1. CONTROL AND CRISIS IN A CITY SEEN AS A REACTOR OF ECONOMIC TRANSACTIONS

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Abstract

The distribution of persons in each economy by their income is similar to the Maxwell-Boltzmann equilibrium distribution encountered in various physical systems. The transactions the persons are making to buy things that make them survive (in all sort of ways) are making them 'poorer', diminishing the amount of money they have. The 'work and get paid' type of transactions are rebuilding the financial capacity of the persons. Describing this process as a diffusion equation, in a cylindrical geometry, results in a Bessel function $J_0(r)$ solution, which matches the density distribution of persons in Paris (as a typical circular pattern city). Moreover, a simple equation for the dynamic behavior of a city, on which a 365 days period is imposed, results in one week as the time after which persons have to be paid to restart transactions, in the case of prompt pay. Thus, the extension of the basic model of the city as a reactor of economic transactions, for the cases of control through insertion or absorption of money, lead to the description of the crisis with a strikingly good accuracy for the case of Romania in 2009. Moreover, considering the prompt and delayed pay case gives a change in the average time to replenish financial resources of persons from 7 to 30 days. Using neutron physics methods in describing the economic transactions environment opens an alternative view on the forecasting models of economic systems' behavior, and shows that the geographical dimension of a city is determined by the economic transaction behavior/environment in that city.

Keywords: nonlinear models, decision, financial crisis
JEL Classification: C3, C61, C62, D7, D87

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Introduction

This paper continues the analysis of the economic behavior of a city seen as a reactor of economic transactions (Purica, 2004) with the case that introduces the control of the dynamic regime. The new feature of the model consists in the analysis of the

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system's response to a reduction in or an addition of the amount of money available for making transactions. This case describes the crisis situation, especially focusing on questions such as 'what is the response to a reduction in money' or 'how much money a decision maker needs to inject into the system to generate a given response'. Below, we review the basic model and then, extend it to the case of the control dynamics.

The basic model

We start by considering that a person is in the situation of having spent his/her money and going to work to get more money. He/she gets paid and raises by a factor K (which may be greater or smaller than one) his/her financial capacity. Also, from his/her 'birth', for transactions, to his/her 'disappearance' it travels a length M as defined in Purica (2004) and Purica (2010).

If the number of persons is constant in time: $\partial n/\partial t = 0$, then the persons who pass, with money, through the transactions space will come directly from work or from diffusion and will 'disappear' – going back to work and gaining more money - after having spent their initial amount of money. Writing this balance, which is constant over time, as we said above, we have:

$$S + \lambda/3\ \nabla^2 I - (1/\lambda_a)\ I = 0$$

where: S are the persons that 'disappear' then go to work and then come back with new money. As per the consideration above:

$$S = K\ I/\lambda_a$$

where from

$$\nabla^2 I + 3(K-1)/(\lambda\lambda_a)\ I = 0 \quad \text{or, with } M^2 = (\lambda\lambda_a)/3$$

$$\nabla^2 I + (K-1)/M^2\ I = 0$$

We see that the coefficient of I depends only on the structure of the work.
If we put $B^2_m = (K-1)/M^2$ the equation becomes:

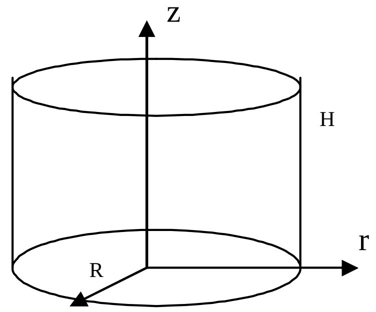
$$\nabla^2 I + B^2_m\ I = 0$$

This type of diffusion equation (Soutif, 1962) has to satisfy zero limit conditions on the border of the transactions space, i.e. nobody goes outside the city to make transactions (this condition implies that the city limits act like a reflector for the transactions space, but we keep things simple for the moment). Under these conditions the geometry of the city gives increasing eigen values of B. We call B_g the minimum of these geometry determined values. The condition for the existence of solutions is then:

$$B_m = B_g$$

Based on the above relations, let us calculate the distribution of persons in a transaction space having a cylindrical geometry. If one looks at a map of great cities, several of them have a circular base (remember that we said that the height is taken as constant). We denote by R and H the dimensions of the cylinder (see Figure1)

Figure 1



and the diffusion equation becomes, in cylindrical coordinates (r,z) :

$$\partial^2 I/\partial r^2 + (1/r)\partial I/\partial r + \partial^2 I/\partial z^2 + B^2_g\, I = 0$$

We put: $I = Z(z) * T(r)$ and have :

$$(1/T)d^2T/dr^2 + (1/r)\, dT/dr + (1/Z)\, d^2Z/dz^2 + B^2_g = 0$$

Each of the two terms in Z and T must be independently constant and so we write:

$$-a^2 - b^2 + B^2_g = 0$$

The term in T becomes:

$$d^2T/dr^2 + (1/r)\, dT/dr + a^2\, T = 0 \quad \text{or :}$$

$$d^2T/d(ar)^2 + (1/(ar))\, dT/d(ar) + T = 0$$

while the one in Z is :

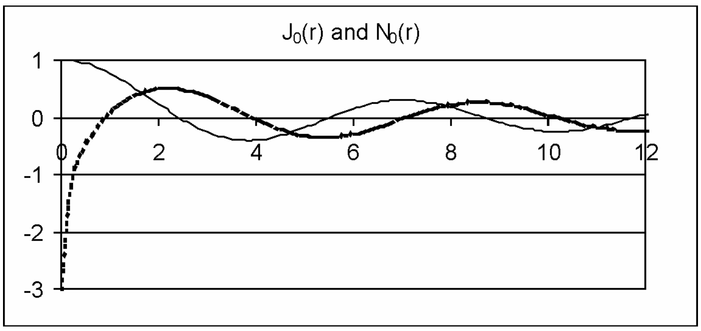
$$d^2Z/dr^2 + b^2\, Z = 0$$

We have, for T, the Bessel differential equation of order 0. The solution is given by:

$$T = A\, J_0(ar) + C\, N_0(ar)$$

The two Bessel function of order 0, i.e. J_0 and N_0 are showed in Figure 2. We see that N_0 is infinite for r=0 which is physically inacceptable to describe the distribution of persons in the city, hence C=0.

Figure 2



The equation in Z is easily solved to: $Z = A'\cos(bz)$, so we have :

$$I = A''\, J_0(ar)\, .\, \cos(bz)$$

The limit conditions are:

1. $\cos(bH/2) = 0$, wherefrom $b = \pi/H$;

2. $J_0(aR) = 0$; the first zero of the Bessel function J_0 is at $aR = 2.405$ hence $a = 2.405/R$.

Thus, the Laplacian is determined by:

$$B^2_g = (2.405/R)^2 + (\pi/H)^2$$

From the above calculations, we obtain that the radial distribution of persons in a city (a transactions space) is given by $I = J_0(2.405\ r/R)$. What would be the critical dimension R of a city that is characterized by a given B^2_m?

From the equality $B^2_g = B^2_m$ we can determine the value of R. First, we calculate the minimum critical volume. From the above relation we may write:

$$R^2 = ((2.405)^2 H^2) / (B^2_m H^2 - \pi^2)$$

The volume V results:

$$V = \pi\ ((2.405)^2 H^3) / (B^2_m H^2 - \pi^2)$$

Deriving in relation to H we find the minimum volume V for:

$$B^2_m H^2 = 3\ \pi^2$$

With these values of H and R, we have the minimum critical volume given by:

$$V_c = 148/ B^3_m$$

The volume may also be written as a function of R (considering the relation between R and H resulting from the above relations, i.e. $R = (2.405/\pi/\sqrt{2})\ H = 0.54\ H$)

$$\pi\ ((2.405)^2 (R/0.54)^3) / (3\ \pi^2 - \pi^2) = 148/ B^3_m \text{, i.e.}$$

$$5.85\ R^3 = 148 / (K-1)^{1.5} * M^3$$

Considering that we may measure M^2 from the relation:

$$M^2 = \lambda^2/3 \quad \text{where} \quad \lambda = - 2 / \ln(I/I_0) = 2 / \ln(I_0/I)$$

we have:

$$R^3 = 38.97 / ((K-1)^{1.5} (\ln(I_0/I))^3)$$

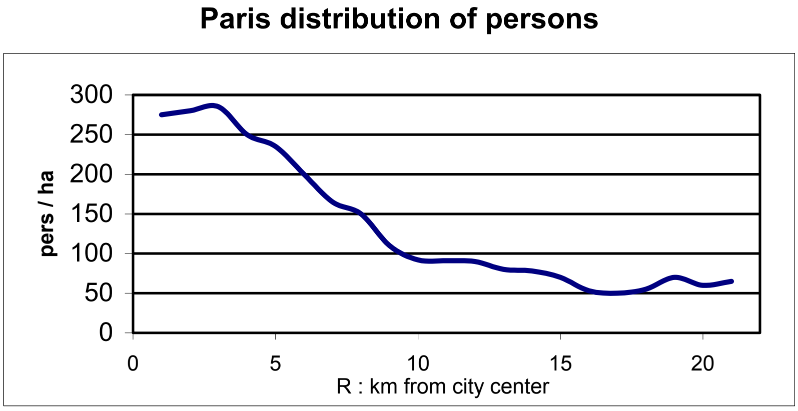
We consider for the moment that H has no significance for the transaction space and consider only the radial behavior (distribution) of persons.

Application for Paris

To have a value for R we need measured values of I_0 and I, which are specific to various cities. If we take Paris, for example, and consider that the distribution of the number of the persons doing transactions is proportional to the density distribution of the population of the city shown in Figure 3, below, (World Bank 2000), and that K is 1.02 (World Bank 1999), then, we may determine the critical dimension of the city, from the formula above (where we take I_0=290pers/ha and I=50pers/ha) (World Bank 2000), as being:

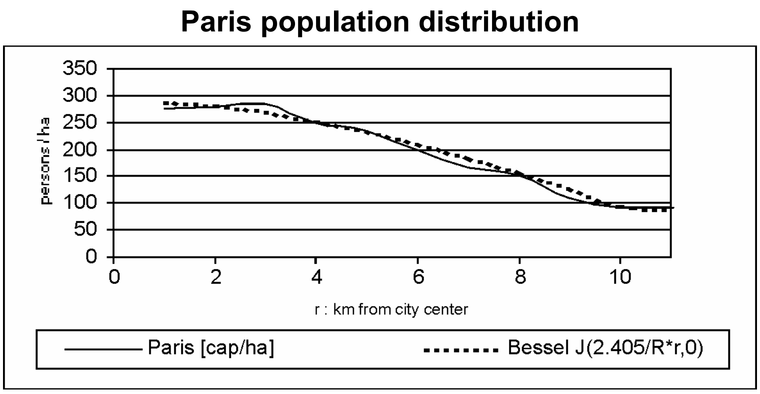
R = 13.64 km.

Figure 3



We are now in the position to draw the radial distribution of persons in Paris resulting from the calculations above and to compare it with the real data of distribution of persons, (World Bank 2000). The Bessel function is scaled by I_0 = 290 persons/ha. A striking matching of the real and the calculated distributions are obtained, as shown in Figure 4.

Figure 4



Actually, the above figure is only showing the distribution for the first 11 km from the city center (the critical radius is actually giving the distance at which the theoretical distribution becomes zero). The size of Paris is greater than the critical radius, as it may be seen in Figure 4 above. However, we should notice that after approximately 11 km the distribution is dropping very smoothly.

Situation of Bucharest

We will consider now the same formulae as above for Bucharest, where: K=1.04; I_0=8107.6 persons/km^2 and I=174 pers/km^2. The surface of the municipality is 238km^2, which results in a geometric radius of 8.7 km.

From the formulae above, one may calculate the critical radius for the economic transactions and the reflector radius, whose total sum is: 5.42 km. This value is significantly smaller that the geographical one. We can easily see that one source of difference could be the large population density that is considering all the population of the city. If one takes into consideration only the employed persons, with I_0=2987.1 pers/km^2, then the economic radius and reflector thickness becomes 7.8 km. This value is much closer to the geometrical radius. The difference occurring above is suggesting that the disposable income is not yet at the level to justify a population of the city being larger than the level sustained by the economic activity, which leads to further work on topics such as the impact of ageing on the economic activity, the hidden work not accounted for and the disposable income and savings policy.

Dynamic evolution equation

The calculations made above for the solution of the diffusion equation assumed that it was a stationary state situation, i.e. $\partial I/\partial t = 0$. Let us see now a simple dynamic behavior situation.

We start with the diffusion equation:

$$\partial I/\partial t = S + (\lambda/3)\, \nabla^2 I - \Sigma_a I$$

and assume that all transaction environment evolves in phase – no local perturbations of I will be considered. Under these conditions we may write:

$$I(x,y,z,t) = I(t)\, \iota(x,y,z)$$

The function $\iota(x,y,z)$ is changing very little for a slow variation in the steady state distribution; thus, we have at any time :

$$\nabla^2 \iota(x,y,z) = - B^2 \iota(x,y,z)$$

We may, thus, replace the value of ∇^2 I in the dynamic diffusion equation with the one above. The remaining equation will be in I(t) only since the source S is also proportional to $\iota(x,y,z)$. After dividing by $\iota(x,y,z)$ we get :

$$dI/dt = S - (\lambda/3)\, B^2 I - \Sigma_a I$$

We will make now another consideration which is important when we consider the dynamic case: not all the persons are paid in the same day (prompt pay). We made this assumption for the steady state case, but, in the dynamic situation the fact that a proportion β of persons is paid later (delayed pay) is not negligible, as we shall see bellow.

The source of prompt paid persons is:

$$S = (1-\beta)\, K\, \Sigma_a I$$

The source of delayed pay persons is given considering that the concentration of persons having a delayed pay is C(I) (we take only one group of delay paid persons for simplicity). The decrease in this concentration, in time, is done with a constant λ_C.

The variation of this concentration is:

$$dC/dt = \beta K \Sigma_a I - \lambda_C C$$

Considering a solution of the type: $C = C_0\exp(\omega t)$ and $I = I_0\exp(\omega t)$, where ω is a real or imaginary constant, we have:

$$C = K\Sigma_a I (\beta\lambda_C/(\omega+\lambda_C))$$

that is the source of delayed pay persons.
With the above expressions for the sources, and after some algebra, the diffusion equation becomes:

$$\theta\omega = K(1-\beta) - 1 + K(\beta\lambda/(\omega+\lambda_C))$$

or with $\delta K = K-1$:

$$\delta K/K = \theta\omega/K + (\beta\omega/(\omega+\lambda_C))$$

We denoted by θ the time used by a person, having spent his money doing transactions, till the moment he goes to work to rebuild his financial capacity. This is the disappearance (death) of the person (although it does not mean physical death in the real world). We may say that every person is entitled to be paid only once, but, after it gets paid it becomes a 'different person' which, then, repeats the process.

The way we defined I(t) shows that there is a period of time variation of the transactions environment, which is given by:

$$T = 1/\omega$$

From the expression above, we obtain a second order equation for ω with two real roots, one positive and small and the other negative and relatively big. The evolution of I is determined only by the positive exponential, i.e. after a short time the negative ω exponential component becomes negligible.

We come back to the case of Paris, for which we have: K=1.02; δK=0.02; and consider the limit value of $\beta = 0$ (all persons paid at the same date). We get from the expression above:

$$\theta = \delta K/\omega = \delta K * T$$

Let us consider that the period T of the city is 365 days (one year for which the values of K are actually calculated), then, the value of θ is:

$$\theta = 0.02*365 = 7.3 \text{ days.}$$

Is 7 days the time after which the persons in Paris have to come back to work and replenish their source of money? This result is serving one more time to enhance the need for an in-depth program to develop the details of the physics of economic transactions systems (see also Purica, 2004).

Absorption of money – Crisis and control

Considering the above case of a cylindrical (circular) city, we are interested in the variation of the factor K on the actions of insertion or absorption of money in the system. The action takes place in the entire city, but we consider it concentrated in the centre of the city, covering a region of radius R_0.

Let us consider first the case of prompt payments. In the case of no action, as above, the equation of the economic reactor is given in the cylindrical geometry (r,z) by:

$$\partial^2 I/\partial r^2 + (1/r)\partial I/\partial r + \partial^2 I/\partial z^2 + B^2_g I = 0$$

whose solution is of the form:

$$I = Z(z) * T(r)$$

where: $Z(z)= A \cos(bz)$, the longitudinal contribution of the flux of persons with money, $b=\pi/H$;

$T(r)= C J_0(ar)$, with a the radial contribution, $a=2.405/R$;

$$B^2_g=a^2+b^2$$

Let us consider we are absorbing money from the system. In this case, the limit conditions of the solution $T(r) = CJ_0(ar)+C'N_0(ar)$ are changed leading to a new value of a and, consequently, of B_g.

We will treat the problem as a perturbation case. Thus we set:

$a=a_0+\delta a$ and since $B^2_g = (K-1)/M^2$ we have that $\delta K/K=-2M^2a_0\delta a$

Now the limit conditions are written as:

Zero flux at external radius i.e. $0= CJ_0(aR)+C'N_0(aR)$, wherefrom we get C'.

Zero flux at the interior radius i.e. $0= CJ_0(aR_0)+C'N_0(aR_0)$, wherefrom with the value of C' from above we have:

$$0== C[J_0(aR_0)-(J_0(aR)/N_0(aR))N_0(aR_0)].$$

From the expression above, the new value of δa results (see Appendix 1 for the case of a close to a_0) and introducing it into expression $\delta K/K=-2M^2a_0\delta a$ we get the decrease in K from the absorption of money as:

$$\delta K=-2M^2a_0\delta a$$

and with $a_0= 2.405/R$ and $M^2 = \lambda^2/3$, where $\lambda = - 2 / \ln(I/I_0) = 2 / \ln(I_0/I)$ we have:

$$\delta K= \pi 2.405\ \lambda^2/(3R^2)[0.116+\ln(1/(2.405R_0/R))]^{-1}$$

which, with $I_0=290$ and $I=50$ results in $\lambda=1.138$ and with $R_0= 0.5$, $R=13.64$ and $M^2=\lambda^2/3$ gives the reduction in K as:

$$\delta K=0.00231$$

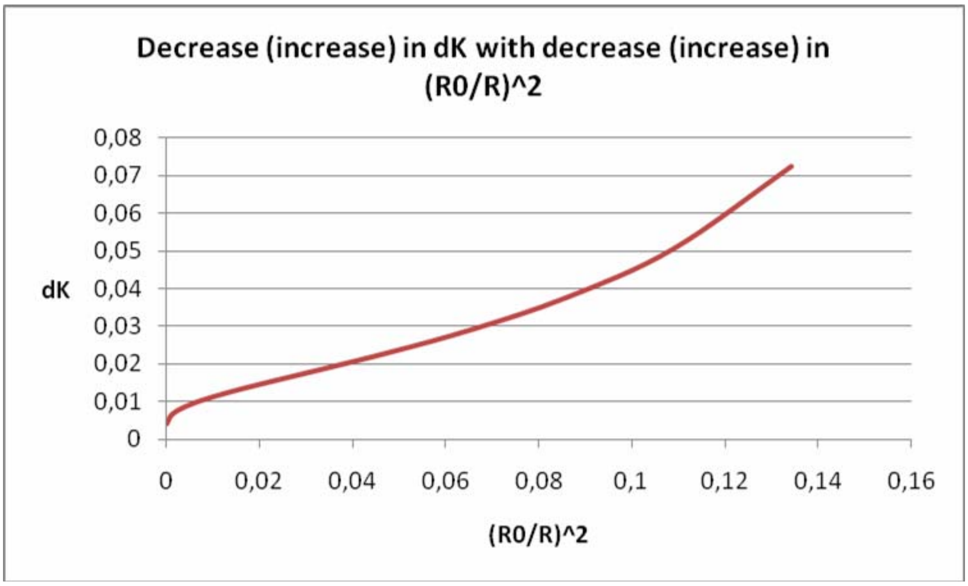
Considering the stationary K is 1, it results that an absorption assumed concentrated in the centre of the city in an area of R_0 (the adsorbed money is proportional to R_0^2/R^2, i.e. in our case 0.13%) gives a decrease by 0.23% in K. Since K is a measure of GDP, we have thus a reduction in GDP. It should be mentioned that the absorption of money may be associated with a reduction in activity (of money) in the area of the city, which is considered to be concentrated in the center. The size of this area relative to the overall size of the city is a measure of the potential crisis (seen as a reduction in money). Reversely, an insertion of money also considered as occurring from an area in the centre of the city is producing an increase in K, i.e. in GDP.

Figure 5 gives the variation of δK with the (R_0/R)^2 for small δa.

Considering that the amount of money in the system is measured by (R_0/R)^2 and the GDP is measured by δK, one may see that for a decrease by approx. 13% in the amount of money the GDP decreases by approx. 7%. Considering that the amount of money is of the order of 170 billion euros in a given economy, a reduction of about 22 billion euros will lead to a decrease in GDP by 7%. Reversely, injecting that amount of money would lead to an increase in GDP.

Figure 5

Variation of δK with the (R_0/R)^2 for small δa



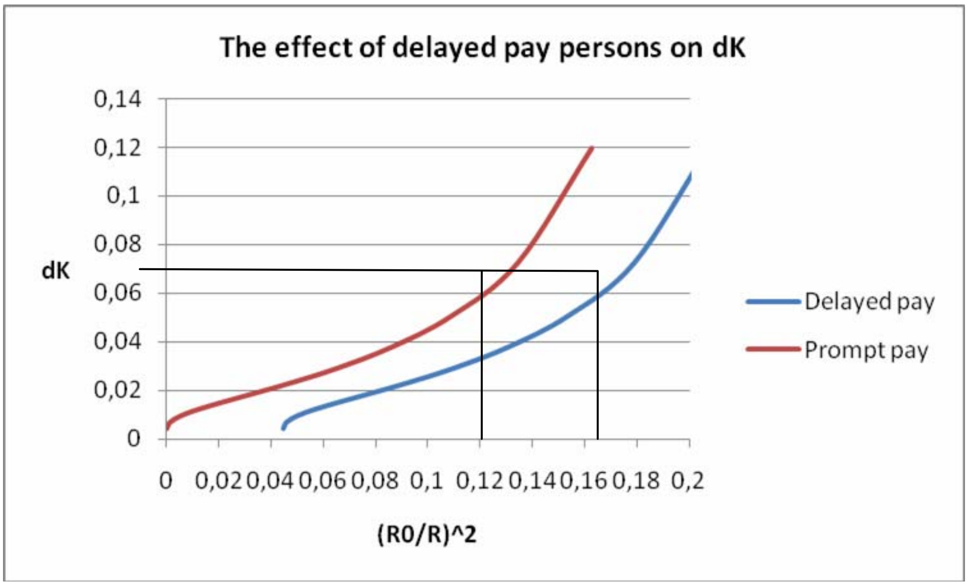
These figures are strikingly close to the situation of Romania at the end of 2009, when the GDP dropped by 7% and the country had borrowed about 20 billion euros from IMF and the EU to compensate for the decrease in money in the economy due to the financial crisis. We think a thorough analysis may be started here by applying the type of model presented above in describing the dynamics of countries' economies in crisis situations.

Data from CIA World Factbook (https://www.cia.gov/library/publications/the-world-factbook/geos/ro.html) for Romania show in 2009 a reduction by 7.1% in GDP as compared to the previous year (to 161.5 billion USD), a reduction by 17.45% in narrow money (to 27 billion USD). The larger reduction value of the money in the economy sends us to the approximation made where we have considered only prompt pay persons. The effect of delayed pay persons is actually to slow down the decrease in dK with the decrease in money. The difference of 4.45% in (R_0/R)^2 is actually repositioning the curve as shown in Figure 6, below.

Actually, the effect of delayed pay persons is seen in a change in dK/K where along with the term in $\theta\omega/K$ an extra term of the form $\beta\omega/(\omega+\lambda_C)$ is added. Considering ω=1/365 days (0.00274) and K=1 and knowing from above that the extra term in dK=0.086, we have for β=0.1 (10% delayed pay persons) that λ_C= 0.017. There is a substantial difference from λ (1.13) of the prompt pay persons.

On a different line, this triggers a change in θ that brings the value of the person pay lifetime from 7 days (prompt pay) to 30 days (in the case of prompt and delayed pay).

Figure 6



Conclusions

If a city with persons doing transactions, in which they lose or gain money, is regarded as an environment where transactions (reactions) are taking place among enterprises and persons, then, the critical dimensions of the city may be calculated, as well as the distribution of persons in the city. The 'experimental' data from Paris – a city that complies with the condition of a uniform distribution of transaction offers (enterprises) – result in a very good matching of the population distribution data with the calculated theoretical distribution, in a cylindrical geometry, i.e. a J_0 Bessel function. A similar calculation was done for Bucharest.

The dynamic diffusion equation is providing also a value for the average time to replenish financial resources of the persons in Paris, which is very close to one week. This time value results if a period of 365 days is considered for the mentioned city, in the case when all persons have a prompt pay in the same day.

Further on, by considering the situation of a reduction in money in the economic interactions reactor that was assumed to be concentrated in the center of the cylindrical geometry city, we have determined the reduction in the magnitude that measures the GDP. Reversely, an insertion of money triggers an increase in GDP. In the graphs of this correlation the values for the crisis situation in Romania for the year 2009 are found. Moreover, the extension of the basic model for the city as a reactor of economic transactions for the cases of control through insertion or absorption of money lead to the description of the crisis with a strikingly good accuracy for the case of Romania. Moreover, considering the prompt and delayed pay case gives a change in the average time to replenish financial resources of persons from 7 to 30 days.

Obviously, there is more to analyze in relation to the dynamic behavior of such systems, especially related to the domains of stable regimes of the parameters. Also, data should be redefined in the framework of this model, such as to go beyond the usual economic parameters, allowing more subtle and accurate forecasting of evolution.

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Appendix 1

Considering that a is close to a_0, we expand the Bessel Functions in series up to order 1:

$$J_0[(a_0+\delta a)R]=J_0(a_0R)+(dJ_0/d(aR))R\delta a$$

The first term is zero since J_0 is zero at R. On the other hand we know that:

$$dJ_0(x)/dx=-J_1(x)$$

where from we have:

$$J_0[(a_0+\delta a)R]=-J_1(aR)R\delta a$$

Also:

$$N_0[(a_0+\delta a)R]=N_0(a_0R)-N_1(a_0R)R\delta a$$

In the tables of Bessel Functions one may see that close to $J_0=0$ the value of N_1 is very small so the second term above is negligible.

Inserting the two expanded functions in the formula for the flux we have:

$$0=J_0(aR_0)+(J_1(a_0R)/N_0(a_0R))R\delta aN_0(aR_0)$$

This formula may be further simplified noticing that:

$-J_1/N_0$ is close to 1 in the vicinity of the value that makes $J_0=0$;

R_0 is small and that $J_0(aR)$ is close to 1;

And, finally that:

$$N_0(aR_0)=-(2/\pi)[0.116+\ln(1/a_0R_0)] \text{ (asymptotic formula)}$$

The equation for the flux becomes:

$$0=1-(2/\pi)R\delta a[0.116+\ln(1/a_0R_0)]$$

If we insert δ a into the formula for $\delta K/K$, we have:

$$\delta K= \pi a_0M^2/R[0.116+\ln(1/aR_0)]^{-1}$$

This formula is further used in the main text.