9. Fractal Analysis of the Gold Market in China[1]

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Abstract

Based on fractal theory, this paper studies the fluctuation characteristics of the gold market in China. Using R/S analysis and fractal dimension analysis this paper demonstrates that the gold market possesses fractal characteristics and determines the length of the aperiodic circulation on the basis of the nonstationarity of the daily yield series of gold validation. With the MF-DFA method, the paper confirms the multi-fractal nature of the gold market and analyzes the factors that influence the multi-fractal nature. Based on the research, forecasts of the fractal interpolation curve can be brought into agreement with the original sequence by constructing a fractal interpolation model. Therefore, the paper provides intuitive evidence regarding the price movement predictions of gold market for investors and managers.

Keywords: gold market; fractal; R/S analysis; MF-DFA
JEL Classification: G17, C53

1. Introduction

As the theoretical foundation of modern financial research, the basic assumption of the efficient market hypothesis is constructed on the basis of "rational" analysis framework, in which market price reflects a timely and accurate response to new information. However, the theory's applicability has been questioned, because in the financial market phenomena appear that the theory finds difficult to explain and that are contrary to assumptions. In the 1960s, the fractal theory was introduced as a new branch of nonlinear science, which was used to analyze irregular or non-smooth

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phenomena in nonlinear systems. As a result of its system characteristics, such as self-similarity and long-term memory, fractal theory better explains the characteristics and behavior of complex systems and of the financial market and is more general as compared to the efficient market hypothesis. Thus, fractal theory is applied extensively.

The idea of fractals and fractal theory was initially introduced by Mandelbrot (1967). In the 1990s, researchers gradually expanded fractal theory to physical, geographical, financial, agricultural and other natural and social fields to analyze complex phenomena. Peters (1994) proposed the fractal market hypothesis on the basis of Mandelbrot's fractal theory. He analyzed the operation mechanism of the market function from a new perspective and described the characteristics of capital market products, which opened a new direction for modern financial research. At the beginning, research based on fractal theory used the single fractal feature as the primary test, mainly adopting the method of rescaled range analysis (R/S analysis) presented by Hurst (1951) and the fractal dimension method proposed by Hausdorff (1918). For in-depth study, which presented the microscopic local fractal features, Mandelbrot introduced the multi-fractal method. This method used multi-fractal detrended fluctuation analysis (MF-DFA), as proposed by Kantelhardt *et al*.

Domestic and foreign scholars have sought to apply fractal theory in the economic field. Their achievements are focused on the nonlinear characteristic test for the stock market, foreign exchange and futures. Yin-Wong Cheung and Kons Lai (1995), Kawaka K. Oponga *et al*. (1999), Robert F. Mulligan (2004), Enrico Onali and John Goddard (2011) investigated the US and European stock markets, among others, and analyzed the fractal features of the relevant market stock price time series. Jonathan Batten and Craig Ellis (1996) and Gordon R. Richards (2000) confirmed the exchange rate time series using fractal structure. In China, Zhou Yan and Yu Ke (1993), Hu Xueming and Song Xuefeng (2003), Yu Jun, Fang Aili and Xiong Wenhai (2008) conducted a fractal analysis of Chinese stock prices and the stock index. Huang Feixue and Zhao Yan (2008), Zhu Xinling and Li Peng (2011) and Song Xia (2012) examined the fractal characteristics of the RMB foreign exchange market. Hou Xiaohong and Li Yizhi (1999), Huang Guangxiao and Chen Guojin (2006) and Sun Wei (2009) analyzed the fractal characteristics of various futures prices on the basis of fractal theory.

As a rare metal, gold exhibits both commodity and currency investment qualities. Recently, against the background of economic globalization, financial crises have recurred. To avoid inflationary losses, gold, because it represents an efficient method of asset hedging and risk prevention, has been keenly sought after by investors. However, the gold market is subject to the Noah Effect. The price yield follows the leptokurtosis distribution with a long-term correlation and demonstrates clearly nonlinear characteristics. These anomalies cannot be explained only by traditional efficient market theory. Rather, explanations of the gold market's complexity should draw support from nonlinear methods and theories. In comparison with the stock and foreign exchange markets, domestic and overseas scholars have paid little attention to the gold market. Wang Zheng and Liang Linfang (2006) and Huang Tengfei, Li Bangyi and Xiong Jixia (2011) determined that the London gold market price series and the daily closing price of the Shanghai gold exchange gold are nonlinear by using the R/S analysis and the Hurst exponent to prove the single fractal character of gold prices. Cao Jianjun and Gu Rongbao (2011) used multi-fractal detrended fluctuation analysis to examine the daily yield of the London and Shanghai gold markets. Their results suggested that both markets had multi-fractal characteristics. Wang Ji and Ma Junhai (2011) used the Hurst exponent, fractal dimension analysis and the Lyapunov index to analyze the non-linear structure of American gold market prices and established an RBF artificial neural network to predict the future trend of the gold price.

The research described above primarily applies fractal theory to stock, foreign exchange and futures markets. Most scholars are inclined to analyze the characteristics of one of the fractal or multi-fractal. Therefore, systematic analyses and forecasts in this field are lacking in China. Thus, starting with a nonlinear approach, this paper comprehensively examines the gold market fractal and multi-fractal structure while investigating the market on the macro level and specific fractal characters on the micro level. Based on this approach, a fractal interpolation model to forecast gold prices is constructed, which lends the paper's results practical value.

The paper is arranged as follows: The second section presents the fractal theory and introduces the fractal and multi-fractal methods. The third section examines the fractal characteristics of the gold daily return in China, whereas the fourth section examines the multi-fractal characteristics, comprehensively investigates the gold market's systematic characteristics, describes its specific analytical structure and provides a reason for nonlinear change. The fifth section uses the fractal interpolation model to predict the future movements of the gold price. The sixth section is the conclusion.

2. The Method

2.1 The Single Fractal Model

After summarizing unstructured geometry, Mandelbrot introduced the fractal theory. He noted that fractal theory as a relationship referred to the similarity between the whole and the parts, has self-similar characteristics, is overall stable and locally random and cannot be described by traditional Euclidean geometry. According to the theory of the Mandelbrot fractal test, this paper proposes a new test method that integrates fractals and multi-fractals.

The fractal test, which is an integrated method, comprehensively applies the R/S analysis and the fractal dimension analysis from different angles to test empirically the fractal characteristics of time series. Based on the Hurst exponent and non-periodic cycle length advanced by the R/S analysis, the long-term memory of the time series is further verified by disruptive inspection.

2.1.1 The Rescaled Range Analysis

The rescaled range analysis (Rescaled Range, R/S) is a widely used non-parametric statistical method applied in fractal analysis. The procedure is as follows:

The time series $\{R_t\}$ of length N must be divided into A (integer) groups of continuous subseries. Subseries of length n are labeled as $I_a (a = 1,2,3,..., A)$. The data of the subseries are $R_{k,a}(k=1,2,3,...,n)$. After finding the mean (e_a) of the subseries, the cumulative deviation of internal elements from the mean in each part is calculated:

$$X_{k,a}=\sum_{i=1}^{k}(R_{i,a}-e_a), k=1,2,3,...,n \quad (1)$$

The range (R_a) is found:

$$R_a=\max(X_{k,a})-\min(X_{k,a}), k=1,2,3,...,n \quad (2)$$

The standard deviation of each subseries is calculated:

$$S_a=\sqrt{(1/n)\sum_{k=1}^{n}(R_{k,a}-e_a)^2} \quad (3)$$

The mean value of the rescaled range is computed for all of the subseries:

$$(R/S)_n=(1/A)\sum_{a=1}^{A}(R_a/S_a) \quad (4)$$

The Hurst Exponent

The basic form of the Hurst exponent is as follows:

$$(R/S)_n=C*n^H \quad (5)$$

where: R/S is the rescaled range, n is the length of the time intervals, C is a constant, and H is the Hurst exponent in equation (5).

The functions of the Hurst exponent are established as follows:

Determine whether the sequence is random. When H equals 0.5, the time series is an independent and identically distributed Gaussian random sequence. When H is different from 0.5, the series follows fractional Brownian motion or is a biased random walk.

Determine the continuity and the anti-continuity. If $0<H<0.5$, the time series has an anti-continuity characteristic, which means that the time series is more likely to have an upward (downward) movement if the sequence had a downward (upward) movement during the past and that the series reflects a mean reversion characteristic and has strong volatility. The closer H is to 0, the stronger the anti-continuity is. If $0.5<H<1$, the time series has a continuity characteristic. The closer H is to 1, the stronger the continuity is. This continuity is reflected in long-term memory, which means that the time series is not random but is fractal.

Measure the correlation of the time series. The correlation index C is a description of the influence of today on the future, and its expression is as follows:

$$C=2^{2H-1}-1 \quad (6)$$

where: H is the Hurst exponent in equation (6).

When the series has a normal distribution and satisfies the random walk, H=0.5, C=0, which means that the increments of the time series are not relevant to one another. If the series is anti-continuous, $0<H<0.5$, $-0.5<C<0$, the increments have a negative correlation. If the series is continuous, $0.5<H<1$, $0< C<1$, and the increments have a positive correlation.

Length of Aperiodic Circulation

The length of aperiodic circulation is the time series required for the initial condition to disappear completely. Aperiodic circulation is usually applied to the mean value. It is also used this way in the course of this study through an R/S regression analysis diagram. In addition, the V-statistic can be used for determination:

$$V_n = \frac{(R/S)_n}{\sqrt{n}} = \frac{Cn^h}{\sqrt{n}} = Cn^{h-0.5} \tag{7}$$

If H=0.5, the R/S statistic is scaled by the square root of time, the process is random and independent, and the $\log V_n$-$\log(n)$ diagram is a level line. If $0.5<H<1$, the movement of R/S is faster than that of the square root of time, and the $\log V_n$-$\log(n)$ diagram is upwardly inclined. If $0<H<0.5$, the movement of R/S is slower than that of the square root of time, and the $\log V_n$- $\log(n)$ diagram is downwardly inclined. During the actual calculation, more than one potential point of circulation may appear. Therefore, to select an optimal circulation length, a comprehensive consideration of the above two methods is required.

Disruption Test

The disruption test is a calculation of the Hurst exponent of the original and disrupted time series. Through an observation of the difference between the two H values, if the original time series is a random walk, the H value of the disrupted independent series should be the same. If the original data exhibit long-term memory, the disruption test influences the diffusion of the original information and influences the long-term memory to a certain extent. Therefore, the newly generated H value should be closer to 0.5 or lower.

2.1.2 The Dimension Analysis

The fractal dimension analysis characterizes the fractal characteristics as a quantitative indicator and depicts the non-smoothness, irregularity, complexity and other characteristics of the time series. The dimensions include the Hausdorff, box, and information dimensions. Because the Hausdorff dimension is relatively complex and the application is narrower, other methods are more widely used during the actual calculation. This paper primarily uses the box dimension analysis.

The primary steps of the box dimension analysis are as follows：Assume a black box with side lengths (δ) of N/2, N/4, N/8, …, N/m （$m = \mathrm{int}[\log_2(N)]$, where N is the number of observations of the sample. Boxes of different lateral lengths are used for measurement, and the number of boxes required is different. Therefore, a different number $N(\delta)$ is obtained. Calculate the logarithmic value of $N(\delta)$ and $1/\delta$, much the same as R/S analysis depicts the corresponding points in the $\log N(\delta)$-$\log(1/\delta)$ double logarithmic coordinate system. The value of the slope is obtained by fitting the points, and the value indicates the dimensions of the box.

2.2 The Multi-fractal Model

Some systems are not only a single fractal structure but also a compound, multiple single fractal structure, in which singularity exists at the micro level. Thus, the multi-fractal model can reflect the complexity and more local fractal features. The multi-fractal, also known as the complex fractal, results from the superposition of multiple single fractals, and every scale of the single fractal and the fractal dimension are not identical. The multi-fractal depicts the local scale of distribution in the subset. By studying the multi-fractal, the complex object can be divided into different parts of singular degrees for study. Then, to master the overall volatility, the local structure characteristics are examined.

Multi-fractal detrended fluctuation analysis (MF-DFA) is the most commonly used method to study multi-fractal features. It is primarily applied to test whether there is a multiple fractal structure in a non-stationary time series.

Given a time series $R(t), t = 1,2,...N$, the MF-DFA proceeds as follows:

Calculate the cumulative deviation of the given time series:

$$\mu(t) = \sum_{i=1}^{t} [R(i) - \bar{R}], t = 1,2,...N \tag{8}$$

Among them, $\bar{R}$ is the average of the time series $R(t)$.

Deviation sequences can be divided into N_s non-overlapping intervals, the length of each interval is s, $N_s = \text{int}(N/s)$, and N may not be divided exactly by s. Therefore, dividing the end of the sequence as the initial segmentation yields $2N_s$ intervals.

In each interval, using the least squares fitting $\mu_v(i)$, obtain the residual sequence as follows:

$$\varepsilon_v(i) = \mu_v(i) - \tilde{\mu}_v(i), 1 \le i \le s \tag{9}$$

The fitting function $\tilde{\mu}_v$ is a polynomial of degree m.

Calculate the detrended fluctuation equation of each interval:

$$[F(v,s)]^2 = \frac{1}{s}\sum_{i=1}^{s}[\varepsilon_v(i)]^2 = \frac{1}{s}\{\sum_{i=1}^{s}[\mu[N-(v-N_s)s+i] - \tilde{\mu}_v(i)\}^2, v = N_s+1,...,2N_s \tag{10}$$

$$[F(v,s)]^2 = \frac{1}{s}\sum_{i=1}^{s}[\varepsilon_v(i)]^2 = \frac{1}{s}\sum_{i=1}^{s}\{\mu[(v-1)s+i] - \tilde{\mu}_v(i)\}^2, v = 1,2,..., N_s \tag{11}$$

Then, calculate the fluctuation equation for the entire period:

$$F_q(s) = \{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,s)]^{q/2}\}^{1/q} \tag{12}$$

Among them, when q = 0, for the entire period of the fluctuation equation $F_0(s) = \exp\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(v,s)]\}$. When q > 0, the volatility function $F_q(s)$ value depends on the value of the large deviation $[F(v,s)]^2$, and when q < 0, the volatility function $F_q(s)$ value depends on the deviation of the size of the small fluctuations $[F(v,s)]^2$.

In the form of power-law increases, $F_q(s) \sim s^{h(q)}$, $h(q)$ is a generalized Hurst exponent. When the value of s changes, $h(q)$ is obtained. To obtain a high stability of $F_q(s)$, the s value is generally not more than a quarter of N.

Rewrite $F_q(s) \sim s^{h(q)}$ as $F_q(s) = Ms^{h(q)}$, and calculate the logarithm on both sides of the equation simultaneously, resulting in $\log F_q(s) = \log M + h(q)\log s$. The different lengths of the subintervals correspond to different fluctuation function values. Return to $\log F_q(s)$ by about $\log s$, and the value of $\log F_q(s)$ can be estimated.

2.3 Construction of the Fractal Interpolation Model

2.3.1 The Iterative Function System

The Iterative Function System (IFS) was introduced by M.F. Barnsley in 1985 and represents an important part of fractal geometry. An iterative function consists of two parts: a metric space (X,d) and a limited compression mapping set $\{\omega_i : X \to X\}$ $0 \le d_i < 1, i = 0,1,2,\cdots,N$, which has the longitudinal compression factor d_i and is defined in metric space. Define the conversion $\omega_i : H(X) \to H(X)$ on $(H(X), h(d)$.

$$\omega(B) = \bigcup_{i=1}^{N}\omega_i(B), \forall B \in H(X) \tag{13}$$

Among them, $\forall B \in H(X)$ is a complete compression mapping with the compression factor d. Then

$$h(\omega(A), \omega(B)) \le dh(A,B), \forall A,B \in H(X) \tag{14}$$

Additionally, there must be only one fixed point on the map (invariant set) P. P is also referred to as the attractor of IFS:

$$P = \omega(P) = \bigcup_{i=1}^{N}\omega_i(P) \tag{15}$$

At any point $B \in H(X)$, $P = \lim_{i \to \infty} \omega^i(B)$.

2.3.2. The Fractal Interpolation Method

Fractal interpolation is a method that constructs an iteration function system (IFS) to transform its attractor into an interpolation function. The dataset $\{(x_i, y_i) \in R^2, i = 0,1,2,...,N\}$, $x_0 < x_1 < x_2 < \cdots < x_N$, $y_i = f(x_i) \in [a,b]$ is known. Now, construct an IFS $\{R^2, \omega_i : 1,2,...,N\}$, which makes the attractor and the interpolation function $f(x)$ the same. Among them, ω_i meets the type under affine transformation :

$$\omega_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_i(x) \\ F_i(x,y) \end{bmatrix} = \begin{bmatrix} a_i & 0 \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} \tag{16}$$

Among them, a_i, c_i, d_i, e_i, f_i are real numbers. Additionally, the article satisfies :

$$\omega_i \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix}, \omega_i \begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{17}$$

d_i indicate the parameters for freedom, and $|d_i| < 1$. Because of the fluctuations of the affine transform of the fractal interpolation function value, it is defined as the longitudinal compression factor. The parameter influences the complexity of the curve. If the longitudinal compression factor is known, four parameters of expressions are obtained:

$$\begin{cases} a_i = (x_i - x_{i-1}) / (x_N - x_0) \\ e_i = (x_N x_{i-1} - x_0 x_i) / (x_N - x_0) \\ c_i = \left[y_i - y_{i-1} - d_i \left(y_N - y_0 \right) \right] / (x_N - x_0) \\ f_i = \left[x_N y_{i-1} - x_0 y_i - d_i \left(x_N y_0 - x_0 y_N \right) \right] / (x_N - x_0) \end{cases} \tag{18}$$

After the various parameters are determined, an iterative algorithm is used to obtain the affine IFS. Additionally, the affine fractal interpolation fitting curve is obtained. The interpolation of the curve fitting increases with the increase in the number of iterations, and the attractor that is the image of the fractal difference function points is finally obtained by interpolation. Then, the original curve approximation can be fully closed.

Mazel and Hayes introduced two methods for calculating the longitudinal compression factor: the analytic method and the geometry method. For a time series in the financial markets, the analytical method is adopted to calculate its longitudinal compression factor. The specific calculation formula is as follows:

For the selected set of fractal interpolation data points $\{(x_i, y_i) : i = 0,1,2,...,N\}$, set

$$\xi_i = (x_N - x_i) / (x_N - x_0) \tag{19}$$

$$A_i = y_i - [\xi_i y_0 + (1-\xi_i) y_N], B_i = y_m - [\xi_i y_{i-1} + (1-\xi_i) y_i] \quad (20)$$

x_m is the abscissa (and y_m is the ordinate) of the interpolation point (x_m, y_m) in the preceding formula, and the abscissa satisfies $x_m \le a_i x_i + e_i \le x_{m+1}$. Then, the longitudinal compression factor can be calculated using the following formula under the condition of the minimum mean square error.

$$d_i = (\sum_{i=1}^{N} B_i A_i) / \sum_{i=1}^{N} A_i^2, i = 1, 2, ..., N \quad (21)$$

2.3.3 Construction of a Fractal Interpolation Model

Because there is a substantial quantity of financial time series data, a certain number of interpolation points must be selected as the basis of the interpolation when using fractal interpolation models to predict prices. Interval equal points are selected as the interpolation points according to the equidistant interpolation method. For the selected N interpolation data from the entire dataset, the space between each datum is m. That is, an interpolation datum for every m is adopted from the first data, which represents the interpolation point set $\{(x_i, y_i)\}: i = 1, 2, ..., N$, where x_i is the sampling time interval form and y_i are the corresponding values of the sample points. The iteration function system is obtained using the affine fractal interpolation method. The IFS attractor is the image of the solved fractal interpolation function $f: [x_0, x_N]$ based on the interpolation point.

To obtain the price value of an unknown interval $[x_N, x_{N+1}]$, the data are used to extrapolate with the help of trend extrapolation after the interpolation graphics are obtained. Trend extrapolation is a method that determines the future based on past and present trends.

The affine transformation is defined on the interval $[x_N, x_{N+1}]$ $\omega_{N+1}(x, y) = (L_{N+1}(x), F_{N+1}(x, y))$ and satisfies

$$\begin{cases} L_{N+1}(x) = x_N + (x_{N+1} - x_N)(x - x_0) / (x_{N+1} - x_0) \\ F_{N+1}(x, y) = c_{N+1} x + d_{N+1} y + f_{N+1} \end{cases} \quad (22)$$

The affine transformation coefficients c_{N+1} and f_{N+1} are obtained by equation (22).

$$\begin{cases} c_{N+1} = [y_{N+1} - y_N - d_{N+1}(y_{N+1} - y_0)](x_{N+1} - x_0) \\ f_{N+1} = [x_{N+1} y_N - x_0 y_{N+1} - d_{N+1}(x_{N+1} y_0 - x_0 y_{N+1})] / (x_{N+1} - x_0) \end{cases} \quad (23)$$

The longitudinal compression factor is acquired according to $d_{N+1} = \sum_{i=1}^{N} d_i / N$. According to the self-similar fractal characteristics and the change trend, the expectation value of the price is given and predicts the number on the interval $[x_N, x_{N+1}]$ by the previously described model and method.

3. The Fractal Inspection of the Gold Market in China

3.1 Data Selection and Inspection

The fractal test requires a substantial quantity of data. However, the monthly data are limited and cannot provide the correctness and rigor required by the fractal analysis. Thus, to test gold market fractal characteristics in China, this paper selects the AU99.99 closing price data of the Shanghai gold exchange between October 30, 2002 and November 30, 2012 (2324 days in total). The specific data distributions are shown in Figure 1.

Figure 1

AU99.99 closing price fluctuation at Shanghai gold exchange

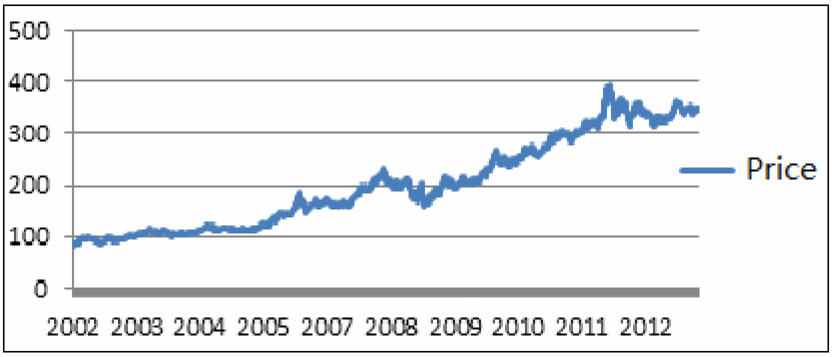


Figure 1 shows that the gold price presents an upward trend and that the data are not stable. Thus, to test the statistical characteristics of the data, a stationarity method is required. Gold's daily return calculation formula is as follows:

$$R_t = \ln(P_t / P_{t-1}) = \ln P_t - \ln P_{t-1} \quad (24)$$

Among them, R_t is the logarithmic yield of Au99.99 in the phase of t and P_t, P_{t-1} is the price of the t stage and the t-1 stage.

Apply the stationarity test for yield. After applying the JB normal distribution test and the nonlinear test, it can be found that the daily yield series of the gold price is stable, that the series does not display normal distribution characteristics, and that there is a nonlinear structure.

3.2 The R/S Analysis

1. The R/S analysis of the gold daily return sequence

If a rescaled range analysis of the gold daily return at the Shanghai Gold Exchange is performed, the rescaled range logarithmic sequence log(R/S)n can be obtained. The double logarithmic figure of log(R/S)-log(n) and the V statistic figure are shown as follows:

Figure 2
log(R/S)-log (n) double logarithmic of daily yield

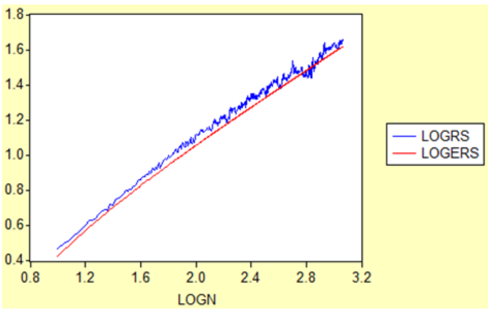
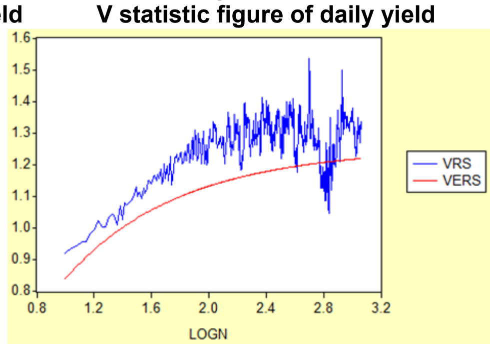


Figure 3.
V statistic figure of daily yield



One may see from Figure 2 that a breaking point appears where log (n) =2.05. However, this point is not highly obvious. On the contrary, the V-log (n) curve in Figure 3 indicates the breaking point more evidently, and the V statistic is upward-sloping, which means that R/S changes at a rate more rapid than the square root of time, and that the time sequence has persistence. Additionally, when log(n)=2.05 (even though this value is not the maximum), the V statistic continues to increase before the point, but exhibits a flat trend after the point, which indicates that the point is a mutation point. These observations suggest the dissipation time of the “memory” of the gold price information at the Shanghai Gold Exchange. The system starts to lose dependence on the initial conditions or information after 112 days on average.

If the point where n=112 is taken as the border, a regression analysis of the sequence before and after the point can be performed, and the Hurst exponent in Table 1 can be obtained.

The Hurst exponent is the regression index of Log(R/S) with respect to log (n). The regression indices pass the t test and display a good degree of fit. The slope rate according to regression analysis on the interval before the point is the real value of the Hurst exponent. Obviously, this value is larger than 0.5, which indicates that the daily yield series has a long-term correlation and a fractal structure. The index obtained by the regression analysis of the interval after the point is 0.49648, which is close to 0.5, and indicates that after a non-circulated period of 112 days, the daily yield sequence tends to be a random walk. The index before the point is $0<C<1$, which suggests that each incremental is positively related.

Table 1

R/S Regression analysis of the daily yield of gold

R/S regression	$10 \leq n \leq 112$	$113 \leq n \leq 1161$	$10 \leq n \leq 1161$
Hurst exponent	0.6367	0.4965	0.5259
t-Statistic	171.7259	163.3139	255.7929
Prob.	0.0000	0.0000	0.0000
R-squared	0.9967	0.9622	0.9827
F-Statistic	29490	26671	65430
Probability	0.0000	0.0000	0.0000
Correlation C	0.2087	-0.0049	0.0365

2. Disorder the test

To demonstrate that the Hurst exponent is reliable, the data will be disordered so that they are random and their statistical characteristics do not change. To reduce contingency, the initial sequence is disordered three times to form random sequence 1, random sequence 2 and random sequence 3. Then, the R/S analysis of the new time series is performed to calculate the Hurst exponent, and the Hurst exponents before and after the disordering are compared. The results are shown in Table 2.

Table 2

Comparison of the Hurst exponents of the gold daily return before and after disordering

	Hurst exponent	t-Statistic	R-squared	F-Statistic	Prob.
Before	036740	171.7259	0.996687	29490.000	0.0000
Sequence 1	0.521893	313.5760	0.988439	98329.918	0.0000
Sequence 2	0.529156	265.7831	0.983981	70640.646	0.0000
Sequence 3	0.541498	200.8607	0.972286	40345.010	0.0000

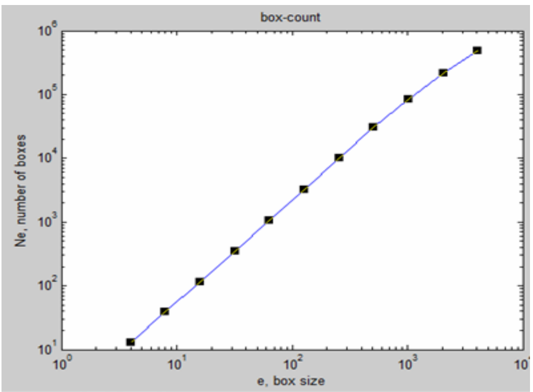
Under the different time intervals, the Hurst exponents of gold at the Shanghai Gold Exchange are higher than 0.5, which indicates that the gold market has a fractal structure, the price fluctuations are of long term, and the daily yield of gold follows a biased random walk; that is, it is a trend with noise. In addition, the Hurst exponent of the gold market is greater than the normal distribution, which means that it has a stronger trend and weaker noise than the normal distribution.

3.3 Fractal Dimension Analysis

In view of the Shanghai gold market AU99.99 daily yield, researcher chooses the box dimension measurement method. The program MATLAB is used to calculate how many boxes correspond to the different sizes of the measuring box. Then, the double logarithmic graphics are fit to calculate the gradient, which represents the gold daily return of box dimensions (Figure 4).

Figure 4

Gold daily return of box dimensions



Based on the previously described analysis, the result of the daily yield of box dimensions is 1.5513, which is larger than one topological dimension. This outcome indicates that the daily gold data do not follow a random walk, but rather fluctuate continuously. The gold daily return has obvious fractal characteristics.

It can be confirmed that the research data are not stable and possess a nonlinear structure by examining the gold daily return using stationarity, normality, linear and nonlinear tests. By comprehensively performing the rescaled range analysis and dimensional analysis to demonstrate the fluctuation characteristics of the gold daily return, it is found that the time series of the Hurst exponent is greater than 0.5, the dimension is greater than 1, and the average cycle length is 112 days. These results demonstrate that the gold market exhibits fractal characteristics on the whole.

4. The Multi-fractal Inspection of the Gold Market in China

The fractal characteristics of the gold market in China were demonstrated in section three. However, the single fractal does not adequately describe the signal's characteristics. For the time series, the single fractal can only describe the system's condition of preliminary macroeconomic fluctuation. It cannot provide detailed information regarding the internal structure. Therefore, the multi-fractal characteristics of the gold market in China should be studied, which would be useful for describing the fluctuations and characteristics of the gold market more accurately.

4.1 Application of Multi-fractal Detrended Fluctuation Analysis

By eliminating the trend component in the time series, the analysis of the gold daily return can monitor a long-range correlation that contains the unpitched sound and superposed signal in polynomials, as well as determine the multi-fractal characteristics of the gold daily return. As a result of the application of the MF-DFA algorithm, the generalized Hurst exponent and scale function are shown in Figure 5 and Figure 6.

Figure 5

Relation of gold daily return $h(q)$ and q

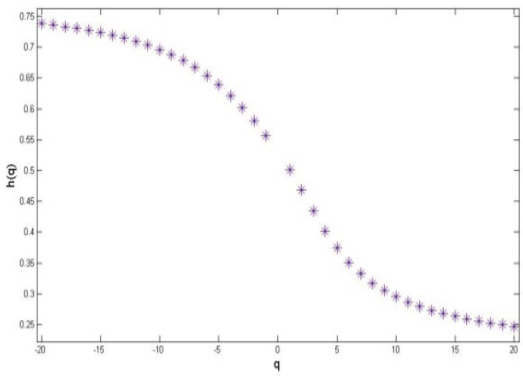


Figure 6

Relation of gold daily return $\tau(q)$ and q

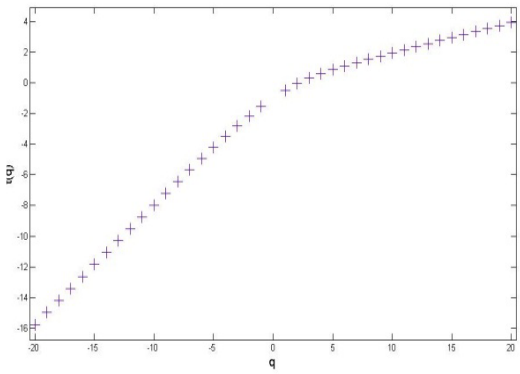
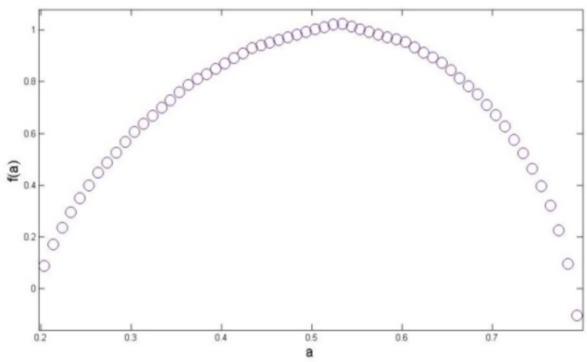


Figure 5 shows that the generalized Hurst exponent of gold daily return decreases with the increase in multi-fractal order q, which confirms that the generalized Hurst exponent obviously changes by variation in order q. This outcome indicates that order q fluctuates differently after the elimination of the trend and that the local structure is not uniform. Therefore, this time series is multi-fractal. When q is negative or small, the generalized Hurst exponent $h(q)$ > 0.5. This indicates that the gold daily return prioritizes small-scope fluctuations, which indicates a long memory and persistence. The change is primarily affected by internal factors in the gold market. When q is positive or large, the generalized Hurst exponent $h(q)$ < 0.5. This result suggests that the fluctuation of the gold daily return occupies the primary position and is anti-durative. Figure 6 shows that $\tau(q)$ and that order q is nonlinear. The gold daily return has a multi-fractal structure.

The multi-fractal spectrum $f(\alpha)$ is calculated using MATLAB programs. Figure 7 shows the multi-fractal spectrum with respect to the singularity exponent.

Figure 7

Multi-fractal spectrum of gold daily returns



In Figure 7, $f(\alpha)$ constantly changes with the alteration of the singularity exponent. The form of α and $f(\alpha)$ is a unimodal bell-shaped curve, which indicates that different singularity exponents correspond to different fractal spectrums. That is, the parts with different singularity degrees possess different fractal dimensions. The gold daily return is multi-fractal.

4.2 The Analysis of Multi-fractal Influence Factors

MF–DFA is used to calculate the Hurst exponent $h_1(q)$ of the new randomized sequence phase $\tau_1(q)$, Hurst exponent $h_2(q)$, of the new sequence after being disordered and the scale function $\tau_2(q)$, as shown in Figures 8 and 9.

Figure 8
Generalized Hurst exponent contrast figure

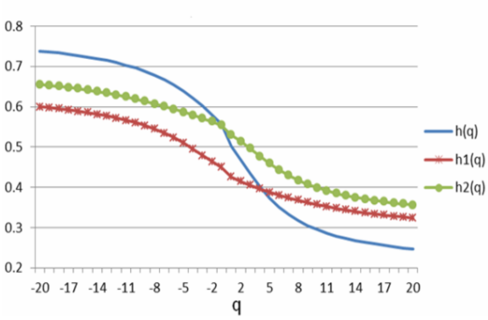
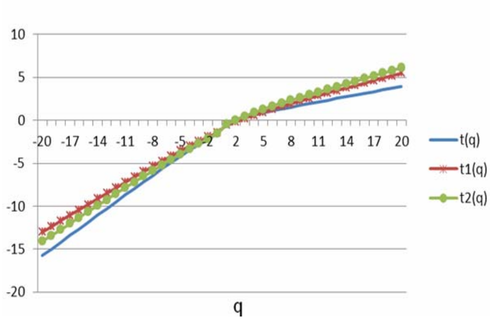


Figure 9
Scale function contrast



In Figure 8, the Hurst exponent $h_1(q)$ of the phase sequence after randomization is not a fixed value, but changes with q. It declines from 0.5998 to 0.3237. However, when q increases, the speed of decline of $h_1(q)$ is significantly slower than that of $h(q)$.

It can be observed that the multi-fractal strength of the new sequence decreases significantly to 0.2761 after the thick tail probability distribution features of the original time series are eliminated. The reduction of the strength of the multi-fractal indicates that the thick tail distribution features that are an inconformity of the normal distribution and are also a cause of the multi-fractality. However, the range of change of the gold daily return generalized Hurst exponent $h_2(q)$ decreases insignificantly. It only decreases from 0.6553 to 0.3561. The strength of the multi-fractal is 0.2992, which is substantially smaller than 0.4917, the former strength of the multi-fractal. This outcome indicates that after being disrupted the multi-fractal features of the gold daily return become less significant. It also indicates that long memorability is a factor that influences the multi-fractal characteristics of the gold yield.

As Figure 9 shows, the range of change in the scale function of new sequence after being randomized is smaller. When q<0, the range resembles that of the original sequence. However, when q<0, the scale function of the new sequence is obviously smaller than that of the original sequence. As described above, the multi-fractal features of the daily yield become less significant after changing. The long-term memorability of the sequence and the fat-tailed distribution are important factors of multi-fractals. In addition, the scale function of the sequence that was disordered to destroy memorability does not possess linear features. It indicates that each factor cannot function without other factors. The multi-fractal feature of the gold daily return consists of two factors.

Thus, the multi-fractal feature of the gold daily return is influenced by long-term memorability of the sequence and the fat-tailed distribution. Additionally, when these two factors are compared, the influence on the fat-tailed feature of the gold daily return time series multi-fractal is larger than the long-term memorability.

5. Gold Price Forecasting Based on the Fractal Interpolation Model

In the efficient market theory, a time series is a random walk. A random walk's price movements are unpredictable and independent. However, based on the analysis described above, the time series of gold in China exhibits nonlinear structure, and the market displays statistically self-similar characteristics with a long-term memory. That is, current information has an impact on the future price, and the fluctuations in the price of gold are multi-fractal.

5.1 The Fractal Interpolation Model

The classical interpolation method uses a set of basic functions to achieve linear combination. Typically, the basis function is selected from the elementary functions, such as polynomial, trigonometric or rational functions. However, traditional methods cannot describe fractal characteristics. Therefore, this paper uses a fractal interpolation based on the iteration function system for simulations and forecasts. Based on this iterated function system, this paper divides the daily closing price between October 30, 2002 and November 30, 2012, into 45 subintervals equidistantly.

The length of each interval is 52(N=45, m=52), and $x_i = 52i, i \in \{0,1,2,...,45\}$. Given the point set of fractal interpolation, the value of the longitudinal compression factor is shown in Table 3.

Table 3

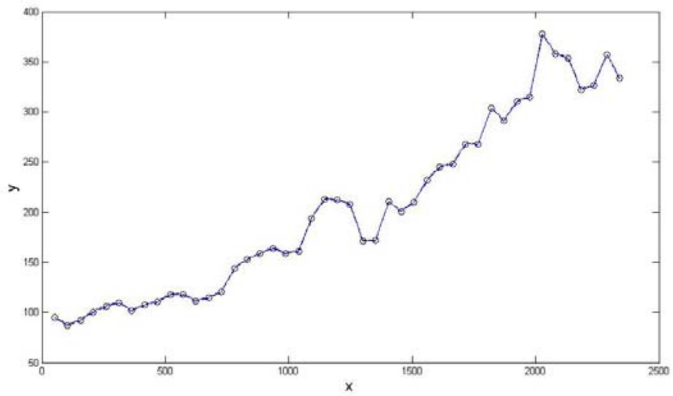
Longitudinal compression factor values

i	d_i	i	d_i	i	d_i	i	d_i	i	d_i
1	-0.0814	10	-0.03	19	0.0195	28	0.0362	37	-0.0706
2	0.0414	11	-0.0001	20	-0.0079	29	-0.0349	38	-0.0165
3	-0.0249	12	0.0262	21	-0.1237	30	-0.0812	39	-0.2304
4	-0.0359	13	-0.0127	22	-0.072	31	-0.0484	40	0.0726
5	-0.0244	14	-0.0232	23	0.0023	32	-0.0121	41	0.0148
6	-0.0153	15	-0.0913	24	0.0168	33	-0.0716	42	0.1145
7	0.032	16	-0.0315	25	0.1365	34	-0.0011	43	-0.0155
8	-0.0219	17	-0.0228	26	-0.0004	35	-0.1312	44	-0.1121
9	-0.0125	18	-0.0212	27	-0.1446	36	0.0464	45	-0.02418

From the MATLAB programs, based on the existing interpolation points and the longitudinal compression ratio, this paper creates a deterministic iteration function system that contains the affine transformation and obtains the fractal interpolation curve after iteration.

Figure 10

The fractal interpolation fitted curve of gold daily closing price



In the comparison of the primitive data curve with the fractal interpolation fitted curve, certain errors occur. However, the fractal interpolation graphics reflect well the changing trend of the gold market prices, which means that the fractal interpolation model provides a better forecast of the gold daily closing prices. The analysis presented above indicates that the gold time series does not follow a random walk and that market price fluctuations are integrated. In addition, market information not only has an impact on the current prices but also influences the future market price performance and has a long-term correlation.

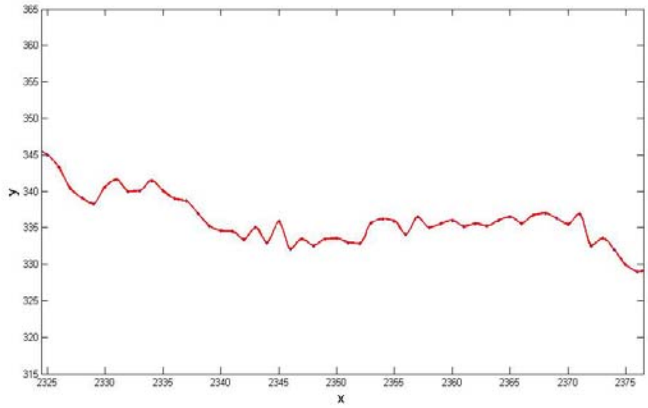
5.2 Analysis of Gold Price Forecast Based on the Fractal Interpolation Model

The extrapolation affine and affine transformation constitutes the new iteration. Through the program operation after the iteration, the fractal interpolation forecast chart is obtained for November 30, 2012 to February 22, 2013 (Figure 11).

According to the previous price range forecast, the overall gold prices continue on a downward trend within the range of 2325-2376 trading days, which is nearly the same as the trend of the raw data tail interval. China's gold market is influenced by many factors at home and abroad; for instance, inflation, economic development and price change trends in the European and the US gold markets. Within the prediction interval, China's gold market closing price decline is primarily the result of the relaxation of the US Federal Reserve Bank's quantitative easing policy. As the market expectations with respect to gold decline, the US gold market may display a downward trend.

Figure 11

Fractal interpolation forecast chart of gold daily closing price



Meanwhile, new economic reports will soon be published that may result in large-scale regulation of the real estate market, among other effects. Additionally, the poor performance of the Shanghai and Shenzhen stock markets would promote the decline in the price of gold. The gold market has a fractal feature with fluctuations in the connection and long-term memorability. Therefore, the factors described above would continue to influence the gold market for some time. After the non-circulating period, the gold market would improve.

6. Conclusion

As the theoretical basis of modern finance theory, the efficient market hypothesis is too rigorous and, therefore, it does not describe the objective circumstances in many cases. Therefore, the nonlinear method is introduced to explain objective phenomena when the traditional linear method is called into question. Through an integrated use of the theories and methods of fractal sets, finance, econometrics and other related disciplines, this paper analyzed the fluctuations and the internal structure of the gold price in time series based on fractal theory, while seeking to describe the characteristics and causes of a fractal system. The primary conclusions are as follows.

(1) The distribution of the gold daily return follows a leptokurtosis distribution. Using rescaled range analysis and the box dimension method, an obvious fractal characteristic of the sequence and a long-term memory characteristic with duration of approximately 112 days are described. After the long-term memory of the original sequence is destroyed, persistent price fluctuations may follow a biased random walk.

(2) By examining the multi-fractal features of the daily return series, the empirical results indicate that the gold time series is multi-fractal and that the main fractal influences are the uniform appearance of information and the positive feedback by investors on the information. In addition, investors are irrational, and the main structure of the market for internal participation remains imperfect. There is also a deficiency in three aspects, such as the lack of a market maker.

(3) Based on fractal theory and the iterated function system, a fractal interpolation model was fitted with the interval daily closing price. The forecast results of the trend extrapolation for the gold price substantially agree with the actual values, which could provide a reference for investors and managers seeking to understand and predict the market.

The systematic analysis of the gold market with fractal theory and methods heralds a new direction for the study of the gold market and a new nonlinear perspective on the problem. As a nonlinear dynamical system, the gold market's self-similarity and long-term memory exhibited a continuous decrease or increase in gold price with accumulative fluctuations. The sharp fluctuations, which are caused when endogenous variables gather during certain hours, resulted in consequential risks, and the strong sensitivity to initial conditions means that even small changes in the financial environment results in large price differences. Therefore, gold market prediction is not effective in the long term. Additionally, this process provides a degree of policy significance for gold market investors.

The gold market is a complex system that contains randomness and uncertainty. In this system, in which the price dynamics of the gold market are considered to have a multi-fractal structure, there is a plurality of interacting price generators. The price follows different scaling laws during different time-scale ranges, which describes the different correlations of different fluctuations. On the one hand, these circumstances remind investors to pay attention to the different factors that influence the gold price at different times. On the other hand, these circumstances suggest that government regulation should fully consider the different scaling characteristics of different scale ranges.

References

Batten, J., Ellis, C., 1996. Fractal structures and naive trading systems: Evidence from the spot US dollar/Japanese yen. *Japan and the World Economy*, 8(4), pp. 411-421.

Cao Jianjun, Gu Rongbao, 2011. Multi-fractal analysis on gold market with MF-DFA method. *Gold*, 9(3), pp.5-7.

Cheung, Y. W., Lai, K. S., 1995. A search for long memory in international stock market returns. *Journal of International Money and Finance*, 14(4), pp. 597-615.

Hou Xiaohong, Li Yizhi, Song Xiaoyan, Xiao Zhiying, 1999. A study of the behaviors of futures with the application of Fractal Theory. *Society Science Journal for Central South University of Technology*, 11(2), pp. 124-127.

Hu Xueming, Song Xuefeng, 2003. The multi-fractal analysis on Shanghai and Shenzhen stock market. *The Journal of Quantitative & Technical Economics*, 9(8), pp. 124-127.

Huang Guangxiao, Chen Guojin, 2006. R/S analysis of Copper Futures Prices Based on Fractal Theory. *Contemporary Finance & Economics*, (3), pp. 60-64.

Huang Feixue, Zhao Yan, 2008. Empirical Research on Fractal Characteristics Based on R/S Analysis in RMB Foreign Exchange Market. *Journal of Harbin Institute of Technology (Social Sciences Edition)*, 10(6), pp. 66-71.

Huang Tengfei, Li Bangyi, Xiong Jixia, 2011. Nonlinearity and uncertainty inspection of the Chinese gold market. *Statistics and Decision*, 11(14), pp. 142-144.

Hurst, H. E., 1951. Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers*, 116(1), pp.770-799.

Kantelhardt, Jan W., *et al*., 2002. Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications*, 316(1), pp.87-114.

Mandelbrot, B. B., 1967. How long is the coast of Britain. *Science*, 156(3775), pp. 636-638.

Mulligan, R. F., 2004. Fractal analysis of highly volatile markets: an application to technology equities. *The Quarterly Review of Economics and Finance*, 44(1), pp. 155-179.

Onali, E., Goddard, J., 2011. Are European equity markets efficient? New evidence from fractal analysis. *International Review of Financial Analysis*, 20(2), pp. 59-67.

Opong, K. K., Mulholland, G., Fox, A. F., and Farahmand, K., 1999. The behaviour of some UK equity indices: An application of Hurst and BDS tests. *Journal of Empirical Finance*, 6(3), pp. 267-282.

Peters, E. E., 1994. *Fractal market analysis: applying chaos theory to investment and economics (Vol. 24)*. New York: Wiley.

Richards, G. R., 2000. The fractal structure of exchange rates: Measurement and forecasting. *Journal of international Financial Markets, Institutions and Money*, 10(2), pp. 163-180.

Song Xia, 2012. The multi-fractal analysis based on MF-DFA of Chinese foreign exchange market. *Securities & Futures of China*, 6(10), pp. 68-69.

Sun Wei, 2009. *A study of the effectiveness of China's futures market based on fractal market hypothesis*. Shanghai: Fu Dan University Press.

Wang Jia, Ma Junhai, 2011. Gold Markets Price Analysis and Application Studies Based on Complexity Theories. *Complex Systems and Complexity Science*, 5(3), pp. 54-59.

Wang Zheng, Liang Linfang, 2006. The analysis of the London gold market based on the R/S analysis, *Science & Technology Information*, 10(12), pp. 211-212.

Yu Jun, Fang Aili, Xiong Wenhai, 2008. Research on the Long Memory in International Stock Returns. *Chinese Journal of Management Science*, 10(16), pp. 24-29.

Zhou Yan, Yu Ke, 1993. Fractal dimension, R/S analysis and stock price movements. *The Journal of Quantitative & Technical Economics*, 10(7), pp. 28-32.

Zhu Xinling, Li Peng, 2011. Test of long memory characteristics of RMB exchange rate by R/S statistics, ARFIMA model and wavelet variance. *Journal of Wuhan University of Science and Technology*, 7(2), pp. 157-160.