financial assets implies the hypothesis of a non-Euclidian space – therefore, the economic space and particularly the financial space, are non-Euclidian spaces. For the time being we ignore these difficulties, but they will be dealt with fully in the doctorate thesis).*

b) Variation of the cost of opportunity (t) of the sustainable destination of financial placement

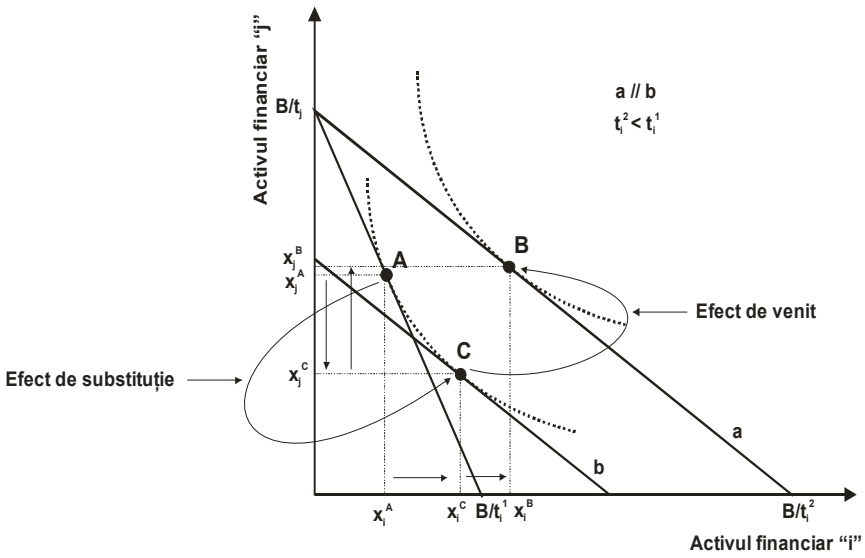
If we consider as given the size of the budget and the level of the marks of sustainability of the sustainable destinations of financial placement and if we consider that we have variations of the cost of opportunity of the sustainable destinations of financial placement that will form the sustainable portfolio of financial assets, then we obtain the two known effects, the effect of income and the effect of substitution. To simplify the graphic representation of these effects, we will suppose (without affecting the general character of the demonstration) that the cost of opportunity of the sustainable destination of financial placement „j" remains constant (that is, t_j = constant) and that only the cost of opportunity of the other sustainable destination of financial placement „i", t_i varies (for instance, we will suppose that this cost decreases[6]).

Taking into account the equation of the budget for the construction of the sustainable portfolio of financial assets, $x_j = -\frac{t_i}{t_j} \cdot x_i + \frac{B}{t_j}$, it results that for a constant B and for a constant t_j, the free term of the straight line representing the budget will remain fixed. What changes is the slope of the straight line representing the budget (i.e. t_i/t_j ratio). In order to maintain the slope of the initial budget, a line of budget is plotted parallel with the initial line of budget (this signifies the return to the relative initial cost of opportunity of financial placement for the two sustainable destinations) and tangent to the initial curve of indifference. This enables us to identify the two remarkable effects of the variation of the cost of opportunity of the sustainable destination of financial placement „i", namely the effect of substitution and the effect of income (*Figure 2*):

[6] *Obviously, a similar reasoning is possible for the situation when this cost of opportunity increases.*

Figure 2

Effect of income and effect of substitution for the decrease of the cost of opportunity of the sustainable financial placement „i"



c) Variation of the mark of sustainability (m) of the sustainable destination of financial placement

If so far we had changes in the budget allocates for the construction of the sustainable portfolio of financial assets (either by the direct variation of the size of the budget, or by the variation of its slope caused by the variation of the cost of opportunity of the sustainable destinations of financial placement), this time we will have changes of the curve of indifference.

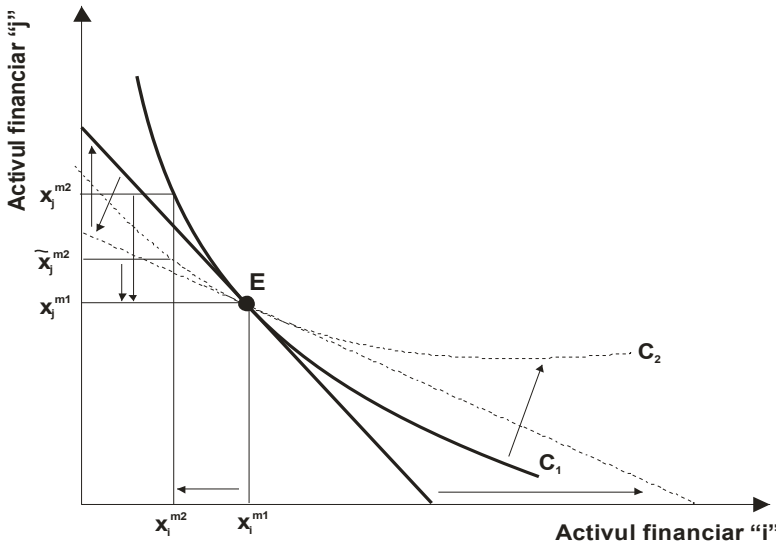
Let us resume the equation that describes the curve of indifference of the two sustainable destinations of financial placement, „i" and „j":

$\frac{\mu_i}{\mu_j} = \frac{\frac{t_i}{t_j}}{\frac{m_i}{m_j}} = \frac{\tilde{t}_{i/j}}{\tilde{m}_{i/j}}$. If we suppose that the relative mark of sustainability of the two sustainable destinations of financial placement decreases, that is $m_i^2 < m_i^1$, and $m_j^2 \geq m_j^1$, it results then that the marginal rate of substitution between the two sustainable destinations of financial placement decreases. This means that the same size of decrease of the "amount" of the sustainable destination of financial placement „i" will need a smaller increment (than at the initial moment) of the increase of the "amount" of the sustainable destination of financial placement „j" so that the mark of sustainability of the sustainable portfolio of financial assets preserves.

Figure 3 shows this situation.

Figure 3

Effect of the variation of the relative mark of sustainability of the sustainable destinations of financial placement „i" and „j"



It can be noticed that the variation of the relative mark of sustainability of the sustainable destinations of financial placement „i"

and „j" is equivalent to the simultaneous variation of the costs of opportunity[7] of the financial placement for the two specific destinations. This result is very important because it allows a neutral behaviour of the decision maker interested to construct a sustainable portfolio of financial assets: if the evaluation of the marks of sustainability changes, he/she can counteract this by negotiating an adequate variation of the coefficient of profit of the purchase of the two destinations of financial placement[8].

[7] *It is noteworthy that, logically, the selection of the cost of opportunity is equivalent to the selection of the coefficient of profit of the purchase of financial assets.*

[8] *We believe that, would this conclusion be developed further up to the final consequences, we might reach a result of neutrality absolutely similar to that obtained by Miller-Modigliani (laureates of the Nobel prise for economy) and, very interesting, also concerning financing designs, but on a higher paradigm than the one used by the two, namely on the paradigm of sustainability (they considered the paradigm of optimality).*