3. SECOND ORDER DYNAMICS OF ECONOMIC CYCLES

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Abstract

The monthly data of the industrial production in Romania after the structural discontinuity occurring at the end of 1989 show an under-damped oscillatory behavior that suggests an evolution of second order systems excited by a step function. Since this behavior is well described in control systems we are doing what the literature usually calls a reversed engineering of the data in order to identify the specific parameters for the economic cycle of industrial production. The final goal is to determine the second order differential equation that may be associated to the economic process related to industrial production evolution. This paper is a first contribution that opens an alternative approach to describe the economic dynamics.

Keywords: business cycles, simulations, nonlinear methods, transition economies, mathematical methods

JEL Classification: C02, E32, E37

1. Introduction

Throughout the development of economic thinking the cycle was a debated notion, stemming from the dynamics of the surrounding world and the way the human society was coping with it. Nature is providing us with cycles be they circadian, seasons, years, etc. The basic activities that we develop such as production of goods and information have the notion of cycle imbedded in our way to consider them. We are frequently talking about production cycles, life cycles, as well as transaction cycles and include the notion in our description of economic activity, even starting to consider it already understood without a need to be mentioned, e.g., when we talk about depreciation or the time value of money there is a cycle behavior implied that is not explicitly mentioned.

Approaching the economic system behavior from the cycle viewpoint we may consider the findings of scientists such as Kondratiev (1981), Grubler and Nowotny (1990), etc.,

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that point towards the existence of cycles in the evolution of prices, in penetration of technologies, etc.

Various dynamic models have been built in recent years for analyzing specific conditions for the occurrence of cycles in the evolution of economic systems. We refer to Gandolfo (1997) for a good synthesis of the main models of: Kaldor, Goodwin and Lotka-Volterra general model, and to Purica (2002) for cyclical behavior in energy systems and their interaction with the economy. The approach by authors above either identifies cyclical behavior from analyzing various sets of data or builds dynamic models and analyzes the conditions of parameters that generate cyclical behavior.

Previous analysis of business cycles in Romania treated the subject in a more standard manner. Caraiani (2004) analyzed the stylized facts of business cycles in Romania by using the HP filter, which was applied on the main macroeconomic time series. The filtered series were analyzed on the basis of the second order moments, like cross-correlations and standard deviations.

A more complex approach was done in Caraiani (2007a), who tested a real business cycle model for the case of Romania with respect to the predictions of second order moments relative to the stylized facts in real data. While the real business cycles model solution results in a nonlinear system, he used the linearized version to discuss the model. Caraiani (2007b) extended the previous results to the case of a new Keynesian model, but the model was again linearized.

As for nonlinear analysis of the macroeconomic dynamics, several significant contributions were made by Albu; see, for example, his study on the nonlinear and complex relationship between unemployment and inflation using small dynamic models, in Albu (2001).

Our approach is to go backward from a set of data showing a behavior that suggest a given model and determine the parameters of that model. Thus, the associated second order differential equation becomes personalized for the evolution of the industrial production in the Romanian economy after the change having occurred in 1990.

2. Methodology

We review here the methods and techniques that we will use in the next section. First, we review the standard business cycles approach, and then we present a nonlinear framework.

2.1. The Standard Business Cycles Analysis

The problem of the optimal method to filter a data series is still much debated. In the context of the business cycles analysis, this problem is much more important as the entire analysis that follows is founded on the results of the filtering method.

Canova (1998) underlined very well the dilemma by showing that the problem of detrending contains two controversies. On the one hand, there is a lack of consensus regarding the definition of the economic cycles. Although at a theoretical level there is an agreement in identifying the cycles as deviations from the trend of the economic process, in the applied work there is no similar agreement about the properties of the trend and the relation of the trend with the cyclical component of the series.

The second controversy refers to the dilemma of choosing a pure statistical approach and a pure founded economic approach. The opinion that the selection of variables and the report of results should be based on an economic theory is often met. Following this argument, an economic theory was proposed to be constructed for serving as a foundation for an economic based time series decomposition.

There are a few methods of extraction which are mostly applied in the reference literature. We present in what follows several of the most important filters and we test them on the Romanian economy. The discussion follows the direction suggested by Canova (1998), namely the formal presentation of each filter and a description of the advantages and disadvantages of each filter.

The simplest and oldest method of filtering is that of the polynomial filter. We assume that the trend and the cycle of the logarithm of the series are not correlated; x_t is a deterministic-type process which could be estimated by using a polynomial function.

Thus, we can decompose the y_t series as:

$$y_t = x_t + c_t \tag{1}$$

$$x_t = \sum_{i=0}^{K} \alpha_i t^i \tag{2}$$

$$c_t = u_t \tag{3}$$

where: x_t is the trend, c_t denotes the cycle, K is a finite integer number, while u is a stationary process.

The most common choice is K=1, so that:

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 t + u_t \\ \psi(L)u_t &= \phi(L)\varepsilon_t \end{aligned} \tag{4}$$

where: ε_t is a white process, ψ(L) is a stationary polynomial AR, while φ(L) is a MA type polynomial process.

One of the problems associated to this filter is that, due to its simplicity, probably it distorts the dynamics of the cyclical component.

Another often used filter is that of the first order differentiating. The fundamental hypothesis of this model is that the trend is a random walk with no drift; the cycle is stationary while the two components are not correlated. We can thus represent y_t as:

$$y_t = y_{t-1} + \varepsilon_t \tag{5}$$

where: the trend is defined as $x_t=y_{t-1}$, while we estimate the cyclical component by:

$$\hat{c}_t = y_t - y_{t-1} \tag{6}$$

One clear advantage of this approach is that it represents a natural transformation of the logarithm of many of the macroeconomic time series, through which we get the growth rate. One important criticism to this method is that it leads to too noisy cycles.

The Hodrick and Prescott (1980) filter (HP, hereafter) has a double justification, both an intuitive one and an economic one. Through the HP filter we extract in an optimal way a tendency which is both stochastic and smooth, while uncorrelated with the cyclical component. In order to estimate the trend we minimize the following expression:

$$\min_{[x_t]_{t=1}^{T}}[\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=2}^{T}((x_{t+1} - x_t) - (x_t - x_{t-1}))]^2 \quad (7)$$

where: T is the number of observations, λ>0 is a parameter which penalizes the variation in the trend.

As λ grows, the degree of correction of cycles grows, while $\hat{x}_t$ becomes smoother.

The HP filter suffered many criticisms from several perspectives. Among the most important studies which approached the problem of the disadvantages of the HP filter we find: Harvey and Jaeger (1993), Cogley and Nason (1995).

Harvey and Jaeger (1993) criticized the HP filter because it induces spurious cycles. Moreover, the cyclical components obtained through this filter are distorted, and thus they could lead to false conclusions regarding the short-run relations between the macroeconomic series.

Cogley and Nason (1995) extended the previous research regarding the effect of the HP filter. They analyzed the effects of the HP filter on the time series, which are difference-stationary and trend-stationary. They showed that when we apply the HP filter to integrated processes, HP can generate periodicities in the economic cycles as well as co-movements even if they are not present.

In spite of the deficiencies, the HP remained the principal method for cyclical components in the reference literature. A significant contribution to the application of the HP filter was provided by Ravn and Uhlig (2002).

Conventionally, the researchers choose a value for λ of 14000 for monthly frequencies, 1600 for quarterly frequencies and of 100 for annual data. The key value is the quarterly frequency, one from which the values for the other frequencies are chosen. The choice of λ=1600 corresponds to the choice of a certain definition of business cycles, namely to the choice of a hypothesis regarding the duration and volatility of the cycles. Thus, Hodrick and Prescott (1980) derived the value of λ for quarterly data by showing that λ can be interpreted as the variance of the cyclical component divided by the variance of the trend, under the hypothesis that both the cyclical component and the second order difference of the trend component are normally distributed. Formally we can write that:

$$\lambda = \frac{\sigma_c^2}{\sigma_x^2} \quad (8)$$

where: σ_x are σ_c are the standard deviations for the trend and cycle component.

Based on the characteristics of the dynamics of American economy, Hodrick and Prescott derived the value of λ for quarterly data as:

$$\lambda=5^2/(1/8)^2=1600. \quad (9)$$

In their study, Ravn and Uhlig (2002) showed that the HP filter should be adjusted by the fourth power of the frequency of observations. Thus, the most proper choices for the λ parameter are:

λ=1600 for quarterly data;

λ=129600 for monthly data;

λ=6.25 for annual data.

2.2. A Nonlinear Framework

As indicated in the literature, the behavior of many oscillatory systems can be approximated by the differential equation:

$$\frac{d^2y}{dt^2} - 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = \omega_n^2 u \tag{10}$$

The positive coefficient ω_n is the natural (un-damped) frequency while ζ is the damping ratio.

The Laplace transformation (L) of y(t) when the initial conditions are zero is

$$Y(s) = \left[\frac{\omega_n^2}{s^2 - 2\zeta\omega_n s + \omega_n^2}\right] U(s) \tag{11}$$

where: U(s)=L(u(t)).

The poles of the transfer function Y(s)/U(s) = $[\omega_n/(s^2-2\zeta\omega_n s+\omega_n^2)]$ are:

$$s = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \tag{12}$$

Note that:

1. if $\zeta>1$, both poles are negative and real;
2. if $\zeta=1$, both poles are equally negative and real ($s=-\omega_n$);
3. if $-1<\zeta<1$, the poles are complex conjugated with negative real parts $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$;
4. if $\zeta=-1$, the poles are imaginary and complex conjugate ($s = \pm j\omega_n$);
5. if $\zeta<-1$, the poles are in the right half of the s-plane.

Of particular interest in our work is case 3, representing an under-damped second-order system. The poles are complex conjugates with negative real parts and are located at

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \tag{13}$$

or

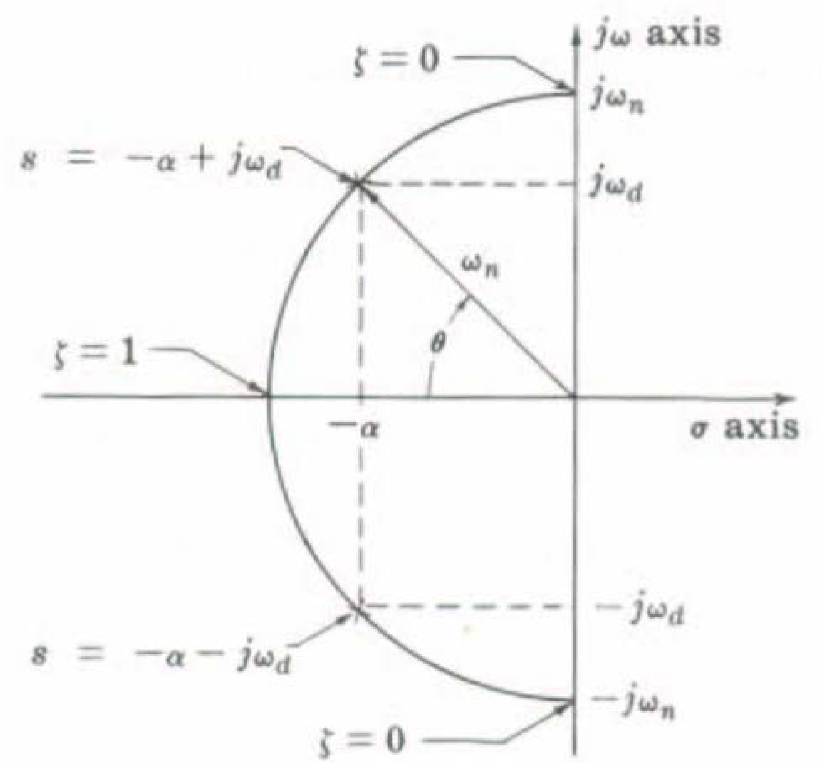
$$s = -a \pm j\omega_d$$

where: $1/a = 1/\zeta\omega_n$ is the time constant of the system and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency of the system. For fixed ω_n the figure below shows these poles as a function of ζ, $-1<\zeta<1$.

The locus is a semicircle of radius ω_n. The angle θ is related to the damping ratio by θ= arccos ζ.

Figure 1

Locus of the poles



Source: DiStefano, III, A.R. Stuberrud and I.J. Williams (1990).

The characteristic function of such an equation is

$$y(t) = (1/\omega_d)e^{-at}\sin(\omega_d t + \phi) \tag{14}$$

where: ϕ=arctan(ω_d/a).

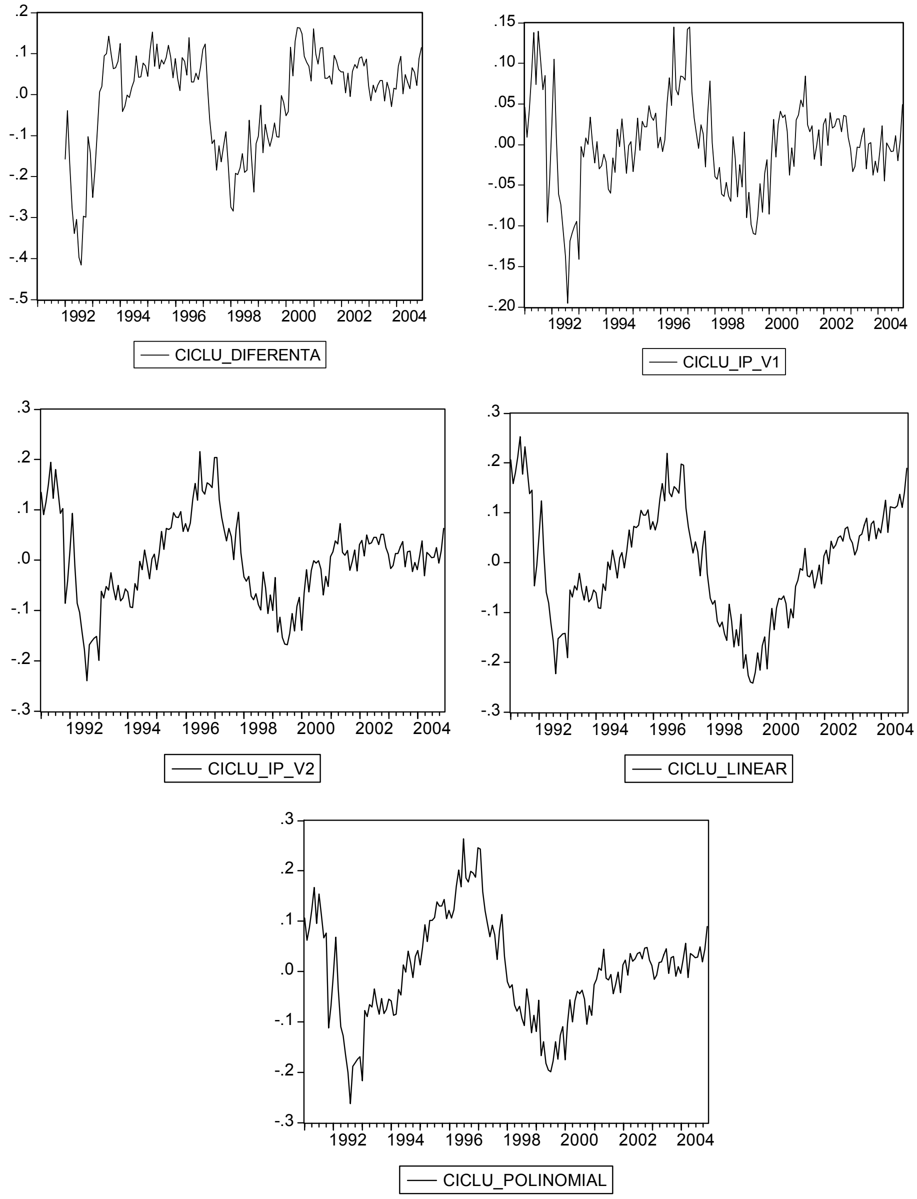
3. Computations and Numerical Results

3.1. Business Cycles Analysis in the Standard Framework

In the following, we apply a few methods to extract and analyze the business cycles in Romania. We will compare the different methods in order to reveal which one of them characterizes properly the business cycles during the transition period. The time series used is the industrial production from January 1991 to December 2004 at a monthly frequency. The initial series was logged, and then deseasonalized using the Census X12 procedure in Eviews. We applied afterwards the above described filters to this transformed series.

Figure 2

Business Cycles in the Industrial Production



Source: Authors' computations.

Figure 2 shows the results of the application of different filters to the Index of Industrial Production, namely the difference filter for "ciclu_diferență", the HP filter with λ=14400 for "ciclu_ipv1", the HP filter with λ=129600 for "ciclu_ipv2", the linear filter for "ciclu_linear", and the polynomial filter for "filtru_polinomial".

As a first evaluation criterion, we can use the volatility, as measured by the standard deviation. The cycle extracted from the annual GDP produces a volatility of 6.71. At the same time, in the developed economies, the volatility of the economic fluctuations is 2-4%. For the emergent economies, the study of Agenor, McDermott and Prasad (2000) quantifies a higher degree of volatility than for the industrialized countries, but also a higher variability of this coefficient (for the countries in their sample they got standard deviations between 4 and 12%). The filters we used show a range for the standard deviations from 4.22 to 11.22%. Naturally, we eliminate the approaches that lead to extreme volatility values, like the cycle produced by differentiating or the cycles produced by either the linear filter or the polynomial filter.

A second criterion is related to the similarity with the fluctuations in GDP. It is expected that the extracted cyclical component correctly reveals the two recessions in the Romanian economy, both the initial shock in the 1991-1993 period, and the 1997-1999 recessions, but also the expansion periods between 1993-1996 and 1999-2004, respectively.

The HP filter in both versions correctly identifies the phases of the economic cycles, but some differences appear for the post-2000 period, a period of sustained economic growth. We consider that the second version of the HP filter, with λ=129600, fits better the growth period after the year 2000, while the first version falsely induces the impression of a peak in the 2001 period, which is actually denied by the real data evidence. Moreover, the superiority of the second version of the HP filter is also obvious from the theoretical point of view, as discussed in Ravn and Uhlig (2002).

Table 1

Characteristics of the cycles produced by different filters

Filter type	Volatility (%)	(1)	(2)	(3)
Differentiating	12.1	0.87	0.80	0.75
H-P version 1	5.65	0.72	0.61	0.54
H-P version 2	8.7	0.87	0.81	0.77
Linear Filter	11.22	0.91	0.86	0.82
2nd degree polynomial filter	10.3	0.91	0.86	0.83
Ciclu PIB anual	6.18	-	-	-

Source: Authors' computations.

It follows that, as the above table shows, two of the filters appear as the most proper for the analysis of the economic fluctuations in Romania, based on the criteria of volatility, persistence, duration or amplitude: the HP filter for λ=14400 and λ=129600. However, both for comparative reasons (as in the literature the HP filter with λ=129600 is used) and also theoretical reasons presented above - especially the higher persistence obtained from the HP filter in the second version - we decided that this HP filter fits best our purposes.

In order to identify in a correct way the business cycles in Romania, we use the standard method in the literature. The cycles are identified starting from the four characteristic phases:

- The expansion, as the economy is characterized by a sustained growth;
- The peak, when the economy reaches the maximum level, relative to the current business cycle;
- The contraction, when the economic activity enters in a phase of decreasing economic growth rates;
- The trough, when the economy reaches the lowest level, relative to the whole duration of the cycle.

In order to be identified as business cycles, the fluctuations must fulfill two important conditions: the duration and the persistence condition. The recent literature considers that a business cycle has a duration between 15 and 84 months. As for the persistence, this is measured through the amplitudes of the expansion (from trough to peak), and of the recession (from peak to trough). Currently, the normal amplitude is considered to be of at least two quarters or six months.

A fundamental problem in dating the business cycles is the choice of turning points, namely the peaks and troughs through which the economic activity passes from an expansion (contraction) phase to a contraction (expansion, respectively). In order to select the turning points, we first consider the local minimum and maximum points, after which we eliminate those points which lead us to the identification of spurious cycles (those cycles that do not respect the conditions of duration and persistence). Table 2 presents the results for Romania.

Table 2

Dating Business Cycles in Romania

Peaks	Duration from Peak to Peak	Troughs	Duration from Trough to Trough
May ‘91		August ‘92	
January ‘97	68 months	July ‘99	83 months
December ‘04	95 months		

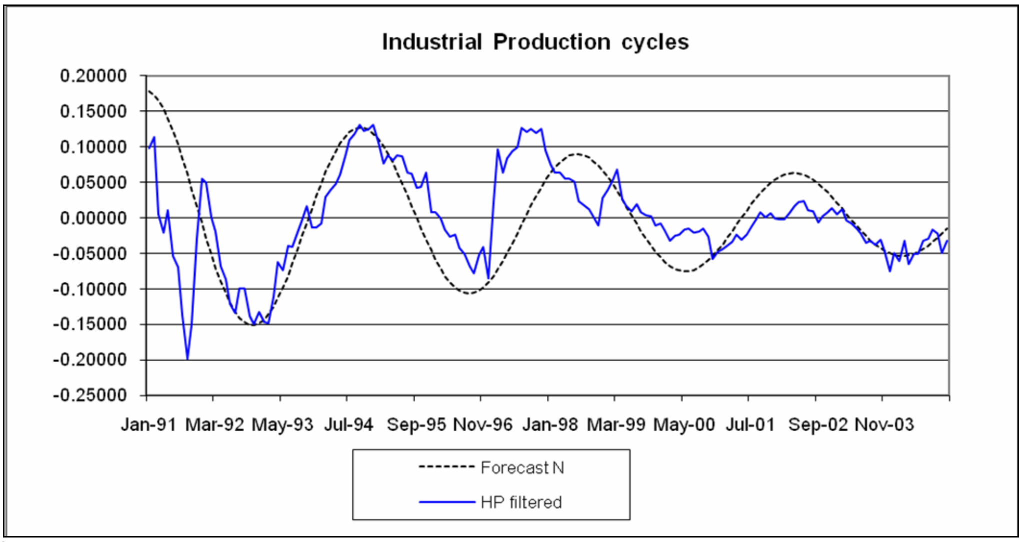
Source: Authors’ computations.

3.2. A Nonlinear Approach to Business Cycles Analysis

The situation of Romania after the perturbation in late 1989, which triggered a transient in the following years resulted in some values showing an oscillatory behavior. What we do next is to try some reverse engineering on this behavior (exponentially damped sinusoidal) to extract the parameters of interest for the process (i.e. the natural frequency, the time constant and the damping constant). The evolution of the data is given in Figure 3 below:

Figure 3

Damped oscillatory behavior of the Industrial Production



Source: Authors' computations.

Further on, we give the characteristic function for such a behavior with the parameters resulted from fitting the real data and the calculation of the main parameters.

Table 3

Parameters from fitted data above

Parameters	A	ϕ	T_d	1/a	ω_d
Calibrated vales	0.18	1.52	45.3	130	0.13

The other parameters can be computed as shown below:

$EXP(-ax)*A*SIN(2*\pi/T_d*x+\phi)$ -characteristic function

where: $\omega_n=A*\omega_d$ undamped natural frequency;

$\phi=\arctan(\omega_d/a)$ phase;

$\omega_d=2*\pi/T_d$ damped natural frequency;

$\zeta_i=a/\omega_n$ damping ratio.

Deduction of the values of interest is given below:

$$a=\zeta_i*\omega_n$$

$$\omega_d=\omega_n\sqrt{(1-\zeta_i^2}$$

$$a=\omega_d\left(\left(\frac{\zeta_i}{1-\zeta_i^2}\right)\right)$$

$$\zeta_i = \sqrt{\left(\left(\frac{1}{1+\left(\frac{\omega_d}{a}\right)^2}\right)\right)}$$

ζ_I = 0.0553

ω_n= a/ξ_i = 0.1389

T_n= 45.2304 months (years n = 3.7692)

Since the damping constant is a positive sub-unit one, the poles of the transfer function in the Laplace transformation space are complex conjugated with negative real parts; thus the system has an under-damped oscillatory behavior.

The formulae and values above suggest that the evolution of the industrial production has a cyclical behavior with a natural frequency,. □$_n$ =0.138 (i.e. a period of 45.2 months) and also, has a transition period with a time constant, 1/a, of 130 months. Also, there is a damping constant, □$_i$ =0.055, whose value is positive and smaller than one.

Thus, the associated differential equation of the process is given below with the denotation above and with y = industrial production.

$$\frac{d^2y}{dt^2} - 0.015\frac{dy}{dt} + 0.019y = 0.019u$$

Comparing the results from the two methods we reached the following conclusions:

- The nonlinear process can simulate the dynamics of the industrial production cycles in a fair way;
- The frequency of the nonlinear process is around half the actual measured cycles, so that the nonlinear process simulates in an acceptable way the contraction and the expansions phases, but cannot reveal the asymmetries of the real economic cycles.

4. Conclusions

The specific equation determined above for the evolution of the industrial production in Romania after the change that occurred in 1990 represents an alternative way to consider the evolution of the main economic parameters.

Starting with the data that show a behavior associated with a specific second order oscillatory model and working our way back to identify the coefficients of the associated differential equation have lead us to describe the observed behavior in a coherent way.

The applications presented in this paper offer an opportunity to represent an economy as a general oscillating system, where various characteristic parameters are associated with specific dynamic equations. This is a first step for a broad potential systematic analysis.

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