STATIONARITY, STABILITY AND SUSTAINABILITY IN THE ECONOMIC PROCESS

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Abstract

The paper has as basic objective to revisit, in an unitary way, the three notions regarding the economic process dynamics: stationarity, stability, and sustainability. This analysis will be performed from three points of view: 1) conceptually; 2) methodologically; 3) instrumentally (including quantifications). The paper will try to discern both the differences and the similarities among the three concepts, in order to extract the practical possibilities to use them in designing and piloting the macroeconomic processes. Some formalisms will be also delivered and argued, based on the accepted definitions of the concepts involved.

Keywords: stability, stationarity, sustainability, dynamics mix

JEL Classification: B41, E32, E63

The economic policies are primarily interested for sustainability of the economic systems, since this concept takes into consideration the long term and, in a significant degree, the self-stabilizing and re-stabilizing of these systems. The paper will discuss the stationary, stable and sustainable systems (especially the economic such systems), in order to formulate the general conditions of the sustainable economic systems, so of the public adjustments policies aimed to generate and maintain these sustainability conditions. Figure 1 suggests the general scheme of the discussion.

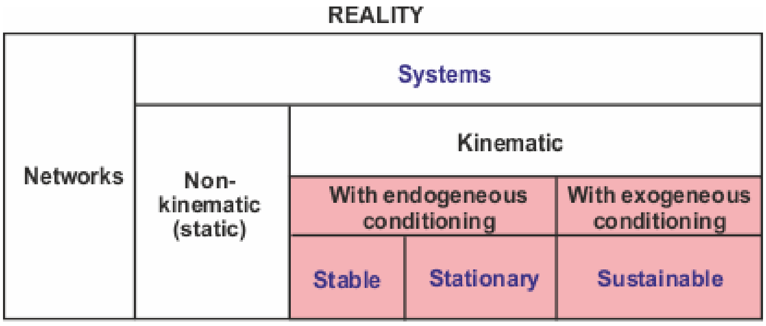

Figure 1. Ontological categories of the reality

1. Economic stationarity

1.1. Preamble

By stationarity[96] of a system we understand the property according to the system's output is a deterministic[97] function of its input, and its state, respectively. Let's describe a system as a set of elements having at least a common property, e_q, with $q = \overline{1,s}$ and let's note also: a) the system's states at the time t with s_p^t, where $p = \overline{1,w}$; b) the system's input (from the system's environment) at the time t with x_i^t, where $i = \overline{1,m}$,; c) the system's output at the time t with y_j^t, where $j = \overline{1,n}$. We shall accept that the system's output can be written as: $Y^t = h(X^t, S^t)$, where $X^t = (x_1^t, x_2^t, \ldots, x_m^t)'$, $Y^t = (y_1^t, y_2^t, \ldots, y_n^t)'$ and $S^t(s_1^t, s_2^t, \ldots, s_w^t)'$. Since any inputs will generate (via S^t) a strictly causal

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96 The concept o stationarity, used here, is larger than the concept of stationarity used in mathematics (or in mathematical physiscs, and in quantum mechanics) where it refers the fact that a solution of a differential equation (that describes a kinematics) is not depending from the variable of time.

97 By determinism we understand o dynamical causality. The dynamical causality is the causality homogeneously recorded for every individual of an observed class of objects, i.e., describeable through the same mathematical function for every individual. By the contrary, we could have a statistical causality, recorded as such only for the class of objects, but not for every individual of the class. It is very polemically if we can "legally" discuss about determinism in the case of the statistical causality.

determined output and since this happens certainly and exactly for any input[98] and any time, we can say we have a stationary system.

By their definitions, the concept of "stationary system" and the concept of "predictable system" are mutually substitutable (they are completely equivalent, because they have the same denotation). But, why a predictable system should be considered as being a stationary system? Our answer is the following: the non-stationarity of a system measures the surprise degree which its dynamics could exhibit for the observer (or analyst, or policy decision maker). Since a predicable system offers a null degree of surprise (any input will generate, in a necessary way, a determined output), then we obtain that a predictable system is equivalent with a stationary one. Here we are far away from the "civil" signification of the concept of stationarity that refers rather the property of a system of maintaining its parameters (of state, and of output, respectively) at quantifiable levels (or relatively constant). In our opinion, such last systems must be named as static not stationary[99]. Consequently, the formal condition for the stationarity of a system is that the analytical expression of the function h be a constant for different moment of times for the time horizon implied (either from a scientific interest, or for a practice interest). Since the system state is modified following the impact of the inputs on the system, and taking into consideration that the system state is equivalent with its structure[100], we expect the analytical expression of the function h be modified over the time. So, we need to discuss about the system stationarity on the time horizon where the analytical expression of the function$h$ remains invariable. In other words, we can also accept a "stairs" stationarity (figure 2).

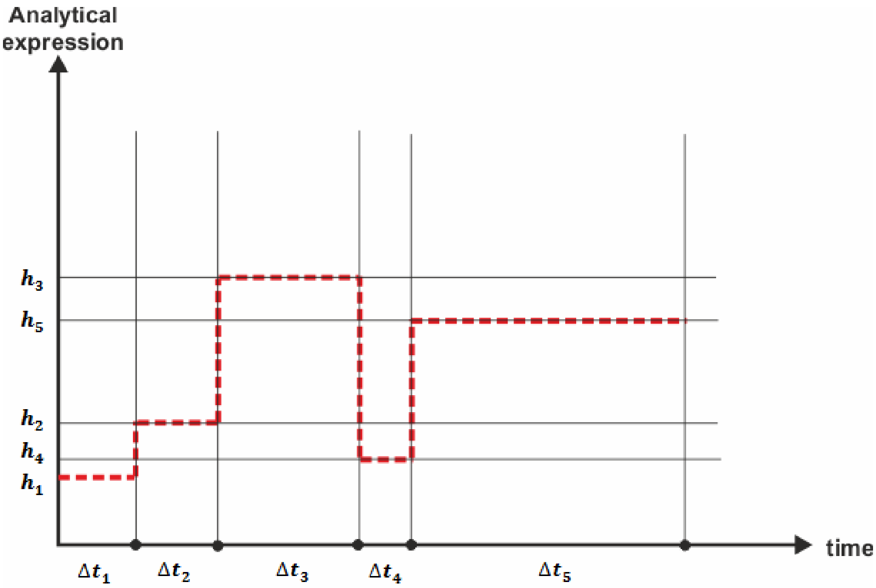

Figure 2. The "stairs" stationarity of a system

1.2. The concept of economic stationarity

We assume an economic system is a system characterized by a state vector, an input vector, and an output vector. In this case, by an stationary economic system we understand a property of such

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[98] As precautionary measure we shall consider, of course, that $x^t \in \mathcal{M}^t(x)$, where with $\mathcal{M}^t(x)$ is noted the set of admissible inputs, at the moment t, for the system in case.

[99] In fact, the distinction between the static systems and the stationary systems is much more radical: the stationary systems are species of the kinematic systems (i.e., with evolution depending from the time), while the static systems are systems that do not suffer changes over time. Although this clarification is of a great methodological support, it is required to still say that today are known only two catgeories of systems verifying this features of the static systems: a) the system of Divinity (the Divinity is considered by definition as being immutable); b) the system of the entire Univers that get its thermic equilibrium (according to the second law of thermodynamics).

[100] The state parameters of a system can be considered as being "allocated", biunivocally, to the elements of thye system, as their properties. Even if this "allocation" could take more complex forms, by violating the mentioned biunivocity, the conclusion that the structure of the system can be altered by the inputs (i.e., can modify the analytical expression of the function h) is still held.

a system so the output vector is a deterministic function from the input vector and the state vector. We can describe an economic system as a set of elements which hold at least a common property, e_q, with $q = \overline{1,s}$. Let's also note: a) the states of the system at the moment t with s_p^t, where $p = \overline{1,w}$; b) inputs into the system at the moment t with x_i^t, where $i = \overline{1,m}$; c) outputs from the system at the moment t with y_j^t, where $j = \overline{1,n}$. We shall accept that the output is a function depending from the input and the state : $Y^t = h(X^t, S^t)$. To be additionally noted that: 1) even if the inputs, and outputs, respectivelly, are vectors, the deterministic causality is conserved, so we do not enter the statistical causality; 2) the equivalence between the "predictable system" and the "stationary system" is held also in the economic system case. Other some mentiones have to be made: a) the states of the real economy refers the "technological recipes" by which the goods and services are produced, on the one hand, and the "normative recipes" which governes the production of goods ande services[101], on the other hand; b) the states of the financial economy refers the financial flows, based on specific technological coefficients of types: prices, wages/salaries, taxes, etc., which help to calculate the level of this coefficients needed by the real economy as counter-part to the real economic flows; c) the states of the nominal economy refers the technological coefficients of types: inflation, interest rates, exchange rates, etc. which help to calculate the money needed by the financial economy as counter-part for the financial economic flows.

1.3. The concept of real economy stationarity

Inside the economic systems, the causality plays through the economic flows, which generate the inputs and the outputs of the considered system. The economic flows are of three categories: real economic flows, financial economic flows, and nominal economic flows. The *real economic flows* refer the goods and services of merchandise type (i.e., pass through the market). The *financial economic flows* refer the monetary flows which act only as counter-parts for the real economic flows. The *nominal economic flows* refer the monetary flows which act only as counter-parts for the financial economic flows. Based on these definitions, we can define the real economy as representing the segment of the economic activity that correlates the real economic flows with the financial economic flows, and the nominal economy as representing the segment of the economic activity that correlates the financial economic flows with the nominal economic flows[102] (figure 3).

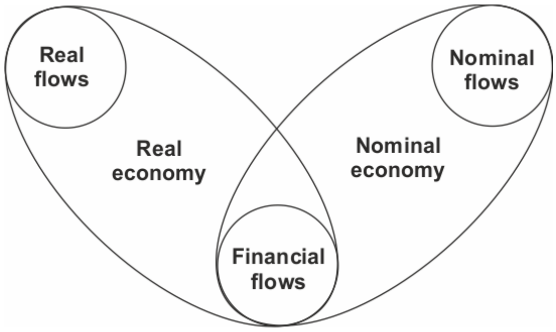

Figura 3. The real and the nominal economy

By stationarity of the real economywe shall understand the stationarity associated to the binominal *real flows – financial flows* in the figure 3. We shall introduce some supplementary notations, for the graph in figure 4 describing the real economy: $x(r)$ is the input into the real economic flows system; $y(r) = h_r(x(r), S_r)$ is the output from the real economic flows system; $x(f)$ is the input into the

[101] Although the norms, in their most general sense, could be considered also as inputs, we would prefer to include them into the concept of state (so, as parameters characterizing the system components) based on the following reasons: 1) they do not enter any economic cycle, but once only, so, for the most economic cycles they are "there" when the proper inputs enter the system, acting as state parameters; 2) they are not "consumed" in the economic cycle of the system in case, but rather act as conditions for the consuming the inputs.

[102] So, the financial economy is only an intermediate term between the real and the nominal economies.

financial economic flows system[103]; $y(f) = h_f(x(f), S_f)$ is the output from the financial economic flows system (Figure 4).

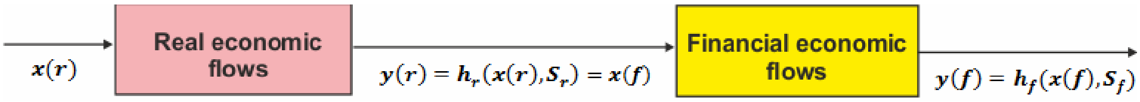

Figure 4. Functioning of the real economy

So, the stationarity of the economy implies the following condition: the analytical expression of the composed function: $y(f) = h_f(x(f), S_f) = h_f((h_r(x(r), S_r), S_f)$ is invariant on the horizon of time of interest (either theoretical, or practical).

1.4. The concept of nominal economic stationarity

By stationarity of the nominal economy we shall understand the stationarity of the binomial *financial flows – nominal flows* in figure 3. Analogously with the real economy case, in figure 5 is described the nominal economy functioning:

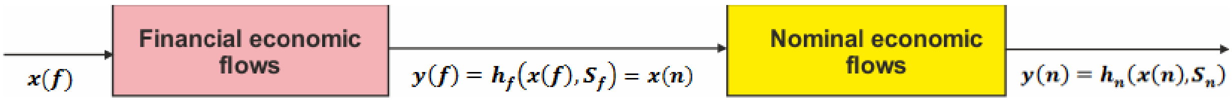

Figura 5. Functioning of the nominal economy

According to the definition above mentioned, we shall say that, in the case of the nominal economy, the stationarity condition is: $y(n) = h_n(x(n), S_n) = h_n((h_f(x(f), S_f), S_n)$ must have an invariant analytical expression on the horizon time of interest (either theoretical, or practical).

1.1.1.5. The concept integrated economic stationarity

By the concept of integrated economy we shall understand the trinomial real flows – financial flows – nominal flows, so the reunion between the real economy and the nominal economy. A graphical image of the concept of integrated economy is shown in figure 6.

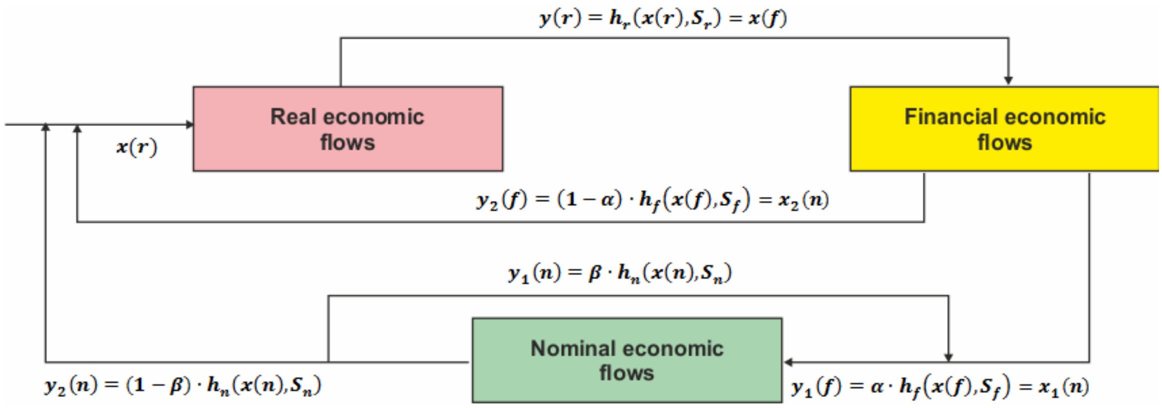

Figure 6. Functioning of the integrated economy

where $\alpha, \beta \in \mathbb{R}$, and $0 \leq \alpha, \beta \leq 1$.

2. The economic stability

2.1. The concept of stability

Based on the concept of stationarity of a system, we can now introduce the concept of stability of the systems. Essentially, by stability we shall understand also an invariance. This invariance is not absolute (of stationary type) but relative (of dynamic type). More exactly, we shall say a system is stable if its outputs are predictable, not in a punctual way (like in the stationarity case) but in an

[103] To be mentioned that, based on the definition of the financial economic flows – as monetary contre-parts to the real economic flows – we have: $y(r) = x(f)$.

interval one[104]. This means the analytical expression of the behavioral function of the system (the outputs as function of inputs and states) maintains its invariance for a given (expected) interval of time. One can now say that the stable systems are predictable not regarding its output per se, but regarding the parameters of the transformation function. It is, somehow, a predictability of second (or indirect) order (see the figure 7).

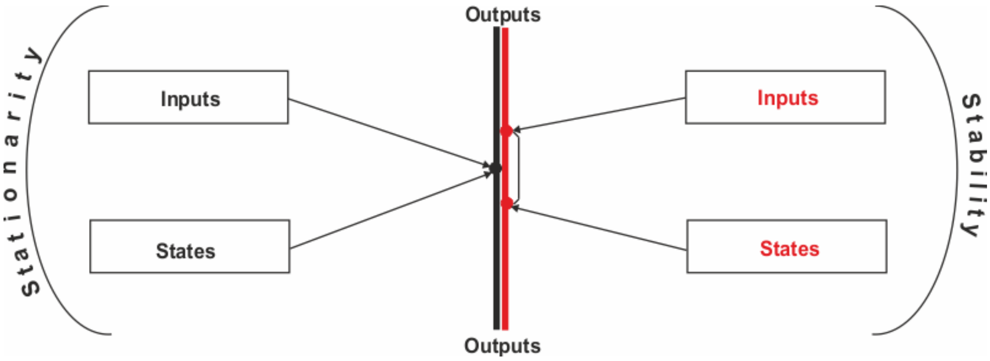

Figure 7. The distinction stationarity - stability

It is obviously that the fundamental issue here is of the variability of the transformation function parameters. How exactly are we ensured regarding the maintaining of this variability inside an acceptable (expected) interval? We shall deliver some considerations:

a. the variation of the transformation function parameters is an objective that emerges in a non-normative way (for example, the relation of complementarity or of substitutability among the production factors could modify itself based on inputs or outputs variation only);

b. the variation of the transformation function is predictable (for example, knowing the analytical expression of the transformation function allows to the observer/analyst to determine quantitatively the possible changes of the coefficients regarding the complementarity, substitutability, marginality, elasticity, etc.);

c. the predictability of the transformation function is not punctual, but of the interval type[105]; if this predictability is presumed to be punctual, then we could acquire the punctual predictability of the output itself, so we could get the stationarity: indeed, if a punctual prediction for the transformation function parameters is possible, then we have simply a new transformation function, with variables and parameters univocally determined, so we have a punctual predictability of the output, so we a have stationarity;

d. if the parameters values of the explanatory variables in the transformation function get out from the established intervals, then the system becomes un-stable.

2.2. The concept of the economic stability

Let's recall the abstract description of an economic system: a set of component elements sharing at least a common property, e_q, with $q = \overline{1,s}$, as: a) the system states at the moment t, s_p^t, where $p = \overline{1,w}$; b) the system inputs (from the system environment), at the moment t, x_i^t, where $i = \overline{1,m}$; c) the system outputs at the moment t, y_j^t, where $j = \overline{1,n}$; d) the transformation function h so $Y^t = h(\varepsilon_k^t, X^t, S^t)$, where: $Y^t = (y_1^t, y_2^t, \dots, y_n^t)'$, $X^t = (x_1^t, x_2^t, \dots, x_m^t)'$, $S^t = (s_1^t, s_2^t, \dots, s_w^t)'$, and ε_k^t stay for the transformation function parameters at the moment t ($k = \overline{1,g}$). So, the input, output, state vector as well as the analytical expression of the transformation function remain invariant on the time horizon of interest, only the numerical values of the transformation function parameters are changing. Moreover, this change allows maintaining of the output inside the established interval (as

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[104] Here an important mention must be made: in the case of stationarity, the output varies, of course, when the input and/or the state vary, but this variation is completely delivered by the invariant analytical expression of the transformation function, so it is predictable. In the case of stability, we have a certain modification of the analytical expression of the transformation function, at the parameters level (so, not at the causal variables level, that remain the same), preserving its analytical invariance, but inside the established interval.

[105] Here a fuzzy approach could be of usefulness. For the moment, we are interested to only postulate this vaqueness, but not to measure it.

the figure 7 indicates). Now, an extremely important mention must be put, namely the variation of the transformation function parameters is non-normative, it emerges simply by the economic system functioning itself. Conceptually, we have an endogenous variation that is self-limitative, generated by the inputs and outputs vectors features, as well as by the analytical expression of the transformation function (the technological "grid").

As result, we can write: $h_{t_1}(\varepsilon_k^{t_1}, X^{t_1}, S^{t_1})$, respectively $h_{t_2}(\varepsilon_k^{t_2}, X^{t_2}, S^{t_2})$. The stability condition could be formalized as: $|\Delta h| = |h_{t_2} - h_{t_1}| \leq \lambda$, with $\lambda > 0$ fixed. So, we have to verify: $|\varepsilon_k^{t_2} - \varepsilon_k^{t_1}| \leq \delta_k$, with $\delta_k > 0$ and $k = \overline{1,g}$ or, as vectors: $\begin{pmatrix} |\varepsilon_1^{t_2} - \varepsilon_1^{t_1}| \\ |\varepsilon_2^{t_2} - \varepsilon_2^{t_1}| \\ \dots \\ |\varepsilon_g^{t_2} - \varepsilon_g^{t_1}| \end{pmatrix} \leq \begin{pmatrix} \delta_1 \\ \delta_2 \\ \dots \\ \delta_g \end{pmatrix}$

Obviously, the political decision maker establishes only the threshold λ, since the thresholds δ_k could be get by calculus, once we have the analytical expression of the transformation function. Must be mentioned, however, this way the numerical sets for the transformation function parameters are not unique for a given value of λ. Nevertheless, the number of such sets, for every value of λ, is finite and, very probable, small, since the analytical expression of the transformation function (presumed reflecting the functional aspects of the explanatory variables[106]) is strongly restrictive.

3. The economic sustainability

3.1. The concept of sustainability

The concept of sustainability passes beyond the existence and functioning of an isolated system. Some considerations are of usefulness: a) about the sustainability we cannot discuss than inside the systems "endowed" with cultural subjects[107]; b) the cultural subjects concomitantly hold three hypostases in the system: 1) cognitive/observational subject; 2) praxiological/actional subject; 3) praxiological object.

In order to clarify the concept of sustainability it is needed to be said that it is not the same with the concept of durability: the durable systems do not contain cultural subjects, although they could contain, of course, subjects. So, although in the common language (even more, in many cases, in the speciality language) the terms durable and sustainable are semantically equivalent, we shall made here the difference between them, from the structural point of view (based on the presence or not of the cultural subjects inside the system in case). More exactly, we consider we face here not a simple terms confusion, but we have properly two different concepts, so two different referents.

We have seen, before, that both the stationary systems and the stable systems are capable of autonomous functioning. By autonomous functioning we understand a functioning that exclusively needs inputs variables, state variables, and transformation function. The state variables must be of two kinds: a) state *functional* variables: the state variables (the technological "grid" based on which the inputs become outputs) of endogenous nature that define the autonomous personality of the system[108], having a persistent, even inertial, feature; b) state *normative* variables: the state variables (teleological "grid" of the system) of, generally, exogenous nature and that have not a persistent feature, being of a non-inertial type. These normative variables give, in fact, the

[106] For example, the variation interval of the complementary or substitutability of the inputs is relatively small. In addition, the structure of the economic system (packed in its state vector) imposes, on its turn, some constraints on the parameters variability inside the transformation function.

[107] The cultural subjects are the subjects capable of conscience (so, of grasping the alterity or, that is the same, of the himself of herself, in mirror with the environment). Or, in other words, the cultural subjects are the subjects capable of representation (the non-cultural subjects are capable of perceptions only). The distinction between the representation and the perception is the following: while the perception needs the perceived object being in actu in front of the subject, the representation needs not the actuality of the represented object.

[108] A system always is only its persistent structure says us it is.

difference between the stationary or stable systems, on the one hand, and the sustainable systems, on the other hand. Shortly, the sustainable systems are those stationary or stable systems that contain channels of receiving normative state variables (see the figure 8).

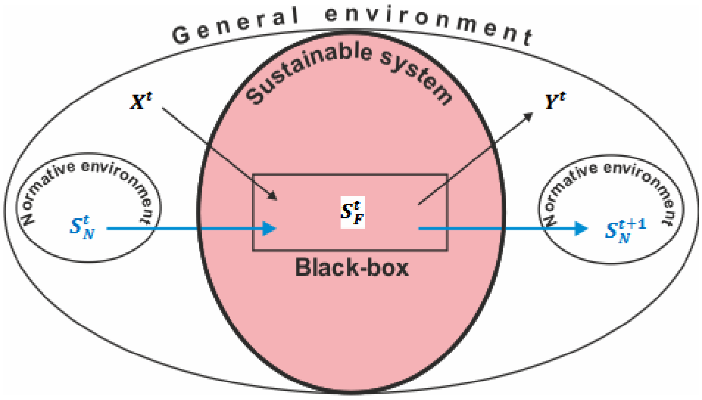

Figure 8. The normative state vector in the sustainable systems

3.2. The concept of economic sustainability

In our opinion, the economic systems cannot be evaluated than as sustainable (or, of course, un-sustainable) systems, so only in the paradigm of sustainability. Of course, this does not mean any economic system is sustainable, but only that any economic system has normative variables coming in from its environment. Maybe a more precise sentence could be here: any economic system is possible to get the sustainability, or that any economic system is a normative system[109]. This means that the normative state variables can or cannot be adequate to provide effectively the sustainable character of the economic system in case: if they are adequate, then the sustainable character is provided, and if they are not adequate, then the sustainable character is failed.

3.3. The relation stationarity – stability – sustainability

Putting together the three concepts discussed above, we can show that the stationarity implies a kinematics towards a point, the stability implies a kinematics inside a one-dimensional interval, and the sustainability implies a kinematics inside a bi-dimensional[110] interval (area), as the figure 9 shows.

[109] In fact, we can say any social system is a normative one.

[110] The expressions “one-dimensional interval” and “bi-dimensional interval”, respectively have not a geometric signification, but an algebric one: in the case of stability, a there is a single degree of liberty (the parameters that change), and in the case of sustainability thare are two degree of liberty (the parameters, and the system state because the normative considerations).

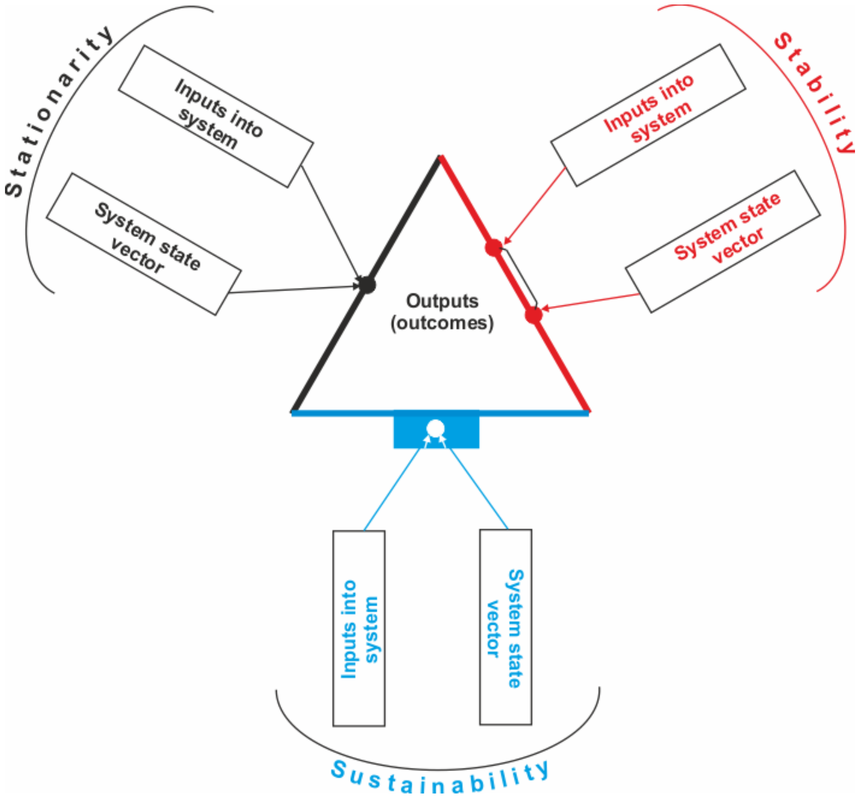

Figure 9. The general distinction among stationarity, stability, and sustainability

A kinematic sinoptic of the general relation among stationarity, stabaility, and sustainability could be captured as in the figure 10.

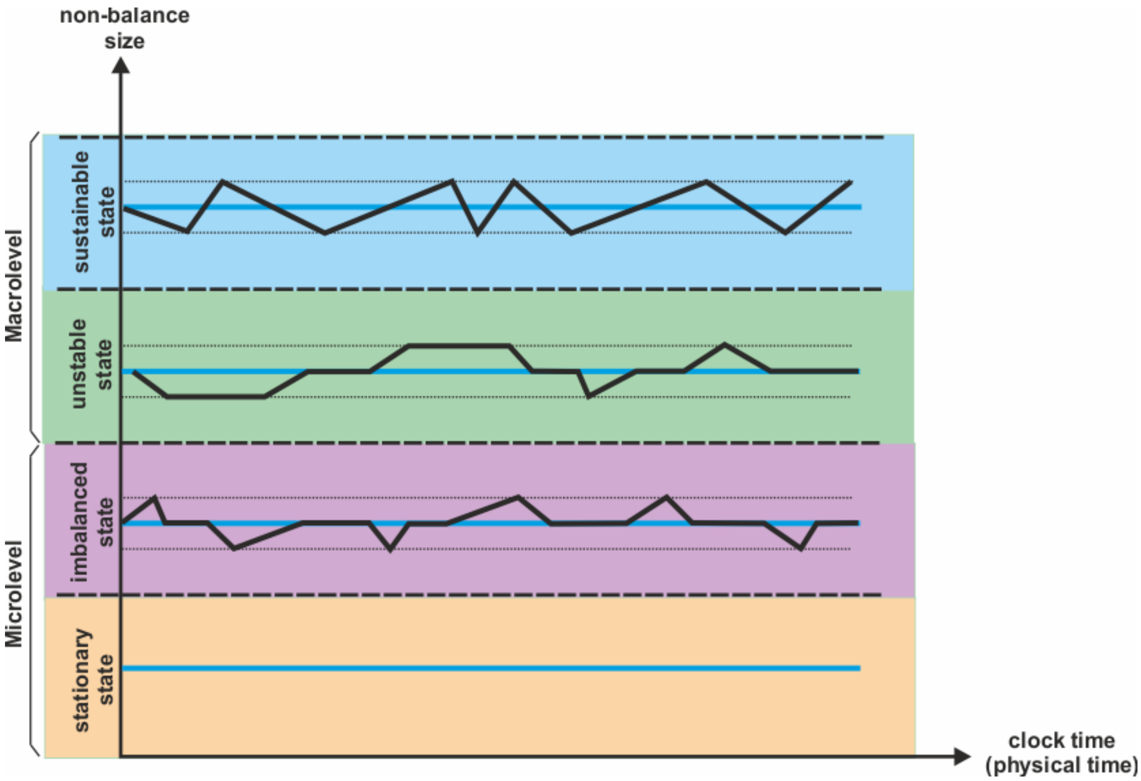

Figure 10. The kinematic distinction among stationarity, stability, and sustainability