12 MEASURING THE SOCIO-ECONOMIC BIPOLARIZATION PHENOMENON

Ştefan ŞTEFĂNESCU*

Abstract

The present paper emphasizes the differences between the Gini concentration coefficient [13] and a new bipolarization index proposed by [20], [21]. In practice, the Gini index is applied frequently to determine the poverty degree of the persons of a given population P. Although the pauperization and the polarization social phenomena are often very strong related, we shall prove that the bipolarization level of the individuals of P cannot be always accurately estimated by applying the classical Gini measure. Therefore, for evaluating the intensity of a polarization event is not adequate to use the Gini-type coefficients based on the Lorenz order.

Keywords: Gini coefficient, bipolarization index, Lorenz order, antithetic variables, bounded exponential distributions.

JEL Classification: C43, C46, C51, C52.

1. THE FORMULATION OF THE PROBLEM

In the literature, the Gini indicator $\gamma_*(W)$, $0 \le \gamma_*(W) \le 1$, was designated to estimate the concentration degree of the "small" values produced by the variable W, $W > 0$ ([1], [2], [6], [9], [13], [15], [16], [19], [25]). Thus, if the random variable W characterizes the income values of the individuals of the population P , the Gini index $\gamma_*(W)$ measures the poverty level in P.

On the contrary, the coefficient $\Delta_*(W)$, $0 \le \Delta_*(W) \le 1$, proposed in [20], [21], expresses the bipolarization degree for the values of the random variable W. In the papers [7], [11] or [26] other new measures of the polarization phenomenon are analyzed.

By using many practical examples, the specialized literature ([1]-[27]) emphasizes often the presence of a strong interdependence between the socio-economic pauperization of the individuals in P and the bipolarization effect. Based on this dependence relation it is frequently accredited in practice the wrong idea that Gini's concentration coefficient

* Associate Professor, Ph.D., Faculty of Mathematics and Informatics, University of Bucharest, e-mail : stefanst@fmi.unibuc.ro.

$\gamma_*(W)$ could also be used to estimate the intensity of the income polarization phenomenon of the persons of the population P.

In the present paper we intend to invalidate such a methodological approach, which could impose unsuitable conclusions and inadequate decisions (see also the opinions mentioned in [14] and [26]).

Pragmatically, taking into consideration a family of distributions for the random variable W, we shall attempt to reveal essential behavioral differences between both indicators, $\gamma_*(W)$ and $\Delta_*(W)$. Such a result is in accordance with the fundamental sociological research since the polarization action of the individuals of P is not always identical with the poverty concentration phenomenon in P (see also [2], [6], [8], [9], [13]-[17], [23]-[25]) .

2. THE BOUNDED EXPONENTIAL DISTRIBUTION

Let the parameter $\lambda \in R$.

Definition 1. The random variable (r.v.) X with the support $[0,1]$ is λ exponentially distributed, $X \sim \mathrm{Exp1}(\lambda)$, if its probability density function (p.d.f.) $f(x;\lambda)$ has the form:

$$f(x;\lambda) = \begin{cases} \lambda e^{\lambda x} / (e^{\lambda} - 1) & ; \quad for \;\; \lambda \neq 0 \\ 1 & ; \quad for \;\; \lambda = 0 \end{cases} \quad , \quad 0 \le x \le 1 \qquad (1)$$

We get the following cumulative function (c.d.f.) $F(x;\lambda)$ for $X \sim$ Exp1( ) ,

$$F(x;\lambda) = \begin{cases} (e^{\lambda x} - 1) / (e^{\lambda} - 1) & ; \quad when \;\; \lambda \neq 0 \\ x & ; \quad when \;\; \lambda = 0 \end{cases} , \quad 0 \le x \le 1 \qquad (2)$$

We shall designate by U([0, 1]) the uniform distribution on the interval $[0,1]$.

Remark 1. The distribution U([0 , 1]) belongs to the distribution family Exp1(λ) , $\lambda \in R$, since Exp1(0) ≡ U([0 , 1]) .

Proposition 1. If $\lambda > 0$ (respectively $\lambda < 0$) then the p.d.f. $f(x;\lambda)$ is a strict increasing (decreasing) function.

Proof. Indeed the derivative function $f(x;\lambda)$,

$$\frac{\partial f(x;\lambda)}{\partial x} = \frac{\lambda^2 e^{\lambda x}}{e^{\lambda} - 1}$$

is strictly positive (negative) for $\lambda > 0$ (respectively $\lambda < 0$).

Depending on the real values of the parameter λ , the graphics *G1*, *G2* give the shape of the p.d.f.-s $f(x;\lambda)$.

Proposition 2. For any $0 \le x \le 1$ and $\lambda \in R$ we have the equality

$$F(x;\lambda)+F(1-x;-\lambda)=1 \qquad 3)$$

Proof. Taking into consideration the variants $\lambda=0$, and $\lambda\neq 0$ respectively, and applying the formula (2) we deduce the relation (3).

When $0\leq q\leq 1$ and $\lambda\in R$ we define the expression

$$Q(q;\lambda)=\begin{cases} ln\left(1+q(e^{\lambda}-1)\right)/\lambda & ; \quad for\ \lambda\neq 0 \\ q & ; \quad for\ \lambda=0 \end{cases} \qquad (4)$$

Proposition 3. For every fixed $\lambda\in R$ the expression $Q(q;\lambda)$, $0\leq q\leq 1$, is just the inverse of the c.d.f. $F(x;\lambda)$, $0\leq x\leq 1$.

Proof. In the case $\lambda=0$ this property is true since, for an arbitrary $0\leq x\leq 1$, we have

$$Q(q;\lambda)=q \qquad F(x;\lambda)=x\ .$$

After a direct calculation, when $\lambda\neq 0$ we obtain the relations :

$$F(Q(q;\lambda);\lambda)=\frac{1+q(e^{\lambda}-1)-1}{e^{\lambda}-1}=q$$

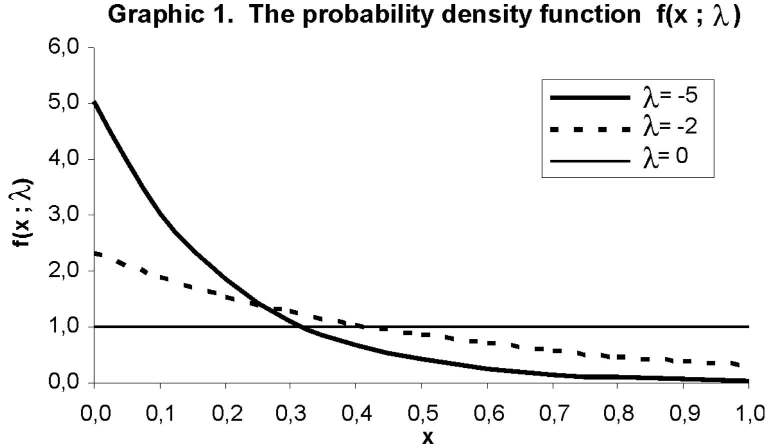
$$Q(F(x;\lambda);\lambda)=\frac{ln(e^{\lambda x})}{\lambda}=x$$

with $0\leq x\leq 1$.

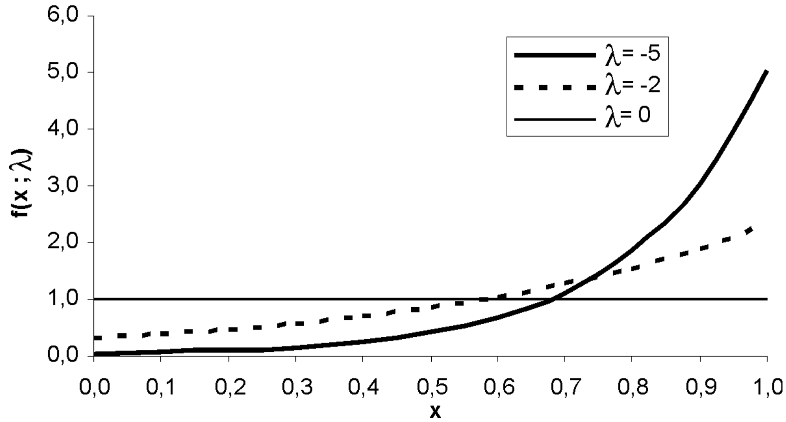
In the following, we shall denote by $\mu(\lambda)$ the expectation value of the r.v. $X\sim Exp1(\lambda)$, that is

$$\mu(\lambda)=Mean(X),\quad \lambda\in R\ .$$

Graphic 1. The probability density function f(x ; λ)



Graphic G. The probability density function f(x; λ)



Proposition 4. For any $\lambda \in R$ we have

$$\mu(\lambda)=\begin{cases}(\lambda e^{\lambda}-e^{\lambda}+1)/(\lambda e^{\lambda}-\lambda);\, for\, \lambda \neq 0 \\ 1/2 \ \ ; for\, \lambda = 0\end{cases} \tag{5}$$

Proof. When $\lambda = 0$ we deduce:

$$\mu(0)=\int_{-\infty}^{+\infty} x\, f(x;0)\, dx = \int_{0}^{1} x\, dx = \frac{1}{2}$$

If $\lambda \neq 0$ then integrating by parts we get:

$$\mu(\lambda)=\int_{-\infty}^{+\infty} x\, f(x;0)\, dx = \int_{0}^{1} \lambda x e^{\lambda x} / (e^{\lambda}-1)\, dx = \frac{e^{\lambda}}{e^{\lambda}-1}-\frac{1}{\lambda}$$

Proposition 5. The following equality is true for an arbitrary $\lambda \in R$,

$$\mu(\lambda)+\mu(-\lambda)=1 \tag{6}$$

Proof. Indeed

$$\mu(0)+\mu(-0)=\frac{1}{2}+\frac{1}{2}=1$$

In the case $\lambda \neq 0$ it results

$$\mu(\lambda)+\mu(-\lambda)=\left(\frac{e^{\lambda}}{e^{\lambda}-1}-\frac{1}{\lambda}\right)+\left(\frac{e^{-\lambda}}{e^{-\lambda}-1}+\frac{1}{\lambda}\right)=1$$

3. GINI'S MEASURE

We shall designate by $L(u;\lambda)$, $0 \le u \le 1$, $\lambda \in R$, the classical Lorenz curve attached to the random variable $X \sim \text{Exp1}(\lambda)$, [13], [27].

Proposition 6. The function $L(u;\lambda)$, $0 \le u \le 1$, $\lambda \in R$ has the expression:

$$L(u;\lambda) = \begin{cases} \dfrac{(1+u(e^{\lambda}-1))\, ln(1+u(e^{\lambda}-1)) - u(e^{\lambda}-1)}{\lambda e^{\lambda} - e^{\lambda} + 1} & ; \quad for \ \lambda \neq 0 \\ u^2 & ; \quad for \ \lambda = 0 \end{cases} \tag{7}$$

Proof. Respecting the previous notations, the Lorenz curve $L(u;\lambda)$ is given by the following formula (see [13]),

$$L(u;\lambda) = \left(\int_0^u Q(q;\lambda)\, dq \right) / \left(\int_0^1 Q(q;\lambda)\, dq \right) = \frac{h_1(u;\lambda)}{h_1(1;\lambda)}$$

where $h_1(u;\lambda) = \int_0^u Q(q;\lambda)\, dq$.

If we establish the specific form of the function $h_1(u;\lambda)$ we shall deduce after a straightforward calculus the expression (7). So, we deduce:

$$h_1(u;0) = \int_0^u Q(q;0)\, dq = \int_0^u q\, dq = \frac{u^2}{2}$$

When $\lambda \neq 0$ it results:

$$h_1(u;\lambda) = \int_0^u Q(q;)\, dq = \int_0^u \frac{ln(1+q(e^{\lambda}-1)}{\lambda}\, dq =$$

$$= \frac{(1+u(e^{\lambda}-1))\, ln(1+u(e^{\lambda}-1)) - u(e^{\lambda}-1)}{\lambda e^{\lambda} - e^{\lambda} + 1}$$

To simplify the notation, we shall denote by $\gamma(\lambda)$, $\lambda \in R$, the Gini coefficient associated to the r.v. $X \sim \text{Exp1}(\lambda)$, more exactly $\gamma(\lambda) = \gamma_*(X)$.

Proposition 7. The Gini coefficient $\gamma(\lambda)$, $\lambda \in R$, has the form:

$$\gamma(\lambda) = \begin{cases} \left(e^{2\lambda} - 2\lambda e^{\lambda} - 1\right) / \left(2(\lambda e^{\lambda} - e^{\lambda} + 1)(e^{\lambda} - 1)\right) & ; \quad for \ \lambda \neq 0 \\ 1/3 & ; \quad for \ \lambda = 0 \end{cases} \tag{8}$$

Proof. Based on the Lorenz curve, the indicator Gini has the expression ([13]):

$$\gamma(\lambda) = 1 - 2\int_0^1 L(u;\lambda)\, du$$

Using the explicit form (7) for the Lorenz curve we deduce:

$$\gamma(0) = 1 - 2\int_0^1 L(u;0)\, du = 1 - 2\int_0^1 u^2\, du = \frac{1}{3}$$

Moreover, in the case $\lambda \neq 0$ we get:

$$\gamma(\lambda) = 1 - 2\int_0^1 L(u;\lambda)\, du = 1 - 2\int_0^1 \frac{(1+u(e^{\lambda}-1))\, ln(1+u(e^{\lambda}-1)) - u(e^{\lambda}-1)}{\lambda e^{\lambda} - e^{\lambda} + 1}\, du =$$

$$= 1 - \frac{2\lambda e^{2\lambda} - e^{2\lambda} + 1 - 2(e^{\lambda}-1)^2}{2(\lambda e^{\lambda} - e^{\lambda} + 1)(e^{\lambda}-1)} = \frac{e^{2\lambda} - 2\lambda e^{\lambda} - 1}{2(\lambda e^{\lambda} - e^{\lambda} + 1)(e^{\lambda}-1)}$$

Proposition 8. If $\lambda \neq 0$, then:

$$\frac{\gamma(\lambda)}{\gamma(-\lambda)} = \frac{e^{\lambda} - \lambda - 1}{\lambda e^{\lambda} - e^{\lambda} + 1} \tag{9}$$

Proof. Indeed, applying Proposition 7 it results:

$$\frac{\gamma(\lambda)}{\gamma(-\lambda)} = \frac{e^{2\lambda} - 2\lambda e^{\lambda} - 1}{2(\lambda e^{\lambda} - e^{\lambda} + 1)(e^{\lambda}-1)} \cdot \frac{2(-\lambda e^{-\lambda} - e^{-\lambda} + 1)(e^{-\lambda}-1)}{e^{-2\lambda} + 2\lambda e^{-\lambda} - 1} = \frac{e^{\lambda} - \lambda - 1}{\lambda e^{\lambda} - e^{\lambda} + 1}$$

4. A BIPOLARIZATION INDEX

In the papers [20] and [21] we proposed the indicator $\Delta(W)$ to measure the bipolarization level for any r.v. W which has a bounded support $[a\, ,\, b]$.

The main idea for designing the index $\Delta(W)$ was to define an appropriate partition $\{I_1\, ,\, I_2\}$ of the domain $[a\, ,\, b]$,

$$I_1 = \{\, x \mid a \le x \le \mu_0 \,\} \qquad\qquad I_2 = \{\, x \mid \mu_0 < x \le b \}$$

and, at the same time, to establish the suitable separation threshold μ_0 between the two disjointed groups I_1 and I_2.

In [20], [21] we considered the average of the r.v. W as a cut point, that is $\mu_0 = Mean(W)$.

The indicator $\Delta(W)$ will measure the distance between the "poles" μ_1 and μ_2 of the groups I_1 and I_2, taking also into consideration the "weights" p, respectively $1-p$, of these groups.

More exactly:

$$\mu_1 = Mean(W \mid I_1) = \int_a^{\mu_0} \frac{w g(w)}{p}\, dw \qquad \mu_2 = Mean(W \mid I_2) = \int_{\mu_0}^{b} \frac{w g(w)}{1-p}\, dw$$

$$p = Pr(W \le \mu_0) = \int_a^{\mu_0} g(w)\, dw \qquad 1-p = Pr(W > \mu_0) = \int_{\mu_0}^{b} g(w)\, dw$$

where $g(w)$, $a \le w \le b$, is the p.d.f. of the r.v. W.

Definition 2 ([20], [21])**.** The bipolarization indicator $\Delta_*(W)$ of the r.v. W is given by the expression:

$$\Delta_*(W) = \frac{4p(1-p)(\mu_2 - \mu_1)}{b-a} \tag{10}$$

In the following, we shall analyze the behavior of the polarization index $\Delta(\lambda)$, $\lambda \in R$, where

$$\Delta(\lambda) = \Delta_*(X)$$

with $X \sim$ Exp1(λ) .

To simplify the calculus, we shall use the function $\delta(x;\lambda)$, $0 \le x \le 1$, $\lambda \in R$, defined by

$$\delta(x;\lambda) = \begin{cases} (\lambda x - 1)e^{\lambda x} / (\lambda e^{\lambda} - \lambda) & ; \quad for\ \lambda \ne 0 \\ x^2/2 & ; \quad for\ \lambda = 0 \end{cases} \tag{11}$$

Proposition 9. The expression $\delta(x;\lambda)$, $0 \le x \le 1$, is a primitive of the function $x f(x;\lambda)$.

Proof. Indeed, keeping the previous notations and taking into consideration the distinct cases $\lambda = 0$ and $\lambda \ne 0$ respectively, after a straightforward calculation we get:

$$\frac{\partial \delta(x;\lambda)}{\partial x} = x f(x;\lambda) \tag{12}$$

Proposition 10. For every $\lambda \ne 0$ and $0 \le x \le 1$ the equality:

$$\delta(x;\lambda) - \delta(1-x;-\lambda) = \frac{e^{\lambda x}}{e^{\lambda} - 1} \quad \text{is true} \tag{13}$$

Proof. If $\lambda \neq 0$ then

$$\delta(x;\lambda) - \delta(1-x;-\lambda) = \frac{(\lambda x - 1)e^{\lambda x}}{\lambda(e^{\lambda} - 1)} - \frac{(\lambda(1-x)+1)e^{-\lambda(1-x)}}{\lambda(e^{-\lambda} - 1)} = \frac{e^{\lambda x}}{e^{\lambda} - 1}$$

Proposition 11. Respecting the previous notations, the bipolarization indicator $\Delta(\lambda)$, $\lambda \in R$, has the form:

$$\Delta(\lambda) = 4F(\mu(\lambda);\lambda)\big(\delta(1;\lambda) - \delta(0;\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(0;\lambda)\big) \qquad (14)$$

Proof. We shall apply the formula (10) for $W \equiv X$ with $X \sim$ Exp1( ), that is

$$a = 0 \qquad b = 1 \qquad \mu_0 = \mu(\lambda) \qquad p = F(\mu(\lambda);\lambda) \qquad g(x) = f(x;\lambda)$$

Using Proposition 9, it results that:

$$\Delta(\lambda) = \Delta_*(X) = 4p\int_{\mu(\lambda)}^{1} x\,f(x;\lambda)\,dx - 4(1-p)\int_{0}^{\mu(\lambda)} x\,f(x;\lambda)\,dx =$$

$$= 4p\int_{0}^{1} x\,f(x;\lambda)\,dx - 4\int_{0}^{\mu(\lambda)} x\,f(x;\lambda)\,dx =$$

$$= 4F(\mu(\lambda);\lambda)\big(\delta(1;\lambda) - \delta(0;\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(0;\lambda)\big)$$

Proposition 12. The bipolarization $\Delta(\lambda)$ is a symmetrical function, that is

$$\Delta(\lambda) = \Delta(-\lambda) \quad , \quad \forall\ \lambda \in R \qquad (15)$$

Proof. For any fixed $\lambda \neq 0$ it is sufficient to prove the relation $\Delta(\lambda) - \Delta(-\lambda) = 0$.

Indeed, by applying Propositions 2, 5, 10 and 11 we obtain in order

$$\Delta(\lambda) - \Delta(-\lambda) = 4F(\mu(\lambda);\lambda)\big(\delta(1;\lambda) - \delta(0;\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(0;\lambda)\big) -$$

$$- 4F(1-\mu(\lambda);-\lambda)\big(\delta(1;-\lambda) - \delta(0;-\lambda)\big) + 4\big(\delta(1-\mu(\lambda);-\lambda) - \delta(0;-\lambda)\big) =$$

$$= 4F(\mu(\lambda);\lambda)\Big(\big(\delta(1;\lambda) - \delta(0;\lambda)\big) + \big(\delta(1;-\lambda) - \delta(0;-\lambda)\big)\Big) +$$

$$+ 4\big(\delta(0;\lambda) - \delta(1;-\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(1-\mu(\lambda);-\lambda)\big) =$$

$$= 4F(\mu(\lambda);\lambda)\Big(\big(\delta(1;\lambda) - \delta(0;-\lambda)\big) - \big(\delta(0;\lambda) - \delta(1;-\lambda)\big)\Big) + \frac{4}{e^{\lambda} - 1} - \frac{4e^{\lambda\mu(\lambda)}}{e^{\lambda} - 1} =$$

$$= 4F(\mu(\lambda);\lambda)\left(\frac{e^{\lambda}}{e^{\lambda} - 1} - \frac{1}{e^{\lambda} - 1}\right) + \frac{4}{e^{\lambda} - 1} - \frac{4e^{\lambda\mu(\lambda)}}{e^{\lambda} - 1} =$$

$$= 4F(\mu(\lambda);\lambda) - 4\frac{e^{\lambda\mu(\lambda)} - 1}{e^{\lambda} - 1} = 4F(\mu(\lambda);\lambda) - 4F(\mu(\lambda);\lambda) = 0$$

5. COMPARING THE INDICES $\gamma(\lambda)$ AND $\Delta(\lambda)$

Proposition 13. All the functions $f(x;\lambda)$, $F(x;\lambda)$, $\mu(\lambda)$, $\gamma(\lambda)$, $\delta(x;\lambda)$, $Q(q;\lambda)$, $L(u;\lambda)$, $\Delta(\lambda)$ are continuous, depending on the λ variable, $\lambda \in R$.

Proof. Having in mind the particular form of the mentioned functions, it remains to show that the continuity property is maintained in the point $\lambda = 0$, too.

So, let $h_2(\lambda)$ be one of the expressions $f(x;\lambda)$, $F(x;\lambda)$, $\mu(\lambda)$, $\gamma(\lambda)$, $\delta(x;\lambda)$, $Q(q;\lambda)$, $L(u;\lambda)$. Applying a direct derivative calculus based on the l'Hospital rule it results that:

$$\lim_{\lambda \to 0} h_2(\lambda) = h_2(0)$$

In addition, the continuity of the function $\Delta(\lambda)$ results easily by analyzing the particular form of the expression (14).

Proposition 14. The following equalities are true:

$$\gamma(-\infty) = \frac{1}{2} \qquad \gamma(0) = \frac{1}{3} \qquad \gamma(\infty) = 0$$

$$\Delta(-\infty) = 0 \qquad \Delta(0) = \frac{1}{2} \qquad \Delta(\infty) = 0 \tag{16}$$

Proof. Using the formula (8), we get $\gamma(0) = 1/3$ and, in addition:

$$\gamma(-\infty) = \lim_{\lambda \to -\infty} \gamma(\lambda) = \lim_{\lambda \to -\infty} \frac{e^{2\lambda} - 2\lambda e^{\lambda} - 1}{2(\lambda e^{\lambda} - e^{\lambda} + 1)(e^{\lambda} - 1)} = \frac{-1}{-2} = \frac{1}{2}$$

$$\gamma(\infty) = \lim_{\lambda \to \infty} \gamma(\lambda) = \lim_{\lambda \to \infty} \frac{1 - 2\lambda e^{-\lambda} - e^{-2\lambda}}{2(\lambda - 1 + e^{-\lambda})(1 - e^{-\lambda})} = \lim_{\lambda \to \infty} \frac{1}{2(\lambda - 1)} = 0$$

Applying Propositions 11 and 12, we have successively

$$\Delta(0) = 4F(\mu(0);0)\big(\delta(1;0) - \delta(0;0)\big) - 4\big(\delta(\mu(0);0) - \delta(0;0)\big) =$$

$$= 4F(1/2);0)\big(1/2\big) - 4\big(1/2\big)^2 / 2 = \frac{4}{4} - \frac{1}{2} = \frac{1}{2}$$

$$\Delta(-\infty) = \Delta(\infty) = \lim_{\lambda \to \infty} 4F(\mu(\lambda);\lambda)\big(\delta(1;\lambda) - \delta(0;\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(0;\lambda)\big) =$$

$$= \lim_{\lambda \to \infty} 4\big(\delta(1;\lambda) - \delta(0;\lambda)\big) - 4\big(\delta(\mu(\lambda);\lambda) - \delta(0;\lambda)\big) = 0$$

Proposition 15. If $X \sim$ Exp1(λ) and $Y = 1 - X$ then it results that $Y \sim$ Exp1($-\lambda$).

Proof. Obviously, if $\lambda = 0$ then we have $X \sim$ U([0 , 1]) and $Y \sim$ U([0 , 1]).

In case that $\lambda \neq 0$ it is sufficient to show that the equality $g(y;\lambda) = f(y;-\lambda)$ is valid for any $0 \leq y \leq 1$ where $g(y;\lambda)$ is just the p.d.f. of the r.v. Y.

Indeed, using the transformation $y = 1 - x$, $0 \leq x \leq 1$, we get, in order:

$$g(y;\lambda) = f(x;\lambda)\left|\frac{\partial x}{\partial y}\right| = f(1-y;\lambda) = \lambda e^{\lambda(1-y)} / (e^{\lambda} - 1) = f(y;-\lambda)$$

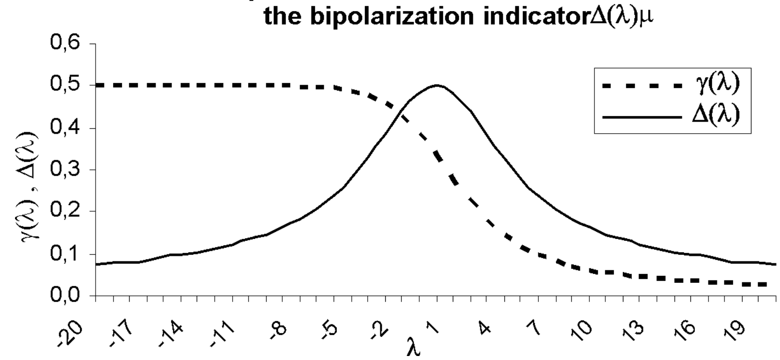
Remark 2. The r.v. $Y = 1 - X$ is the antithetic of the variable X .

If the variable X defines the income distribution of the individuals of the population P, then Y will characterize the income of the same persons when their status changes, namely the "rich" individuals become "poor" and vice versa. In fact, the antithetic type operation inverts the "rich" with the "poor" persons.

Remark 3. From Proposition 1 and the graphics *G1* and *G2* it results that at the same time with the increase in the values of λ the number of "poor" people is diminished in P (the probability to have individuals in P with "small income" decreases). Taking into consideration the theoretical significance of Gini's coefficient ([1], [2], [6], [12], [13], [15]), we deduce that $\gamma(\lambda)$ must be a non-increasing function. This aspect is in accordance with the interpretation of the graphic *G3* or with *Table 1* values.

Remark 4. The augmentation of the λ values implies a reduction in the percent of "small" values produced by the r.v. *X* ~ Exp1(λ) (to interpret the graphics *G1*, *G2* and Propositions 1, 14). But this fact does not necessarily impose a monotony property for the bipolarization coefficient $\Delta(\lambda)$ (see, for example, the graphic *G3* or *Table 1*). In practice, reducing the percentage of the "poorest" individuals in P does not substantially affect a strong bipolarization phenomenon. This perception is also confirmed by the graphic *G3* and by the *Table 1* values, too.

Graphic 3. Gini's coefficient $\gamma(\lambda)$ and the bipolarization indicator $\Delta(\lambda)\mu$



Remark 5. If the income X of the individuals of P has an exponential Exp1(λ) distribution, then the sign of the parameter λ is changed by inverting the socio-economic

status of the people in P (Proposition 15). Logically, the application of an antithetic transformation does not affect the bipolarization level in P. This intuitive perception is theoretically sustained by the relation $\Delta(\lambda) = \Delta(-\lambda)$ (see Proposition 12 and the symmetrical shape of the function $\Delta(\lambda)$ in graphic *G3*).

Usually we have $\gamma(\lambda) \neq \gamma(-\lambda)$ (Proposition 8, graphic *G3*, *Table 1*), since the concentration Gini index cannot measure, except for some particular situations, a polarization phenomenon.

Table 1

The values of the indices γ(λ) , Δ(λ) depending on λ (the formulas (8), (14))

λ	-100	-70	-50	-40	-30	20	15	10	9	8
γ(λ)	0.500	0.500	0.500	0.500	0.500	0.026	0.036	0.056	0.062	0.071
Δ(λ)	0.015	0.021	0.029	0.037	0.049	0.074	0.098	0.147	0.163	0.183
λ	7	6	5	4	3	2	1	-0.5	-0.25	0.000
γ(λ)	0.082	0.097	0.117	0.144	0.179	0.224	0.277	0.360	0.347	0.333
Δ(λ)	0.208	0.240	0.279	0.328	0.384	0.440	0.483	0.496	0.499	0.500
λ	0.25	0.5	1	2	3	4	5	6	7	8
γ(λ)	0.319	0.305	0.277	0.224	0.179	0.144	0.117	0.097	0.082	0.071
Δ(λ)	0.499	0.496	0.483	0.440	0.384	0.328	0.279	0.240	0.208	0.183
λ	9	10	15	20	30	40	50	70	100	
γ(λ)	0.062	0.056	0.036	0.026	0.017	0.013	0.010	0.007	0.005	
Δ(λ)	0.163	0.147	0.098	0.074	0.049	0.037	0.029	0.021	0.015	

6. CONCLUDING REMARKS

In practice, the pauperization and the socio-economic phenomena of bipolarization are generally strongly interconnected, but they are not identical. The classical Gini coefficient $\gamma_*(X)$ measures the "concentration" poverty level in the population P and the index $\Delta_*(X)$, proposed in the papers [20], [21], shows a bipolarization property of the X values.

The results presented in the previous sections confirm that both coefficients, $\gamma_*(X)$ and $\Delta_*(X)$, are suitable measures for different aspects of the social reality (compare, in the graphic *G3*, the shape of the functions $\gamma(\lambda)$, $\Delta(\lambda)$ or analyse the monotony of the values in *Table 1*).

An Exp1(λ) , $\lambda \in R$, family of distributions was selected to model the income variation in the individuals of the population P. Moreover, the indicators $\gamma(\lambda)$, $\Delta(\lambda)$ vary in a continuous manner (Proposition 13). This property permits us to simulate adequately the income dynamics in P.

In the last years, in Romania it is perceptible the presence of a strong economic bipolarization trend. This aspect could not be practically emphasized by using only the classical Gini measure. Therefore, it is insufficient to describe the socio-economic reality, in time and in space, based exclusively on the poverty concentration coefficient $\gamma_*(X)$.

A parallel study concerning the variation in the bipolarization indicator $\Delta_*(X)$ or any other polarization indices is absolutely necessary, too (see [7], [11], [26]).

The present paper proved that, in general, it is inadequate to use the Gini coefficient $\gamma_*(X)$ for measuring the bipolarization degree inside the population P (Remarks 2-3; graphic *G3*).

Thus, we recommend applying also the $\Delta_*(X)$ indicator to emphasize a grouping phenomenon in P (Remarks 4-5). In fact, the Gini index $\gamma_*(X)$ estimates only the "poverty level" for a given population.

References

C. D'Ambrosio, (2001), “Household characteristics and the distribution of income in Italy - An application of social distance measures”, *The Review of Income and Wealth*, 47 (1): 43-64.

A.B. Atkinson, (1970), “On the measurement of inequality”, *Journal of Economic Theory*, 2: 244-263.

F. Bourguignon, (1979), “Decomposable income inequality measures”, *Econometrica*, 47: 901-920.

Alain Chateauneuf, Thibault Gajdos and Pierre-Henry Wilthien, (2002), “The principle of strong diminishing transfer”, *Journal of Economic Theory*, 103: 311-333.

F.A. Cowell, (1980), “On the structure of additive inequality measures”, *Review of Economic Studies*, 47: 521-531.

F.A. Cowell, S.P. Jenkins, (1995), “How much inequality can we explain ? A methodology and an application to the United States”, *The Economic Journal,* 105: 421-430.

Joan Esteban, Debraj Ray, (1994), “On the measurement of polarization”, *Econometrica*, 62 (4): 819-852.

Joan Esteban, Debraj Ray, (1999), “Conflict and distribution”, *Journal of Economic Theory*, 87: 379-415.

J. Foster, A.K. Sen, (1997), *On economic inequality,* Clarendon Press, Oxford.

J. Gastwirth, (1975), “The estimation of a family of measures of economic inequality”, *Journal of Econometrics*, 3: 61-70.

Carlos Gradin, (December 2000), “Polarization by sub-populations in Spain”, 1973-1991, *Review of Income and Wealth*, 46 (4): 457-474.

N.C. Kakwani, (1993), "Statistical inference in the measurement of poverty", *Review of Economics and Statistics*, 75 (3): 632-639.

Christian Kleiber and Samuel Kotz, (2003), *Statistical size distributions in economics and actuarial sciences,* John Wiley & Sons - Interscience, New Jersey.

J.O. Lanjouw and P. Lanjouw, (2001), "How to compare apples and oranges - Poverty measurement based on different definitions of consumption", *Review of Income and Wealth*, 47 (1): 25-42.

G. Pyatt, (1976), "On the interpretation and disaggregation of Gini coefficient", *The Economic Journal,* 86: 243-255.

Friedrich Schmid, (1993), "A general class of poverty measures", *Statistical Papers*, 34: 189-211.

A.F. Shorrocks, (1980), "The class additively decomposable inequality measures", Econometrica, 48: 613-625.

A.F. Shorrocks, (1984), "Inequality decomposition by population subgroups", Econometrica, 52:1369-1385.

Poliana Ştefanescu, Ştefan Ştefanescu, (2001), "Extending the Gini index to measure inequality and poverty", Economic Computation and Economic Cybernetics Studies and Research, 35 (1-4) : 145-154.

Poliana Ştefanescu, Ştefan Ştefanescu, (2006), "The polarization index for bounded exponential distributions", Economic Computation and Economic Cybernetics Studies and Research, 40 (3-4): 211-218.

Poliana Ştefanescu, Ştefan Ştefanescu, (2006), "The properties of a polarization index for bounded exponential distributions", UPB - Scientific Bulletin, Series A: Applied Mathematics and Physics, 68 (4): 9-20.

Bernd Wilfling, (1996), "Lorenz ordering of power function order statistics", Statistics & Probability Letters, 30: 313-319.

M. Wolfson, (1994), "When inequalities diverge", American Economic Review, 84 (2): 353-358.

M. Wolfson, (1997), "Divergent inequalities - Theory and empirical results", The Review of Income and Wealth, 43 (4): 401-422.

S. Yitzhaki, (1994), "Economic distance and overlapping of distributions", Journal of Econometrics, 61: 147-159.

X. Zhang, R. Kanbur, (2001), "What difference do polarization measures make? - An application to China", Journal of Development Studies, 37: 85-98.

Claudio Zoli, (1999), *"Intersecting generalized Lorenz curves and the Gini index", Social Choice and Welfare*, 16: 183-196.