

# 6 MODELING THE ECONOMIC GROWTH IN ROMANIA WITH THE SOLOW MODEL

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## Abstract

In this study I make an estimation of the Solow model for the Romanian economy. Starting from the estimates of the parameters from other studies, I simulate the model both for the 1990-2004 period and in the long run. The study shows that the Solow model provides a good approximation of the dynamics of the Romanian economy for the 1990-2004 period, with respect to the dynamics of the aggregate GDP and to the ratios of the main macroeconomic variables, like production per worker, capital-output ratio or capital per worker. The simulation for the 2030 time horizon indicates a potential of growth of over 3%.

**Keywords:** economic growth, productivity, transition.

**JEL classification:** O47, P47.

## 1. Introduction

The beginning of the transition of the former socialist countries caused a massive fall of output which rivaled, and in several countries even surpassed, the dimension of the United States recession during the Great Depression. After a few years, the fall in output stopped, and a few countries, mostly Central and Eastern European ones (CEE), started a process of economic growth. The success of reforms and the sustained economic growth spurred a whole line of research on the determinants of economic growth and on the long run economic growth perspectives of the CEE countries.

Thus, Fischer, Sahay and Vegh (1998) used the regressions developed by Barro (1991), and Levine and Renelt (1992), respectively, in order to forecast both the long run economic growth and the convergence towards the income levels in OECD. Barro estimated the following regression:

$$\text{Growth rate per capita} = 0.0302 - 0.0075*Y_{1960} + 0.025*PRIM + 0.0305*SEC - 0.119*GOV$$

While Levine and Renelt estimated the next relation:

$$\text{Growth rate per capita} = -0.83 - 0.35*Y_{1960} - 0.38*POP + 3.17*SEC + 17.5*INV$$

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Where  $Y_{1960}$  is per capita income in 1960 expressed at the purchase power parity, PRIM is the school enrollment at the primary level, SEC is the school enrollment at the secondary level, INV are the gross investments as a percentage of GDP, GOV represents the percentage of governmental expenditures as a percentage of GDP, and POP is the population growth rate.

Using these regressions they estimated forecasts for the transition economies, based on estimated dynamics of the exogenous variables, that is, of the enrollments, investments, and public expenditures. For the former socialist countries Fischer, Sahay and Vegh (1998) obtained optimistic estimates, with a median value of 5.26%. For Romania, they estimated a long run growth rate of 5.35% based on Barro regression, and of 5.99% based on Levine and Renelt regression.

Crafts and Kaiser (2004) reevaluated Fischer, Sahay and Vegh (1998) results using data that became available later. They reestimated the Levine and Renelt regression which they used for projecting the long run growth rates for the transition economies, while using also as an alternative regressions that included the quality of the institution under various specifications. They obtained less optimistic results as compared to Fischer, Sahay and Vegh (1998), the average growth rate ranging between 3 and 4%. For Romania, they estimated a growth rate between 1.89% and 4.75% (the latter one for the reestimated regression of Levine and Renelt).

A more recent contribution which studies the economic growth in Romania is that of Croitoru and Târhoacă (2003). They calibrated a CGE model for the Romanian economy which they used for estimating the growth rate of the economy in the medium run. The two estimated a growth rate with an increasing trend, which tends to 6% towards the end of the period of forecasting, namely 2010.

The present study makes an estimation of the Solow model, both for the past transition period, relatively to the real dynamics of the economy and in the long run. The Solow model is one of the foundations of the modern macroeconomics, not only of the neoclassical and of the new theory of economic growth, but also of the modern theory of business cycles. Although the Solow model was formulated during the 50's (1956), it remains one of the fundamental instruments for analyzing the economic growth in a country, by offering essential information regarding the mechanism of economic growth.

This article is structured as follows: the second section introduces the model and explains its fundamental characteristics; in the third section I estimate this model, starting from estimations of the parameters of the model, on the basis of statistical data and previous studies, and I also make a qualitative and quantitative analysis of the model. The fourth section derives and discusses the main results of this paper.

## **2. A Short Presentation of the Model**

In this section, I present the standard Solow model, emphasizing the main hypotheses that underlie this model and also those specifications I decided to use.

The model is expressed in discrete time. I consider the case of a closed economy in which there is a single homogenous good produced in each period. The aggregate production in each period is denoted by  $Y_t$ . I simplified the model by considering that



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the government is absent. Thus, the identity equation for expenditures becomes:

$$Y_t = C_t + I_t \quad (1)$$

where  $C_t$  is the aggregate consumption and  $I_t$  is the aggregate investments.

The aggregate production is characterized with the help of a production function of Cobb Douglas type. The production function is characterized by constant returns to scale:

$$Y_t = F(K_t, N_t) = K_t^\alpha (A_t N_t)^{1-\alpha} \quad (2)$$

where:  $A_t$  is the level of technology in period  $t$ , that is the efficiency with which the production factors are utilized;  $K_t$  and  $L_t$  represent the two factors of production, the physical capital stock, and the labor force;  $1-\alpha$  parameter represents the elasticity of the production relative to the labor force, while  $\alpha$  is the elasticity of the production relative to the capital. The production function incorporates a Harrod-neutral technology.

The three variables of the production function are modeled with the help of the dynamic equations, as follows:

$$A_{t+1} = (1 + g_t) * A_t \quad (3)$$

where  $g_t$  is the exogenous growth rate of the total factor productivity (TFP hereafter).

$$N_{t+1} = (1 + n) * N_t \quad (4)$$

where  $n_t$  is the exogenous growth rate of the labor force.

$$K_{t+1} = (1 - \delta) * K_t + I_t \quad (5)$$

where  $\delta$  is the depreciation rate of capital, and  $I_t$  stands for the gross investments.

In the Solow model, the investments result from a Keynesian type equation:

$$I_t = s * Y_t \quad (6)$$

where  $s$  is the savings rate, and  $s$  is also an exogenous parameter of the model. Obviously, the consumption results from equations (1) and (6):

$$C_t = (1 - s) * Y_t \quad (7)$$

In order to find the solution of the model it is necessary to formulate and then to solve a difference equation in the capital stock  $K_t$ . The model is however more significant when solved in terms of production and capital stock per capita. And since the model also incorporates the technological progress, I develop the difference equation in terms of efficiency unit of labor.

First, I define the production and capital stock in terms of variables per efficiency unit of labor:

$$y_t = \frac{Y_t}{A_t N_t} \quad (8)$$

$$k_t = \frac{K_t}{A_t N_t} \quad (9)$$



where  $y_t$  is the output per efficiency unit of labor and  $k_t$  is the capital stock per efficiency unit of labor.

For the Cobb Douglas production function, one can write  $y_t$  as:

$$y_t = \frac{Y_t}{A_t N_t} = \left(\frac{1}{A_t N_t}\right) K_t^\alpha (A_t N_t)^{1-\alpha} = \left(\frac{K_t}{A_t N_t}\right)^\alpha \left(\frac{A_t N_t}{A_t N_t}\right) = k_t^\alpha \quad (10)$$

I write now the difference equation in  $k_{t+1}$ :

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{A_{t+1} N_{t+1}} = \frac{(1-\delta)K_t + I_t}{(1+n)(1+g)N_t A_t} = \frac{(1-\delta)K_t + sK_t^\alpha (A_t N_t)^{1-\alpha}}{(1+n)(1+g)N_t A_t} = \\ &= \frac{(1-\delta)}{(1+n)(1+g)} \frac{K_t}{N_t A_t} + \frac{s}{(1+n)(1+g)} \frac{K_t^\alpha}{(N_t A_t)^\alpha} = \\ &= \frac{1}{(1+n)(1+g)} [(1-\delta)k_t + s k_t^\alpha] \end{aligned} \quad (11)$$

This equation shows that  $k_{t+1}$  depends positively on the savings rate and on the capital stock, and it depends negatively on the growth rate of population  $n$ , and on the depreciation rate of capital.

With this equation, one can determine the equilibrium  $k_t$  and  $y_t$ .

In order to determine  $k_t$  I solve equation (11) for  $k_{t+1}=k_t$ , while  $y_t$  is solved from equation (10). The below equations give the expressions for these variables:

$$k_t = \left( \frac{s}{n + g + \delta + ng} \right)^{1/1-\alpha} \quad (12)$$

$$y_t = \left( \frac{s}{n + g + \delta + ng} \right)^{\alpha/1-\alpha} \quad (13)$$

### 3. The Simulation of the Model

The simulation of the model implies, first of all, the so-called process of model calibration. The calibration of the dynamic models has become recently one of the fundamental stages in macroeconomic simulations. The calibration consists in setting certain values to the exogenous parameters of the model, by using empirical estimations either by oneself or from previous studies in the area. In order to take into account the two different periods through which the economy has passed, a first one during the '90s, which was dominated by severe recessions, and the second one, which is characterized by a sustained process of economic growth, I calibrate the model using different values for some of the exogenous parameters. Thus, I use one value for the first period, 1990-1999, and another value for the period of economic growth which starts in the year 2000.

For the depreciation rate,  $\delta$ , I use a depreciation 10% for the period 1990-1999, a value which takes into account the rapid depreciation in capital stock during the first



period of the transition of the Romanian economy. For the period after 2000, I use a depreciation considered as normal for a stable economy, namely 5%.

For the retribution of capital and of the labor force, I started from other estimates of these parameters from previous studies. Caraiani (2004) estimated a 0.42 retribution of labor factor for the period 1990-2002, using the data for the wages for this period from the national accounts. A closer value has found Croitoru and Târhoacă (2003), one of 0.46, by using data from the national accounts regarding the labor retribution in 2000. I decided to use the results from Caraiani (2004), as it is a known fact that the retribution of the labor force in the emergent economies is much below 0.5.

For the growth rate of the labor force, I use for the first period a value for  $n$  equal to -2.6%, which is the actual average growth rate of the labor force for the period 1990-1999, while for the period after 2000, I use a value of -0.5%. The use of a negative growth rate in the first period takes into account the sharp decline in the labor force during the first part of the transition. After 2000, in line with the demographical research, I consider that the labor force will slowly decline.

The estimation of the savings rate takes into account the hypothesis of a closed economy. Thus, I estimate the saving rate by the rate of gross fixed capital formation. Again, I use two different values for the two periods. For the interval 1990-1999, I use a value of 0.17, which, although lower than the average of gross fixed capital formation, corrects the actual value in relation to the quality of investments. For the second period, I use a saving rate equal to 0.3, which is a long-term rate. This value is based on Croitoru and Tarhoaca (2003) estimates, which found and increasing saving rate after 2000, which justifies the possibility of a long-term saving rate of 0.3.

The exogenous rate of TFP, namely  $g$ , is estimated on the basis of previous estimates of TFP based on the growth accounting approach, such as in Caraiani (2004). In order to justify the negative impact upon the TFP of the two recessions during the '90s, I use a negative value for  $g$ , of -2.6, as the results in Caraiani (2004) suggest. For the period after 2000, I use a  $g$  equal to 2.5%, which is a reasonable growth rate of the TFP, taking into account the characteristics of the economic growth after 2000. Croitoru and Târhoacă (2003) also consider a TFP growth of 2.5% in their forecast regarding economic growth between 2003 and 2010.

There are some arguments that a growth rate of the TFP higher than 2% is not sustainable. Thus, Bosworth and Collins (1996) show that even the South-East Asian economies did not have growth rates of the TFP higher than 2%. However, there is also an argument against this conclusion. Thus, it is a well-known fact that the South-East Asian Economies developed rather in an extensive way. Moreover, the estimates of TFP for the Eastern European transition economies indicate, for the more dynamic economies, TFP values of around 2.5%. If we also take into account the growth rates of TFP in the Romanian economy after 2000, it seems that the choice for the period after 2000 is reasonable.

The estimation of the model implies also the computation of the initial values for the GDP, capital stock, labor force and technological level  $A_1$ . For GDP I use the GDP in 1990 constant prices, that is  $Y_1=858$  billion 1990 lei. For the labor force I use the average labor force in 1990, which is  $L_1= 10.200$  million. As in the previous studies,

the initial capital stock is the one estimated in Albu and Roudoi (2003), that is  $K_1=1439$  billion 1990 lei constant prices.

The estimate of the  $A_1$  was based on the other variables of the production function in period 1, using the formula:

$$A_1 = \left( \frac{Y_1}{K_1^\alpha L_1^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} \quad (12)$$

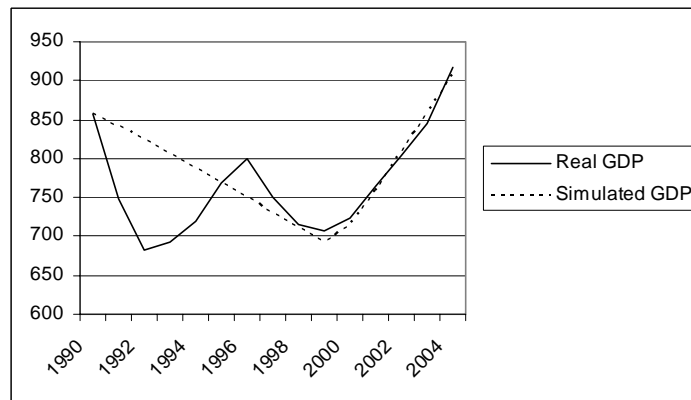
I made a simulation of the model in two steps: in the first step, I performed a simulation for the period 1990-2004, in order to compare the dynamics of the model with that of the real economy, while in the second step I simulated the model in the long run.

### 3.1. A Simulation for the Period 1990-2004

In this section, I present both the simulated model and a comparison between the dynamics of the main variables of the model and that of the similar variables in the real economy. This comparison allows an analysis of how realistic the model is, and constitutes a basis for the long run simulation of the model in the next section. The comparison in this section has some degree of relativity as compared to the long run dynamics, since the transition period was characterized by severe fluctuations and changes in the structural parameters, while the long run dynamics implies a constancy of the structural parameters.

Figure 1

**A comparison between the dynamics of real and simulated GDP**



The above figure compares the dynamics of the real and simulated GDP. It is a well-known fact that the GDP had a fluctuating dynamics during the transition period. Besides the initial correction, which was a common phenomenon in all the transition economies, the Romanian economy passed through another recession, only Bulgaria and the Czech Republic being the other transition economies that had experienced two recessions during the '90s.

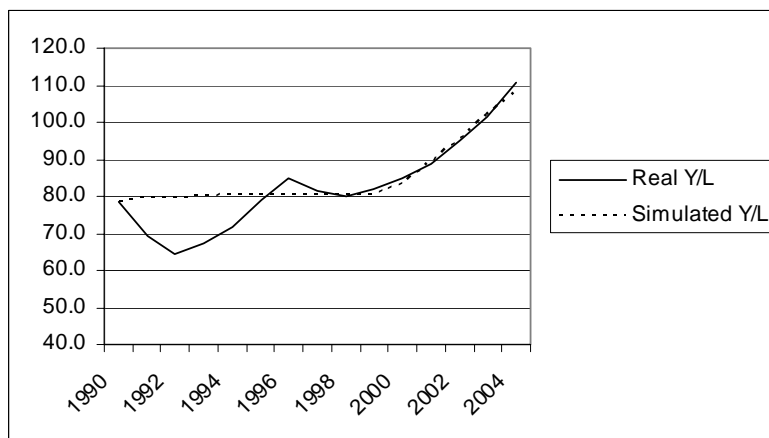
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The simulated GDP has a decreasing trend which is determined by the two essential parameters with a strong negative dynamics, namely the decrease in the labor force and the negative dynamics of the TFP. Even if the economy grew between 1993 and 1996, the simulated GDP dynamics is significant in the sense that it reveals the corrective trend of the dynamics of the economy in that period.

Starting in 2000, we can see a break relative to the dynamics of the period 1990-1999. In the real economy one could have witnessed a period of economic growth, which, this time, was based on a restructured and much less distorted economy. The simulated model captures this growth process too, with the growth coming mainly from the growth in the gross rate of capital formation and from the positive growth rate of the TFP.

**Figure 2**

**The actual and simulated labor productivity**



Another key variable not only of the model but also of the real economy is the productivity. As a measure of productivity, I use the output in constant prices per workers. The above figure reveals some differences between the dynamics of the real and simulated variables, mostly in the first part of the transition. The main explanation of the differences comes from the deterministic feature of the model. In the real economy, this variable fluctuated mostly because of the aggregate production fluctuations.

As the labor force passed through a strong correction the productivity started to grow after 1993, with a minor slowdown during the 1997-1999 recession. After 2000, the simulated and the real variable converge, and have similar dynamics.

The above figure shows the dynamics of the capital stock (in constant prices) per worker, in the real economy and in the model. In this case, as the figure reveals, in the period 1993-2002, a gap appeared between the simulated and the real K/L, which vanishes afterwards toward the end of the analyzed period.

Figure 3

K/L ratio, actual and simulated

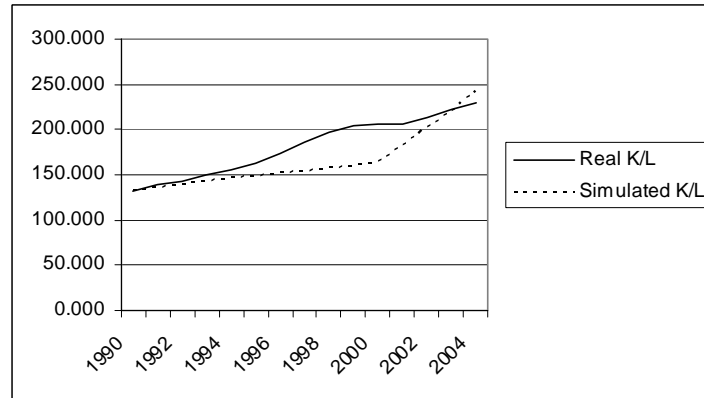
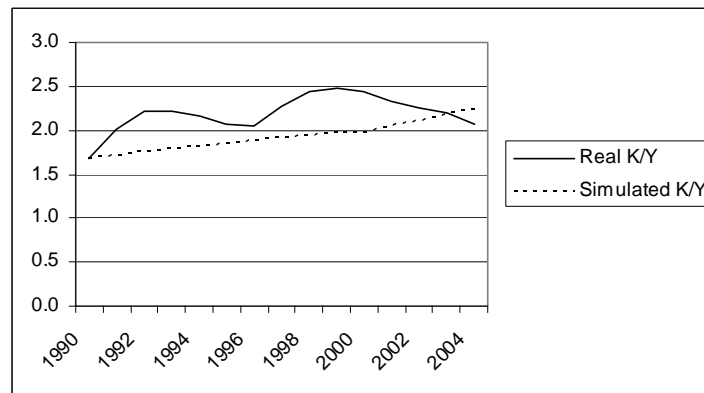


Figure 4

K/Y ratio, actual and simulated



A key variable of the analysis of the economic growth is the capital-output ratio (both terms expressed in constant prices). A well-known fact is that this ratio takes values which depend on the level of the development of the economy. Thus, Dhareshwar and Nehru (2003) estimated a K/Y ratio for the developing economies of about 1.5. This result requires some discussion regarding its significance for a transition economy. As we know, the transition economies started their reform from a high level of capital stock, due to the high level of investments during the communist period. Because of the poor allocation of resources during the communist period and, also due to the negative shocks during transition, much of the capital stock was lost during the transition, a loss which was reflected by the option for a high rate of depreciation.

The dynamics of the real capital-output ratio during 1990-2004 reveals a certain overdevelopment of this ratio, mostly because of the two severe recessions, which



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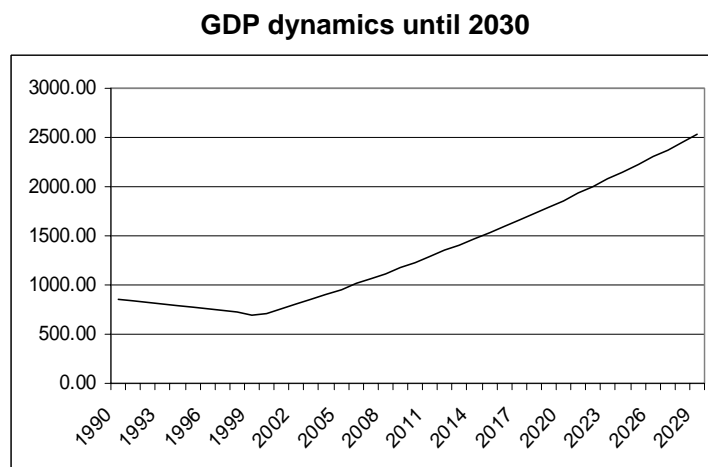
negatively affected the output. Over this period, there is convergence between the two ratios, the variable converging towards a value of 2.2, which is realistic if we consider the level of development in 2004.

As one may notice, the model provides a satisfactory simulation of the economy, both relative to the dynamics of the aggregate production and relative to the dynamics of the main variables. The fact that there are certain gaps in the dynamics of the variables in the model relative to the dynamics of the real variables is caused mostly by the shocks in the real economy, which are harder to simulate with the help of the Solow model, which is fit for long run simulations of the economic growth.

#### **3.2. A Simulation of the Model in the Long Run**

In this section I make a projection of the model for the period 1990-2030 and analyze the GDP dynamics and the growth rates of GDP and GDP per capita. This estimation implies the recalibration of the model by using the new values of various parameters after 2000.

**Figure 5**

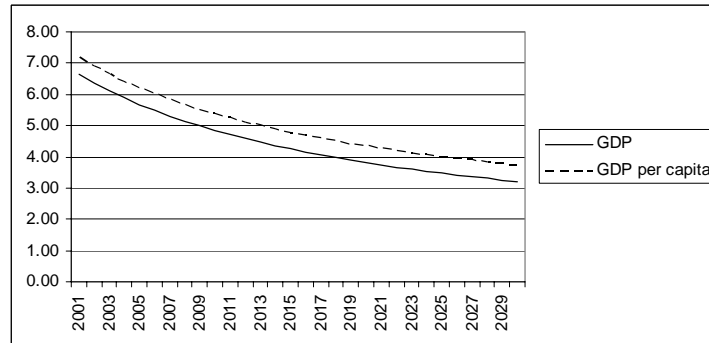


One may notice, first of all, the decreasing trend over the period 1990-2000, which was previously analyzed. After the year 2000, the model predicts a sustained economic growth. Towards the year 2025 the economy already becomes three times larger relative to the year 1999.

More interesting to discuss are actually the growth rates of GDP and GDP per capita. The Romanian economy had during the past years growth rates of over 5%. However, I consider that these growth rates are unsustainable in the long run. Historically, only the South East Asian economies had high growth rates for long periods but under very different circumstances than the Romanian economy. The Solow model predicts that these growth rates will slowly decline. In the long run, these growth rates will converge, in the case of per capita GDP, to the growth rate of TFP.

Figure 6

The dynamics of the growth rate of GDP and per capita GDP



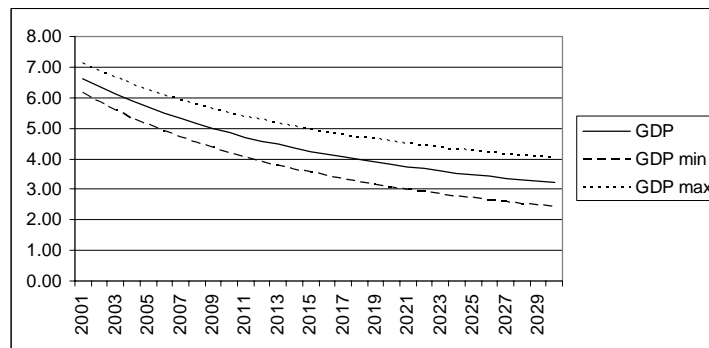
The above figure presents the growth rates of GDP, and per capita GDP, respectively. One may notice the adjustment in the growth rates in the period 2000-2030. If at the beginning of the period the potential of growth is of over 6%, towards the end of this period the potential decreases to a growth rate of 3%. This rate is, nevertheless, a high potential growth rate if we consider that it is a long run potential.

There is, as one may see, a difference between the growth rate of GDP and that of GDP per capita. This difference comes from the fact that, in the long run, the labor force slowly declines, implying a higher growth rate of per capita GDP, which is 3.7% towards 2030.

In order to illustrate the sensitivity of the model relative to the hypothesis regarding the technical progress, I present below the dynamics of GDP in the case of two alternatives regarding the TFP growth.

Figure 7

The dynamics of GDP under various scenarios for TFP



The GDP max. scenario incorporates a 3.5% growth rate of TFP. This growth rate may seem too high, but it takes into account the actual dynamics of the Romanian

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economy, which registered high growth rates of TFP after 2000. It also takes into account the plausible hypothesis that, under the circumstances of actual labor force and capital stock dynamics, it is only the productivity growth that can sustain high growth rates. The GDP min. scenario considers a TFP growth rate of only 1.5%.

One may notice that in the long run the gaps between the growth rates of GDP under the various specifications are growing towards the year 2030. Thus, the growth of GDP in the optimistic scenario is over 4%, while the pessimistic scenario predicts a growth of only 2.4% towards the end of the period.

#### **4. Conclusions**

This article estimated and simulated the Solow model for the Romanian economy. The simulation was done in two stages. In a first period the model was estimated and analyzed for the period 1990-2004, and the results were compared to the real economy dynamics. In the second stage, I simulated the model in the very long run and analyzed the main results relative to the economic growth literature.

As a preliminary stage before simulating, I calibrated the parameters of the model. In order to calibrate I used data from previous studies on the Romanian economy or my own estimations, such as Albu and Roudoi (2003) for the capital stock, Caraiani (2004) for the growth rate of TFP and the retribution of labor and capital factors, and Croitoru and Târhoacă (2003) for the saving rate. The different dynamics of the economy in the two analyzed periods implied the recalibration of the model after the year 2000.

The simulation for the period 1990-2004 reveals that the model estimates relatively well the dynamic of the real economy, except for the fluctuations implied by the negative shocks during the period 1990-1998. The simple explanation of these differences comes from the fact that the Solow model is a long run model of economic growth, and it is less fit to simulate the short run dynamics, especially the dynamics of the Romanian economy during this period. Nevertheless, the comparison of the simulated model with the real economy reveals important information regarding this period.

The long run projection indicates a high potential of economic growth in Romania, which is about 3% in the long run. For the time horizon 2000-2010, the growth potential is higher, passing through an adjustment from over 6.5% in 2000 to 5% towards 2010. For the period 2010-2030 the model predicts a growth rate that declines slowly, from 5% to over 3%. When we consider the long run growth rates of the industrialized economies (about 2%), we conclude that the development perspectives of Romania are positive, and under some further improvement of the fundamental parameters, like the saving rate or the growth rate of productivity, the potential of growth could become higher.



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