6 EXPLORING THE GARCH (1,1) MODEL WITH STUDENT-T INNOVATIONS IN A BAYESIAN FRAMEWORK: A CASE STUDY ON VIX VOLATILITY FORECASTING

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Abstract

The nonlinearity of financial time series is reflected in "stylized facts "such as the leverage effect, volatility clustering, and fat-tailed distributions. In this context, the following paper aims to test a new approach and regime regarding the volatility forecasting process and evaluate its robustness. We employed a Bayesian estimation technique coupled with the GARCH (1,1) model with Student-t innovations. The model was applied to the daily log returns of the Cboe Volatility Index (VIX) spanning 14 years (2011-2024), using both rolling and non-rolling windows. The results revealed that our GARCH model, with a Bayesian approach for parameter estimation, can provide a plausible forecast. Moreover, for the robustness of forecast accuracy, we compare the results of the Bayesian approach with those of frequentist models, both symmetric (GARCH) and asymmetric (EGARCH, APARCH). The DM test reveals that the Bayesian approach generally outperforms the frequentist models both in non-rolling window and in rolling window, except for the APARCH and EGARCH models under the rolling window approach.

Keywords: *Bayesian inference, conditional variance, volatility forecasting, stylized facts, VIX.*

JEL Classification: *B23, G12, G17, C5, C53.*

Highlights:

- Volatility forecasting helps in understanding the impact of uncertainty on asset prices
- A good volatility model must be able to capture the 'stylized facts' about financial markets
- The use of truncated normal priors on GARCH parameters provides realistic posterior values

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- Bayesian estimation of a GARCH (1,1) with Student-t innovations leads to reasonable predictions
- Comparison between the forecasting accuracy of various models under different settings, such as non-rolling and rolling window settings.

1. Introduction

The ability to forecast future volatility within a working model represents a crucial *endeavor for practitioners and theoreticians alike. For a risk manager, options trader, portfolio manager, or market maker, it is essential to understand the likelihood that the market will move in one direction or another in the short run (Engle and Patton, 2007)*. Volatility metrics help acquire insights into how market participants perceive uncertainty regarding economic fundamentals and how this translates into financial asset prices (Drechsler and Yaron, 2011). Hence, measuring volatility becomes necessary for those who adhere to a decrease in uncertainty (Bhowmik and Wang, 2020).

Therefore, as a direct consequence that stems from the necessity of understanding the underlying mechanisms of volatility and the ability to forecast future volatility, combined with sharp increases in stock price volatility stemming from various endogenous and exogenous shocks, have reinvigorated the interest of investors, researchers and stakeholders alike, in various volatility forecasting tools and volatility timing techniques (Chun et al., 2025).

In practice, financial markets dealing with derivatives, such as the options market, can be valuable resources for understanding the connection between the fundamental factors driving asset values, the level of uncertainty associated with these factors, and the resulting impact on asset prices. Drechsler and Yaron (2011) suggest that analyzing the behavior and pricing of derivatives can provide insights into market participants' expectations and perceptions of risk, allowing for a more comprehensive understanding of the relationships between these key elements.

One of the most known indexes that captures the implied volatility of the options and reflects both the uncertainty of the market and a variance risk premium is the VIX (Volatility Index) (Bekaert and Hoerova, 2014). The Chicago Board of Options Exchange (CBOE) introduced VIX in 1993 and was continuously revamped as the industry suggested (Carr and Wu, 2006; Whaley, 2009). Moreover, it is important to understand that, as indicated by Whaley (2009), VIX is a forward-looking indicator, showing the level of volatility that investors expect to see regarding the underlying asset (SPX - S&P 500).

On the other hand, the classical theory regarding the linear behaviour of financial time series is more and more scrutinized by researchers and investors alike. Jiang et al. (2019) noted that econophysics planted the seeds in the research space in the early 1990s and came with the model promise and prediction (Huber and Sornette, 2016), the non-Gaussian features of the financial markets. One of the streams, which is considered the early developments in econophysics, focused on the so-called stylized facts of financial variables (see also Huber and Sornette, 2016), the majority of them identified in the field of finance, but deliberately preferred to treat them marginally as an anomaly (Lux, 2009).

An extensive body of literature describes a set of ‘stylized facts’ of financial asset returns time series. Cont (2001) in his work, identified eleven stylized facts that are common for a wide range of financial assets, out of which, the most relevant regarding volatility forecasting are: 1) volatility clustering, which refers to a process under which large upward or downward movements in volatility are followed by a similar movement in returns. The same movement can be observed for small-amplitude movements of volatility, which are followed by a similar trend in asset returns. 2) Leptokurtic distributions or, more generally, non-normal distributions are associated with the 'leverage effect' under which volatility remains at higher levels after the impact of a negative exogenous or endogenous shock. Nevertheless, the latter is not true when observing positive shocks or surges in the market. This fact can be argued within the work of Whaley (2009), who argues that the general presentation of volatility after negative shocks leads to higher levels of fear and uncertainty among investors. Thus, contributing to the misrepresentation of the VIX itself as being a so-called "investor fear-gauge" and ultimately leading to higher levels of volatility. And lastly, 3) the steady decay of autocorrelation in absolute returns or asymmetry in time scales (Cont, 2001; Hommes, 2002; Malmsten and Teräsvirta, 2010; Iqbal and Shahana, 2019; Sen and Subramaniam, 2019). It can be argued that their presence in this type of data may indicate a short-term informational inefficiency of the market. Thus, the existing literature considers the presence of "stylized facts" of financial time series to be more relevant for high-frequency financial data (Lux, 2009; Cont, 2001; Sewell, 2011). At the same time, expanding on the pioneering work of Cont (2001), several authors have led to argue that the presence and effects stemming from the "stylized facts" of financial assets are exclusive to high-frequency financial data. As Shakeel and Srivastava (2021) note, the presence and effects of "stylized facts" can be considered a salient feature of high-frequency financial data. Hence, suggesting that intraday trading patterns, such as the "U" shaped trading pattern with higher volumes at the opening and closing of a trading session, increase the level of volatility. Thus, high-frequency data exhibits volatility clusters during a single trading session or over a larger time span represented by multiple trading sessions. Arguably, the volatility clustering and the leverage effect "stylized facts" are the most relevant when examining their potential impact on volatility spillover and contagion effects. Nevertheless, the volatility clustering "stylized fact" is the most relevant and significant in this aspect. This point can be further supported by the fact that financial markets are to some extent globally integrated and operate under a global risk-sharing mechanism, which has significant implications for asset pricing in general and asset allocation decisions in particular (Shen, 2018). Thus, it can be argued that volatility clustering episodes, in general, have a short-term horizon, such as a few trading sessions at most. Therefore, the implementation of a model from the GARCH family could provide relevant insight (Fransson and Lafrenz, 2024). Given this complex globalized system under which financial markets operate, it can be implied that the influence that stems from the presence of "stylized facts" can facilitate the flow of excess volatility from one market to another. According to Vuong et. al (2022), the COVID-19 pandemic was illustrative of this development, as the excess volatility that originated within the Chinese stock market spilled over into the US stock market. The results of this contagion, as it occurred on a global scale regardless of geographical setting, are represented by large losses recorded by both individual and institutional investors. Furthermore, the example suggested by Vuong et. al (2022) also showcases the trade relations between the countries mentioned above, and their given level of integration can influence and direct the flow of excess volatility. Thus, given the aforementioned considerations, we argue that during turbulent times, the presence and effects of "stylized facts" can lead to volatility spillover and the contagion effect on a global scale. Especially, among integrated financial markets. Therefore, a proper model of financial assets' returns volatility should be able to capture at least some of these empirical properties. And, equally important, a good volatility model must be able to forecast volatility (Engle and Patton, 2007). Based on the above arguments, we propose a two-fold contribution. First, we explore the Bayesian framework for estimating a GARCH (1,1) model with Student-t Innovations parameters and its applicability for modelling the market log-returns volatility. Second, we performed a comparative assessment of the Bayesian GARCH model's (rolling and no-rolling window) forecasting accuracy against frequentist models, such as GARCH, EGARCH, and APARCH, using the Diebold-Mariano test. We. As such, we have taken into account the work of Chancharat and Valadkhani (2007), who suggest that the presence of structural breaks may lead to spurious results. The issue of structural breaks generally plagues time series that cover large time horizons and is generally a direct result of various endogenous and exogenous shocks, such as episodes of economic and financial crises, sudden policy and regime changes, and other turbulent events such as wars or epidemics. Therefore, to appropriately deal with the issue related to the presence of structural breakpoints, and given the fact that the time series employed covers a large observation period, we have decided to implement the Zivot-Andrews test. Thus, we argue that this particular approach provides a potent tool concerning volatility forecasting while providing additional relevant insight with respect to the presence of structural breakpoints and their impact.

The rest of the paper is organized as follows. Section 2 is dedicated to the literature review of the Bayesian and frequentist GARCH family models. Section 3 covers the methodology and data used. Section 4 presents and comments on the results. Section 5 is dedicated to the findings and conclusions of our study.

2. Literature review

Given the aforementioned considerations, we investigate the existing literature regarding the current level of methodological approaches selected for volatility forecasting. Usually, the Maximum Likelihood (ML) estimation method is involved in implementing frequentist GARCH-type models(Virbickaite et al., 2015). The argument is that this method provides estimates with desirable asymptotic properties (Bollerslev et al., 1994). Nevertheless, the likelihood function may fail to achieve a global maximum. Additionally, a frequentist GARCH-type model estimation requires two types of constraints, i.e., positive conditional variance and covariance stationarity condition (Ardia & Hoogerheide, 2010). However, when the actual parameters are close to the boundary space, the numerical method may fail to converge (Jerrell & Campione, 2001; Ardia, 2008). The Bayesian approach is an alternative to the frequentist estimation that can tackle most of the related issues. Namely, this approach is not affected by the presence of multiple local maxima constraints on parameters that can be incorporated through a proper prior specification, and it allows direct inferences. Moreover, even in the case of long time series, the posterior distribution of the model parameters is frequently following a non-normal distribution. Such a fact leads to the need for a Bayesian approach and an efficient posterior sampling method (Li et al., 2021). The key advantages of the Bayesian approach may be related, inter alia, to the fact that this approach enables small sample results, robust estimation, and model discrimination (Ardia & Hoogerheide,2010). Nakatsuma and Tsurumi (1999) proposed three Bayesian methods for estimating the ARMA-GARCH model, namely: Markov chain Monte Carlo, Laplace approximation, and quadrature formula. Several attempts to create optimal forecasting models can be discussed based on the relevance of volatility forecasting. For instance, Li et. al (2024) develop an extended generalized autoregressive conditional heteroscedasticity (GARCH) with mixing data sampling (MIDAS), and employ this variation of the GARCH model on volatility forecasting for the Shanghai Stock Exchange. The novelty element explored in this approach is linked to the inclusion of the economic policy uncertainty and geopolitical risk uncertainty variables. Thus, the inclusion of such variables could allow for a more accurate volatility forecasting in the context of endogenous and exogenous shocks. Thus, it can be suggested that the GARCH family of models with various configurations and improvements is generally employed with respect to volatility forecasting. Similarly, Qiao et al. (2024) propose a hybrid forecasting framework that combines GARCH-MIDAS with a HAR-DJI-GARCH structure, incorporating both the short-term and long-term components of volatility. While at the same time, it augments the approach with high-frequency jumps in the volatility itself. The empirical findings confirm that accounting for distinct volatility components—especially through wavelet-based decomposition-can enhance the forecast accuracy across various time horizons. It can be considered that these enhancements reflect the continued evolution of the GARCH modeling framework, particularly when augmented with a forward-looking uncertainty measure and high-frequency dynamics.

Nevertheless, given the rise of neural networks and other relevant developments concerning AI and machine learning, new methods and tools are gaining traction. Among these new methods, the GARCH-Informed Neural Network or GINN for short, constructed by Xu et. al (2024), stands out as a hybrid combination between the reliability of a GARCH model and the Long-Short Term Memory workability provided by a deep neural network. Thus, providing a reliable model that is a hybrid methodology for volatility forecasting.

Sahiner (2022) provides a comparative analysis between various specifications, such as the classical GARCH (1,1), the GARCH-M, the EGARCH, TGARCH, and even PGARCH. Additionally, the models specified have been implemented in both rolling window and non-rolling window settings. The results obtained in the comparative analysis suggest that the asymmetric models provide a higher degree of accuracy, yielding lower forecasting errors. At the same time, the choice of a rolling or no-rolling setting did not influence the results significantly, as under this approach, both the EGARCH and PGARCH performed well. It can also be argued that the choice of estimation window may also play a key role in examining the performance of volatility forecasting models. Some models may perform better in a rolling window setting, while others may provide stronger results in a non-rolling window environment. While the classical models, such as the GARCH (1,1) and the remainder of models that pertain to the GARCH family, remain a favored choice, some authors have decided to further augment these approaches. For instance, Liu et al. (2023) propose a Bayesian inference approach to augment the GARCH family of models. As such, the authors test the volatility forecasting abilities of the Bayesian augmented GARCH, EGARCH, HAR-GARCH, and TGARCH models on the S&P 500 data. The study covers an out-of-sample period of 5 years, including the exogenous shocks such as the COVID-19 Pandemic and the 2022 War in Ukraine. At the same time, other authors propose further augmentations or settings for the existing models, such as Wu, Xia & Zhang (2022), who propose the implementation of a two-component realized EGARCH model for joint returns/realized-volatility. While De Khoo et al. (2024) developed GARCH and EGARCH models augmented with a combined weighted volatility measure that also integrates return-based and range-based volatility information. Lastly, the estimation window selection issue and its impact on the forecasting ability of the models are discussed by Feng et al. (2023), who argue in favor of introducing a hybrid approach that dynamically switches between rolling and expanding windows based on recent performance or momentum (Chung et al., 2021) They noticed that the Bayesian method has become a substitute in modelling the datasets in different fields such as psychology or public health (Ogundeji et al., 2021; Wagenmakers et al., 2008). Moreover, the comparison between frequentist and Bayesian models has drawn the attention of researchers. In their paper, Chung et al. (2021) performed a comparison between frequentist and Bayesian GARCH models on six major foreign exchange rates and found that the frequentist models outperformed the Bayesian prediction models.

Another relevant implementation of the Bayesian approach is provided by Sigauke (2016), who applies a varied methodological toolset consisting of Bayesian and frequentist GARCH models for volatility modelling on the Johannesburg Stock Exchange (JSE). The results obtained suggest that the Bayesian approach provides more relevant and more accurate results concerning both conditional and unconditional volatility. While another relevant contribution to the field of volatility forecasting is explored by Majmudar and Banerjee (2004), who investigate the volatility forecasting ability of various models from the GARCH family, in regards to the derivatives market. The results obtained can be considered as a bridge between theoretical considerations and practical applications, by showcasing how refined models may support volatility trading strategies based on the VIX forecasts.

Thus, as a direct consequence of the issue of model and specification selected that can be observed in the existing literature, we have decided to implement a dual approach in regards to volatility forecasting. As such, we have decided to implement a large selection of models consisting of: GARCH (1,1), EGARCH (1,1), APARCH (1,1), and the Bayesian augmented GARCH (1,1). All the selected models employ the Student-t distribution. Lastly, to address the issue concerning the estimation window selected and to also have the possibility of directly comparing the results, we have also decided to implement the aforementioned models in both a non-rolling window setting and in an overlapping rolling window setting. Thus, by employing this particular choice, we can derive relevant insights regarding the forecasting ability of the Bayesian estimation, in various settings and also in direct comparison with its peers.

3. Methodology and data

The literature surrounding the identification of "stylized facts" of financial assets provides a broad and diverse set of tools that can capture and identify most "stylized facts". Nevertheless, concerning the identification of the leverage effect and volatility clustering, two main approaches can be employed to model and forecast volatility. One such approach involves conditional variance directly as a function of observables. The most famous cases are the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models (Engle, 1982; Bollerslev,1986). The second one deals with models of volatility that are not functions purely of observables (see Engle & Patton, 2007, for such a distinction). Here, we focus on GARCH-type models' ability to capture the 'stylized facts' and predict volatility. More precisely, we consider a *GARCH (1,1)* model with *Student-t* innovations. These innovations are designed to capture conditional excess kurtosis (Geweke & Amisano, 2010). For the log-returns {*yt*} of a financial asset, such a model can be described as (Geweke, 1993):

$$y_t = \varepsilon_t \sqrt{\left(\frac{\upsilon-2}{\upsilon}\Big| \quad \Big|\omega_t h_t\right)}, t = 1,2, \ldots . T$$
$$\varepsilon_t \; {}^{iid} N(0,1)$$
$$\omega_t \; {}^{iid} IG\left(\frac{\upsilon}{2}, \frac{\upsilon}{2}\right)(1)$$
$$h_t = \alpha_0 + \alpha_1 y_t^2 + \beta h_{t-1}$$
$$\alpha_0 > 0, \alpha_1, \beta \geq 0, \upsilon > 2 \tag{1}$$

N (0,1) represents the standard normal distribution, and *IG* denotes the inverted gamma distribution. The condition on the degrees of freedom *υ* guarantees a finite conditional variance. While the restrictions on α_0, α_1, and β ensure its positivity.

Let:

$$y = (y_1 \ldots . y_t)', \omega = (\omega_1 \ldots \ldots \omega_t)', \alpha = (\alpha_1 \ldots \ldots \alpha_t)', \psi = (\alpha, \beta, \upsilon),$$
$$ht(\alpha, \beta) = \alpha_0 + \alpha_1 y_t^2 + \beta_{h_{t-1}}(\alpha, \beta), \Sigma = \Sigma(\psi, \omega) = diag\left(\{\omega_t \frac{\upsilon-2}{\upsilon} h_t(\alpha, \beta)\}_{t=1}^{T}\right)$$

With these notations, the likelihood of (ψ, ω), is:

$$L(\psi, \omega y)\mu \tag{2}$$

Applying GARCH models for the probabilistic prediction of stock index returns, Hoogerheide et al. (2012) compare frequentist and Bayesian models' results. The comparison finds that the Bayesian estimation shows superior results in the case of predicting extreme risks. However, no significant differences were found between the models regarding how well they capture the entire distribution of the forecasted variable. Similarly, Xia et al. (2017) introduced a Bayesian method based on the Griddy Gibbs sampler for TGARCH and PGARCH models, and found that it is both effective for estimating parameters and forecasting future volatility. Moreover, we found that Sumalinab & Supe (2018) also compare the performance of the Bayesian estimator with the frequentist model using mean squared error (MSE). Their findings demonstrated that the Bayesian method outperformed the MLE method regarding estimation accuracy. However, as Ardia & Hoogerheide (2010:42) argue: "Moreover, depending on the researcher's prior information, this density can be more or less informative". Therefore, a critical issue here is related to the choice of a proper prior. In more general terms: “High data certainty resulting from high statistical power (i.e., large effect sizes, large sample sizes, low noise) strongly updates the prior… Conversely, the posterior distribution is relatively insensitive to low certainty in either the prior (i.e., noninformative priors) or the data (i.e., low statistical power)” (Lemoine, 2019: 913).

For instance, Ardia & Hoogerheide (2010) involve the truncated normal priors on the GARCH parameters *α, β:*

$$p(\psi, \omega)\mu\phi N_2(\alpha|\mu_\alpha, \Sigma_\alpha)1$$
$$p(\beta)\mu\phi N_1(\beta|\mu_\beta, \Sigma_\beta)1\{\beta \in \beta^2\} \quad (3)$$

Here μ and Σ are the ‘hyper-parameters’, 1{} is the indicator function, and N_d is the *d*-dimensional normal density.

Further, the prior distribution for the degrees of freedom parameter used by Ardia and Hoogerheide (2010) follows the specification of Deschamps (2006). More precisely, in their approach, the distribution is a shifted exponential with parameters $\lambda>0$ and $\delta \geq 2$, such as:

$$p(\upsilon) = \lambda exp[-\lambda(\upsilon - \delta)]1\{\upsilon > \delta\} \quad (4)$$

It is important to note that, for significant values of λ, the mass of the prior is concentrated in the neighborhood of δ. In this case, the degrees of freedom parameter may be subject of a constraint. Assuming a sufficiently large value for δ, the error terms are treated as normally distributed. Finally, the joint prior distribution is formed by assuming prior independence between the parameters (for more details on the Bayesian approach implementation that is followed here, see Ardia & Hoogerheide, 2010). Lastly, in order to compare the forecasting accuracy of the Bayesian approach, we have also decided to implement the GARCH (1,1), EGARCH (1,1), and APARCH (1,1) models, with Student t distribution. Moreover, in order to further investigate their performance in different settings, we have implemented a non-rolling window specification and also an overlapping 1-step ahead rolling window setting. For both the rolling and non-rolling settings, we have decided to employ a training window of 2500 observations, which are approximately equal to a full business cycle. Furthermore, the benchmark metric employed to test the forecasting accuracy of the model comes in the form of an amplitude metric, computed on the VIX data, which is defined by the following relation:

$$\text{Realized volatility proxy} = \frac{High - Low}{Open} \quad (5)$$

Therefore, by implementing this realized volatility proxy as defined by Equation 5, we obtain a reliable benchmark that will be used to directly compare the forecasting accuracy and performance for the GARCH (1,1), EGARCH (1,1), and APARCH (1,1) models.

VIX data

To illustrate the features of a Bayesian estimate for the GARCH (1,1) with *Student-t* innovations, we consider the case of the Cboe Volatility Index (VIX Index). This index is a financial benchmark designed to estimate the expected volatility of the S&P 500 Index. Cboe Options Exchange calculates the VIX Index using standard SPX options and weekly SPX options listed for trading on Cboe Options (see the Cboe Global Markets website for more details: https://www.cboe.com/tradable_products/vix/faqs/). We collect daily data for VIX log returns (based on closing prices) for a period between 2011- 01-04 and 2024-12-31 by using the R package “yfR (Perlin, 2023). The log returns are computed from closing prices. A total of 3521 daily observations are available. We further split the entire dataset into a ‘training set’ with 2500 observations (between 2011- 01-04 and 2020-12-07) over which we will derive the estimates of the model parameters and, respectively, a ‘test set’ with 1021 observations (between 2020-12-08 and 2024-12-31) over which the volatility will be forecasted in a non-rolling window setting. While at the same time, in the case of the rolling window setting, we maintain the same dataset division. The basic statistics for the full sample and ‘training’ and 'test' sets are reported in Table 1. The VIX log-returns display fat-tail effects (with right-skewed and leptokurtic distribution) and several structural breakpoints. Therefore, the volatility modelling should account for such characteristics and address the consequences of the non-normal distribution of the market evolutionary path.

4. Results and comments

Main results

We fit the GARCH (1,1) model with Student-t innovations to the 'training set'. With this aim, we consider μ_α as a *2 X 1* vector of zeros, Σ_α as a *2 X 2* diagonal matrix whose variances are set

to 1000, i.e., a diffuse prior, $\mu_\beta = 0$, Σ_β=1000,i.e., a diffuse prior, λ=0.01and δ=2.

We generate two MCMC chains for 11000 passes each. The starting values of the chains are a vector with the following values: $(0.01, 0.1, 0.7, 20)$.

With this in mind, we further proceed to employ the dual methodology presented above. Thus, we employ the GARCH, EGARCH, and APARCH models in a (1,1) specification with Student-t distributions, which are nested within the `rugarch` R language package developed by Galanos (2024). For the Bayesian approach, we have employed the R language package 'bayesGARCH' (Ardia, 2008; Ardia, 2009; Ardia & Hoogerheide, 2010; Ardia, 2022), which is based on Nakatsuma's (1998) previous work. A Metropolis-Hastings (MH) algorithm with proposal distributions based on auxiliary ARMA models fitted to squared observations. With these settings, Figure 1 displays a trace plot of the MCMC chains.

Hence, in order to compare the forecasting accuracy between the Bayesian implementation of the GARCH (1,1) with Student t distribution and the non-Bayesian GARCH, EGARCH, and APARCH models, we have decided to use the Diebold-Mariano test (Diebold and Mariano, 1995). The DM test has been implemented by using the `forecast` R language package, developed by Hyndman et al. (2025), we further argue that the main advantages of this test lie in its flexibility regarding the use of loss functions, its inherent model-agnostic nature, and its applicability to both nested and non-nested models across different forecast horizons.

Figure 1. Trace plot of the two MCMC chains (in black and red) for the four model parameters generated by the MH algorithm.

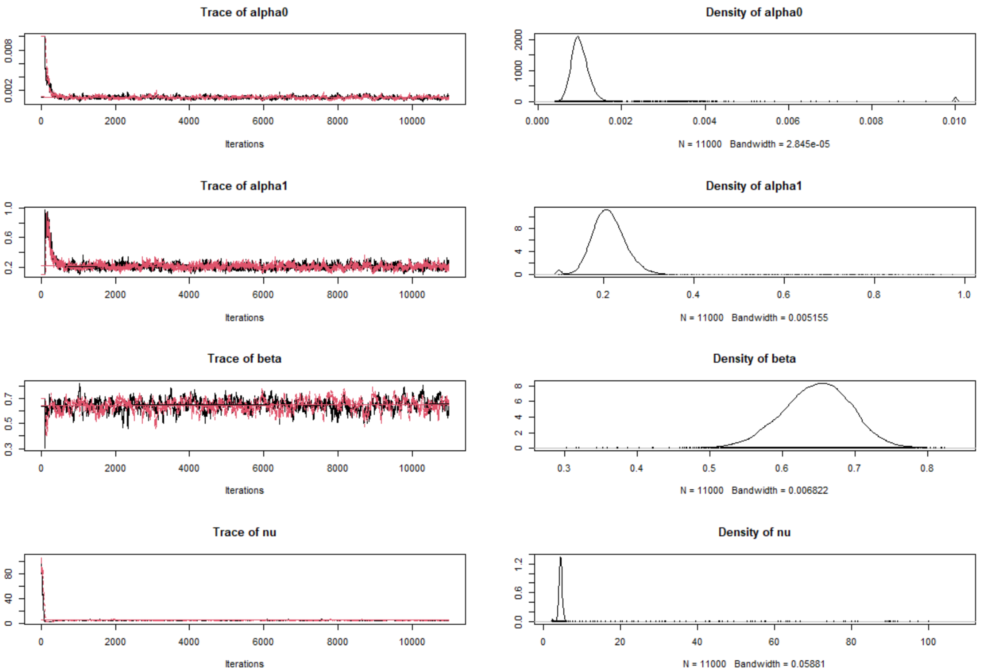


Table 1 reports the posterior statistics of the model parameters.

Table 1. Posterior statistics of the model parameters

Parameter	Mean	Standard deviation	Naive SE	50% quantile	97.5% quantile
α0	**0.0011**	**0.0010**	**0.0000**	**0.0010**	**0.0018**
α1	**0.218**	**0.068**	**0.000**	**0.210**	**0.314**
β	**0.647**	**0.049**	**0.000**	**0.649**	**0.737**
υ	**4.952**	**5.357**	**0.030**	**4.524**	**5.531**

The convergence of the sampler (using the diagnostic test of Gelman & Rubin (1992)) is reported in Table 2.

Table 2. The diagnostic test of Gelman & Rubin (1992) (based on the second half of the chain)

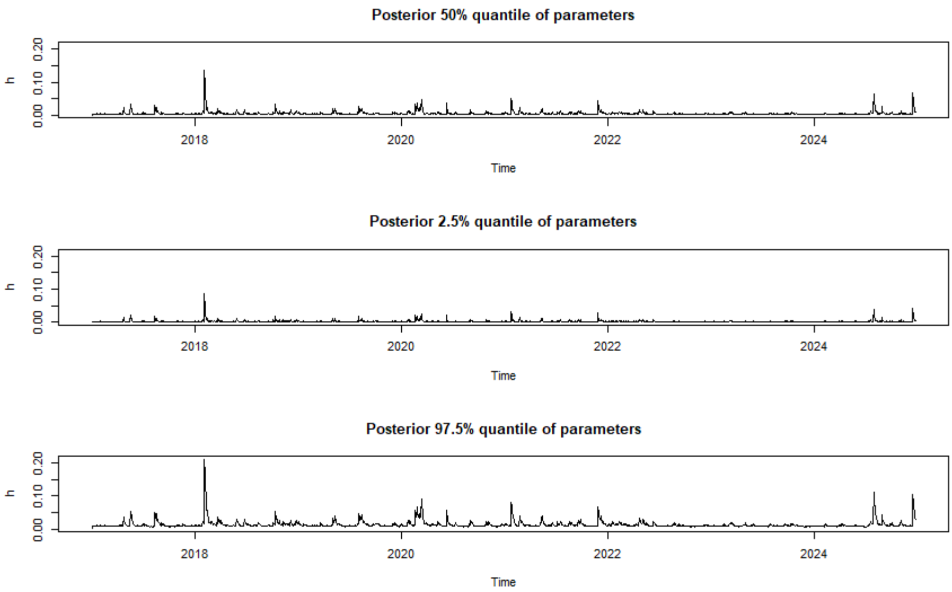
Parameter	Point estimation	Upper confidence interval (97.5% quantile)

α0	**1.000**	**1.010**
α1	**1.000**	**1.000**
β	**1.000**	**1.010**
υ	**1.010**	**1.010**

As long as the scale reduction factor remains below 1.2, the convergence diagnostic shows no evidence against convergence. Meanwhile, the MCMC algorithm yields high acceptance rates ranging from 92% for vector α to 96% for β. This outcome suggests that the proposal distributions are close to the full conditionals. Finally, the one-lag autocorrelations in the chains range from 0.862 for parameter α_1 to 0.985 for parameter α_0. Reassured by these diagnoses, we further forecast the conditional variance of VIX log-returns, h_t , for the 'test set'.

Reassured by these diagnoses, we further forecast the conditional variance of VIX log-returns, h_t, for the 'test set'. The results are reported in Figure 2.

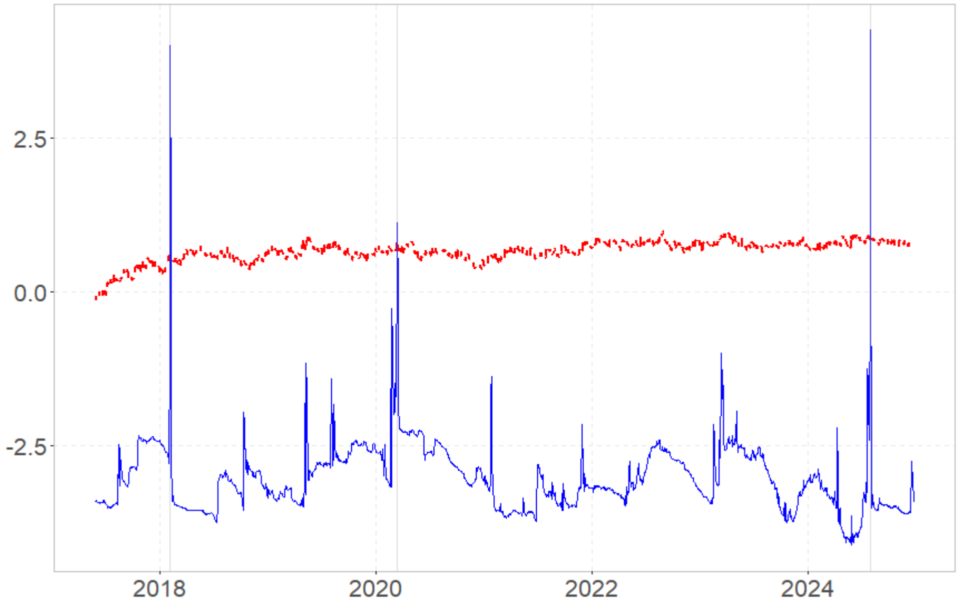
Figure 2. The conditional variance of VIX log-returns for the 'test set' (based on various posterior quantiles of parameters as estimated over the 'training set')



Interestingly, the Phillips et al. (2015a, b) *Generalized Supremum ADF* (GSADF) test can detect multiple changes in the forecasted conditional variance regime. More precisely, this test has the null hypothesis of a unit root versus the alternative hypothesis of the presence of 'exuberance' in different subsamples. We follow the implementation of this test from the R language package 'exuber' (Vasilopoulos et al., 2022; Vasilopoulos, 2023). The Phillips & Shi (2020) wild bootstrap re-sampling scheme, which is asymptotically robust to non-stationary volatility, generates the critical values for the recursive unit root tests.

Figure 3 displays the results of this test for the estimates based on the posterior 97.5% quantile of parameters derived over the 'training set'.

Figure 3. Date stamping with the GSADF test for the volatility forecast



Additionally, we have compared the forecasting accuracy between the Bayesian implementation of the GARCH (1,1) model with a Student t distribution and the regular GARCH (1,1), EGARCH (1,1), and APARCH (1,1) models that also follow a Student t distribution. This comparison was conducted using the Diebold-Mariano (DM) test. The results obtained are presented for both the non-rolling and rolling window specifications that have been employed. The results obtained for the non-rolling window setting are reported in **Table A2.**

The results obtained after the implementation of the models in a non-rolling setting and after the employment of the DM test indicate that the Bayesian specification outperforms the GARCH, EGARCH, and APARCH models. Especially when considering the lower forecasting errors computed and obtained with the help of the volatility or amplitude proxy, which was previously defined. Therefore, the results confirm the accuracy of the Bayesian specification of the GARCH (1,1) model with Student-t distribution, which is a more reliable and accurate choice regarding to volatility forecasting in a non-rolling window setting. We also argue that the key advantages of the Bayesian specifications, in a non-rolling window setting over its peers, lie in the flexibility of the model, the stability, which is also influenced by the large data set, and lastly, by the implementation of various priors that prevent the negative impact that may arise from overfitting. This result can also be observed in the graphical representations shown in **Figure A1,** where the forecasts for each model, alongside the volatility proxy employed, are plotted.

As can be seen in **Figure A1,** from a visual point of view, the models may seem to perform similarly. Nevertheless, we argue that the Bayesian estimation is less prone to excessively "jumping" at short-term disruptions or volatility spillovers, thus providing investors and researchers with a more accurate gauge of the volatility. Additionally, as per the DM test, the forecasting errors are smaller than the forecasting errors of its peers. We also argue that the only non-Bayesian specification that shows a given level of potential comes in the form of the Asymmetric GARCH or APARCH. Which can be considered slightly more accurate and less sensitive than the EGARCH and GARCH models. With this in mind, we will further proceed to augment and shift our methodology to the overlapping 1-step ahead rolling window approach.

The results obtained after implementing the volatility forecasting models in the overlapping 1-step ahead rolling window approach stand in stark contrast with the results obtained for the DM test in the non-rolling window approach. While in the case of the non-rolling window setting, the Bayesian estimation of the GARCH model provided more accurate forecasting results than its peers, the same cannot be said in the case of the 1-step ahead rolling window setting. The results obtained for the DM test can be seen in **Table A3.** As can be observed, the Bayesian specification manages to outperform only the GARCH model, while in the case of asymmetric variants of the GARCH, namely the APARCH and EGARCH, we observe that the models are more or less equal in forecasting results and accuracy of the Bayesian estimation. It can also be argued that, as was previously observed in the non-rolling window setting, the APARCH model tends to offer a better performance than the regular GARCH and the EGARCH models, ultimately being on par with the Bayesian specification, if not better. We also argue that the results are heavily influenced by several factors that are related to the intrinsic characteristics of the models employed. To this end, we consider that in the case of the simple GARCH estimation, the rigidity of the model itself and the lack of an asymmetrical specification tend to hamper the forecasting ability of this particular model. In the case of the EGARCH and APARCH specifications, their asymmetrical characteristics, combined with their general ability to re-estimate optimally after each rolling step, can ensure a higher forecasting accuracy. Hence, in the case of the Bayesian specification of the GARCH model, we argue that the primary characteristic of the model, such as the recursive forecasting chain, may negatively influence the forecasting accuracy in this particular rolling window setting.

Thus, given the impact of the aforementioned characteristics of the models, it can be argued that certain issues may arise when employing a one-step ahead rolling window setting. Nevertheless, before further discussing these elements, we also argue that the results obtained can be visualized in a graphical approach in **Figure A2,** in which we can observe the forecasts generated for each model, alongside the volatility proxy employed.

We further argue that the rather lackluster performance of the Bayesian estimation approach, in the rolling window setting, can be attributed to several key characteristics and limitations of the model. First and foremost, the 1-step ahead rolling window forces the models to re-estimate after each forecasted step; thus, in the case of the Bayesian estimation, this reduced sample of 1 step or 1 observation leads to instability and a slow estimation that can also lead to convergence issues for each window. Moreover, the recursive forecasting chain, which is unique to the Bayesian analysis, can become noisy, given unstable posterior parameters. In contrast, the EGARCH and APARCH models tend to perform better in this scenario. This performance is largely owed to their faster adaptations to sudden changes and also to re-estimations that are conducted in short-term horizons, such as the 1-step ahead forecast employed. Thus, we argue that their specific structure may allow the models to "learn" and derive relatively accurate forecasts based on the incorporation of recent data. Furthermore, the general weakness of the regular GARCH (1,1) model can be attributed to the general rigidity of the model, which assumes a symmetric response, while at the same time, failing to incorporate and model the leverage effects or heavy tails that can be observed in financial time series, as argued by Cont (2003). Thus, owing to the aforementioned characteristics of the GARCH (1,1) estimation, its employment in the 1-step ahead rolling window setting provides poor results. Lastly, concerning the general implementation of a Bayesian estimation within a rolling window setting, we also argue that another downside is represented by the high computational power and by its time-consuming nature. While at the same time, it provides mixed results and an improper implementation of the Bayesian setting, which is largely owed to the unique characteristics of this model. Thus, we argue that overall, the results obtained confirm the ability of the Bayesian estimation to provide accurate volatility forecasting estimates. This is especially true in the non-rolling window setting, in which the Bayesian estimation manages to outperform the GARCH, APARCH, and EGARH models. While in the 1-step ahead rolling window setting, the Bayesian estimation manages to provide reasonable and accurate results that are on par with the forecasting results provided by the EGARCH and APARCH models, while at the same time, outperform the simple GARCH approach.

5. Comments

The Bayesian estimation of a *GARCH (1,1)* model with *Student-t innovations* can reasonably predict the volatility of VIX market log-returns. However, is this finding plausible? Several comments can be highlighted here.

First, the involvement of the truncated normal priors on the GARCH parameters provides realistic posterior values of the parameters and does not raise convergence issues.

Second, it appears that the spikes in the forecasted variance can be associated with some identifiable events leading to the emergence of endogenous or exogenous shocks in the market.

For example, the model predicted the strong volatility episode from February 2018, the key event known as “Volmageddon,” generated by endogenous factors such as investors' overreaction to possible interest rate increases. Further, the model signals well exogenous impacts such as the pandemic crisis since March 2020 or the Russian invasion of Ukraine. More recently, the model predicted the end of July and beginning of August 2024 volatility spike as a result of exogenous and endogenous influences. The main driver of the movement was the FED’s decision regarding interest rates. Notably, the GSADF test is early signaling a potential spike in volatility, linked with the FED decisions starting on the 30th of July, while the central bank communication that was generated on the 31st of July, when the VIX had a spike in real life.

Third, the volatility spikes are also identified by the formal application of the GSADF test. Nevertheless, the length of the three boost episodes identified by this test is short (1-2 days). The first episode occurred in February 2018, while the second was in mid-March 2020.

Fourth, it can be argued that the Bayesian *GARCH (1,1)* model with *Student-t innovations*, while a simplistic framework it is still able to capture a significant portion of the tail risk present within a particular financial market, due to the heavy-tailed shape of the distribution used. This feature makes it valuable in contexts where modelling extreme shocks is essential for risk assessment. However, even if more flexible GARCH models are used, alternative approaches to identifying market volatility, such as stochastic volatility models, might provide better results (Chan & Grant, 2016). Still, our aim here was instead a limited one. We intended to show that even a GARCH model can provide a plausible forecast when a Bayesian approach for estimating its parameters is involved. However, to capture other stylized facts of financial volatility, such as leverage effect and volatility clustering, this approach can be augmented by the use of asymmetric frequentist models, such as EGARCH and APARCH. For example, the EGARCH model does not impose a sign restriction on its coefficients, which gives it the flexibility to capture both the leverage effect and the volatility clustering “stylized facts” (Sahiner, 2022). On the other hand, the APARCH model has a superior flexibility by generalizing more models and by introducing a power parameter δ, which determines how shocks are transformed (Ding et al., 1993) thus capturing asymmetry and other complex nonlinearities.

At the same time, we consider that as a future research direction, the investigation of the underlying mechanisms that drive market volatility, coupled with a more detailed explanation of such elements, should be explored, along with an ex-post assessment of the forecasting outcomes. Especially when considering that the simple identification of the endogenous or exogenous shocks that impact market volatility is insufficient; the transmission channels through which these shocks are transmitted should also be identified, and their evolving characteristics should be further explored and described in detail. Hence, this potential expansion of the current research topic could provide valuable insights to investors, policymakers, and the general interested public alike, by providing additional insights regarding the transmission mechanism and processes that are especially relevant in a globalized setting in which the process of market integration can be present at a certain level.

Even though the model looks to be a simplistic framework, it identifies the major volatility episodes and, in our opinion, can have some important implications. Firstly, at the level of financial market stability and risk management, the model can improve risk assessment by monitoring and early identification of exogenous and endogenous shocks that can spill over through financial markets. Additionally, by creating reliable early warning signals, investors can adjust their risk strategy based on how volatility reacts to structural breaks or shocks. Finally, the financial system can utilize the findings to enhance its stress testing models, taking into account the Bayesian approach in the context of the non-linear features of financial market data.

Secondly, our study can help policy stakeholders be more effective in the regulation framework (e.g., central banks make more informed decisions based on the identified linkage between volatility patterns and macroeconomic shocks).

Thirdly, our findings can help traders improve their volatility-based strategies, enhance algorithmic trading models, and more accurately price derivatives and other financial instruments.

Furthermore, considering the potential policy implications and observations mentioned above, while at the same time acknowledging the current highly uncertain and rather volatile trading environment, we suggest further research directions for the scope of volatility forecasting. Thus, we consider that a potential research direction for our methodology could be an expansion of the data employed in order to cover the highly uncertain and volatile period of the March-May 2025 period. The period in question is characterized by a high degree of uncertainty stemming from various geopolitical events and also from various economic and policy decisions with a globalized impact. Among them, we note the trade tariffs imposed by the current Trump administration on the main trading partners of the US, ultimately resulting in a full-fledged trade war with China. While at the same time, various geopolitical developments were ongoing, thus resulting in a highly uncertain and volatile trade environment, under which a volatility forecasting tool would provide additional insights for investors and policymakers alike.

6. Conclusions

In conclusion, the article addressed the problem of estimating and forecasting the volatility of the VIX index for the period 2011-2024, using a Bayesian GARCH (1,1) model with Student-t distribution. In our analysis, we have compared the performance of this model in both fixed window and rolling window configurations, concerning the realized volatility. The reference models used for the comparison were the frequentist models, both symmetric (GARCH) and asymmetric (EGARCH and APARCH). The results obtained were mixed; in the fixed window estimation, the Bayesian model recorded significantly lower errors than all frequentist models, highlighting the superiority of the Bayesian approach in this context. This result is in line with recent literature, which emphasizes the ability of the Bayesian model to capture extreme risks and tail-risk behaviors efficiently. Moreover, the use of Student-t innovations contributed to modeling heavy-tailed behaviour while the truncated normal priors ensured realistic posterior estimates and robust convergence. This aspect is also supported by the GSADF test, which revealed periods of explosive volatility behavior corresponding to known episodes of financial instability. In contrast, in the estimation based on rolling windows, the performance of the Bayesian model was outperformed only compared to the standard GARCH (1,1) model. The EGARCH (1,1) and APARCH (1,1) models generated smaller errors than the Bayesian model in this variant. This outcome highlights an important limitation: while the Bayesian approach provides stability, robustness, and superior results under the fixed window setting, it may lack structural flexibility needed to capture asymmetric volatilities and volatility clustering in the rolling window configuration. In particular, EGARCH benefits from its ability to accommodate negative coefficients, thus capturing the leverage effect, while APARCH adds further flexibility by introducing a power parameter *δ,* allowing for a broader modelling of stylized facts. Another important aspect to mention is the high computational cost of the Bayesian model in the rolling window scenario. The constant re-estimation of the parameters by MCMC methods makes this approach very time-consuming over long periods of analysis, as in our case (14 years of daily data).

Hence, the results obtained suggest that despite its simplicity, the Bayesian GARCH (1,1) model proved capable of identifying key volatility events and providing a plausible forecast under structural shifts. Moreover, we consider that our findings have some practical implications, such as: for risk management, the model can support early detection of volatility spikes, improving the monitoring of systemic risk. For policymakers, it offers useful signals for anticipating the impact of macroeconomic decisions (e.g., monetary policy changes) on market volatility. For traders and financial institutions, it provides a solid foundation for volatility-sensitive strategies, derivatives pricing, and stress testing frameworks.

Looking forward, one promising future research direction is to combine the Bayesian estimation framework with more flexible structures in a hybrid approach that preserves the robustness of Bayesian inference while capturing other dynamics of the volatility process.

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