9. Role of Realized Skewness and Kurtosis in Predicting Volatility

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Abstract

This study employs tick-by-tick data to estimate realized volatility (RV), realized skewness (RSK) and realized kurtosis (RKU) measures of 452 individual firms listed at Pakistan Stock Exchange (PSX). Using standard HAR model and its extensions, role of realized skewness and realized kurtosis is examined for predicting realized volatility. Both in-sample and out-of-sample forecasts strongly support the predictability power of realized kurtosis in one, five and 22 days ahead forecasts of realized volatility. This research provides a comprehensive empirical evaluation to guide practitioners and applied researchers discerning the selection of variables, lag criteria and measurement models to acquire reliable volatility predictions.

Keywords: HAR model; Emerging market; Predictability power; Realized kurtosis; Realized volatility

JEL classification: G10; G12; G17; O16

1. Introduction

Measuring volatility and understanding its dynamics play a crucial role in dealing with many fundamental issues in the field of finance. As a basic gauge of risk in modern financial practices, volatility is an underlying factor while constructing optimal portfolios, in pricing options and other derivative instruments or determining the exposure of a firm to various risks and its expectation to earn for compensating from those risk exposures. It is also critical in finding new trading and investment opportunities that may offer appealing risk return trade off (Bollerslev *et al*., 2016; Bollerslev *et al.,* 2018). It is well accepted that even though daily and monthly stock market returns are generally not predictable but their volatility is a highly predictable phenomena having meaningful implications for finance economists and risk managers (Bekaert and Wu, 2000). Of course, volatility has an inherent property that it cannot be observed directly and much of its characteristics are known through fitted parametric econometric techniques like generalized autoregressive conditional heteroscedasticity (GARCH). GARCH like models and its modifications are still popular among academicians and practitioners (e.g., Klein and Walther 2016). Financial theorists offer numerous explanations to negatively price the risk related to changes in market volatility. Chen (2002) documents that excessive volatility present deteriorating investment opportunities therefore investors want to hedge against such fluctuations. Risk averse traders desire securities for hedging against this risk. Time intervals of higher volatility are generally related with downside market fluctuations. Securities having higher sensitivities to market volatility risk supply hedge against market downward risk. The high desirability of stocks having higher systematic volatility loadings leads to increase in their price and decrease in their mean return. Finally, assets doing badly during periods of high volatility tend to have negative

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skewness in returns across intermediate horizons, however assets doing well during periods of increasing volatility tend to have positive skewness in returns. According to Harvey and Siddique (2000) if agents prefer co-skewness, assets having higher sensitivities to innovations in market volatility are desirable and therefore have lower returns (Ang, Chen, *et al.,* 2006).

Though, there are studies where the topic of modelling volatility has been discussed mostly for indices level data, such as Lyócsa and Todorova (2020) who study the potentiality of market volatility of trading and non-trading periods in predicting the individual stock volatility and find significant improvement in forecast accuracy attributed to the sensitivity of the stocks to market risk component. Recently, Wilms *et al.* (2021) analyze the impacts of volatility components on stock market volatility of major international indices. Fei *et al.* (2019) consider the information content of cross-sectional return dispersion in forecasting the volatility of Chinese equity market. Kumari and Mahakud (2016) examine the role of investor sentiment to determine the Indian stock market volatility and Mei *et al.* (2017) check if realized skewness and kurtosis are helpful in enhancing volatility forecast accuracy. In fact, Mei *et al.* (2017)'s study is the first one to empirically test the predictive ability of realized skewness and kurtosis. Building on their study, Gkillas *et al.* (2019) document the role of realized skewness and kurtosis in forecasting volatility of major currencies. Extending this line of research, Bonato *et al.* (2021) apply this concept to international REITs markets. There is no study in the existing literature that checks the role of realized skewness and kurtosis to predict volatility of individual firms. This research brings a major contribution to the existent literature by evaluating the role of realized skewness and kurtosis to forecast the volatility of individual stocks listed at PSX.

This study is similar to Mei *et al.* (2017) who utilize realized skewness and kurtosis as extensions to HAR-RV model to predict realized volatility for SSEC and S & P 500 indices. Though this research differs in several aspects: (i) this study works on individual stock level data of firms listed at PSX; (ii) examines the degree to which realized skewness and kurtosis enhance the forecasting ability of benchmark HAR-RV model consisting of of the realized volatility constituents; (iii) finds strong support for the forecasting power of realized kurtosis but could not find any significant proof for predictive ability of realized skewness. A fresh and important finding of this research is the ability of realized kurtosis to capture additional information related to a large number of predictors and this ability may help investors to attach weight to a risky asset and to switch between a risky asset and a risk-free asset which may lead to superior gains. In addition, investors may time the market in a better way.

The remaining paper is framed in the following way. The second section comprises of literature review, third section describes data and methodology, fourth section analyzes findings and fifth section documents conclusions.

2. Literature Review

The importance of characterizing the magnitude and patterns in time series variance of volatility to determine the suitable stylized facts that could be useful in evaluating the asset pricing models, cannot be ignored. Paye (2012) discover the countercyclical behavior of stock return volatility. He explains the positive skewness and leptokurtosis of aggregate stock return volatility as a partial outcome of numerous extreme episodes of stock return volatility that also includes October 1987 market crash and the histrionic plunge in stock prices attributable to 2008 financial crisis. He uses linear approach to predict volatility taking ordinary least squares (OLS). OLS technique may be inferior to nonlinear measures if the regression errors are not normal and have fat tails. However, an approximately Gaussian sample can be generated if the natural log of realized volatility is taken as documented in Andersen *et al.* (2001). Higher stock return volatility is observed through recessions as compared to expansions.

There is strong evidence in favor of predictable volatility, observed through short run persistent patterns and long run mean reversion processes (e.g., Ahmed *et al.,* 2018; Auer, 2016). This predictable constituent should be important for investors while taking portfolio selection decisions as investors should consider suitable time for investment decisions by increasing allocation during periods of higher expected returns and lower volatility (Johannes *et al.,* 2014). The vast literature in the field of finance implies the time varying nature of stock market return volatility (Ang, Hodrick, *et al.* 2006). The time variation property of stock return volatility brings about changes in investment opportunities by changing the expectancy of future stock return or by modifying the risk return trade off. If the stock market return volatility is considered as a systematic risk-component then according to arbitrage pricing theory, there should be a price for the aggregate volatility factor in the cross section of stock returns. Thus, stocks facing different market volatility concoctions should demand different expectations in returns (Ang, Hodrick, *et al.,* 2006).

2.1. Potential Rationale for Temporal Changes in Stock Return Volatility

Theoretically, the conditional variance of asset return is dependent on the conditional variance of expected cash flows, the conditional variance of discount rate and conditional covariance of both these variables. If the discount rate is unchanged, the conditional variance of total market return is dependent only on conditional variance of total expected cash flows. This demonstrates a channel for time varying countercyclical return variance. Absolutely, such properties are displayed by shocks to fundamentals. Bansal and Yaron (2004) develop an approach in this regard that hypothesize that the volatility related to dividend and consumption progression is countercyclical and is responsible for subsequent countercyclical stock market movements.

Mele (2007) asserts that besides fundamentals (dividends) channel, countercyclical movements in return variance can also be generated by time varying discount rate. This mechanism operates only if future returns are convex in a state variable that realizes business circumstances. Intuitively, in recession periods future returns must show sensitiveness to movements in the state variable, however in expansion periods, future returns show less sensitiveness to such movements. At times of strong asymmetries, Mele (2007) suggest that the price/dividend ratio is rising concave function of the state variable resulting in subsequent increasing downward return volatility.

The third clarification for time variations in stock market volatility incorporates the role of information. In such type of approaches, agents learn through public signals for analyzing upcoming events in the economy. Timmermann (1993) considers a context within which security price is equal to total future expected dividends having exogenous signified discount rate and demonstrates that increasing return volatility can be an outcome of learning effects in relation to a benchmark context where growth in dividend could be observed. This observation is further extended by researchers to a stochastic equilibrium model accounting for rational learning. For example, in asset pricing, Veronesi (1999) elaborates a dynamic general equilibrium model with rational expectations with the major assumption that the dividend growth rate moves in a random manner between two states which cannot be observed directly. During equilibrium state, the desire of investors for hedging against variations in their uncertainty levels cause security prices having more sensitiveness to unfavorable news in favorable conditions in relation to favorable news in unfavorable conditions. Accordingly, stock return volatility acts counter cyclically.

Finally, the financial crisis of 2007-2008 spur academic researchers to examine the ability of financial intermediation in amplifying shocks to security markets. Brunnermeier and Pedersen (2009) examine the interconnection of funding liquidity of traders with stock market liquidity. Within the context, balance sheet effects of a borrowing firm may result in amplifying comparatively smaller primary shocks in a loss spiral and bolstering margins spiral. For leveraged firms, decrease in security prices erodes net worth speedier than gross worth. These investors may decide on selling assets to keep up with leverage resulting in further drop in asset prices. The explanation of the term margin spiral is that margins tend to rise when there are extreme declines in prices, exacerbating the stress on leveraged investors for selling off assets, resulting in further decline of prices and rise in margins and so forth. Brunnermeier and Pedersen (2009) predict ensuing of a vicious cycle with existence of multiple equilibriums. Moreover, capital limitation of lenders, network effect and bank run could lead to amplifying shock in asset markets.

2.2. Realized Volatility

Merton (1980) noticed that increasing sampling frequency leads to an accurate way of measuring volatility arbitrarily. Later research on realized volatility apply his discernment for measuring time varying volatility by constructing daily measures of realized volatility calculated using intraday squared returns. Technically realized volatility is defined as the standard deviation of an asset log return within a defined time interval by assuming zero mean, no degrees of freedom and a fixed annualized factor of 252 days in spite of the actual number of trading days over the year. Although the implied volatility is related to the market's evaluation of future fluctuations, the realized volatility is a measure of what actually occurred in the past (Andersen and Bollerslev, 1998). Compared to the parametric techniques such as GARCH type models that utilize data of daily or monthly frequency, the realized volatility approach generates model free unbiased measures of ex-post return variation under specific provisions (Barndorff-Nielsen and Shephard, 2002). Bailey and Steeley (2019) conclude that squared returns perform poorly for volatility forecasts and that realized volatility is a good proxy for forecast evaluation. Based on the now long-standing conception of realized volatility, Amaya *et al.* (2015) computed realized skewness and kurtosis using intraday cubed and quartic returns and showed that relying on continuous time specificity of stock price dynamics that accounts for stochastic component and jumps, the realized moments convene to true moments validating that Merton's discernment also relates to higher moments. Mei *et al.* (2017) provide evidence that realized skewness and kurtosis can predict realized volatility of Shanghai Stock Exchange Composite Index (SSEC) and S & P 500 index.

2.3. Excess Kurtosis and Stock Returns

The presence of leptokurtosis in time series data is evidenced by Lux and Marchesi (2000). They also evidence that volatility clustering is positively related to the fourth moment i.e., kurtosis. A similar study on financial time series by Tseng and Li (2012) tries to detect the existence of volatility clustering and discovers that higher volatility clustering leads to greater kurtosis risk. During the last decade, numerous studies have paid attention to kurtosis because of the integral part, it plays in portfolio formation. The studies by eminent scholars such as Beardsley *et al.* (2012), Hong *et al.* (2007), Mitton and Vorkink (2007) and Liu *et al.* (2013) suggest the important role of kurtosis inclusion for optimal portfolio construction. The stock returns are not normally distributed and exhibit excess kurtosis. According to the finding of Fama (1965), large positive stock returns happen to be followed by negative stock returns of the similar magnitude leading to volatility clustering effect that relates to the information arrival and market reaction (Campbell and Hentschel, 1992). Kurtosis measures extreme episodes of returns in the tails. A large value for kurtosis stipulates higher risk in investment.

2.4. Lepto Kurtosis and Emerging Markets

The relevance of kurtosis within the context of emerging economies was asserted by existing literature (e.g., Hwang and Satchell 1999). According to Azher and Iqbal (2018), for emerging market returns the departure from normality is primarily driven by kurtosis and not skewness. In a group of 17 emerging markets including Pakistan, Hwang and Satchell (1999) show that cokurtosis has at least as much explanatory power as coskewness. Further, the wide spread evidence of outliers in emerging market returns suggest that the extreme outcomes have a higher probability of occurrence in emerging markets. Iqbal *et al.* (2010) discover the importance of kurtosis factor in explaining Pakistan stock market returns in excess of Fama and French three factor model, even though excess kurtosis does not represent a revelatory economic risk.

There are two widely accepted explanations for Kurtosis. First, it is considered as a peakedness of return distribution and second, the observation that irregular but sizable deviations are more prevalent in return distributions than the regular smallish deviations. Azher and Iqbal (2018) show the relevancy of co-kurtosis in the emerging economy of Pakistan as such markets are characterized by thin trading as an outcome of illiquidity and also presence of downside risk linked to sizable extreme deviations. In emerging economies, infrequent trading for most stocks results in excessive zero returns leading to large kurtosis. The co-kurtosis factor than can explain the portfolios' returns comprised of such stocks. Hence, illiquidity produced by thin trading may be one feasible possibility behind the previous evidences of the significance of co-moment based factor, i.e., kurtosis. A contemporary way to deal with kurtosis is incorporating the association of co- kurtosis factor with higher tail risk due to the probability of extremely large fluctuations in emerging economies.

Based on the aforementioned discussion, this study employs HAR model to analyze the informative properties of realized higher moments computed using intraday returns in predicting realized volatility of individual stocks at the emerging stock market of Pakistan. Though HAR is not formally regarded as a long memory model, it is capable of reproducing the strong persistent patterns of financial volatility by summing the realized volatility constituents accrued at different time durations. Furthermore, implementing, interpreting and forecasting HAR is comparatively easier (Yao *et al.,* 2019).

3. Methodology

3.1. Data

The tick-by-tick data of stock prices for listed firms at Pakistan Stock Exchange (PSX) are acquired from PSX for the time period covering 1 July 2008 till 31 August 2018. Five minutes prices are extracted from tick-by-tick data by using 'previous' interpolation technique, such as if there is no observation for some time slot, the previous observation is utilized. Based on the inclusion criteria i.e., stock prices of Rs. 5 and more and stocks having at least daily 80 transactions (e.g., Amaya *et al.,* 2015; Choi and Lee, 2014), the data initially comprised of 559 firms but after deleting the firms with thin trading, the final sample contains 452 firms. Sampling prices of five minutes interval is in accordance with the extant literature on realized variation measures. A recent study of Liu *et al.* (2015) compares the performance of five minutes sampling interval with various sampling frameworks and conclude that it is difficult to beat five minutes periodicity for in-sample as well as out sample volatility forecasts.

Realized volatility (RV) is calculated by summing squared five minutes returns (e.g., Andersen et al., 2001) and following Amaya *et al.* (2015), realized skewness (RSK) and kurtosis (RKU) are formulated by aggregating five minutes cubed and quartic returns.

3.2. Computation of Realized Higher Moments

The daily firm level log returns are initially depicted for *i*th daily return of day *t* as follows:

$$r_{t,i} = P_{t,\frac{i}{N}} - P_{t,\frac{(i-1)}{N}}, \tag{1}$$

where *P* denotes the log stock price and *N* stands for number of return observations for a trading day. The symbol $P_{t,0}$denotes the opening log price and $P_{t,1}$denotes the closing log price for day *t*.

For getting better insights, the regular trading times at PSX are accounted for i.e., from Monday till Thursday, 9:30am to 3:30pm and for Friday 9:15am to 4:30pm. Friday breaks at PSX are addressed as well, such as there are *N*=72 observations of five minutes interval for normal trading days and N=57 observations for Fridays. The five minutes returns are acquired by taking log difference of prices (current and prior period price) as follows:

$$r_{t,i} = \ln(P_t / P_{t-1}) \tag{2}$$

The theory of quadratic variation defines the cumulative return framework as:

$$[r, r]_t = \int_0^t \sigma^2 (s) ds + \sum_{0<s\underline{<t}} \kappa^2(s). \tag{3}$$

Such as the quadratic variation could be decomposed into its continuous and discontinuous constituents. The time interval [0, 1] is divided into *N* sub intervals to measure these constituents. Therefore, the famous intraday realized volatility of Andersen and Bollerslev (1998) is computed by aggregating the squares of the high frequency returns:

$$RV_t = \sum_{i=1}^{N} r_{t,i}^2 \tag{4}$$

It is not conventional to compute mean of high frequency returns as it is affected by variance of such high frequency returns. The model free characteristics of this measure of volatility makes it distinguishable from other measurement models (Andersen *et al.,* 2001; Barndorff-Nielsen and Shephard, 2002). Moreover, increase in the sampling frequency results in convergence of realized volatility to a clearly defined quadratic variation limit.

The average realized volatility over the prior *h* days is denoted by:

$$RV_t^h = \frac{1}{h} \sum_{i=1}^{h} RV_{t-i+1} \tag{5}$$

Thereby weekly average of realized volatility is given by:

$$RV_t^w = \frac{1}{5} \sum_{i=1}^{5} RV_{t-i+1} \tag{6}$$

And its monthly component is:

$$RV_t^m = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i+1} \tag{7}$$

The realized skewness of the stock return distribution is computed by aggregating cubic five minutes returns to measure its asymmetry. This measure is divided by realized volatility to standardize it as given by the following equation:

$$RSK_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RV_t^{3/2}} \tag{8}$$

The negative values of realized skewness show left skewed data and positive values of realized skewness show right skewed data. The realized daily kurtosis is computed by adding quartic five minutes returns divided by realized volatility for standardizing purpose as follows:

$$RKU_t = \frac{N\sum_{i=1}^{N} r_{t,i}^4}{RV_t^2} \tag{9}$$

The RSK_t and RKU_t measures are scaled by $\sqrt{N}$ and *N* to ensure that their magnitudes cohere to daily skewness and kurtosis.

3.3. Model Specification

The standard heterogeneous autoregressive model of realized volatility (HAR-RV) as proposed by Corsi (2009) is employed in numerous recent studies as a bench mark model The HAR-RV model is known to measure the stylized facts of stock return volatility e.g., long memory and multi scaling behavior. The popularity of HAR model lies in its parsimony, however employing it with high frequency data enhances its predictive power relative to standard GARCH models (e.g., Horpestad *et al.,* 2019). The first model termed as standard HAR is presented as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_{d,t} + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \varepsilon_{t+h}, \tag{10}$$

where the independent variables are the average realized previous day, previous week and previous month volatility respectively and the error term ε_{t+h} presents the unpredictable constituent. This research analyze volatility forecasts for h=1, 5 and 22 days. According to Müller *et al.* (1997), these volatility constituents' proxy for investors' heterogeneous investment horizons. The autoregressive models can exploit the persistence in volatilities as Corsi (2009) indicates that the HAR model, presented by equation (6) is parallel to an AR (22) model having levied equality restrictions on the AR coefficients:

$$RV_{t+1} = \beta_0 + \sum_{i=1}^{22} \phi_i^{HAR} RV_{t-i+1} + \varepsilon_{t+1} \tag{11}$$

The constraints on the realized volatility coefficients imposed by the HAR are presented as:

$$\phi_i^{HAR} = \begin{cases} \beta_d + \frac{1}{5}\beta_w + \frac{1}{22}\beta_m \; for\ i = 1 \\ \frac{1}{5}\beta_w + \frac{1}{22}\beta_m \quad for\ i = 2, \dots, 5 \\ \frac{1}{22}\beta_m \qquad\qquad for\ i = 6, \dots, 22 \end{cases} \tag{12}$$

To observe how the lagged realized skewness (RSK) of individual stocks is related to their realized volatility, the second model i.e., another HAR model is specified, augmented with realized skewness, referred to as HAR-RV-RSK:

$$RV_{t+h} = \beta_0 + \beta_d RV_{d,t} + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \theta RSK_t + \varepsilon_{t+h} \tag{13}$$

Next, in the third model, plain HAR-RV model is augmented with lagged realized kurtosis (RKU) to examine if it improves its forecasting performance, HAR-RV-RKU as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_{d,t} + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \eta RKU_t + \varepsilon_{t+h} \tag{14}$$

The final model includes both realized skewness and realized kurtosis to examine if the effect of one factor subsumes the effect of the other and is given by:

$$RV_{t+h} = \beta_0 + \beta_d RV_{d,t} + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \theta RSK_t + \eta RKU_t + \varepsilon_{t+h}. \tag{15}$$

4. Results and Discussion

As the data comprises of an extensive sample of 452 stocks with a forecasting horizon of over 22 trading days (452 firms × 4 models × 3 horizons = 5,424 regressions), the results for three of the firms (out of 452 firms in the sample) that are part of KSE-100 index are provided in this section, i.e., Highnoon Laboratories Ltd. (HINOON), Ibrahim Fibre Ltd. (IBFL) and Pakistan Tobacco Co Ltd. (PAKT). HINOON has been the fastest growing pharmaceutical company, incorporated in Pakistan in 1984. The company is a major manufacturer, importer, seller and marketer of pharmaceutical and related consumer products. IBFL was incorporated as a Public limited company in Pakistan. The company is principally involved in manufacturing and selling polyester staple fiber and yarn. PAKT, a public limited company was incorporated in Pakistan in 1947. The principal business of the company is manufacturing and selling cigarettes/ tobacco.

4.1. Descriptive Statistics

The descriptive statistics of realized volatility, realized skewness and realized kurtosis of HINOON, IBFL and PAKT is reported in Table 1. Based on the statistics in Table 1, the well-known stylized facts of volatility can be detected, such as right skewed distributions and extreme values of kurtosis in accordance with the occurrences of days of excess volatility (e.g., Lyócsa *et al.,* 2021). Most values for the three firms are close to values reported in literature (e.g., Mei *et al.,* 2017; Bonato *et al.,* 2021) however the values of kurtosis for realized volatility are way higher than values for indices level data, which is intuitive as stock level data is usually noisy. The high kurtosis values indicate the departure from normality as confirmed by Jarque Bera test statistics, therefore emphasizing the role of the incorporation of realized skewness and realized kurtosis in modelling the behavior of realized volatility of stock returns distribution.

Table 1. Descriptive statistics of RV, RSK and RKU

	Mean	Max	Min		Std. dev	Skewness	Kurt (excess)	Jarque Bera
	Highnoon Laboratories Ltd. (HINOON)							
RV	0.003	0.301	0		0.013	14.077	257.939	5040888
RSK	0.17	7.55	-7.55		3.369	-0.127	3.367	15
RKU	20.835	57	2.438		15.321	1.139	3.314	396
	Ibrahim Fibre Ltd. (IBFL)							
RV	0.002	0.046	0		0.003	5.917	70.126	169382
RSK	-0.097	7.55	-7.55		4.263	0.014	2.437	11
RKU	29.308	57	3.596		16.454	0.574	1.996	79
	Pakistan Tobacco Co Ltd. (PAKT)							
RV	0.003	0.399	0		0.011	26.498	907.741	59651582
RSK	-0.003	7.55	-7.55		4.88	-0.077	1.97	66
RKU	33.301	57	3.834		17.342	0.221	1.576	135

Note: The Jarque–Bera statistic tests are for the null hypothesis of normality for the distribution of the series.

4.2. In-sample Results

Following conventional approach, the ordinary least squares (OLS) technique is used to estimate all of sample HAR models. The results for in-sample analysis of the three firms for h=1 are reported in Table 2, for h=5 in Table 3 and for h=22 in Table 4 respectively.

In sample analysis for the 452 stocks show that the coefficient on daily lagged volatility ranges between -0.063 and 0.6479 with a positive mean value of 0.163 and standard deviation of 0.121 across all stocks while using benchmark HAR-RV model. The coefficients are positive for 435 stocks, which is 96% of the sample. However, 287 of the coefficients are significant as evidenced by Newey-West t-statistics which allows for serial correlation. Thus, the daily realized volatility influences the stocks' own future volatility for one day ahead forecast horizon. Similarly lagged weekly realized volatility has significant coefficients for 262 stocks and positive values for 94% of stocks indicating the impact of weekly volatility on its future counterpart. An increase in the magnitude of the coefficients can also be observed, confirming that the weekly realized volatility carries meaningful information in explaining the stocks' volatility. The coefficient loadings on lagged monthly realized volatility are significant for 322 stocks and positive for 97% of the sample confirming the part of monthly realized volatility in explaining future levels of the stocks' own volatility systematically.

Table 2. All of sample evaluation of the HAR-RV model and its extensions (h=1)

	β_0	β_d	β_w	β_m	Θ	η	Adj.R2
Highnoon Laboratories Ltd. (HINOON)							
HAR-RV	0.0004***	0.0757**	0.2083***	0.3652***			0.0916
t-values	*5.38*	*2.455*	*3.72*	*5.627*			
HAR-RV-RSK	0.0004***	0.0761**	0.2077***	0.3654***	0.0445		0.0913
t-values	*5.37*	*2.456*	*3.732*	*5.642*	*0.224*		
HAR-RV-RKU	0.0002***	0.0656**	0.1904***	0.2902***		0.1714***	0.1048
t-values	*3.42*	*2.099*	*3.667*	*4.462*		*5.099*	
HAR-RV-RSK-RKU	0.0002***	0.0659**	0.1899***	0.2904***	0.0355	0.1714***	0.1044
t-values	*3.39*	*2.101*	*3.675*	*4.482*	*0.183*	*5.121*	
Ibrahim Fibre Limited. (IBFL)							
HAR-RV	0.0002***	0.1374**	0.3835***	0.1708***			0.195
t-values	*5.556*	*2.3*	*6.698*	*2.706*			
HAR-RV-RSK	0.0002***	0.1376**	0.3835***	0.1708***	-0.1822		0.1951
t-values	*5.557*	*2.302*	*6.708*	*2.709*	*-0.835*		
HAR-RV-RKU	0.0001***	0.1294**	0.293***	0.1208***		0.2319***	0.2237
t-values	*4.556*	*2.346*	*5.299*	*2.48*		*5.606*	
HAR-RV-RSK-RKU	0.0001***	0.1296**	0.2933***	0.121***	-0.1253	0.231***	0.2236
t-values	*4.541*	*2.347*	*5.315*	*2.484*	*-0.596*	*5.668*	
Pakistan Tobacco Co Ltd. (PAKT)							
HAR-RV	0.0003***	0.1773***	0.3167***	0.1969***			0.1638
t-values	*5.939*	*4.986*	*5.518*	*3.417*			
HAR-RV-RSK	0.0003***	0.1769***	0.316***	0.1968***	0.2589		0.1652
t-values	*5.943*	*4.991*	*5.524*	*3.41*	*1.756*		
HAR-RV-RKU	0.0002***	0.1728***	0.287***	0.151***		0.1338***	0.177
t-values	*3.263*	*4.983*	*4.999*	*2.901*		*4.954*	
HAR-RV-RSK-RKU	0.0002***	0.1722***	0.2853***	0.1496***	0.306**	0.1375***	0.1791
t-values	*3.219*	*4.987*	*4.99*	*2.871*	*2.04*	*5.036*	

Note: t-values are Newey-West. RSK and RKU are divided by 10 000.
*** Denote rejections of null hypothesis at 5% significance level.*
**** Denote rejections of null hypothesis at 1% significance level.*

The results for the second model i.e., HAR-RV-RSK for forecast horizon h=1 are not encouraging as no meaningful improvement is seen in the model characteristics such as adjusted R-squares. The factor loadings on realized skewness are mostly positive (for 332 firms out of 452 firms) but significant for just 49 of the stocks. The HAR-RV-RKU model is an HAR-RV model augmented with only realized kurtosis, meaning that it could be compared directly to HAR-RV-RSK model. The coefficients of realized kurtosis factor are mostly positive (i.e., 442 out of 452), have higher magnitudes (between -0.998 and 4.735) and are significant for 343 of the firms. The fit of the model (e.g., Figure 1) is better across all forecast horizons. These findings supply empirically strong evidence that the realized kurtosis factor contains meaningful information content for forecasting stocks' volatility. The fourth model HAR-RV-RSK-RKU which comprises of both realized skewness and realized kurtosis as extensions to the benchmark model provides similar results as the three prior models in terms of magnitudes and significance of the coefficients and also no incremental improvement in adjusted R-squares is observed as compared to the third model HAR-RV-RKU.

Table 3. All of sample evaluation of the HAR-RV model and its extensions (h=5)

	β_0	β_d	β_w	β_m	Θ	H	Adj. R2
Highnoon Laboratories Ltd. (HINOON)							
HAR-RV	0.0007***	0.1279***	0.2066***	-0.0095***			0.0436
t-values	*10.372*	*3.398*	*4.13*	*-7.668*			
HAR-RV-RSK	0.0007***	0.1291***	0.2056***	-0.0095***	0.1699		0.0437
t-values	*10.362*	*3.467*	*4.086*	*-7.662*	*0.926*		
HAR-RV-RKU	0.0005***	0.1106***	0.1668***	-0.0069***		0.1585***	0.0554
t-values	*7.518*	*3.032*	*3.484*	*-5.716*		*5.09*	
HAR-RV-RSK-RKU	0.0005***	0.1117***	0.1659***	-0.0069***	0.1585	0.1581***	0.0554
t-values	*7.505*	*3.095*	*3.455*	*-5.705*	*0.915*	*5.03*	
Ibrahim Fibre Limited. (IBFL)							
HAR-RV	0.0003***	0.1911***	0.2962***	0.1295***			0.1327
t-values	*6.143*	*3.256*	*4.922*	*3.19*			
HAR-RV-RSK	0.0003***	0.1914***	0.2964***	0.1291***	-0.1913		0.1329
t-values	*6.125*	*3.254*	*4.92*	*3.19*	*-0.618*		
HAR-RV-RKU	0.0001***	0.1452***	0.1714***	0.0861**		0.34***	0.1973
t-values	*3.063*	*2.94*	*3.539*	*2.309*		*7.159*	
HAR-RV-RSK-RKU	0.0001***	0.1455***	0.1718***	0.0859**	-0.1167	0.3393***	0.1972
t-values	*3.068*	*2.94*	*3.532*	*2.307*	*-0.387*	*7.252*	
Pakistan Tobacco Co Ltd. (PAKT)							
HAR-RV	0.0007***	0.2128***	0.1617***	0.0057			0.0682
t-values	*8.068*	*5.071*	*3.372*	*0.344*			
HAR-RV-RSK	0.0007***	0.2127***	0.1608***	0.0053	0.111		0.0681
t-values	*8.107*	*5.055*	*3.345*	*0.322*	*0.713*		
HAR-RV-RKU	0.0005***	0.195***	0.1191***	0.0049		0.1527***	0.0862
t-values	*6.7*	*4.54*	*2.59*	*0.31*		*5.23*	
HAR-RV-RSK-RKU	0.0005***	0.1946***	0.1172***	0.0043	0.1684	0.1548***	0.0865
t-values	*6.738*	*4.51*	*2.541*	*0.275*	*1.102*	*5.366*	

Note: t-values are Newey-West. RSK and RKU are divided by 10 000.
*** Denote rejections of null hypothesis at 5% significance level.*
**** Denote rejections of null hypothesis at 1% significance level.*

Findings for HAR models at forecasting horizon h=5 are mainly similar to the findings for h=1, however the declining role of lagged monthly realized volatility is evident as it loses its size and significance. The number of significant coefficients reduced to 170 from 322 with increasing horizon. The magnitudes of factor loadings on realized skewness show slight improvement over the forecast horizon of h=5 but lose their significance further from 49 to 29 coefficients as explained by Newey-West t-statistics. On the other hand, the results for realized kurtosis show enhancement in terms of magnitudes and significance of factor loadings. The range of factor loadings on realized kurtosis increased from (-0.998 - 4.735) to (-0.379 - 10.013) for h=1 and h=5 respectively and the number of significant coefficients increase from 343 to 355 out of total sample of 452 firms. In parallel to results for h=1, the fourth model does not provide any incremental enhancement over the third model.

Table 4. All of sample evaluation of the HAR-RV model and its extensions (h=22)

	β_0	β_d	β_w	β_m	Θ	H	Adj. R2
Highnoon Laboratories Ltd. (HINOON)							
HAR-RV	0.001***	0.0856***	0.0131	-0.0148***			0.0108
t-values	*11.704*	*3.032*	*0.408*	*-5.415*			
HAR-RV-RSK	0.001***	0.087***	0.0113	-0.0148***	0.2323		0.0113
t-values	*11.639*	*3.158*	*0.354*	*-5.4*	*1.636*		
HAR-RV-RKU	0.0007***	0.0618**	0.0001	-0.0103***		0.1925***	0.0288
t-values	*10.075*	*2.321*	*0.002*	*-5.998*		*6.224*	
HAR-RV-RSK-RKU	0.0007***	0.0632***	-0.0015	-0.0103***	0.2202	0.1921***	0.0292
t-values	*10.026*	*2.438*	*-0.051*	*-5.978*	*1.482*	*6.233*	
Ibrahim Fibre Limited. (IBFL)							
HAR-RV	0.0003***	0.0936**	0.221***	0.2066***			0.0733
t-values	*4.443*	*2.314*	*3.764*	*3.702*			
HAR-RV-RSK	0.0003***	0.0937**	0.2208***	0.2069***	-0.209		0.0736
t-values	*4.432*	*2.315*	*3.791*	*3.71*	*-0.821*		
HAR-RV-RKU	0.0001***	0.0454	0.1696***	0.1516***		0.2623***	0.1152
t-values	*2.981*	*1.475*	*2.951*	*2.846*		*6.357*	
HAR-RV-RSK-RKU	0.0001***	0.0456	0.1697***	0.152***	-0.1319	0.2613***	0.1151
t-values	*2.982*	*1.483*	*2.967*	*2.847*	*-0.545*	*6.36*	
Pakistan Tobacco Co Ltd. (PAKT)							
HAR-RV	0.001***	0.0949***	-0.0214	0.0326			0.0123
t-values	*7.761*	*3.247*	*-0.589*	*1.362*			
HAR-RV-RSK	0.001***	0.0949***	-0.0216	0.0326	0.025		0.0119
t-values	*7.777*	*3.246*	*-0.597*	*1.364*	*0.168*		
HAR-RV-RKU	0.0006***	0.0646***	-0.022	0.0307		0.1894***	0.0408
t-values	*5.999*	*2.486*	*-0.66*	*1.219*		*5.047*	
HAR-RV-RSK-RKU	0.0006***	0.0642***	-0.0228	0.0309	0.0992	0.1906***	0.0407
t-values	*5.983*	*2.483*	*-0.688*	*1.224*	*0.678*	*5.115*	

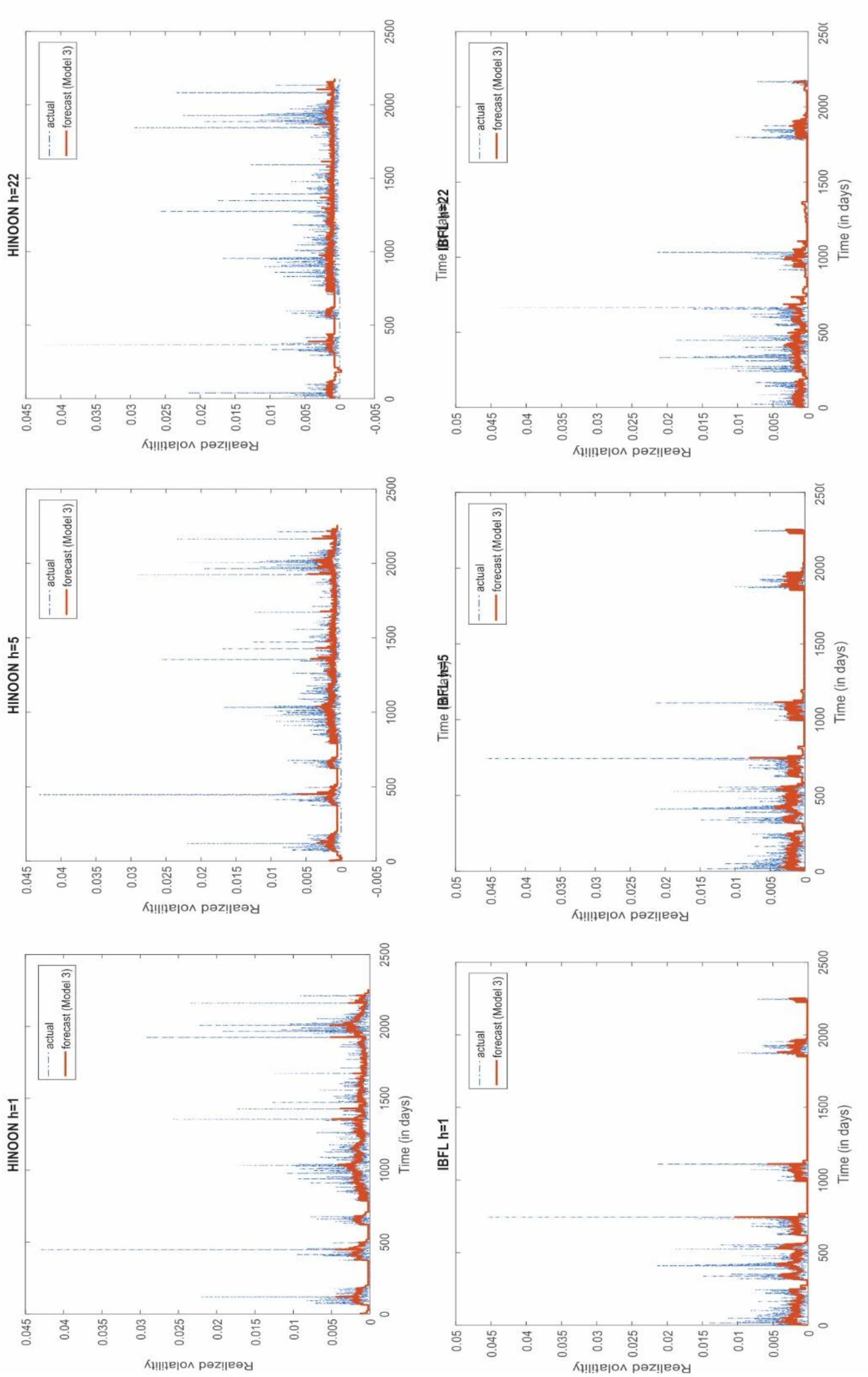
Note: t-values are Newey-West. RSK and RKU are divided by 10 000.
*** Denote rejections of null hypothesis at 5% significance level.*
**** Denote rejections of null hypothesis at 1% significance level.*

The findings for 22 days ahead volatility forecasts are reported in Table 4. A gradual decrease could be observed in the weekly and monthly realized volatility coefficients, however the daily coefficients do not show much change. The significance and size of realized kurtosis is persistent across forecast horizons, though realized skewness stays insignificant for most of the firms across horizons and for all models containing realized skewness.

The daily, weekly and monthly volatilities play a role in forecasting volatilities of individual stocks, however the significance and magnitude of their coefficients change with individual forecast horizons. For one day ahead volatility forecasts, the most distant realized volatility matters the most, with the daily and weekly factor loadings declining fractionally. Monthly realized volatility outperform daily and weekly volatilities only for short forecast horizons. Moving five days ahead volatility forecast, monthly volatility loses its leading role and the coefficients for most recent realized volatilities are the highest and for the 22 day ahead forecast horizon daily volatility stays at the top with weekly and monthly volatilities decreasing gradually. Such as the lagged daily realized volatility has a persistent effect on stocks' volatility across all models and forecast horizons. Including the stocks' own skewness in the baseline HAR model does not show any improvement in model characteristics across all forecast horizons and even declining adjusted R-squares are witnessed for most firms. However, strong empirical evidence is observed for the third model containing stocks' own realized kurtosis, persistent through h=1, 5 and 22, supportive of the fact that realized kurtosis carries meaningful information content for explaining the stocks' volatility. In the fourth model, all the coefficients for all factors are more or less the same, such as no factor subsumes the effect of the other. The last columns of Tables 2, 3 and 4 report adjusted R-squares of the benchmark HAR model and the alternatives through the whole sampling period for the three firms. The HAR-RV-RKU model doubtlessly outperforms its counterparts concerning in-sample fit across the three horizons.

Figure 1

Realized Volatility and its Forecasts from Model 3.



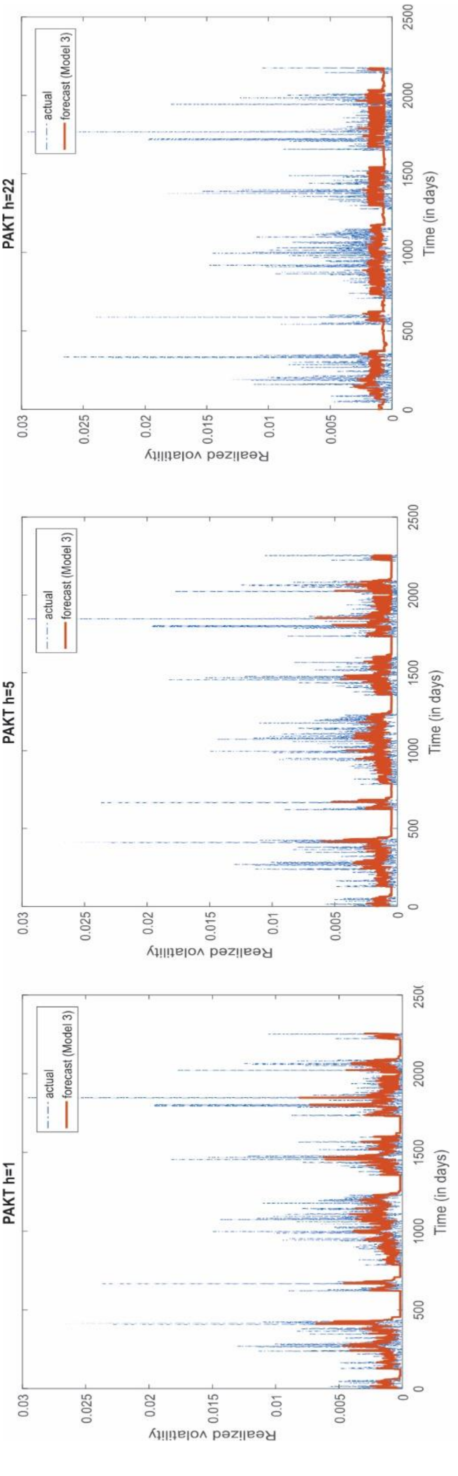
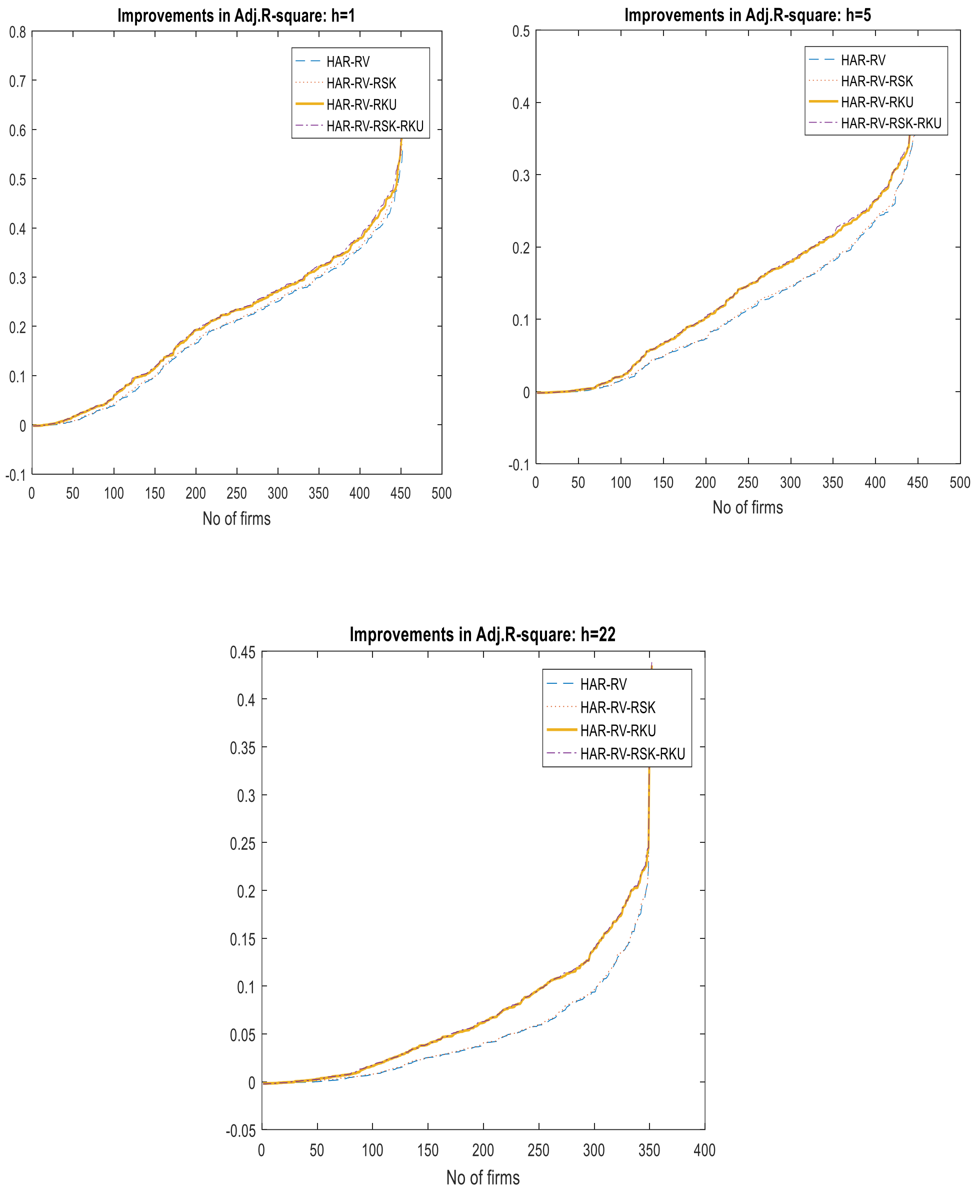

PAKT h=1
PAKT h=5
PAKT h=22
actual
forecast (Model 3)
Realized volatility
Time (in days)
0
0.005
0.01
0.015
0.02
0.025
0.03
500
1000
1500
2000
2500

Figure 2. Comparison of all HAR models in terms of improvement in adjusted R-squares for the time period from July 2008 till August 2018 for 452 firms.



The plots of actual values of realized volatilities together with their predictions obtained from Model 3 are presented in Figure 1. Figure 1 depicts sudden spikes of realized volatility measures exhibiting days of extreme volatility. Lyócsa, Molnár and Výrost (2021) detect the presence of significant outliers and show their detrimental effect on the otherwise high autocorrelation function of the time series at PSX. A close look on Figure 1 demonstrates that predictions from Model 3 are comparatively well in capturing the turning points of extreme volatility episodes, validating the advantages of including realized kurtosis to capture extreme movements in equities at PSX.

Figure 2 supports the in-sample results for the benchmark HAR-RV model and its extensions for all firms in the sample. The adjusted R-square curve for the second model is not indicative of any change over the first baseline model, however, the standard HAR model augmented with realized kurtosis escalates adjusted R-squares over all three forecast horizons as visible in Figure 2. The curve for the fourth model does not show improvement over the third model.

4.3. Out-sample Analysis

According to Wang *et al.* (2016), due to the temporal changes in in-sample forecasting relationships, the market participants are more interested in the out-of-sample predictive performance of a model as it provides more consistent information about the future. For out-of-sample forecasts, the sample is divided into two subgroups: the in-sample data covers the first 1,555 trading days (July 1, 2009 till October 26, 2015) and the out-of-sample data for measuring model's performance covers the last 700 days (October 27, 2015 till August 31, 2018) (e.g., Ma *et al.,* 2018). Out-of-sample forecasts are generated using common practice of rolling window regression i.e., adding one new day and removing the most far-off day (e.g., Lyócsa and Todorova, 2020). In this way the sample size used in estimating the models stays at a constant span with no overlapping forecasts. The out-of-sample covers the last 700 days of the total sample, such as each model is recomputed 700 times having parameters that vary with time, with varying samples.

For analyzing the forecast precision quantitatively, two popular loss functions are employed, mean squared error (MSE) and mean absolute error (MAE), statistically defined as follows:

$$MSE = N^{-1} \sum_{t=1}^{N} \left(RV_t - \widehat{RV}_t\right)^2, \tag{16}$$

$$MAE = N^{-1} \sum_{t=1}^{N} \frac{\left|RV_t - \widehat{RV}_t\right|}{RV_t} \tag{17}$$

where *N* stands for out-of-sample forecasts, RV_t denotes actual realized volatility and $\widehat{RV}_t$ denotes volatility forecasts. Extreme under/over estimations are penalized more deeply by the symmetric mean squared error loss function. According to Patton (2011), it ranks models consistently even in the presence of a noisy proxy. However, mean absolute error loss function does not require penalty for the model complexity in out-of-sample framework and is useful in avoiding extremely large errors (e. g., Hansen and Lunde, 2005).

Table 5

Forecasting results of volatility models in DM test.

	HINOON			IBFL			PAKT		
	h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
Loss functions: MSE									
HAR-RV vs. HAR-RV-RSK	0.3683	0.3033	-0.6619	0.2237	0.3981	0.4535	0.2613	0.8255	0.5333
p-values	*0.7127*	*0.7617*	*0.5082*	*0.8230*	*0.6907*	*0.6504*	*0.7940*	*0.4093*	*0.5940*
HAR-RV vs. HAR-RV-RKU	2.7598	4.7279	5.0214	1.5702	2.4322	1.4278	2.0119	1.3392	1.9837
p-values	*0.0059*	*0.0000*	*0.0000*	*0.1168*	*0.0153*	*0.1538*	*0.0446*	*0.1809*	*0.0477*
HAR-RV vs. HAR-RV-RSK-RKU	2.8667	4.8009	4.8991	1.3597	2.6432	1.2637	2.0730	1.5322	1.8119
p-values	*0.0042*	*0.0000*	*0.0000*	*0.1744*	*0.0084*	*0.2068*	*0.0385*	*0.1259*	*0.0705*
Loss functions: MAE									
HAR-RV vs. HAR-RV-RSK	0.0726	0.8685	-0.1081	-2.8333	2.2822	-1.2561	0.1229	-0.8696	-0.8482
p-values	*0.9421*	*0.3853*	*0.9139*	*0.0047*	*0.0228*	*0.2095*	*0.9022*	*0.3848*	*0.3966*
HAR-RV vs. HAR-RV-RKU	11.5415	11.5458	8.4467	4.3556	6.6112	4.8791	3.9674	2.9649	2.5326
p-values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0031*	*0.0116*
HAR-RV vs. HAR-RV-RSK-RKU	11.4941	11.6086	8.1311	3.8172	6.8467	4.4579	3.9368	1.9358	2.0969
p-values	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0533*	*0.0364*

The conventional paired comparison test, the DM test, introduced by Diebold and Mariano (2002) is used to examine the predictability precision of the two contending models (e.g., Liang *et al.,* 2020). The DM test can be expressed as:

$$DM = \frac{\bar{d}}{\sqrt{LRV_{\bar{d}}/T}} \tag{18}$$

where $\bar{d} = \frac{1}{q}\sum_{t=m+1}^{m+q} d_t$ and d_t equals $L_{A,t} - L_{B,t}$. $L_{A,t}$ and $L_{B,t}$ present the loss functions from the two competing models. *m* and *q* present the in-sample and out-of-sample time period span respectively.

The out-of-sample findings for daily, weekly and monthly forecasting horizon for the three firms are exhibited in Table 5. In line with results for in-sample estimates, the second model HAR-RV-RSK clearly does not show any improvement over the benchmark HAR-RV model for all firms and for all forecast horizons. The third HAR model HAR-RV-RKU beats the benchmark model for all horizons for HINOON at 1% significance level, but for IBFL, it shows improvement only for weekly horizon and for PAKT, the improvement is seen for daily and monthly horizons at 5% level of significance under mean squared error loss function. Turning to the fourth model HAR-RV-RSK-RKU, the results are more or less the same, adding realized skewness factor to the third model does not have any meaningful impact on model characteristics, though slight decline in adjusted R-square values as observed in Tables 2, 3 and 4 makes it evident that realized skewness is useless to explain individual stock level volatility at PSX.

On the other hand, under mean absolute error loss function, the second model outperforms the standard HAR model as DM test shows significant results for the daily and weekly horizons at 1% and 5% levels of significance respectively for IBFL. However, the third and fourth model substantially outperform the first model in all of the different scenarios for the three firms. The performance of the fourth model is mainly attributed to realized kurtosis factor as realized skewness factor shows poor performance in in-sample forecasts and thus fails to subsume the effect of realized kurtosis in out-sample analysis for Model 4, as well. In conclusion, the out of sample forecast precision with regard to absolute forecast errors in a rolling window structure, furnishes the information that the basic HAR model augmented with realized kurtosis generally yields the more forecast precision confirmed by DM tests.

5. Conclusions

The undue stock market movements as witnessed in finance literature, provide motivation to assess the role of realized skewness and kurtosis in forecasting firm level realized volatility at PSX, computed using five minutes intraday returns. Based on the benchmark HAR model and its modifications including realized higher order moments for the time period from July 2008 to August 2018, this study finds strong evidence for predictability performance of realized kurtosis for various forecast horizons. Kinateder and Papavassiliou (2019) find realized kurtosis as the dominant predictor of future returns on European bond market.

The basic assumption of the HAR-RV model of Corsi (2009) is the validation of heterogeneous markets hypothesis (HMH), which states that market participants are heterogeneous with respect to their expectations and behavior. Thus, three different horizons can capture the common pattern of the volatility framework. The daily volatility sampling frequency reflects the needs of short-term investors, weekly volatility of medium-term investors and monthly volatility of traders who focus on long term trends. Though long memory is not imposed externally in modelling volatility by HAR framework, the cascade like model catalyzes slow decay in memory over the forecast horizons (Özbekler *et al.,* 2021).

Given the important role of volatility forecasts accuracy in optimal portfolio designs, this study provides strong evidence that realized kurtosis is most useful when one to 22 day ahead forecasts are of interest by taking large and diverse set of data of 452 listed firms at PSX, and thus could assist in improving asset allocation decisions. Thus, the standard HAR model and its extensions containing realized kurtosis predicts the expected realized volatility as a linear function of yesterday's realized volatility and its mean over prior week and month as well as yesterday's realized kurtosis. Therefore, it is concluded that stocks' own realized kurtosis carries meaningful information for stocks' future volatilities. These findings have great importance for portfolio managers and investors. For example, Ho *et al.* (2005) find that Hong Kong market has comparatively lower efficiency than markets in developed countries. Thus, such research adds value to the literature by providing institutional framework for the country under study. China, an important country of Asia Pacific region, has substantial participation in PSX strategic shares. PSX average returns and price variation patterns have moved more towards regional markets, connectivity and coalitions following collaborations with the Chinese consortium resulting in enhanced regional integration.

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