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TIME-VARYING BEHAVIOUR OF SECTOR BETA RISK – THE CASE OF POLAND

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Abstract

The problem of proper beta (measure of systematic risk) estimation is crucial both for academic considerations and financial market practice purposes. There is a group of empirical studies that questioned the assumption of beta time-invariance, while only some of them tried to model the process of beta time-variation. Basing on previous research, we apply the state-space methodology, which was found to be the most relevant. We focus our attention on the Polish stock market and five sector indices. Unlike other studies, we estimate our models using three different data frequencies (daily, weekly and monthly), while holding the estimation period fixed. The results indeed show the dependence on data frequency; however, in most cases, the persistence parameter is close to unity, which indicates long-lasting shocks to beta.

Keywords: systematic risk, market model, beta hedging, Kalman filter, state-space methodology

JEL Classification: C22, G11, G12

1. Introduction

The Beta parameter is one of the most fundamental concepts of modern finance. This measure of systematic risk proposed by Sharpe (1963) in his seminal work is represented by the β_i parameter of the Diagonal Model³. The model's equation presents as follows:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \quad (1)$$

where: R_{it} is the return of security or portfolio i for period t , α_i and β_i are the parameters of the equation, ε_{it} is the normally distributed error term and R_{mt} is the

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³ The Diagonal Model is the name proposed by Sharpe (1963). The other names for this concept that can be found in the literature are Single-Index Market Model (SIMM) or simply Sharpe's Model.

return of some index, which may be the return of *the stock market as a whole, the Gross National product, some price index or any other factor thought to be the most important single influence on the returns of securities* (Sharpe, 1963, p. 281).

Beta is widely employed both by academics and practitioners of the financial markets. Calculating the cost of equity and stocks valuation (e.g. through CAPM), hedging strategies, assessing fund managers performance (e.g. Treynor's and Jensen's ratios) are the examples where the estimation of beta matters. Therefore, the appropriate method of estimation is crucial for all these purposes.

The standard technique for estimating systematic risk parameter was Ordinary Least Squares (OLS) where the time-invariance of beta was assumed. However, since the seventies some of the studies have undermined this assumption and the research efforts have been directed towards modelling of beta time-variation⁴. In this paper, we tried to challenge this latter question.

To model the time-conditional beta, we employed the state space methodology with four different state equation specifications and compared their forecasting accuracy with the OLS estimation results. For our estimation tasks, we utilized the data on Polish stock market sector indices.

The comparative studies between emerging economies and well-developed markets usually attract the attention of economic policy works, but we believe that this division can be significant also in the case of financial market research. The CEE financial markets are usually characterized by lower market liquidity and depth, which may have a detrimental impact on markets' information efficiency and, consequently, on asset returns volatility. Therefore, we hope to provide two contributions to the existing literature.

First, by comparing with the results of the previous works (Faff *et al.*, 2000; Mergner and Bulla, 2005) we would like to verify if the modelling of beta in case of CEE market should be different from the well-developed markets cases.

Secondly, unlike the existing studies we repeated our procedure for different data frequencies: daily, weekly and monthly. To our best knowledge, there is lack of research where the estimation results of conditional beta for the same dataset, but for different frequencies, would be compared. The results provided in this study should clarify if beta forecasting in the short term horizon (e.g. beta hedging objective) should be the same like forecasting beta for the longer horizons (e.g. mergers & acquisitions target).

The outline of this study presents as follows. In the next section, we make the review of the literature relevant to the developments in conditional beta modelling. Then, we move to Methodology (III) and Data (IV) sections, where the detailed description of the utilized models is provided. Empirical results part (V) presents the summary of the obtained estimates together with their interpretation. In the last section, Conclusion (VI), we share our views about the potential implications of the carried research and signal the direction for further studies in this area.

⁴ For a literature review, see section II "State of the art".

II. State of the Art

The literature review made in this section presents the works that questioned the concept of unconditional beta, which motivates our modelling efforts. Additionally, we refer the studies that provided the theoretical explanation of the observed instability in order to justify that beta time dependence is not only the statistical phenomenon.

In the empirical study of the capital market theory and its implications for the US market, Jacob (1971) found the assumption of constant beta to be very debatable. The single-period relationship between the return and systematic risk was found to be unstable. The regression estimates were changing as the successive horizons were considered, no matter if the single securities or portfolios were examined.

Using still OLS method, Blume (1975) verified the betas of a few 100 securities portfolios for the subsequent estimation periods. In general, he found portfolios' betas to be stationary over time; however, in the case of extreme beta values (very high or very low) he observed a process of reverting to the market beta value, *i.e.* one. According to these results, Blume (1975) formulated two hypotheses of the observed phenomenon. Firstly, the risk of existing projects run by the company may become less extreme over time. This idea, therefore, explains only very high beta cases, where the value of systematic risk was decreasing over time. Hence, the alternative explanation relies on the possibility that the new projects may tend not to have so extreme risk characteristics comparing to the existing ones. As the author stated this statements should be further examined.

Finally, Fabozzi and Francis (1978) verified the random coefficient model (RCM) for the data on US securities. As the above discussed studies identified only the beta's instability phenomenon, Fabozzi and Francis (1978) made a significant contribution by an attempt to model the process of beta's time-variation. In the case of some securities from the US market they found RCM to be valid for beta estimation. In their opinion, the obtained results could clarify why OLS estimates for the New York Stock Exchange (NYSE) securities indicated that less than half of returns variance was explained by common risk factors. OLS estimates residual variance may be upward biased by the beta coefficient rigidity assumption. The other important contribution of their study was the list of reasons, grouped into four categories, of the time-dependend systematic risk phenomenon. Firstly, the microeconomic factors, like changes in dividend payouts or leverage may influence beta over time. Secondly, the macroeconomic conditions, namely business cycle fluctuations and inflation variability were noted. The third group consisted of political reasons, *e.g.* labour legislation, pollution-control legislation and elections. Last but not least, the financial market conditions (bull and bear market, credit crunches) may be also important for beta determination.

In the later research, the attention was focused on the stochastic process of beta time-variation where the alternative modelling techniques were used, but these studies covered only well-established economies.

In the case of the emerging market economies, where the stock markets were established mostly in the nineties, the research in these area is very limited. The literature does not comprehensively address the problem of proper conditional beta

modelling, but focuses on the identification of the instability phenomenon merely. However, it is still worth to mention some papers, as they indicate the importance of the discussed problem also for the emerging economies in the very early years of stock exchanges operation.

Kuziak (1998) applied the Chow test of structural stability for the estimates of the SIMM for nine securities with the longest listing history on the Warsaw Stock Exchange (WSE) that time. Using monthly data she decided to choose May 1993 as the breakpoint, which was the last month of the bull market. After rigorous testing the applicability conditions of Chow test, two companies were cancelled from the research. From the remaining seven securities, in case of three of them the Chow test rejected the hypothesis on SIMM parameters stability.

A few years later, when the number of companies listed on the WSE increased, Byrka-Kita (2004) using again the Chow test verified the same hypothesis like Kuziak (1998), but this time for weekly observations of nine portfolios, each one consisted of 10 securities. The results revealed that only in case of three portfolios the null hypothesis of no structural change was not rejected.

Valuable contribution on beta (in)stability was also made by Wdowiński (2004). What is especially worth to underline in his study is that he analyzed the country's (Poland) beta risk by regressing two WSE indices (WIG and WIG20) on major foreign stock market indices (DJIA, NASDAQ, DAX and FTSE). The research period was 1996-2002, which was further divided into monthly intervals. Then, using daily data, he estimated for every sub-period separate regressions. Comparing the values of beta for the subsequent intervals, he found Poland's beta risk to be highly time-varying ranging from 0.1-0.5 (Hodrick-Prescott filtered values). As at the country's level idiosyncratic risks are diversified away, Wdowiński (2004) analyzed also macro-determinants of Poland's beta risk. Estimating the model with real and monetary factors, he found these latter ones to be relatively more powerful in explaining the country's systematic risk.

III. Methodology

In the empirical studies, one may find different approaches to model the conditional beta: GARCH approach, stochastic volatility (SV) models, Markov switching framework, state space models, and Schwert and Seguin (1990) approach, where the SIMM is augmented by the additional term capturing the time-varying market volatility. Some comparative works⁵ suggest that the state space approach is the most relevant in terms of fitting model to the data and out of sample forecasting accuracy. For this reason, in this study we focus on the state space models.

The state space form comprises two equations: a measurement (or signal) equation and a transition (or state) equation. The measurement equation specifies how the

⁵ The examples of the comparative studies for different modelling techniques are Brooks *et al.* (1998) for Australian industry portfolios, Faff *et al.* (2000) in the case of UK sector indices, Bulla and Mergner (2005) for the Pan-European industry portfolios, Choudhry and Wu (2009) for twenty UK companies.

vector of observed variables is related to a vector of unobserved state variables. The transition equation specifies the time-series process generating the unobservable state variables. In our study, the state space system to be estimated presents as follows:

$$R_{it} = \alpha_i + \beta_{it} R_{mt} + \varepsilon_{it}, \quad (2)$$

$$\beta_{it} = \beta_{i,t-1} + \mu_{it}, \quad (3)$$

where: R_{it} is the return on i -th sector index for period t and R_{mt} is the return on the market index. In such framework, the SIMM with conditional beta becomes the measurement equation (2) and dynamic process of market risk is described by the transition equation (3). The error terms are assumed to be normally distributed and mutually uncorrelated to each other at all leads and lags:

$$\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_i}^2),$$

$$\mu_{it} \sim N(0, \sigma_{\mu_i}^2),$$

$$E(\varepsilon_{it}, \mu_{is}) = 0, \text{ for all } t \text{ and } s.$$

State space models are estimated using a recursive algorithm known as the Kalman Filter, which given new information sequentially updates the one step ahead mean and variance of the state variables estimates. Moreover, the filter allows the computation of the log-likelihood function of the model, which enables the parameters as well as the state variables to be estimated using maximum likelihood method⁶.

The dynamic process of beta represented by state equation can be modelled in a number of ways. The specification presented in equation (3) is actually the *random walk* (RW) specification, where the shocks to the conditional market risk persist indefinitely. It is possible, however, that the effect of shock to conditional beta is not permanent and disappears after some time. To model this type of beta's behaviour we use *mean-reverting* (MR) specification, with a first order autoregressive process, AR(1), and a constant mean:

$$\beta_{it}^{MR} = \bar{\beta}_i + \phi_i (\beta_{i,t-1} - \bar{\beta}_i) + \mu_{it}, \quad (4)$$

where: $\bar{\beta}_i$ is a constant and $|\phi_i| < 1$ is the AR(1) parameter. The ϕ_i is often interpreted as „persistence parameter”. The interpretation is straightforward: the higher the parameter's value the longer the shock persists.

If one would like to assume that the effect of shock disappears in the next period, the *random coefficient* (RC) model applies:

$$\beta_{it}^{RC} = \bar{\beta}_i + \mu_{it}. \quad (5)$$

Zhongzhi and Kryzanowski (2008) proposed the specification combined of trend (RW) and cycle (MR) components, therefore denoting it by RWMR:

$$\beta_{it}^{RWMR} = B_{it} + C_{it}, \quad (6)$$

⁶ For more detailed exposition and discussion concerning state space methodology, please refer, for instance, to Harvey (1989).

$$B_{it} = B_{i,t-1} + w_{it}, \quad (7)$$

$$C_{it} = \phi_i C_{it-1} + v_{it}, \quad (8)$$

where: B_{it} and C_{it} represent the RW and MR components, respectively. In their approach, beta is allowed to revert to a stochastic trend which is itself time-varying.

The RWMR specification is more general than the previously presented, as it encompasses all other cases. With $\sigma_{wr}^2 = 0$ the model reduces to the MR case. If additionally $\phi_i = 0$, the result is the RC specification. When σ_{wr}^2 is positive, while at the same time $\sigma_{vt}^2 = 0$ the model becomes RW. Finally, if both state variations are zero, we simply receive the model with unconditional (fixed) beta.

Condensing the above, in our study we employ four state space models, which share common measurement equation (2), but different state equations governing the evolution of beta, namely: eq. (3) for random walk, eq. (4) for mean-reversion, eq. (5) for random coefficient specifications and eq. (6) – (8) corresponding to the RWMR specification. Additionally, as a point of reference and a benchmark against which the performance of state space models will be measured, we utilize the model (1) estimated by OLS method (hereinafter referred to as OLS model).

Our assessment of model performance starts with calculating the Akaike Information Criterion (*AIC*) for each specification as a measure of the relative goodness of fit. In the next step of the analysis, we attempt to identify the optimal specification for various industries by constructing, evaluating and comparing in-sample and out-of-sample forecasts of the sector returns. The forecast accuracy is measured by Root Mean Squared Error (*RMSE*) and Mean Absolute Error (*MAE*), which are given by the following formulas:

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T (R_{it} - \hat{R}_{it})^2}, \quad (9)$$

$$MAE = \frac{1}{T} \sum_{t=1}^T |R_{it} - \hat{R}_{it}|, \quad (10)$$

where: T is the number of forecast observations and \hat{R}_{it} denote the series of index return forecasts for sector i . The values of \hat{R}_{it} for state space models are calculated as one-step-ahead signal forecasts based on relevant measurement equation (2), assuming that the series of market returns R_{mt} is known in the forecast period. The forecast quality is inversely related to the size of *RMSE* and *MAE*. However, it should be noted that these two error measures account for slightly different aspects of forecast evaluation: *MAE* weights all forecast errors equally, while the use of a squared term in the equation (9) for *RMSE* places a heavier penalty on outliers. To provide a clear comparison, different modelling techniques will be formally ranked based on their *RMSE* or *MAE*. Moreover, since the Kalman filter is likely to produce large outliers in the first stages of estimation, the first ten conditional beta estimates

for all model specifications are not included in the calculations to avoid an unjustified bias.

In order to complement the information delivered by *RMSE* and *MAE* measures and compare the forecasting ability of the different models the formal statistical test is implemented. We use the Diebold-Mariano (1995) test, which allows for using a wide variety of accuracy measures and forecast errors can be non-Gaussian, nonzero mean, serially correlated and contemporaneously correlated. When comparing two forecasts, the question arises of whether the predictions of a given model, *A*, are significantly more accurate, in terms of a loss function $g(\cdot)$, than those of the competing model, *B*. The Diebold-Mariano test aims to test the null hypothesis of equality of expected forecast accuracy against the alternative of different forecasting ability across models. The null hypothesis of the test can be, thus, written as

$$H_0 : E[g(e_t^A) - g(e_t^B)] = 0,$$

against the alternative:

$$H_1 : E[g(e_t^A) - g(e_t^B)] \neq 0,$$

where: e_t^i refers to the forecasting error of model *i* when performing *h*-steps ahead forecasts. The most popular loss functions $g(\cdot)$ includes the square and absolute error loss, namely for $i = \{A, B\}$:

$$g(e_t^i) = (e_t^i)^2, \tag{11}$$

$$g(e_t^i) = |e_t^i|. \tag{12}$$

Defining the loss differential as

$$d_t = g(e_t^A) - g(e_t^B)$$

the Diebold-Mariano test uses the autocorrelation-corrected sample mean of d_t in order to test H_0 . If T observations and forecasts are available, the test statistic is, therefore:

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})}}, \tag{13}$$

where:

$$\hat{V}(\bar{d}) = \frac{1}{T} \left[\gamma_0 + 2 \sum_{k=1}^{\infty} \hat{\gamma}_k \right],$$

$$\hat{\gamma}_k = \frac{1}{T} \sum_{t=k+1}^T (d_t - \bar{d})(d_{t-k} - \bar{d}).$$

Under the null hypothesis, *DM* has an asymptotic standard normal distribution. So, we reject the null of equal predictive accuracy at the 5% significance level if $|DM| > 1,96$.

IV. Data

The choice of the Warsaw Stock Exchange (WSE) among other CEE markets is motivated by pragmatic reasons. WSE has been the leader in the region (Köke and Schröder, 2003) with the relatively long history of listed companies for many years. Nowadays its position is still dominant. According to the information provided by WSE, Polish market: *is the largest national stock exchange in CEE, with 384 listed companies, including 23 foreign companies with a combined domestic market capitalisation exceeding €128 billion (as at 14 Sept. 2010) and aggregated session equity turnover of €32.2 billion (WSE, 2010)*. Important for our research, WSE publishes the number of sector (industry) indices. In the case of other markets such data are usually unavailable⁷, or the history of listing is too short to deliver robust estimates. To our best knowledge, there is still lack of studies focusing on the stochastic process of beta variation in the case of Poland and, perhaps, other CEE countries.

In this study, we utilize data on five WSE sector indices - WIG-Banking (Banks), WIG-Construction (Construction), WIG-Food (Food Production), WIG-IT (IT), WIG-Telecom (Telecom) and the measure of broad market – WIG index (WIG). These five sectors have the longest history of listing from the eleven industry averages published by the WSE. All of the indices (sector + broad market) are of total return type, including dividends and pre-emptive rights. Sector indices are revised quarterly and contain all companies qualified for a specified sector and included in the WIG index portfolio. The number of companies covered by the sector averages ranges from seven (Telecom) to thirty one (Construction)⁸.

We decided to avoid using data on single securities, as the variance of idiosyncratic component may be quite large. Rather we use indices (portfolios) data, where this risk is mostly diversified away and it is easier to capture the changes in systematic risk. Secondly, there is some group of studies where the industry indices of other countries were employed. This fact creates then possibility for international comparisons. Last but not least, research on well-known indices, not the subjectively created portfolios, may provide some valuable conclusions for fund managers operating on the Polish market.

We examine three data frequencies: daily, weekly and monthly. In the case of daily data the closing values from 31.12.1997 to 15.04.2011 are collected from Bossa (2011). The choice of the time sample was motivated only by data availability and our intention was to collect the data from as long as possible time period to guarantee the robustness of the estimation results. The continuously compounded daily rates of return are calculated as the log differences of indices values from the two subsequent working days. The weekly rates are calculated as logarithmic returns between the first and the last day values of the week. The monthly returns are defined as the log difference between the index closing value from the last day of the previous month and the index closing value from the last day of the current month.

⁷ An alternative for non-existing sector indices published by stock exchanges in CEE states may be the MSCI sector indices.

⁸ As on 24th June 2011.

V. Empirical Results

Looking at the results of beta estimates presented in Table 2 of the Appendix one broad finding emerges. While the mean betas of state space models are generally in line with the OLS estimates the range of state space beta estimates is quite large. In many cases, the highest upper bound beta value is two times larger than its mean. This confirms the stylized fact that systematic risk is highly time-dependent. However, this general assessment should be confirmed by the estimates of the forecast errors and the results of DM test.

Table 3 in the Appendix presents the AIC values for five different models in question and their ranks according to this measure. Unquestionably, the highest support is received by MR specification, which exhibits the best average rank for all frequencies. From the AIC perspective the benchmark OLS model is the worst model for weekly and daily data, but for monthly data AIC puts in this role the RWMR specification.

V.1. In-sample Forecasting Accuracy

The in-sample forecasts for all specifications and data frequencies are computed over the whole available sample period, i.e. Jan. 1998 – Apr. 2011. Forecasts are made for the log-returns time series of five sector indexes.

The results presented in Tables 4 and 5 of the Appendix are surprising to some extent. While the OLS forecasts are always dominated by RW, MR and RWMR specifications, the OLS estimates are superior to RC model predictions in five of six cases. We note also the rising rank of MR specification when moving to lower data frequency.

To measure how similar the rank order is when RMSE and MAE criterion is used we apply the Spearman's rank correlation coefficient (ρ^S):

$$\rho^S = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}, \quad (14)$$

where: d_i is the difference between corresponding ranks of different SIMM specifications for a given sector at given data frequency. Comparing the corresponding rows of Tables 4 and 5 we receive 15 coefficients and calculate the mean values for every frequency. The obtained average values (Table 10) indicate high similarity of rankings independent of the used criterion.

The Diebold-Mariano test results are given in Table 6 in the Appendix. Applying this test, the forecasting ability of the different models is compared against the OLS predictions and loss differential takes the form:

$$d_t = |e_t^{OLS}| - |e_t^i|, \quad i = \{RW, MR, RC, RWMR\}.$$

As one may see from above, we use the absolute error loss function (12), however we also arrive at similar inferences when the square loss function is used. Table 6 reveals

the following pattern: the higher the data frequency – the more often the null hypothesis of equal forecast accuracy is rejected (5 times for monthly, 11 times for weekly and 16 times for daily data). In most cases, the DM statistics value is positive, which means that OLS forecast error is higher. This brings us to conclusion that the state space models with conditional beta are statistically superior over the OLS model, especially for higher frequencies. The top number of null rejections across different frequencies is indicated by the MR specification, which confirms to perform better than OLS model in 11 cases.

V.2. Out-of-sample Forecasting Accuracy

With the aim of setting the adequate length of forecast sample across different frequencies, we apply the following rule: if we assume that the number of observations in forecast sample for monthly data is set to 50, then it gives 200 (50 months x 4 weeks) observations for weekly and 1000 (200 weeks x 5 working days) for daily datasets. Table 1 in the Appendix depicts this approach more precisely.

The above solution enables us to maintain the forecast-to-total sample ratio at the constant level of approximately 1/3 and to cover by the out-of-sample forecasts the same historical time periods for each examined data frequency. As in the case of in-sample forecasts, the out-of-sample forecasts are made for the log-returns of sector indexes.

The results of out-of-sample forecast errors (Table 7 in the Appendix) are partly in line with the in-sample estimates. Three of the state space models, namely RW, MR and RWMR dominate the OLS results. The RC model predictions are inferior to the OLS results in two of three cases and the rising rank of MR specification when moving to lower frequency data is also generally confirmed. However, it should be noted that in the out-of-sample case the RWMR model produces the lowest forecasts errors when weekly and daily data is analyzed. The results of Spearman's correlation coefficient estimate (Table 10) confirms also the similarity of results obtained using RMSE or MAE specification.

Table 8 in the Appendix shows the Diebold-Mariano statistics and *p-values* for out-of-sample forecasts. Comparing with analogous in-sample results (Table 6 in the Appendix) the evidence concerning the preference of state space models over the OLS model is more mixed. Even though for monthly frequency the null hypothesis is more often rejected in favour of state space specifications (9 cases vs. only 5 in Table 6), the RC model underperforms the simple OLS framework, because the DM statistics value for RC is lower than zero in 7 out of 8 statistically significant cases.

VI. Conclusions

This study confirmed the general property of time-varying behaviour of systematic risk. We analyzed the data for five sector portfolios, using the returns of daily, weekly and monthly frequency. Finally, we estimated all our models for the same sample period to deliver robust results. This procedure enabled us to make some sensible comparisons.

First of all, we did not observe many differences in results between the previously mentioned studies and our research. Also, in the case of Poland, as the example of the emerging market, the RW, MR and RWMR dominate the other specifications. Notable is also a poor performance of the RC model.

The more appealing conclusions can be derived by comparing the estimates for different frequencies. Faff *et al.*, 2000) or Mergner and Bulla (2005) reported the RW model as dominant specification in terms of minimizing the forecast error. These studies, however, used daily and weekly data, respectively. In our paper, when the monthly frequency was examined the rank of MR model was rising, especially in the in-sample case. The DM tests results were even more favourable to the MR relatively to other state space models, when the data frequency was decreasing. One should take also a quick look at Table 9 in the Appendix, where the values of ϕ (“persistence parameter”) are reported. In the case of every sector, the lowest ϕ values are reported for monthly frequency. Nevertheless, in three of five sectors the monthly parameters are still close to unity, which indicates high persistence of beta shocks, but at the same time the long run process of mean reversion is identified (the highest rank of MR specification for in-sample monthly data). These results, supported by the poor performance of the RC model lead us to the ending conclusion that shocks to systematic risk are rather long-lasting, but finally usually mean-reverting.

Our finding can be utilized for practical considerations. For company’s valuation purposes the time-variance property of beta is not as important as it is mean-reverting. The horizon of mergers&acquisition actions is usually long enough to employ the OLS model as it produces the similar results to the long run mean state-space estimates. In case of portfolio beta hedging strategies, where the investment horizon is definitely shorter, the RW, MR or RWMR state space estimates should be applied. We reported high persistence of beta shocks so the systematic risk estimates should be updated to avoid large hedging errors.

We see also some extensions of the used methodology in future research.

Many of the studies, starting from the seminal paper by Fama and MacBeth (1973), verify the validity of CAPM employing the beta values, which are estimated via OLS by rolling regressions or by extending the sample period. As we demonstrated especially in the out-of-sample case, the use of Kalman filter for beta estimation may shed a new more favourable light on this model, as it better reflects changing expectations about systematic risk. Some initial research using different approaches to time-varying beta estimation have been carried out, e.g. Basu and Stremme (2007); however, the potential of state space methodology is still unexploited in this area.

It is also tempting to find the sources of shocks to beta. Are they rather of macro- or micro- type? In the case of sector portfolios, do country or rather international shocks matter more? Finally, are the identified causal relationships stable over time? These issues definitely need further research attention.

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Appendix

Table 1

Sample Lengths for Out-of-sample Forecasting

Frequency	Number of observations in estimation sample	Number of observations in forecast sample	Total number of observations
Monthly	109	50	159
Weekly	466	200	666
Daily	2335	1000	3335

Table 2

Comparison of Alternative Estimates of Beta

Industry Sector	β^{OLS}	β^{RW}	β^{MR}	β^{RC}	β^{RWMR}
Monthly frequency					
Banks	1.061	1.031 [0.69; 1.53]	1.044 [0.72; 1.67]	1.053 [0.50; 1.92]	1.031 [0.65; 1.71]
Construction	0.914	0.899 [0.61; 1.37]	0.875 [0.52; 1.28]	0.869 [0.58; 1.16]	0.845 [0.51; 1.19]
Food Production	0.697	0.649 [0.54; 0.71]	0.670 [0.46; 0.92]	0.700 [0.47; 0.92]	0.621 [0.31; 0.92]
IT	1.103	1.153 [0.54; 2.09]	1.129 [0.63; 1.90]	1.127 [0.23; 2.83]	1.153 [0.44; 2.40]
Telecom	0.958	1.050 [0.16; 2.15]	1.040 [0.21; 2.09]	1.105 [0.04; 3.14]	1.086 [-0.01; 2.75]
Weekly frequency					
Banks	1.135	1.076 [0.58; 1.78]	1.078 [0.67; 1.78]	1.092 [0.65; 1.98]	1.075 [0.54; 1.98]
Construction	0.811	0.774 [0.24; 1.38]	0.778 [0.39; 1.29]	0.790 [0.34; 1.30]	0.772 [0.21; 1.45]
Food Production	0.587	0.576 [0.43; 0.86]	0.563 [-0.48; 1.34]	0.569 [-0.64; 1.38]	0.567 [-0.51; 1.34]
IT	0.981	1.060 [0.45; 2.06]	1.045 [0.53; 1.98]	1.024 [-0.01; 2.47]	1.053 [0.10; 2.46]
Telecom	0.969	1.100 [0.16; 2.20]	1.089 [0.22; 2.18]	1.057 [-0.17; 2.74]	1.102 [-0.17; 2.65]
Daily frequency					
Banks	1.087	1.075 [0.33; 1.98]	1.075 [0.37; 1.97]	1.075 [-0.14; 2.05]	1.075 [0.27; 2.01]
Construction	0.776	0.713 [0.25; 1.37]	0.716 [0.30; 1.31]	0.744 [0.15; 1.74]	0.712 [0.22; 1.42]
Food Production	0.578	0.510 [-0.09; 1.13]	0.535 [-0.73; 1.80]	0.542 [-0.75; 1.94]	0.509 [-0.71; 1.91]
IT	1.029	1.049 [0.29; 2.16]	1.047 [0.34; 2.08]	1.043 [-0.41; 2.66]	1.056 [-0.13; 2.41]
Telecom	1.064	1.156 [0.25; 2.44]	1.154 [0.29; 2.33]	1.121 [-0.19; 2.58]	1.163 [-0.07; 2.51]

Note: This table summarizes the various beta series by reporting the mean betas and their range (in square brackets).

Table 3

Comparison of Different Models According to Akaike Information Criterion

Industry Sector	OLS	RW	MR	RC	RWMR
Monthly frequency					
Banks	-3.5201 (3)	-3.5151 (4)	-3.6107 (1)	-3.5648 (2)	-3.4597 (5)
Construction	-3.0974 (3)	-2.9775 (4)	-3.1026 (1)	-3.1000 (2)	-2.9172 (5)
Food Production	-2.8244 (1)	-2.6895 (4)	-2.7901 (3)	-2.8023 (2)	-2.6182 (5)
IT	-2.5900 (4)	-2.6008 (3)	-2.6846 (1)	-2.6147 (2)	-2.5303 (5)
Telecom	-2.4861 (5)	-2.6677 (2)	-2.7507 (1)	-2.6452 (3)	-2.6072 (4)
Average Rank	3.2	3.4	1.4	2.2	4.8
Weekly frequency					
Banks	-5.0886 (5)	-5.2451 (3)	-5.2750 (1)	-5.2363 (5)	-5.2592 (2)
Construction	-4.5500 (5)	-4.5876 (2)	-4.6120 (1)	-4.5741 (3)	-4.5701 (4)
Food Production	-4.3837 (4)	-4.3493 (5)	-4.4491 (1)	-4.4430 (2)	-4.3974 (3)
IT	-4.0985 (5)	-4.2236 (2)	-4.2442 (1)	-4.1692 (4)	-4.2219 (3)
Telecom	-4.0183 (5)	-4.2433 (3)	-4.2644 (1)	-4.1535 (4)	-4.2526 (2)
Average Rank	4.8	3.0	1.0	3.4	2.8
Daily frequency					
Banks	-6.8074 (5)	-7.0403 (2)	-7.0493 (1)	-6.9493 (4)	-7.0378 (3)
Construction	-6.2978 (5)	-6.3768 (2)	-6.3819 (1)	-6.3303 (4)	-6.3751 (3)
Food Production	-6.0377 (5)	-6.0678 (4)	-6.0984 (2)	-6.0950 (3)	-6.1013 (1)
IT	-5.8206 (5)	-6.0091 (3)	-6.0147 (2)	-5.9139 (4)	-6.0338 (1)
Telecom	-5.6778 (5)	-5.8762 (3)	-5.8818 (2)	-5.7523 (4)	-5.8897 (1)
Average Rank	5.0	2.8	1.6	3.8	1.8

Note: For each sector, figures in parentheses denote the relative rank of a model's AIC, where the model with the smallest AIC ranks first.

Table 4

In-sample Root Mean Squared Errors

Industry Sector	OLS	RW	MR	RC	RWMR
Monthly frequency					
Banks	0.04013 (4)	0.03752 (2)	0.03733 (1)	0.04029 (5)	0.03760 (3)
Construction	0.04996 (3)	0.05115 (5)	0.04925 (1)	0.04972 (2)	0.05045 (4)
Food Production	0.05838 (1)	0.05882 (4)	0.05839 (2)	0.05840 (3)	0.05907 (5)
IT	0.06371 (5)	0.05956 (3)	0.05916 (1)	0.06365 (4)	0.05936 (2)
Telecom	0.06785 (4)	0.05793 (2)	0.05764 (1)	0.06851 (5)	0.05807 (3)
Average RMSE	0.05601	0.05300	0.05235	0.05611	0.05291
Average Rank	3.4	3.2	1.2	3.8	3.4
Weekly frequency					
Banks	0.01892 (4)	0.01750 (3)	0.01748 (2)	0.01899 (5)	0.01743 (1)
Construction	0.02483 (4)	0.02415 (3)	0.02407 (2)	0.02484 (5)	0.02404 (1)
Food Production	0.02694 (3)	0.02717 (5)	0.02665 (1)	0.02697 (4)	0.02677 (2)
IT	0.03098 (4)	0.02889 (3)	0.02885 (2)	0.03103 (5)	0.02880 (1)
Telecom	0.03228 (4)	0.02872 (3)	0.02869 (2)	0.03243 (5)	0.02863 (1)
Average RMSE	0.02679	0.02528	0.02515	0.02685	0.02513
Average Rank	3.8	3.4	1.8	4.8	1.2
Daily frequency					
Banks	0.00804 (4)	0.00723 (2)	0.00722 (1)	0.00805 (5)	0.00725 (3)
Construction	0.01037 (4)	0.009961 (2)	0.009956 (1)	0.01038 (5)	0.00997 (3)
Food Production	0.01180 (4)	0.01171 (2)	0.01178 (3)	0.01182 (5)	0.011709 (1)
IT	0.01316 (4)	0.012048 (2)	0.012049 (3)	0.01317 (5)	0.01200 (1)
Telecom	0.01414 (4)	0.01286 (3)	0.01285 (2)	0.01417 (5)	0.01284 (1)
Average RMSE	0.01150	0.01076	0.01077	0.01151	0.01075
Average Rank	4.0	2.2	2.0	5.0	1.8

Note: For each sector, figures in parentheses denote the relative rank of a model's RMSE, where the model with the smallest RMSE ranks first. Some results are rounded even up to 6 digits after decimal comma in order to allow unambiguous comparison.

Table 5

In-sample Mean Absolute Errors

Industry Sector	OLS	RW	MR	RC	RWMR
Monthly frequency					
Banks	0.02940 (4)	0.02780 (1)	0.02833 (3)	0.02949 (5)	0.02786 (2)
Construction	0.04017 (4)	0.04041 (5)	0.03952 (1)	0.03968 (2)	0.03984 (3)
Food Production	0.04243 (1)	0.04308 (4)	0.04248 (2)	0.04249 (3)	0.04332 (5)
IT	0.047804 (5)	0.04501 (3)	0.04490 (2)	0.047797 (4)	0.04470 (1)
Telecom	0.06090 (4)	0.04037 (1)	0.04941 (3)	0.06499 (5)	0.04095 (2)
Average RMSE	0.04414	0.03933	0.04093	0.04489	0.03933
Average Rank	3.6	2.8	2.2	3.8	2.6
Weekly frequency					
Banks	0.01394 (5)	0.01335 (3)	0.01333 (2)	0.01383 (4)	0.01326 (1)
Construction	0.01833 (5)	0.01774 (2)	0.01776 (3)	0.01831 (4)	0.01765 (1)
Food Production	0.01957 (3)	0.01966 (5)	0.01941 (1)	0.01959 (4)	0.01944 (2)
IT	0.02266 (4)	0.02133 (3)	0.02125 (1)	0.02277 (5)	0.02130 (2)
Telecom	0.02709 (4)	0.02005 (2)	0.02012 (3)	0.02849 (5)	0.01991 (1)
Average RMSE	0.02032	0.01843	0.01837	0.02060	0.01831
Average Rank	4.2	3.0	2.0	4.4	1.4
Daily frequency					
Banks	0.00602 (5)	0.00553 (1)	0.00554 (2)	0.00602 (4)	0.00556 (3)
Construction	0.00783 (5)	0.007474 (1)	0.00748 (2)	0.00781 (4)	0.007478 (3)
Food Production	0.00834 (5)	0.00821 (1)	0.00829 (3)	0.00832 (4)	0.00823 (2)
IT	0.00959 (4)	0.00878 (2)	0.00879 (3)	0.00960 (5)	0.00874 (1)
Telecom	0.01251 (4)	0.00998 (2)	0.01006 (3)	0.01294 (5)	0.00995 (1)
Average RMSE	0.00886	0.00800	0.00803	0.00894	0.00799
Average Rank	4.6	1.4	2.6	4.4	2.0

Note: For each sector, figures in parentheses denote the relative rank of a model's MAE, where the model with the smallest MAE ranks first. Some results are rounded even up to 6 digits after decimal comma in order to allow unambiguous comparison.

Table 6

Diebold-Mariano test results for in-sample forecasts

Industry Sector	RW	MR	RC	RWMR
Monthly frequency				
Banks	1.5151 (0.1298)	1.3485 (0.1775)	-0.5543 (0.5794)	1.5210 (0.1283)
Construction	-0.2462 (0.8055)	1.4242 (0.1544)	2.0998* (0.0357)	0.6398 (0.5223)
Food Production	-1.5315 (0.1256)	-0.2756 (0.7829)	-0.3731 (0.7091)	-1.4703 (0.1415)
IT	1.4674 (0.1423)	2.0211* (0.0433)	0.0256 (0.9796)	1.6073 (0.1080)
Telecom	2.1536* (0.0313)	2.6506* (0.0080)	-0.2484 (0.8039)	2.1106* (0.0348)
Weekly frequency				
Banks	1.9691* (0.0489)	2.3729* (0.0176)	1.3391 (0.1805)	2.4174* (0.0156)
Construction	1.8582 (0.0631)	2.2048* (0.0275)	0.4635 (0.6430)	2.3521* (0.0187)
Food Production	-0.6987 (0.4847)	1.232 (0.218)	-0.5035 (0.6146)	0.8706 (0.3840)
IT	2.8108* (0.0049)	3.3314* (0.0009)	-1.2825 (0.1997)	2.9476* (0.0032)
Telecom	3.772* (0.0002)	4.1882* (0.0000)	-0.7013 (0.4831)	3.8708* (0.0001)
Daily frequency				
Banks	6.1072* (0.0000)	6.4384* (0.0000)	0.2857 (0.7751)	5.7568* (0.0000)
Construction	6.264* (0.0000)	6.7529* (0.0000)	1.5132 (0.1302)	6.0894* (0.0000)
Food Production	2.2533* (0.0242)	2.6331* (0.0085)	1.996* (0.0459)	2.1189* (0.0341)
IT	8.3144* (0.0000)	8.7256* (0.0000)	-1.8221 (0.0684)	8.7075* (0.0000)
Telecom	8.4035* (0.0000)	8.8405* (0.0000)	-0.7026 (0.4823)	8.6568* (0.0000)

Note: Figures in parentheses denote p-values. The asterisk denotes the rejection of the null hypothesis of equal forecasting accuracy at 5% significance level. In all cases in the role of benchmark the OLS forecasts are employed.

Table 7

Out-of-sample Root Mean Squared Errors

Industry Sector	OLS	RW	MR	RC	RWMR
Monthly frequency					
Banks	0.054469 (5)	0.05164 (1)	0.054468 (4)	0.054464 (3)	0.05174 (2)
Construction	0.04497 (4)	0.04436 (3)	0.04400 (1)	0.04433 (2)	0.04581 (5)
Food Production	0.064324 (2)	0.06438 (3)	0.06441 (4)	0.064322 (1)	0.06463 (5)
IT	0.05644 (4)	0.03997 (3)	0.03908 (1)	0.06062 (5)	0.03947 (2)
Telecom	0.08587 (4)	0.05130 (1)	0.06735 (3)	0.09276 (5)	0.05223 (2)
Average RMSE	0.06121	0.05033	0.05386	0.06330	0.05078
Average Rank	3.8	2.2	2.6	3.2	3.2
Weekly frequency					
Banks	0.02431 (4)	0.01921 (2)	0.01959 (3)	0.02453 (5)	0.01919 (1)
Construction	0.02231 (4)	0.02174 (2)	0.02164 (1)	0.02252 (5)	0.02175 (3)
Food Production	0.02978 (3)	0.02991 (5)	0.02948 (2)	0.02987 (4)	0.02947 (1)
IT	0.02718 (4)	0.02173 (2)	0.02221 (3)	0.02884 (5)	0.02153 (1)
Telecom	0.03652 (4)	0.02648 (2)	0.02680 (3)	0.03836 (5)	0.02640 (1)
Average RMSE	0.02802	0.02381	0.02394	0.02882	0.02367
Average Rank	3.8	2.6	2.4	4.8	1.4
Daily frequency					
Banks	0.009885 (5)	0.00748 (2)	0.009882 (4)	0.009881 (3)	0.00743 (1)
Construction	0.00920 (4)	0.00895 (2)	0.00894 (1)	0.00928 (5)	0.00906 (3)
Food Production	0.01355 (1)	0.01369 (4)	0.01370 (5)	0.01360 (2)	0.01366 (3)
IT	0.01312 (4)	0.01059 (2)	0.01065 (3)	0.01325 (5)	0.01049 (1)
Telecom	0.01633 (4)	0.01316 (2)	0.01321 (3)	0.01693 (5)	0.01311 (1)
Average RMSE	0.01242	0.01077	0.01128	0.01259	0.01075
Average Rank	3.6	2.4	3.2	4.0	1.8

Note: For each sector, figures in parentheses denote the relative rank of a model's RMSE, where the model with the smallest RMSE ranks first. Some results are rounded even up to 6 digits after decimal comma in order to allow unambiguous comparison.

Table 8

Diebold-Mariano Test Results for Out-of-sample Forecasts

Industry Sector	RW	MR	RC	RWMR
Monthly frequency				
Banks	1.778 (0.0754)	2.1168* (0.0343)	2.2808* (0.0226)	1.8700 (0.0615)
Construction	2.0792* (0.0376)	1.0096 (0.3127)	1.1991 (0.2305)	-0.4624 (0.6438)
Food Production	0.6942 (0.4875)	0.5088 (0.6109)	0.0032 (0.9975)	-0.7287 (0.4662)
IT	3.7397* (0.0002)	3.5697* (0.0004)	-3.3587* (0.0008)	3.6753* (0.0002)
Telecom	2.7951* (0.0052)	3.6556* (0.0003)	-3.7555* (0.0002)	2.5909* (0.0096)
Weekly frequency				
Banks	2.2183* (0.0265)	2.6737* (0.0075)	-4.5772* (0.0000)	2.2224* (0.0263)
Construction	1.0234 (0.3061)	0.9023 (0.3669)	-0.4172 (0.6766)	1.1327 (0.2573)
Food Production	-0.5164 (0.6056)	0.5504 (0.5821)	-1.3738 (0.1695)	0.3768 (0.7063)
IT	4.5827* (0.0000)	4.664* (0.0000)	-7.2321* (0.0000)	4.5114* (0.0000)
Telecom	4.0871* (0.0000)	4.2838* (0.0000)	-6.5925* (0.0000)	4.1066* (0.0000)
Daily frequency				
Banks	6.3320* (0.0000)	9.0137* (0.0000)	0.7478 (0.4546)	6.5836* (0.0000)
Construction	2.7016* (0.0069)	2.9103* (0.0036)	-0.8422 (0.3997)	1.5033 (0.1328)
Food Production	-0.5641 (0.5727)	-0.5988 (0.5493)	-0.0692 (0.9448)	-1.2304 (0.2185)
IT	9.1216* (0.0000)	9.2417* (0.0000)	-12.5394* (0.0000)	9.3246* (0.0000)
Telecom	8.164* (0.0000)	8.5852* (0.0000)	-12.1727* (0.0000)	8.2934* (0.0000)

Note: Figures in parentheses denote p-values. The asterisk denotes the rejection of the null hypothesis of equal forecasting accuracy at 5% significance level. In all cases in the role of benchmark the OLS forecasts are employed.

Table 9

Persistence Parameter for MR Specification

Industry Sector	Monthly frequency	Weekly frequency	Daily frequency
Banks	0.9007	0.9666	0.9850
Construction	0.6971	0.9789	0.9956
Food Production	0.1539	0.3776	0.3231
IT	0.9547	0.9892	0.9938
Telecom	0.9509	0.9869	0.9944

Note: Persistence parameter estimates are based on all available data.

Table 10

Spearman's Rank Correlation Coefficient between RMSE and MAE

Industry Sector	Monthly frequency	Weekly frequency	Daily frequency
In-sample forecasts			
Banks	0.70	0.90	0.80
Construction	0.90	0.80	0.80
Food Production	1.00	1.00	0.80
IT	0.90	0.90	1.00
Telecom	0.70	0.90	0.90
Average correlation	<i>0.84</i>	<i>0.90</i>	<i>0.86</i>
Out-of-sample forecasts			
Banks	1.00	1.00	1.00
Construction	0.90	0.60	1.00
Food Production	0.10	0.80	0.60
IT	0.70	1.00	1.00
Telecom	1.00	1.00	1.00
Average correlation	<i>0.74</i>	<i>0.88</i>	<i>0.92</i>