3. THE RELATIONSHIP BETWEEN STOCK AND EXCHANGE RATES FOR BRICS COUNTRIES PRE- AND POST-CRISIS: A MIXED C-VINE COPULA MODEL

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Abstract

We investigate the relationship between stock and foreign exchange rates for BRICS countries pre- and post- U.S. sub-prime crisis and European sovereign debt crisis. With a wide set of exchange rates, the mixed c-vine copula models are used. The results show the correlations are negative for most of the stock/exchange rate pairs. After the U.S. crisis, the stock markets in BRICS countries have stronger negative dependences and risk hedge ability with the USD and JPY currencies. However, after the European crisis, the changes of the correlations are diverse. The risk hedge effectiveness of stock markets in BRICS countries against foreign currencies decreases. These findings suggest that BRICS countries and investors should pay more attention on the multivariate exchange rates and the flows of cross-border capitals with their influence on the local stock markets after the crisis.

Keywords: crisis, relationship, stock, exchange rate, BRICS, mixed c-vine copula model.

JEL Classification: F31, G01

1. Introduction

The BRICS (Brazil, Russia, India, China and South Africa) countries play an important role for the emerging economies and suffer from the recent U.S. sub-prime crisis and European sovereign debt crisis. Capital flows faster among different countries and markets during the crisis period. The managed float policy for foreign exchange rates in emerging economies provides a cushion for the capital outflow (Ghosh *et al*. 2015). After the U.S. crisis, according to the International Finance Association, the amount of

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capital flowing to the emerging economies reaches $825 billion with an increase of 42% in 2010. However, after the European sovereign debt crisis, the volume of total investment flowing to the emerging markets for stock investment and bank loans decreased. The currencies of emerging economies suffered sharp depreciation. The investment rates for BRICS countries are higher. China's investment rates are 48.7% of GDP with India's being almost 35% while the investment rates for developed countries are between 15% and 20% in 2012. Stock is one of the most important investment objectives in BRICS countries. That capital flows in and out has huge impacts on the stock and currency markets in BRICS countries. Therefore, the research on the dependence structure for stock and exchange rates is very important and helpful for the policy makers in the emerging economies and investors to manage the cross-border investment risk.

There are many researches on the relationships between foreign exchange rates and stock, but few studies focus on the co-movements during the crisis period, especially for BRICS countries. The comovements between stock and exchange rates in Asia markets are studied by Granger *et al*. (2000) and Lin (2012). The former shows the stock markets lead the exchange rates during the Asia flu and the following finds the comovements between stock and markets are affected by capital account balance for the Asia emerging countries during the U.S. crisis. Caporale *et al*. (2014) discuss the relationship between stock and foreign exchange rates for the developed countries during the U.S. financial crisis. Grammatikos and Vermeulen (2012) investigate the co-movements among stock, CDS and foreign exchange rates in EMU during the European sovereign debt crisis. Hence, the questions are: what are the correlations between stock and exchange rates of BRICS countries during the crisis period like? Is there any difference between the U.S. crisis and European sovereign debt crisis periods? Under the high uncertainty and risk aversion condition, are BRICS countries a safe haven for cross-border capital during the crisis?

To answer the above questions, we explore the co-movements and dependences between stock and foreign exchange rates in BRICS countries from January 2005 to December 2012 when the U.S. subprime crisis and European sovereign debt crisis happened. We focus on the changing of dependence structures pre and post the two crisis periods and investigate the risk hedge effectiveness of stock and currencies for BRICS countries. We select the foreign exchange rates of BRICS countries with 4 major international USD, EURO, GBP, JPY currencies to local currency. The foreign exchange rates are high-dimensional.

The dependence structures for the joint distribution of high-dimensional variables are complex. With the higher peak and fat-tail phenomenon of financial data, it is difficult to modify the distributions of multivariate distribution. Many researches apply GARCH and DCCGARCH models to solve this problem. In this paper, we use the mixed c-vine copula model to obtain the dependence structures between foreign exchange rates and stock markets in BRICS countries. Vine copula models show the multivariate distributions in a more straightforward way. The vine copula models are constructed by the Markov trees. Trees are built by pair copula models. The mixed c-vine copulas are not constrained by the pair copula families and are more flexible (Czado *et al*. 2012). Based on the mixed c-vine copula model, we calculate the unconditional and conditional dependences between stock and multivariate foreign exchange rates and obtain the optimal dependence structure for the stock and foreign exchange rate pairs.

In general, this paper has three contributions: (1) we study the correlations between stock markets and a wide set of foreign exchange rates including USD, EURO, GBP, JPY to the local currencies. This extends the studies on the dependence structures between foreign exchange rates and stock. The exchange rates are high-dimensional; (2) we introduce the mixed c-vine copula models to study the high-dimensional correlations between stock markets and foreign exchange rates in BRICS countries. The mixed copula models are more flexible and better fit for the multivariate joint distribution; (3) we study the changes for the dependence structures between foreign exchange rates and stock pre and post the crisis periods, not only the U.S. sub-prime crisis but also the European sovereign debt crisis. We also compare the dependence structures pre and post the two periods and show the difference of dependence structures during the two financial crisis periods and analyze the risk hedge effectiveness between stock and foreign exchange rates in BRICS countries.

The rest of this paper is structured as follows: section 2 is the literature review. The data and model are presented in section 3 and section 4. Section 5 shows the empirical results and results analysis. The conclusion is presented in section 6.

2. Literature Review

The recent U.S. subprime crisis and the following sovereign debt crisis make it clear that the financial risks spread quickly and globally nowadays. Frankel and Saravelos (2012) indicate that the appreciation and depreciation of currencies are significant and show the exchange market pressure during the U.S. crisis period. Bigger stock market falls, greater currency weakness can be found during crisis period. Tsangarides (2012) studies the foreign exchange rates performance during the financial crisis and recovery period in the emerging markets and finds the financial channel plays an important role during the crisis. Lane and Milesi-Ferretti (2012) find the foreign exchange rates matters when the emerging markets and developed countries adjust the crisis. Lin (2012) indicates the correlations between foreign exchange rates and stock are stronger after the crisis. Grammatikos and Vermeulen (2012) find the risk transmission from U.S. to Europe while the EURO depreciations are correlated with European stock decreases after the U.S. crisis and the following sovereign debt crisis.

The main theories on the correlations between exchange rates and stock are demonstrated by two models: 'flow-oriented' models indicated by Dornbusch and Fischer (1980) and 'stock oriented' model by Branson (1981) and Frankel (1987). Dornbusch and Fischer (1980) emphasize the changing of foreign exchange on national competitiveness and then the future cash flows of stock markets are influenced by trade and investment. The 'stock oriented' model suggests the innovations of stock markets affect the liquidity demand and in turn have an effect on the exchange rates. Lin (2012) indicates capital account balance drives the co-movements between stock and exchange rates in the Asia emerging markets. Kollias *et al*. (2012) imply that when the stock markets get better, the foreign exchange rates will raise. Thus the correlations between stock and exchange rates can be positive or negative. The results are mixed.

The studies on relationships between exchange rate and stock markets during crisis are very few. Granger *et al*. (2000) study the causal relationship between stock markets and exchange rates during the early and late Great Depression and Asian crisis and find a significant structural break around the crisis period. Aloui and Hkiri (2014) use wavelet analysis to study the short and long term correlations for stock and foreign exchange rates in GCC countries and find the correlation between stock and foreign exchange rates enhanced during the crisis. Pan *et al*. (2007) explore the relationship between the exchange rates and stock prices of seven East Asian countries and find a causal relation from stock market to the foreign exchange rates. Inci and Lee (2014) also find the dynamic correlations between exchange rates and stock enhanced during the recession.

Vine copula model is proposed by Joe (1996) and Bedford and Cooke (2002) . They use vines to specify high-dimensional distributions and couple the marginal distributions of multivariate variables. Aas *et al*. (2009) apply vine copula models to describe the complex tail dependence for high-dimensional copula models and the possibility of copula models to large data space development. Czado *et al*. (2012) introduce mixed c-vine copula models which are not constrained by the types of pair copula models and give the MLE estimation method. They also use the regular vine copula method to study the correlations among foreign exchange rates for six Latin American countries. Allen *et al*. (2013) and Zhang *et al*. (2014) apply vine copula models to the multivariate correlations of the stock markets and show the vine copula models are more accurate than other copula models.

Overall, this paper extends the literature by focusing on the emerging markets: BRICS countries and multivariate exchange rates. And particularly, we study the changing of dependences between stock and multivariate exchange rates before and after the U.S. crisis and European crisis from January 2005 to December 2012. The application of mixed c-vine copula models also gives a new view of understanding the relationship between stock and exchange rates in BRICS countries.

3. Data

We collect the daily closing market data of stock and foreign exchange rates from January 2005 to December 2012 of BRICS countries from Wind database as our sample. For the stock markets, we use the main stock indexes of BRICS countries. The exchange rates include the USD, ERUO, GBP and JPY exchange rates. If the exchange rates increase, the currency of the country depreciates. As we focus on the comparison for the dependence structures pre and post-crisis period, the sample period is divided into 4 sub-periods: January 1, 2005 to September 14, 2008 as the pre U.S. subprime debt crisis period and September 15, 2008 to March 31, 2009 as the post U.S. subprime debt crisis period (Ait-Sahalia *et al*. 2012; Frankel and Saravelos, 2012); April 1, 2009 to December 8, 2009 as the pre European debt crisis period and December 9, 2009 to December 31, 2012 as the post European debt crisis period (Alsakka and Ap Gwilym, 2013; Lane, 2012)

To satisfy the stationarity condition of the variables, we transform the price series into return series. The return series are calculated as: $R_t = 100 \bullet log(P_t / P_{t-1})$. P_t denotes the price index for the stock markets or offer price for the foreign exchange rates at time t .

Table 1 presents the descriptive statistics for the return series. All the return series reject the null hypothesis of Jarque-Bera test and show the leptokurtosis phenomenon. We apply Ljung–Box tests for serial correlation in returns and squared returns and Lagrange multiplier tests for autoregressive conditional heteroscedasticity. The significance levels of Ljung–Box statistics and Lagrange multiplier statistics indicate the existence of noise and GARCH effects. So ARMA-GARCH process is needed for the conditional variance.

Table 1

The Descriptive Statistics for Stock and Exchange Rate Returns

		Mean	Median	Std.	Skew.	Kurt.	J-B	LB1	LB2	LM-F
B	Stock	0.027	0.069	1.811	-0.026	8.653	3290.63***	44.951***	3428.8***	1.625**
	USD	0.000	-0.032	0.942	0.482	15.561	16339.36***	37.881***	2999.8***	1.897***
	EURO	-0.004	-0.028	0.943	0.171	14.207	12942.18***	18.272	1648.4***	1.963***
	GBP	-0.008	-0.023	0.932	-0.015	12.481	9254.72***	14.335	1897.8***	1.313*
	JPY	-0.006	-0.047	1.234	0.729	12.816	10139.19***	41.197***	2731.0***	1.790***
R	Stock	0.011	0.103	2.260	-0.468	14.845	14546.99***	73.871***	2542.2***	2.475***
	USD	0.028	0.000	0.792	0.275	57.597	307181.30***	180.90***	1536.7***	9.893***
	EURO	0.025	0.000	0.743	-0.817	90.505	789282.70***	223.38***	1323.6***	11.092***
	GBP	0.021	0.014	0.850	-0.005	54.485	273131.30***	93.623***	1223.9***	5.281***
	JPY	0.023	-0.029	1.040	-0.038	24.965	49713.73***	81.083***	1308.6***	3.215***
I	Stock	0.059	0.098	1.586	0.139	10.872	6240.43***	45.151***	1068.3***	1.688***
	USD	0.016	0.000	0.526	0.201	7.747	2282.44***	56.974***	1401.2***	2.116***
	EURO	0.011	0.015	0.648	-0.016	6.146	995.90***	39.697***	588.47***	1.896***
	GBP	0.007	0.016	0.673	-0.489	8.378	3005.31***	58.612***	1445.8***	2.009***
	JPY	0.009	-0.026	0.907	0.215	6.489	1242.58***	48.245***	1112.7***	2.035***
C	Stock	0.039	0.084	1.674	-0.329	6.639	1382.29***	51.543***	695.06***	2.049***
	USD	-0.012	-0.002	0.088	-5.292	116.570	1315113.00***	25.815	0.1326	1.567
	EURO	-0.017	0.000	0.623	-0.409	11.090	6683.86***	20.511	383.33	1.260
	GBP	0.007	0.016	0.673	-0.489	8.378	3005.31***	81.750***	1666.8***	3.102***
	JPY	0.009	-0.026	0.907	0.215	6.489	1242.58***	52.786***	389.56***	2.482***
S	Stock	0.054	0.098	1.275	-0.190	6.800	1517.21***	37.417**	3442.0***	1.858***
	USD	0.029	-0.045	1.119	0.576	8.669	3481.87***	47.679***	1360.0***	1.594**
	EURO	0.025	-0.039	0.967	0.766	8.987	3973.39***	28.325	675.88***	1.545**
	GBP	0.021	-0.033	1.001	0.603	8.192	2956.05***	46.302***	1001.8***	1.802***
	JPY	0.023	-0.054	1.380	0.521	8.387	3131.63***	39.123***	3072.5***	1.465**

*Notes: The letters in the first column B, R, I, C, S are short for Brazil, Russia, India, China and South Africa. The symbols ***, ** and * represent the significance levels of 1%, 5% and 10%, the same below; J-B represents the Jarque-Bera statistics; LB1 and LB2 are the Ljunk–Box statistics for returns and squared returns sequentially; The lag of Ljung-Box Q-statistics is 20; LM-F shows the F statistics of Lagrange multiplier test; The F statistics are constructed for maximum lag of 36.*

4. The Model

The two-stage method is used to calculate the mixed c-vine copula models. First, we use the $ARMA\text{-}GARCH(1,1)\text{-}t$ model to modify the marginal distribution of the return series. The conditional variances show the volatilities for the returns. Second, we estimate the parameters for the mixed c-vine copula model based on the uniformed data series calculated by the marginal distribution model.

4.1 Marginal Distribution Model

The $ARMA\text{-}GARCH(1,1)\text{-}t$ model is defined as follows:

$$\mathrm{R}_t = c + \sum_{i=1}^{p} \phi_i R_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{1}$$

$$\varepsilon_t = h_t^{1/2} \xi_t \tag{2}$$

$$h_t = \overline{\omega} + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \text{where} \xi_t \sim t(v) \tag{3}$$

where: R_t denotes the return at t moment, ε_t presents variance, h_t is the conditional variance. ξ_t obeys the Student-t distribution with v degrees of freedom.

4.2 Mixed C-vine Copula Model

According to Skar theory, for the given conditional marginal distributions, there exists a copula function C for the multivariate joint conditional distribution of the variables F.

$$\mathrm{F}(x_1, x_2, \cdots, x_n) = C[F_1(x_1), F_2(x_2), \cdots, F_n(x_n)] \tag{4}$$

As the conditional density functional theory presents, the probability density function of multivariate joint distribution can be broken into the product of multivariate conditional probability terms:

$$f(x_1, x_2, \cdots, x_n) = f_n(x_n) \bullet f_{n-1|n}(x_{n-1} | x_n) \bullet f_{n-2|n-1,n}(x_{n-2} | x_{n-1}, x_n) \bullet \cdots \bullet f_{1,2,\ldots,n}(x_1 | x_2, \cdots, x_n) \tag{5}$$

Based on the conditional density function theory, the probability density function calculated by copula model can be transformed into:

$$\mathrm{f}(x_1, x_2, \cdots, x_n) = c_{1,2,\cdots,n}(F_1(x_1)), \cdots, F_n(x_n)) \bullet f_1(x_1) \cdots f_n(x_n) \tag{6}$$

where: $c_{1,2,\cdots,n}(F_1(x_1)), \cdots, F_n(x_n))$ is the copula density function. And the probability density function based on the conditional density function for multivariate joint distributions can be obtained through equation (5) and equation (6) as follows:

$$f(x|v) = c_{xv_i|v_{\neq i}}(F(x|v_{\neq i}, F(v_i|v_{\neq i})) \cdot f(x|v_{\neq i}) \tag{7}$$

where: $c_{xv_j|v_{\neq j}}(\bullet,\bullet)$ is a bivariate copula model; v is a d-dimension vector. v_i means a randomly selected component of vector v. $v_{\neq i}$ presents the left components in vector v except $v_{\neq i}$.

A complete vine structure is composed of a collection of trees. For the given nodes N and the edges built by paired nodes, a tree is defined as the one that joins the nodes and edges and have no cycles. Regular vines require the edges which join the nodes may not be reused. Joe (1996) proposes the vine copula model based on pair copula models. Vine copula models calculate the multivariate joint distributions by a series of trees. A tree is constructed by a series of pair copula models. C-vine (canonical vine) copula model is proposed by Bedford and Cooke (2002). The characteristic of c-vine copula model is that each vine tree only has one root node and the other nodes of the tree are connected to the root node. The density function for 5-dimensional c-vine copula model is presented as follows:

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5) &= f_1(x_1)\bullet f_2(x_2)\bullet f_3(x_3)\bullet f_4(x_4)\bullet f_5(x_5) \\ &\bullet c_{12}(F_1(x_1),F_2(x_2))\bullet c_{13}(F_1(x_1),F_3(x_3))\bullet c_{14}(F_1(x_1),F_4(x_4))\bullet c_{15}(F_1(x_1),F_5(x_5)) \\ &\bullet c_{23|1}(F(x_2|x_1),F(x_3|x_1))\bullet c_{24|1}(F(x_2|x_1),F(x_4|x_1))\bullet c_{25|1}(F(x_2|x_1),F(x_5|x_1)) \\ &\bullet c_{34|12}(F(x_3|x_1,x_2),F(x_4|x_1,x_2))\bullet c_{35|12}(F(x_3|x_1,x_2),F(x_5|x_1,x_2)) \\ &\bullet c_{45|123}(F(x_4|x_1,x_2,x_3),F(x_5|x_1,x_2,x_3)) \end{aligned} \tag{8}$$

The mixed c-vine copula model proposed by Czado *et al*. (2012) is not constrained by the types of pair copula models. The pair copula families for each term of the mixed c-vine copula models are chosen individually by MLE method. The pair copula families used in this paper are: Gaussian copula, t-copula, Clayton copula, Gumbel copula, Frank copula and Joe copula. Gaussian copula models are usually a benchmark for copula models. For the given marginal distributions u and v . The distribution functions of the former pair copula families are presented in table 2.

Table 2

Pair Copula Family

Copula model	Distribution function
Gaussian	$C(u,v;\rho)=\int_{-\infty}^{\phi^{-1}(u)}\int_{-\infty}^{\phi^{-1}(v)}\frac{1}{2\pi\sqrt{1-\rho^2}}\exp(\frac{-(r^2+s^2-2\rho rs)}{2(1-\rho^2)})drds$
T	$C(u,v;\rho,\upsilon)=\int_{-\infty}^{T_v^{-1}(u)}\int_{-\infty}^{T_v^{-1}(v)}\frac{1}{2\pi\sqrt{1-\rho^2}}\exp(1+\frac{-(s^2+t^2-2\rho st)}{\upsilon(1-\rho)^2})^{-\frac{\upsilon+2}{2}}dsdt$
Gumbel	$C_G(u,v;\alpha)=\exp(-[(-\ln u)^{\frac{1}{\alpha}}+(-\ln v)^{\frac{1}{\alpha}}]^{\alpha})$
Clayton	$C_{cl}(u,v;\theta)=(u^{-\theta}+v^{-\theta}-1)^{-\frac{1}{\theta}}$
Frank	$C_F(u,v;\lambda)=-\frac{1}{\lambda}\ln(1+\frac{(e^{-\lambda u}-1)(e^{-\lambda v}-1)}{e^{-\lambda}-1})$
Joe	$C_J(u,v;\theta)=1-((1-u)^{\theta}+(1-v)^{\theta}-(1-u)^{\theta}(1-v)^{\theta})^{\frac{1}{\theta}}$

Following Claudia Czado *et al*. (2012), we use Kendall's tau to show the dependence structures of the copula models. Kendall's tau is calculated as:

$$\tau = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1 \tag{9}$$

4.3 Estimation Process

We use maximum log-likelihood method to estimate the parameters of mixed c-vine copula models. First, we estimate the parameters of trees layer by layer through pair copula models. Then based on the initial parameters of pair copula models, we use maximum log-likelihood method to obtain the final value of the parameters. The log-likelihood value for c-vine copula models is defined as:

$$L(\theta) = \sum_{J=1}^{n-1}\sum_{i=1}^{n-j}\sum_{t=1}^{T}\log(c_{j,j+i|1,\ldots,j-1}(F(x_{j,t}\left|x_{1,t},\cdots,x_{j-1,t}\right.),F(x_{j+i,t}\left|x_{1,t},\cdots,x_{j-1,t}\right.),\theta)) \tag{10}$$

5. Empirical Results and Results Analysis

5.1 Marginal Distribution and Volatilities

$ARMA\text{-}GARCH(1,1)\text{-}t$ models are used to calculate the marginal distributions of return series. Table 3 shows the parameters and log-likelihood value of the marginal distribution models for the overall sample. As most of the parameters are significant, the $ARMA\text{-}GARCH(1,1)\text{-}t$ models we use are appropriate.

Table 3

The Parameters of ARMA-GARCH(1,1)-t Models: 2005-2012

	c	ϕ_1	$\overline{\omega}$	β	α	ν	LLF
				Brazil			
stock	0.060*	0	0.057***	0.909***	0.071***	11.278***	-4630.918
USD	-0.029**	0.088***	0.010***	0.860***	0.137***	7.356***	-2736.671
EURO	-0.027**	0	0.053***	0.787***	0.146***	7.165***	-2943.807
GBP	-0.025	0	0.029***	0.852***	0.112***	8.001***	-2940.611
JBP	-0.048**	0.051**	0.031***	0.865***	0.116***	6.442***	-3500.256
				Russia			
stock	0.110**	0.081**	0.059**	0.898***	0.093***	5.431***	-4869.856
USD	-0.012	0.065**	0.002**	0.897***	0.103***	6.848***	-1650.197
EURO	0.005	0.091***	0.002*	0.889***	0.111***	6.365***	-1470.049
GBP	0.003	0.042*	0.003*	0.921***	0.078***	5.791***	-2093.927
JBP	-0.041**	-0.011	0.006**	0.902***	0.098***	5.800***	-2893.502
				India			
stock	0.098***	0.080***	0.027**	0.895***	0.095***	7.897***	-4028.621
USD	-0.001	-0.015	0.003***	0.838***	0.163***	7.232***	-1336.426
EURO	0.005	0.028	0.008**	0.921***	0.060***	9.205***	-2178.594
GBP	0.008	0.034	0.008**	0.914***	0.066***	8.920***	-2175.924
JBP	-0.030**	-0.014	0.010**	0.885***	0.110***	6.283***	-2822.745
				China			
stock	0.123***	0.062**	0	0.942***	0.058***	4.679***	-4358.013
USD	0	0	0.000***	0.743***	0.257***	5.238***	4005.541
EURO	-0.01	0	0.001	0.964***	0.035***	8.311***	-1962.581

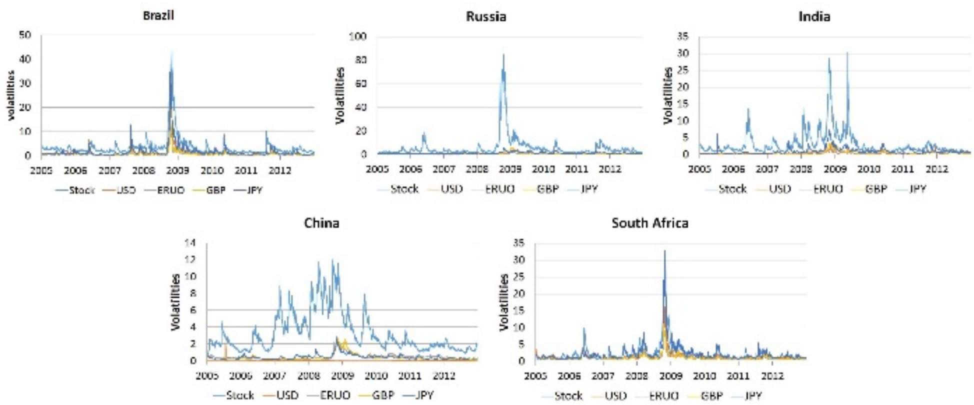
	c	ϕ_1	$\overline{\omega}$	β	α	ν	LLF
GBP	-0.009	-0.004	0.002**	0.949***	0.045***	15.541***	-1818.829
JBP	-0.030**	0	0.003*	0.944***	0.050***	5.727***	-2107.830
South Africa							
stock	0.094***	0	0.016***	0.895***	0.096***	14.351***	-3696.272
USD	-0.001	-0.046**	0.027***	0.901***	0.076***	11.953***	-3563.344
EURO	-0.013	-0.003	0.030***	0.879***	0.088***	7.233***	-3188.887
GBP	-0.003	-0.018	0.025***	0.895***	0.078***	10.180***	-3294.168
JBP	-0.036	-0.048**	0.034***	0.882***	0.102***	7.469***	-3950.073

Notes: LLF is short for the log-likelihood value.

The conditional variance calculated by the GARCH models shows the modified volatilities of the return series. Figure 1 shows the modified volatilities of stock and foreign exchange rate returns for all BRICS countries. The volatilities of stock returns are greater than the foreign exchange rate returns for all BRICS countries. The U.S. subprime crisis and European sovereign debt crisis make the movements of volatilities have two special periods. The volatilities of the stock returns and foreign exchange rate returns have a big rise from October 2008 to April 2009 when the U.S. crisis spread globally. The volatilities also increase around October 2011 when the European sovereign debt crisis booms, but are smaller than that after the U.S. subprime crisis takes place.

Figure 1

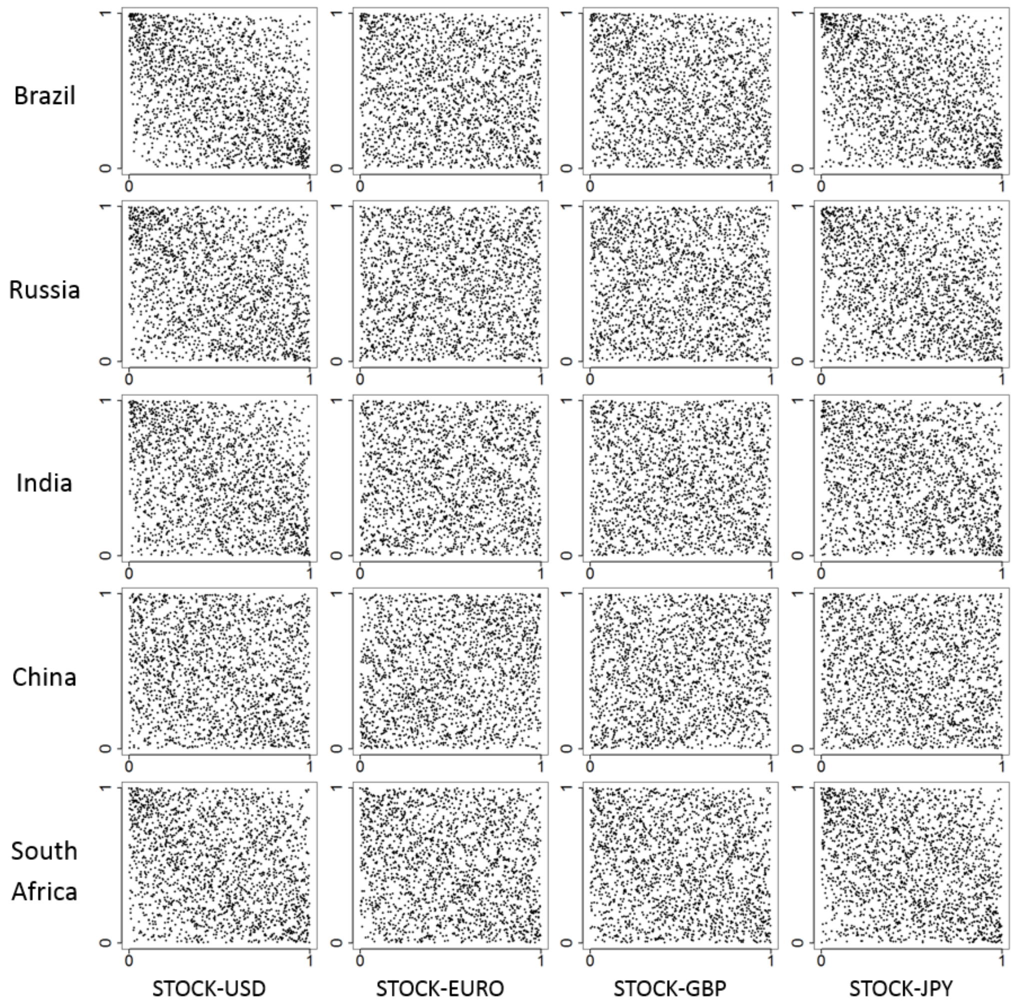
Volatilities of the Stock and Exchange Rate Returns



5.2 Correlation Analysis: The Whole Sample Period

To give a full picture of the relationships between stock and exchange rates, we estimate the dependence structure for stock and exchange returns in BRICS countries during the whole sample period. We compute the unconditional dependence structures between paired stock and exchange returns using the pair copula models in tree1 of the mixed c-vine copula models and use the USD and EURO (currency of the country where the U.S. crisis and European crisis occurs) exchange rates as condition separately and calculate the correlations between stock and the other exchange rate returns in tree 2 of the mixed c-vine copula models. Based on the results of ARMA-GARCH(1, 1)-t model, we uniform the obtained residuals. Figure 2 shows the scatter plots for the paired copula data formed from the transformed standardized residual returns. The contour plots of the paired copula data are displayed in Figure A1 (see Appendix).

Figure 2. Scatter Plots for the Paired Copula Data Formed from the Transformed Standardized Residual Returns



The parameters and selected copula model families for the pair copula models are presented in table 4. The optimal pair copula models are selected by the log-likelihood values, AIC criterion and BIC criterion. For the goodness of fit tests, we use Vuong test and Clarke test (Clarke, 2007; Vuong, 1989) to check which model provide the best fit. Since Vuong test and Clarke test are based on the comparison of two specified copula models, we use a c-vine copula model with all pair-copulas being Gaussian copulas as comparison. The significance levels of Vuong and Clarke statistics shows the mixed c-vine copula model we choose is better. The T, Frank, Clayton and Gumbel pair copula models investigate the tail dependence between stock and exchange rate pairs. The degree of freedom for the T copula models show the extreme value exists during the sample period. The multivariate pair copula families capture the various dependence structures between stock and exchange rates.

Table 4

Parameters for Mixed C-vine Copula Models: 2005-2012

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Brazil										
copula	T	T	Gau	T	Gau	Frank	T	Gau	Frank	T
Par1	-0.409	-0.161	-0.194	-0.380	0.127	0.859	-0.132	-0.389	-0.629	-0.368
Par2	20.643	16.884	0	9.988	0	0	17.236	0	0	10.944
Std1	0.018	0.023	0.021	0.02	0.022	0.134	0.023	0.018	0.137	0.02
Std2	10.469	7.604	0	2.618	0	0	6.682	0	0	2.843
Goodness of fit LLF= 3054.833; AIC= -6071.665; BIC= -5965.487; Vuo = 6.569***; Cla=1099***										
Russia										
copula	T	T	T	T	T	T	T	Gau	T	T
Par1	-0.283	0.010	-0.038	-0.248	-0.057	0.019	-0.149	-0.280	-0.043	-0.254
Par2	20.692	12.271	12.198	12.924	29.645	19.125	19.169	0	14.165	15.104
Std1	0.021	0.024	0.024	0.022	0.023	0.024	0.023	0.020	0.024	0.022
Std2	10.518	3.800	3.775	4.032	17.595	9.007	8.208	0	5.199	5.382
Goodness of fit LLF= 1158.171; AIC= -2280.341; BIC= -2179.688; Vuo = 8.718***; Cla=1290***										
India										
copula	T	T	T	T	Clayton	T	T	T	T	T
Par1	-0.287	-0.010	-0.048	-0.223	0.077	0.043	-0.097	-0.289	-0.048	-0.230
Par2	13.046	14.401	11.176	9.818	0	10.238	17.281	16.420	15.160	12.055
Std1	0.022	0.024	0.024	0.023	0.026	0.025	0.024	0.021	0.024	0.023
Std2	4.425	5.366	3.396	2.547	0	2.950	6.995	7.017	5.512	3.702
Goodness of fit LLF=1234.084; AIC=-2430.168; BIC=-2324.388; Vuo = 6.260***; Cla=1137***										
China										
copula	Frank	Gau	Gau	T	Gumbel	Gumbel	Gau	Frank	Frank	Gaun
Par1	-0.680	0.144	0.081	-0.077	1.061	1.026	-0.101	-0.263	-0.018	-0.096
Par2	0	0	0	17.717	0	0	0	0	0	0
Std1	0.137	0.022	0.023	0.024	0.015	0.012	0.022	0.138	0.136	0.023
Std2	0	0	0	8.058	0	0	0	0	0	0
Goodness of fit LLF= 866.027; AIC= -1702.053; BIC= -1618.473; Vou = 6.081***; Cla= 1109***										
South Africa										
copula	Gau	T	T	T	Gumbel	T	T	Gau	Gau	T
Par1	-0.277	-0.162	-0.153	-0.278	1.051	0.125	-0.097	-0.242	-0.034	-0.250
Par2	0	24.382	24.401	13.188	0	25.633	17.988	0	0	24.761
Std1	0.02	0.022	0.023	0.021	0.015	0.023	0.023	0.020	0.022	0.021
Std2	0	14.512	14.174	4.290	0	15.041	6.863	0	0	12.457
Goodness of fit LLF= 4127.386; AIC= -8218.771; BIC= -8117.973; Vuo = 5.871***; Cla=1121***										

Note: Parameters (1) ~ (10) donates the pairs: S_USD, S_EURO, S_GBP, S_JPY, S_EURO|USD, S_GBP|USD, S_JPY|USD, S_USD|EURO, S_GBP|EURO, S_JPY|EURO. S represents stock return series. S_USD, S_EURO, S_GBP are the stock-foreign exchange rate (USD, EURO, GBP) pairs in tree1 of the given vine copula models. S_EURO|USD, S_GBP|USD, S_JPY|USD, S_USD|EURO, S_GBP|EURO, S_JPY|EURO are the conditional stock-foreign exchange rate pairs in tree2 of the given vine copula models. Gau is short for Gaussian; Gau is short for Gaussian. Vuo and Cla are the statistics of Vuong tests and Clarke tests. This note is also suitable for the tables below.

We use Kendall's tau from formula (15) to calculate the correlations between stock and foreign exchange rates. Table 5 presents the Kendall's tau correlation results for the relationship between stock and exchange rate returns. Almost all of the unconditional correlations between stock and foreign exchange rates for BRICS countries are negative. This means when the currencies for the developed countries become weak, the stock prices for BRICS countries will rise. The stock markets in BRICS countries are risk hedges for the countries of developed countries. The only exceptions are the dependences which are positive for stock-EURO foreign exchange rate returns and stock-GBP returns in China and stock-EURO returns in Russia. But the dependences are not very strong. Using the USD foreign exchange rates as condition, the stock and EURO, GBP foreign exchange rate returns show positive correlations for all BRICS countries. While given the EURO foreign exchange rate as condition, the negative dependence between stock and JPY foreign exchange rate returns decreases.

Table 5

The Correlations: The Whole Sample Period

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Brazil	-0.268	-0.103	-0.124	-0.248	-0.254	-0.070	-0.240	0.081	0.095	-0.084
Russia	-0.183	0.006	-0.024	-0.159	-0.181	-0.028	-0.163	-0.036	0.012	-0.095
India	-0.185	-0.006	-0.031	-0.143	-0.186	-0.031	-0.148	0.037	0.028	-0.062
China	-0.075	0.092	0.051	-0.049	-0.029	-0.002	-0.061	0.057	0.025	-0.065
South Africa	-0.178	-0.103	-0.098	-0.180	-0.155	-0.022	-0.161	0.048	0.080	-0.062

Note: Parameter (1) ~ (10) donate the pairs: S_USD, S_EURO, S_GBP, S_JPY, S_EURO|USD, S_GBP|USD, S_JPY|USD, S_USD|EURO, S_GBP|EURO, S_JPY|EURO. We use Kendall's tau to calculate the unconditional and conditional correlations of the paired return series, the same below.

5.3 Correlation Analysis: Pre- and Post-U.S. Subprime Crisis

As one may see from Figure 1, after the U.S. subprime crisis occurs and intensifies, the exchange rates and stock returns volatile greater than the ones before the crisis period. So following Grammatikos and Vermeulen (2012), we select the pre-U.S. crisis period: January 2005 to July 2007 and the post-U.S. crisis period: August 2007 to October 2008 to investigate how the dependence structure for the stock markets and exchange rates changes before and after the crisis. Similarly, we use the models to obtain the uniformed marginal distributions first. And we establish the mixed c-vine copula models by maximum likelihood estimation and get the optimal parameters and correlations for the mixed c-vine copula models. The estimated parameters and results of goodness of fit tests for the selected models are shown in Table A1. The significance levels of Vuong statistics and F statistics of Clarke tests show the mixed c-vine copula models we choose are appropriate.

Table 6 summarizes the estimated Kendall's tau correlations and tail dependence between stock and foreign exchange rates pre and post the U.S. crisis. As Table 6 shows, all the Kendall's tau correlations between stock and foreign exchange rates in BRICS countries get stronger after the U.S. crisis. The unconditional Kendall's tau correlations between stock and USD, ERUO, GBP and JPY foreign exchange rates are negative for all BRICS countries except China's stock/EURO pairs. These results indicate the risk hedge effectiveness for the stock markets in BRICS countries become stronger and explain the phenomenon why hot money flows to the emerging markets after the U.S. crisis. The negative dependences between stock and JPY foreign exchange rates are the strongest of the stock/foreign exchange rate pairs. The stock markets show the strongest risk hedge ability for JPY currency. China's stock and EURO exchange rate are positively correlated. The risk for stock/EURO and stock/GBP investment is great and get a little stronger in China after the U.S. crisis.

Given the USD foreign exchange rates as condition, the conditional Kendall's tau correlations between stock and foreign exchange rates are estimated. After the U.S. crisis, the negative conditional correlations between stock and EURO exchange rates for BRICS countries except China decrease and the positive conditional correlations in China get stronger. The European markets suffer from the U.S. crisis and the risk hedge effectiveness for stock and EURO exchange pairs decrease. The stock/JPY exchange rates show stronger negative conditional correlations after the U.S. crisis. This indicates the stock/JPY portfolios may be a better risk hedge strategy after the crisis.

The results of tail dependence show the consistent conclusions. The unconditional tail dependence between stock and USD, ERUO, GBP and JPY foreign exchange rates of BRICS countries reduces except stock/EURO and stock/GBP pairs of India after the U.S. crisis. This means the stock markets of BRICS countries show the function of avoiding risk. For conditional tail dependence, China's stock/foreign exchange rate pairs show greater decrease. This explains China plays a significant role of hedging risk after the U.S. crisis. Meanwhile, the conditional tail dependence for stock/GBP and stock/ERUO pairs in India and stock/EURO pairs in South Africa gets larger. As tail dependence shows the co-movements between return series, this means the risk of stock/EURO portfolios in India and South Africa gets stronger after the U.S. crisis.

Table 6

The Correlations: Pre- and Post-U.S. Subprime Crisis

	S_USD	S_EURO	S_GBP	S_JPY	S_EURO|USD	S_GBP|USD	S_JPY|USD
Pre-crisis							
Brazil	-0.244	-0.147	-0.158	-0.170	0.063	0.046	0.009
				(0.001)		(0.001)	(0.021)
Russia	-0.091	0.091	0.029	-0.008	0.053	-0.011	-0.017
	(0.001)		(0.013)	(0.016)			(0.003)
India	-0.143	-0.007	0.023	-0.019	0.024	0.033	-0.004
		(0.002)	(0.031)	(0.007)		(0.045)	(0.004)
China	-0.032	0.036	0.033	0.020	0.032	0.032	0.029
	(0.002)	(0.049)	(0.076)	(0.046)	(0.073)	(0.073)	(0.068)
South Africa	-0.094	-0.088	-0.094	-0.108	-0.016	-0.014	-0.063
	(0.001)	(0.003)	(0.001)	(0.003)			
Post-crisis							

	S_USD	S_EURO	S_GBP	S_JPY	S_EURO|USD	S_GBP|USD	S_JPY|USD
Brazil	-0.320	-0.228	-0.193	-0.367	-0.004	0.050	-0.222
						(0.001)	
Russia	-0.189	0.057	0.026	-0.261	-0.036	0.038	-0.214
	(0.003)	(0.003)					
India	-0.227	-0.075	-0.059	-0.267	-0.007	0.017	-0.162
		(0.012)	(0.055)		(0.013)	(0.062)	
China	-0.061	0.074	0.046	-0.166	0.053	0.029	-0.180
		(0.013)	(0.001)		(0.002)		
South Africa	-0.206	-0.112	-0.089	-0.224	0.071	0.116	-0.115
					(0.096)		

Note: we use Kendall's tau to calculate the unconditional and conditional correlations of the paired return series, the same below; The Values of tail dependence are in brackets, the same below.

As the above results indicate, the dependences between the stock and foreign exchange rate returns during the post the crisis period strengthen greatly. The stock markets in BRICS countries show stronger risk-hedge effectiveness for USD and JPY currencies. The government of BRICS countries should pay more attention to the changes of the dependence structure and the effect of hot money flowing into the local stock markets. As the risk hedge effectiveness for stock/Euro foreign exchange rate pairs become weak, the government should concern more on the macro-economic environment for the European countries and take precautions on the remaining influence of the U.S. crisis on Eurozone. The cross board investors should explore the optimal foreign exchange currencies and stock investment to hedge risk.

5.4 Correlation Analysis: Pre- and Post-European Sovereign Debt Crisis

Similar to the calculation process for the dependence structure pre and post the U.S. crisis, we study the changing of dependence between stock and exchange rates between the pre European sovereign debt crisis period (November 2008 to April 2011) and the post European sovereign debt crisis period (May 2011 to December 2012) by mixed c-vine copula models. Table A2 shows the estimated parameters of the best copula models. With the results of Vuong tests and Clarke tests being significant, we conclude the selected mixed C-vine copula models are suitable.

The estimated Kendall's tau correlations and tail dependence before and after the European sovereign debt crisis are shown in table 7. The influences of European sovereign debt crisis on the dependence structures between stock and exchange rates are different from that of U.S. crisis. The changes in the Kendall's tau dependence structures between stock and multivariate foreign exchange rates are diverse. For the unconditional correlations between stock and USD exchange rate, the dependences in Russia, India and South Africa do not change much while the dependences become weak and the risk hedge effectiveness for stock/USD exchange rate pairs decreases for Brazil. China's stock market shows a stronger negative correlation with USD currency than that before the European sovereign debt crisis. China's stock market becomes a safe haven for the USD currency. For the stock/Euro foreign exchange rate pairs, the positive dependences between China and Brazil get stronger. The stock/Euro foreign exchange portfolio risk in China and Brazil increases, especially in China.

However, the stock market in Russia shows stronger risk hedge effectiveness with the EURO and GBP with greater negative correlations after the European crisis. The negative dependences between JPY foreign exchange rates and stock markets in the BRICS countries decrease after the European crisis. The risk hedge effectiveness of JPY/stock pairs decreases.

Given the EURO foreign exchange rate as condition, we find the conditional Kendall's tau correlations between stock and USD foreign exchange rate for Brazil, Russia, India and South Africa decrease during the post European crisis period. These indicate the risk hedge effectiveness for the stock markets in these countries decreases. However, China's stock market shows stronger negative correlations with the USD foreign exchange rate. The dependences between stock and GBP foreign exchange rates for all BRICS countries rise slightly while the stock and JPY foreign exchange rate pairs show a great decrease.

For tail dependence, as one may see from Table 7, the stock-EURO pair of Brazil and stock-GBP pair of China show a sharp rise in the unconditional tail dependence after the European crisis. The tail dependence for stock-EURO, stock-JPY pairs of India cannot be captured after the European crisis. Although other stock-foreign exchange rate pairs show weaker tail dependence, the changes of unconditional tail dependence between stock and foreign exchange rate pairs of BRICS countries are different after the European crisis. For the conditional tail dependence, all the stock-foreign exchange rate pairs of India, the stock-JPY pair of Brazil and South Africa show weak tail dependence. Tail dependence for other pairs does not exist after the European crisis. One of the possible reasons is the distinct policies of foreign exchange rates in BRICS countries after the European crisis.

Table 7

The Correlations: Pre- and Post-European Crisis

	S_USD	S_EURO	S_GBP	S_JPY	S_USD|EURO	S_GBP|EURO	S_JPY|EURO
				Pre-crisis			
Brazil	-0.317	-0.043	-0.078	-0.302	-0.335	-0.074	-0.329
Russia	-0.264	-0.027	-0.029	-0.221	-0.268	-0.021	-0.239
		(0.001)					
India	-0.231	0.028	-0.060	-0.192	-0.229	-0.081	-0.203
		(0.008)	(0.001)			(0.002)	
China	-0.094	0.130	0.065	-0.051	-0.021	-0.018	-0.049
South Africa	-0.226	-0.102	-0.078	-0.220	-0.224	-0.012	-0.215
			(0.003)			(0.007)	
				Post-crisis			
Brazil	-0.173	0.056	-0.015	-0.157	-0.233	-0.094	-0.210
		(0.076)		(0.008)			(0.005)
Russia	-0.271	-0.150	-0.188	-0.235	-0.230	-0.118	-0.192
India	-0.204	0.024	-0.075	-0.172	-0.217	-0.107	-0.185
	(0.007)			(0.007)	(0.002)	(0.008)	(0.003)
China	-0.112	0.153	0.098	-0.006	-0.043	-0.012	0.024
	(0.003)		(0.035)				
South Africa	-0.210	-0.117	-0.144	-0.196	-0.194	-0.095	-0.167
				(0.002)			(0.006)

According to the above results, the dependence structures between stock and multivariate foreign exchange rates for BRICS countries are diverse after the European sovereign debt crisis. The risk hedge effectiveness for the stock markets in BRICS countries is different. The influences of multivariate exchange rates are growing. And the relationship between stock and exchange rates are time varying. These results have interesting implications on policy in BRICS countries. Monetary policies for BRICS countries should be more flexible. The governments of BRICS countries need to relax the restrictions on foreign exchange transaction subjects, increase the kinds of foreign exchange products and build diverse management of foreign exchange reserves. When considering the risk prevention mechanism of stock and foreign exchange portfolios, the government should build warn systems for cross-border capital flows and multi-level financial system and strengthen the stability of the local stock market by the use of various foreign exchange hedging tools.

6. Conclusion

In this paper, we investigate the dependence structures for the stock markets and exchange rates of BRICS countries pre and post the recent U.S. sub-prime crisis and European sovereign debt crisis. By mixed c-vine copula models, we study the high-dimensional correlations between stock and multivariate exchange rates. The pair copula model families including Gaussian, t, Clatyon, Gumbel, Frank and Joe copulas are selected by the MLE method. The mixed c-vine copula models provide a flexible and accurate dependence structures for the stock and exchange rates pairs. Focusing on the changing of dependences before and after the crisis, this paper answers the question how the stock markets in BRICS countries are correlated with the multivariate exchange rates during the recent crisis periods and investigate the risk hedge effectiveness of stock markets in BRICS countries.

The results suggest that the stock markets in all BRICS countries show stronger negative correlations with USD and JPY currencies after the U.S. crisis and are safe havens for the cross-board capitals. But after the European debt crisis, the dependences between stock and multivariate foreign exchange rates are diverse. The risk hedge ability for stock/USD and JPY pairs decreases for BRICS countries except China. China's stock and USD currency are negatively correlated. And the positive correlations between stock and EURO for Brazil, India, China and South Africa strengthen. The two financial crises affect the stock markets in BRICS countries through foreign exchange differently. The results explain why the cross-border capital flows into BRICS countries after the U.S. crisis and flows out after the European crisis. BRICS countries and investors should pay more attention to the hot money flows and its influence on the local stock markets. This paper provides a new view of understanding the impact of crisis on the correlations between high-dimensional exchange rates and stock returns for BRICS countries by mixed c-vine copula models. The results are also useful for policy makers in BRICS countries and multinational investors to hedge the risk of stock and foreign exchange during the crisis period.

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Appendix

Figure A1

Contour Plots for the Paired Copula Data Formed from the Transformed Standardized Residual Returns

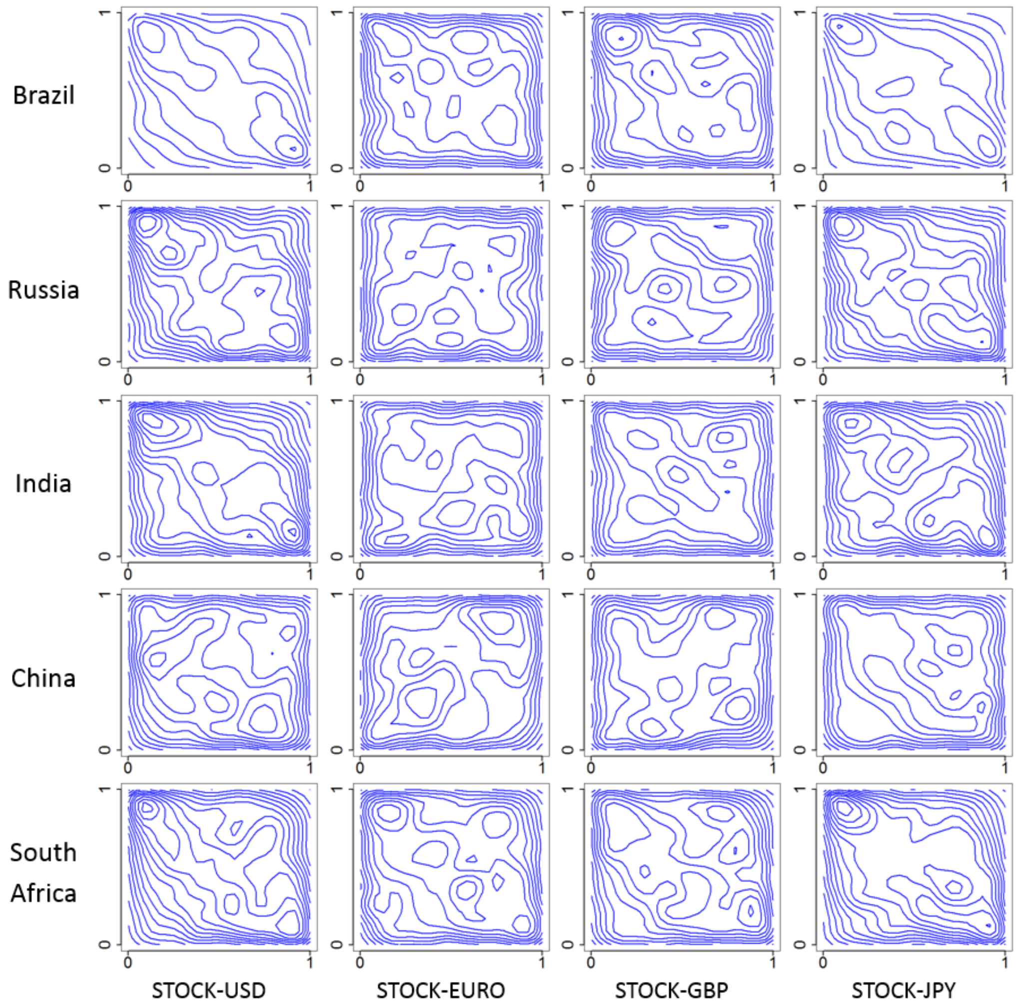

Table A1

Parameters of Mixed C-vine Copula Models: Pre- and Post-U.S. Crisis

	Pre-crisis					Post-crisis				
	type	Par1	Par2	Std1	Std2	type	Par1	Par2	Std1	Std2
Brazil										
(1)	T	-0.373	11.270	0.035	5.725	GN	-0.482	0.000	0.034	0.000
(2)	T	-0.229	14.976	0.039	10.545	GN	-0.351	0.000	0.041	0.000
(3)	T	-0.246	13.822	0.039	8.812	GN	-0.298	0.000	0.043	0.000
(4)	T	-0.263	9.929	0.039	4.804	T	-0.545	9.743	0.034	5.272
(5)	GN	0.099	0.000	0.039	0.000	GN	-0.006	0.000	0.049	0.000
(6)	C	0.096	0.000	0.051	0.000	C	0.105	0.000	0.059	0.000
(7)	J	1.015	0.000	0.023	0.000	F	-2.081	0.000	0.302	0.000
LLF	1348.483					772.680				
AIC	-2662.966					-1517.360				
BIC	-2587.148					-1460.435				
Vuo	2.760***					2.503**				
Cla	357***					261***				
Russia										
(1)	T	-0.143	12.147	0.042	7.227	T	-0.292	7.694	0.047	3.565
(2)	GN	0.143	0.000	0.039	0.000	C	0.120	0.000	0.061	0.000
(3)	T	0.045	9.051	0.044	3.867	C	0.053	0.000	0.055	0.000
(4)	T	-0.013	7.676	0.044	2.884	F	-2.488	0.000	0.306	0.000
(5)	F	0.480	0.000	0.241	0.000	GN	-0.056	0.000	0.047	0.000
(6)	F	-0.098	0.000	0.241	0.000	C	0.079	0.000	0.057	0.000
(7)	T	-0.027	11.347	0.043	6.273	F	-2.003	0.000	0.299	0.000
LLF	660.440					276.444				
AIC	-1288.879					-524.887				
BIC	-1217.621					-467.929				
Vuo	5.088***					4.651***				
Cla	464***					287***				
India										
(1)	T	-0.223	15.158	0.040	11.179	T	-0.349	10.587	0.045	6.795
(2)	T	-0.010	13.360	0.043	8.799	T	-0.118	7.163	0.054	3.375
(3)	GL	1.023	0.000	0.024	0.000	T	-0.092	4.109	0.057	1.136
(4)	T	-0.030	9.365	0.044	4.002	T	-0.407	11.385	0.042	8.494
(5)	Cl	0.050	0.000	0.045	0.000	T	-0.011	8.241	0.054	4.239
(6)	GL	1.034	0.000	0.025	0.000	T	0.027	4.668	0.056	1.458
(7)	T	-0.006	11.641	0.043	6.330	GN	-0.252	0.000	0.045	0.000
LLF	399.303					374.612				
AIC	-766.607					-717.224				
BIC	-695.526					-652.733				
Vuo	2.926***					3.342***				
Cla	364***					253***				
China										
(1)	T	-0.050	11.868	0.043	6.665	F	-0.551	0.000	0.290	0.000
(2)	GL	1.038	0.000	0.024	0.000	C	0.159	0.000	0.061	0.000
(3)	J	1.060	0.000	0.031	0.000	C	0.096	0.000	0.059	0.000
(4)	J	1.034	0.000	0.030	0.000	GN	-0.258	0.000	0.045	0.000
(5)	J	1.057	0.000	0.035	0.000	C	0.111	0.000	0.056	0.000
(6)	J	1.056	0.000	0.030	0.000	C	0.061	0.000	0.056	0.000
(7)	J	1.052	0.000	0.034	0.000	GN	-0.279	0.000	0.044	0.000
LLF	262.073					229.841				

	Pre-crisis					Post-crisis				
	type	Par1	Par2	Std1	Std2	type	Par1	Par2	Std1	Std2
AIC	-498.145					-433.681				
BIC	-440.517					-380.943				
Vuo	3.686***					2.902***				
Cla	346***					243***				
South Africa										
(1)	T	-0.147	11.828	0.042	6.857	GN	-0.317	0.000	0.042	0.000
(2)	T	-0.137	9.724	0.043	4.614	GN	-0.175	0.000	0.047	0.000
(3)	T	-0.146	13.416	0.041	8.194	GN	-0.139	0.000	0.048	0.000
(4)	T	-0.169	9.462	0.042	4.332	GN	-0.345	0.000	0.041	0.000
(5)	G	-0.025	0.000	0.040	0.000	GL	1.076	0.000	0.035	0.000
(6)	G	-0.021	0.000	0.040	0.000	GN	0.182	0.000	0.046	0.000
(7)	F	-0.569	0.000	0.235	0.000	T	-0.179	7.947	0.050	3.592
LLF	1720.976					1024.518				
AIC	-3409.952					-2021.035				
BIC	-3338.469					-1963.948				
Vuo	3.763***					3.875***				
Cla	323					259***				

Notes: Parameters (1) ~ (7) donates the pairs: S_USD, S_EURO, S_GBP, S_JPY, S_EURO|USD, S_GBP|USD, S_JPY|USD. GN, C, F, GL are short for Gaussian, Clayton, Frank and Gumbel copulas. Vuo and Cla represent the statistics of Vuong test and Clarke test.

Table A2

Parameters of Mixed C-vine Copula Models: Pre- and Post-European Crisis

	Pre-crisis					Post-crisis				
	type	Par1	Par2	Std1	Std2	type	Par1	Par2	Std1	Std2
Brazil										
(1)	GN	-0.477	0.000	0.031	0.000	GN	-0.268	0.000	0.046	0.000
(2)	GN	-0.068	0.000	0.045	0.000	GL	1.059	0.000	0.034	0.000
(3)	GN	-0.122	0.000	0.044	0.000	GN	-0.024	0.000	0.052	0.000
(4)	GN	-0.457	0.000	0.032	0.000	T	-0.244	6.651	0.053	2.940
(5)	T	-0.503	11.267	0.033	7.421	GN	-0.358	0.000	0.042	0.000
(6)	F	-0.666	0.000	0.265	0.000	GN	-0.147	0.000	0.050	0.000
(7)	T	-0.494	10.224	0.034	5.822	T	-0.324	6.658	0.050	2.917
LLF	604.241					579.376				
AIC	-1182.482					-1128.752				
BIC	-1127.334					-1069.222				
Vuo	1.789*					2.827***				
Cla	301					224				
Russia										
(1)	GN	-0.403	0.000	0.035	0.000	GN	-0.413	0.000	0.039	0.000
(2)	T	-0.043	13.381	0.048	9.294	GN	-0.233	0.000	0.047	0.000
(3)	GN	-0.046	0.000	0.045	0.000	GN	-0.291	0.000	0.045	0.000
(4)	GN	-0.340	0.000	0.037	0.000	GN	-0.360	0.000	0.042	0.000
(5)	GN	-0.408	0.000	0.034	0.000	GN	-0.353	0.000	0.042	0.000
(6)	F	-0.185	0.000	0.265	0.000	GN	-0.184	0.000	0.048	0.000
(7)	F	-2.255	0.000	0.277	0.000	GN	-0.297	0.000	0.044	0.000
LLF	374.651					626.350				
AIC	-723.301					-1226.701				
BIC	-668.102					-1174.844				
Vuo	2.492**					2.366**				
Cla	312***					240				

	Pre-crisis					Post-crisis				
	type	Par1	Par2	Std1	Std2	type	Par1	Par2	Std1	Std2
India										
(1)	GN	-0.355	0.000	0.037	0.000	T	-0.315	6.320	0.051	2.648
(2)	T	0.044	10.351	0.048	5.916	C	0.048	0.000	0.059	0.000
(3)	T	-0.094	12.208	0.047	8.229	GN	-0.118	0.000	0.051	0.000
(4)	T	-0.297	13.267	0.042	8.817	T	-0.267	6.614	0.052	2.843
(5)	GN	-0.353	0.000	0.037	0.000	T	-0.334	8.054	0.049	4.390
(6)	T	-0.127	11.045	0.047	6.590	T	-0.167	7.379	0.054	3.601
(7)	T	-0.314	15.631	0.041	11.785	T	-0.286	7.747	0.051	3.592
LLF	269.972					500.391				
AIC	-511.944					-966.782				
BIC	-452.745					-899.666				
Vuo	3.198***					2.834***				
Cla	304***					232***				
China										
(1)	GN	-0.147	0.000	0.044	0.000	T	-0.176	9.653	0.053	5.428
(2)	GN	0.202	0.000	0.043	0.000	F	1.402	0.000	0.312	0.000
(3)	GN	0.102	0.000	0.045	0.000	T	0.153	7.538	0.055	3.472
(4)	F	-0.457	0.000	0.271	0.000	GN	-0.009	0.000	0.052	0.000
(5)	GN	-0.032	0.000	0.045	0.000	F	-0.384	0.000	0.312	0.000
(6)	GN	-0.028	0.000	0.045	0.000	GN	-0.019	0.000	0.052	0.000
(7)	F	-0.439	0.000	0.268	0.000	F	0.215	0.000	0.310	0.000
LLF	264.747					238.549				
AIC	-503.493					-447.098				
BIC	-448.548					-387.683				
Vuo	3.283***					2.772***				
Cla	315***					206				
South Africa										
(1)	GN	-0.347	0.000	0.037	0.000	GN	-0.324	0.000	0.044	0.000
(2)	F	-0.928	0.000	0.266	0.000	F	-1.062	0.000	0.304	0.000
(3)	T	-0.122	10.495	0.047	5.447	GN	-0.224	0.000	0.048	0.000
(4)	T	-0.338	15.153	0.040	11.356	T	-0.303	8.609	0.049	4.338
(5)	GN	-0.345	0.000	0.037	0.000	GN	-0.300	0.000	0.044	0.000
(6)	T	-0.019	9.505	0.048	4.820	GN	-0.149	0.000	0.049	0.000
(7)	GN	-0.331	0.000	0.037	0.000	T	-0.260	7.024	0.051	2.736
LLF	758.724					946.127				
AIC	-1489.448					-1864.253				
BIC	-1429.867					-1808.478				
Vuo	1.432					1.778*				
Cla	293***					219**				

Notes: Parameters (1) ~ (7) donates the pairs: S_USD, S_EURO, S_GBP, S_JPY, S_USD|EURO, S_GBP|EURO, S_JPY|EURO. GN, C, F, GL are short for Gaussian, Clayton, Frank and Gumbel copulas. Vuo and Cla represent the statistics of Vuong test and Clarke test.