5 OPTION BOUNDS FOR MULTINOMIAL STOCK RETURNS IN JUMP-DIFFUSION PROCESSES - A MONTE CARLO SIMULATION FOR A MULTI-JUMP PROCESS -

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Abstract

This paper addresses the problem of option bounds computation under the assumption that the price of the underlying asset follows a jump-diffusion Merton process as formulated in Perrakis (1993) extending the number of the jumps from one jump up and one jump down with fixed sizes to a finite number of jumps with sizes drawn from the lognormal distribution. The objective of this paper is to create a Monte Carlo simulation for the estimation of the bounds with various numbers of jumps and periods to maturity.

Key words: Monte Carlo simulation, Jump-Diffusion processes, multi-jump process

Jel Classification: C15, G12

The two major option pricing models used in derivatives pricing – the Black-Scholes (1973) model and the binomial option model of Cox et al. (1979) and Rendleman and Barter (1979) – are derived under relatively restrictive assumptions. The latter assumes that in a short period of time the stock price can take only two possible values. Relaxing this assumption conducted to important results in the theory of incomplete markets such as the computation of the option bounds.

The binomial model is valid in the complete markets which are markets where there are exactly as many securities (with linearly independent payoffs) as there are possible future states. If these security prices do not exhibit arbitrage opportunities, then in such complete markets we can always recover a unique set of risk-neutral probabilities. These probabilities form a distribution which allows us to compute a unique price for the option

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payoff. However, from the fact that possible states of the world tend to be much more numerous than existing securities we can infer that complete markets rarely obtain.

This result conducted to the development of option pricing in incomplete markets. An incomplete market may be defined as a market in which there are not enough securities to hedge all sources of uncertainty. Most of the recent research in this area is focused on the analysis of the observed market prices in search of implied distributions and stochastic processes that could have generated these prices.

If the prices are influenced by additional stochastic factors such as stochastic volatility, stochastic interest rates or stochastic jumps, which are not traded, then there is no possibility for an investor to exactly replicate the payoff of an option state by state.

This means that there are multiple risk-neutral distributions which can price the option correctly. Pricing a new security, currently not being traded, with these multiple risk-neutral distributions will yield a range of possible option prices between a lower and an upper bound.

The computation of these bounds is derived from letting the one-period stock price (or return) distribution to be a general multinomial distribution rather than a binomial one. When the market is comprised only of the stock and the riskless asset we are dealing with an incomplete market which can no longer provide an option price without additional assumptions about investor preferences.

Perrakis (1986) and Ritchken (1985) suggested that, in addition to keeping the pricing kernel positive, one should also require the pricing kernel to be monotonically decreasing in wealth in order to be consistent with risk averse investors. This assumption recognizes the validity of one of the features of the binomial model, the monotone ordering of stock returns and state-contingent discount factors. Thus, under the financial market equilibrium characterized by no-arbitrage opportunities, the risk-neutral distributions of the option bounds were determined by linear programming and no-arbitrage strategies.

Perrakis (1993) showed that the n-period upper and lower bounds of stock returns both converge to the Black-Scholes option price when the number of periods tends to infinity, with the length of each period tending to zero, when the stock returns form a lognormal diffusion.

When the jump-diffusion process was taken into account for a distribution with five states the result was that bounds converged to two distinct values that bracketed both the Black-Scholes and Merton option prices. The limits are correlated with the parameters of the jump components as well as with the mean and variance of the diffusion. The conclusion was that the presence of rare events in stock returns does not provide a single price under only the monotone ordering assumption.

This paper provides a possible numerical estimation method for the bounds when the stock returns follow a jump-diffusion process. We will consider a discrete jump-diffusion model with three states for the diffusion and up to 200 states for the jumps. Comparisons between processes with different jump amplitudes and periods to expiration may be derived from the numerical results.

The jump-diffusion model

The notation used is the following:

S: the price of the stock

X: the strike price of the European call option

2m: the number of jumps

n: the assumed number of the states of the world within a period ΔT (=2m+3)

R: one plus the riskless rate of interest per period ΔT

Z: the random stock return per \$ invested per period ΔT

z: the realized value of Z

We will consider discrete time periods and the standard assumptions in option pricing as in Merton (1973). The bounds were computed on the grounds that the option is a convex function of the stock price which needs the standard assumption (Theorem 10 in Merton 1973) that the distribution of returns per dollar invested in the common stock is independent of the level of the stock price.

It is also assumed that the distribution of the returns is the same for every period ΔT and there is no correlation between successive returns. Further we will assume that there are no transaction costs and taxes or dividends and that R remains constant until the expiration of the option.

The computation of the bounds requires also the standard assumptions concerning the equilibrium in incomplete financial markets as specified by Rubinstein (1976): i) single-price law of markets, ii) nonsatiation, iii) perfect, competitive and Pareto-efficient financial markets as well as iv) rational time-additive tastes. The last assumption allows for the monotone ordering of the discount factors with respect to the stock return Z which permits the construction of a monotone function Y(Z) (the state-contingent discount factor) satisfying the relations $\sum_{i=1}^{n} p_i Y(z_i) z_i = 1$ and $\sum_{i=1}^{n} p_i Y(z_i) = \frac{1}{R}$. Y(Z) was identified in Rubinstein (1976) as the normalized conditional expected marginal utility of consumption of the representative investor.

The distribution of Z is assumed multinomial (z_i,p_i) with the average $\hat{z} = E(Z) = \sum_{i=1}^{n} p_i z_i$.

As in Perrakis and Ryan (1984) and Ritchken (1985) we will assume non-increasing discount factors. The non-decreasing case is similar in the sense that the bounds are inverted. This implies that $\hat{z} \geq R$ and $z_i < R < z_n$.

We are dealing now with an incomplete market where the prices and the state securities are not unique. In terms of risk-neutral probability distributions, this means that there are multiple risk-neutral distributions, all of which can price all existing assets correctly. We will obtain an equation system with m equations and two securities which is an undetermined system with infinite solutions for the risk-neutral probabilities.

Under the mentioned assumptions, the bounds $\underline{C}(S,X)$ and $\overline{C}(S,X)$ were found by solving the optimizations (linear programming) problems:

$$\max_{Y_j}(\min)\left\{\sum_{j=1}^{n} p_j \hat{Y}(z_j) c_j(S,X)\right\}$$

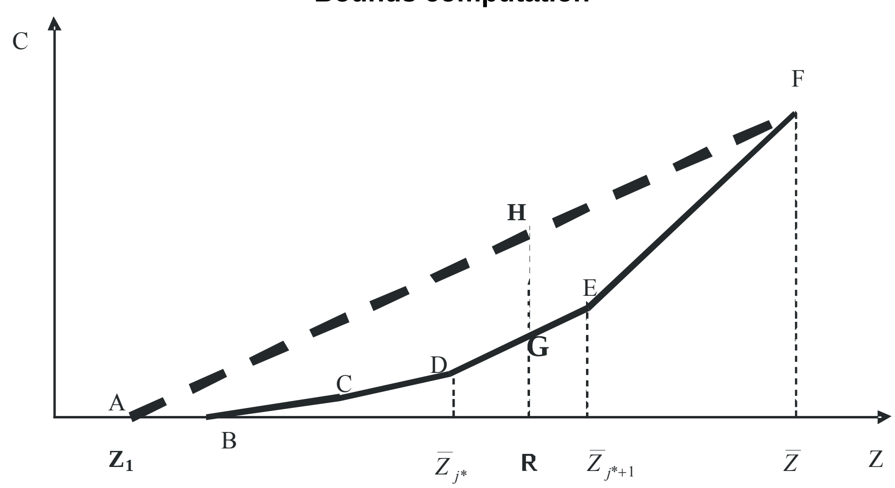
$$\sum_{j=1}^{n} p_j \hat{Y}(z_j) z_j = R$$

$$\sum_{j=1}^{n} p_j \hat{Y}(z_j) = 1$$

This is equivalent to finding the risk-neutral distributions which bound the whole set of possible distributions in terms of the values of the call option.

Figure 1

Bounds computation



Geometrically, taking into account the fact that the call option is a convex function of the returns of the stock we can observe that (under the assumption that the Z are monotonically non-decreasing) all the convex combinations of the possible returns are covered in the area ABCDEF but only the points on the segment GH satisfy the first condition. Clearly, the minimum value of the option is attained at point G and the maximum is attained at H.

The formulae for the bounds in a single period are computed for the respective points G and H as convex combinations of the:

$$\overline{C}(S,X) = Q \cdot C(Sz_1, X) + (1-Q) \cdot C(S\hat{z}_n, X)$$

$$\underline{C}(S,X) = P \cdot C(S\hat{z}_{j^*}, X) + (1-P) \cdot C(S\hat{z}_{j^*+1}, X)$$

where

$$Q = \frac{\hat{z} - R}{\hat{z} - z_1}, \; P = \frac{\hat{z}_{j^*+1} - R}{\hat{z}_{j^*+1} - \hat{z}_{j^*}} \text{ and } \hat{z}_j = \frac{\sum_{i=1}^{n} p_i z_i}{\sum_{i=1}^{n} p_i}$$

Merton's jump-diffusion model was introduced in order to construct a continuous-time process which may allow for the incorporation of the rare events into the evolution of stock price changes. This process was defined to comprise two different types of dynamics: i) the "normal" vibrations which determine marginal change in the price and have constant variance per unit time and continuous sample path and ii) the "abnormal" vibrations determined by arrival of important information about the stock with more than a marginal effect on the price. The "normal" vibrations are modeled by the standard geometric Brownian motion while the "abnormal" ones are modeled by a Poisson-driven process.

Merton developed in this way a formula for a call option with an underlying asset which follows this process at the limit of continuous trading. We will construct a pattern for the evolution of Z assuming that the return may take a finite number of values corresponding to the same number of possible states of the world. This pattern will consist of three different states (up, down and the same) standing for the diffusion evolution of Z to which a finite number of jumps will be added. The main purpose is to construct a framework for the computation of a Monte Carlo simulation of Z under the two risk-neutral distributions revealed by the bounds.

We will first compute the probabilities and the values for a three-state diffusion process corresponding to the "go up, go down and stay the same" pattern of Z. The size will be denoted by *u* and *d* (for up and respectively down states) and *p* will stand for the probability of an up movement. We will consider that the probability that the price will stay the same is 1-γ .Thus we will have the following results for the mean and the variance of *lnS*:

$$E(\ln S) = p.\ln u + (\gamma - p).\ln d = \mu \Delta T$$
$$Var(\ln S) = p.(\ln u)^2 + (\gamma - p).(\ln d)^2 - (p.\ln u + (\gamma - p).\ln d)^2 = \sigma^2 \Delta T$$

We can set ln u = U and ln d = D and also ln u = - ln d (d=1/u). Then

$$U(2p - \gamma) = \mu \Delta T$$
$$U^2(\gamma - 4p^2 + 4p\gamma - \gamma^2) = \sigma^2 \Delta T$$ and

Squaring the first relation and adding to the second we have $\gamma U^2 = \sigma^2 \Delta T + \mu^2 (\Delta T)^2$.
But $(\Delta T)^2 = o(\Delta T)$ which means that

$$U = \sigma \sqrt{\frac{\Delta T}{\gamma}}$$

$$u = e^{\sigma \sqrt{\frac{\Delta T}{\gamma}}}$$

$$d = e^{-\sigma \sqrt{\frac{\Delta T}{\gamma}}}$$

Plugging this in the relation for the mean we have the following results for the probabilities of up and down movements – p_u and respectively p_d:

$$U(2p - \gamma) = \mu \Delta T$$

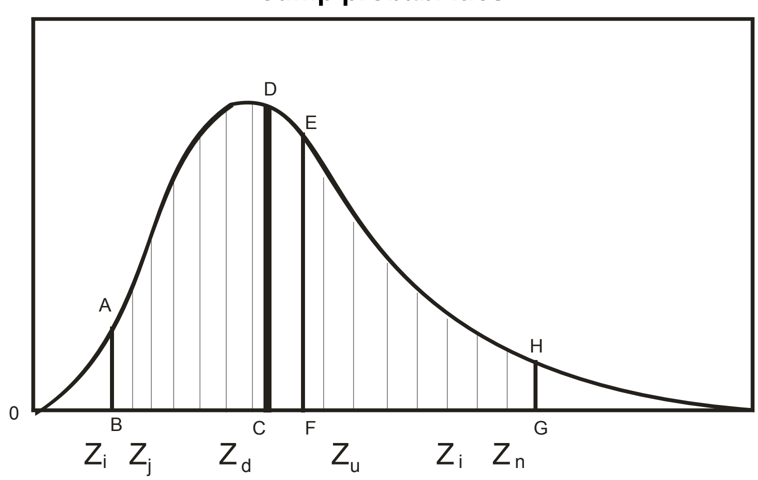
$$p_u = \frac{\gamma}{2} + \frac{\mu \sqrt{\Delta T . \gamma}}{2\sigma}$$

$$p_d = \frac{\gamma}{2} - \frac{\mu \sqrt{\Delta T . \gamma}}{2\sigma}$$

We can now compute the values of Z for the pattern of the jump-diffusion states. We will denote by d_i the sizes of the jumps in the upper states (grater than d) and by u_i the sizes of the jumps in the lower states (less than u).

Figure 2

Jump probabilities



The sizes of the jumps will be drawn from the log-normal distribution. We will assume that there is a probability of λ that a jump will occur and that there are m jumps up (i=m+4,2,…2m+3) and m jumps down (j=1,2,…m). We will also compute a probability for the realization of a certain jump and we will denote this probability by δ_i and δ_j for the up and down jumps respectively. For the computation of δ's the probability densities corresponding to the areas *ABCD* and *EFGH* will be determined. Thus the probability of an up-jump is computed as the weight of the probability density between two consecutive jumps in the total probability designated by *ABCD* and *EFGH*. In other words the area corresponding to the sum of *ABCD* and *EFGH* stands for the whole λ probability of a jump. *ABCD*/(*ABCD+EFGH*) represents the cumulated probability of the down-jumps and *EFGH*/(*ABCD+EFGH*) is the cumulated probability of the up-jumps.

In the following numerical simulation the computation of the sizes and probabilities of the jumps will consider approximately equal probabilities for the realization of an up-jump or a down-jump (i.e. areas ABCD and EFGH are approximately equal). In order to implement this we will start from the normal distribution and we will take two bounding values as given – we will denote them by b^n_{d1} and b^n_{um}. Each of the intervals (b^n_{d1}, z_d) and (z_u,b^n_{um}) is divided in m equal segments and the cumulated probabilities of each of this segments are retained (the area under the normal density function for the segment). For the down-jumps we will have equal segments of the form (b^n_{dj},b^n_{dj+1}) and larger probabilities as j becomes bigger, while for the up-jumps we will have equal segments (b^n_{ui-1},b^n_{ui}) and smaller probabilities as i becomes bigger. These probabilities will then be used to compute the sizes of the jumps under the lognormal distribution.

The retained **up-jump** probabilities will be added to the cumulated probability of z_u in order to determine the quantiles as the corresponding values of b^n_d's and b^n_u's for the lognormal distribution. We can denote these segments by (b^l_{ui-1},b^l_{ui}). Thus the size of a up-jump z_i will be drawn from the lognormal distribution in the interval (b^l_{ui-1},b^l_{ui}) and the probability ascribed to each jump will be computed as $\frac{CLognorm(b^l_{ui})-CLognorm(b^l_{i-1})}{EFGH}\cdot\frac{EFGH}{ABCD+EFGH}$ (where *CLognorm(b)* is the cumulated lognormal probability function of *b*).

The **down-jump** probabilities will be subtracted from the cumulated probability of z_d and quantiles b^l_d's will be computed. The size of a down-jump z_j will then be drawn from the lognormal distribution in the interval (b^l_{uj},b^l_{uj+1}) and the probability of the z_j is $\frac{CLognorm(b^l_{uj+1})-CLognorm(b^l_{uj})}{ABCD}\cdot\frac{ABCD}{ABCD+EFGH}$.

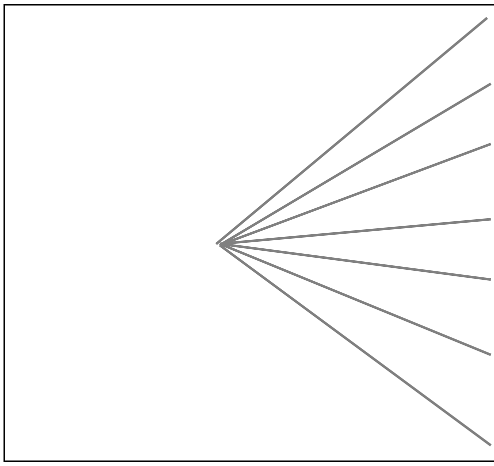
Thus *ABCD* = *CLognorm*(z_d)-*CLognorm*(b^l_{d1}) and *EFGH* = *CLognorm*(b^l_{um})-*CLognorm*(z_u) and of course $\sum_{s=1}^{2m+3}\delta_s=1,$ where $\delta_{m+1}=\delta_{m+2}=\delta_{m+3}=0$.

As m becomes larger more values are covered by the jumps and their respective probabilities tend to the values designated by the lognormal probability density function. Thus, if a jump will occur (the probability λ) then the δ's will decide which size is to be realized. We can say that when m tends to infinity the segments (b^l_{ui-1},b^l_{ui}) and (b^l_{uj},b^l_{uj+1})

will tend to zero and the probability of a jump is the value of the lognormal density function for the size of that jump.

The multinomial distribution of the jump-diffusion pattern will be the following:

Figure 3



$z_n = u_n \rightarrow p(z_n) = \delta_n \lambda \Delta T$

$z_i = u_i \rightarrow p(z_i) = \delta_i \lambda \Delta T$

$z_u = u \rightarrow p_u = \frac{\gamma}{2} + \frac{\mu\sqrt{\Delta T.\gamma}}{2\sigma}(1-\lambda\Delta T)$

$1 \rightarrow p(1) = (1-\gamma)(1-\lambda\Delta T)$

$z_d = d \rightarrow p_d = \frac{\gamma}{2} - \frac{\mu\sqrt{\Delta T.\gamma}}{2\sigma}(1-\lambda\Delta T)$

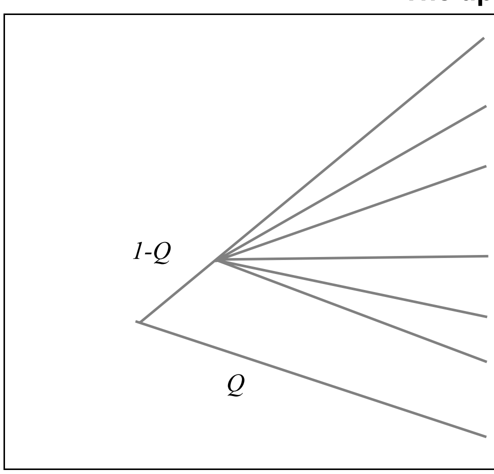
$z_j = d_j \rightarrow p(z_j) = \delta_j \lambda \Delta T$

$z_1 = d_1 \rightarrow p(z_1) = \delta_1 \lambda \Delta T$

We can now present the risk-neutral distributions corresponding to the two bounds. For both of the bounds the distributions were computed as a result of the linear programming:

Figure 4

The upper bound



$z_n \rightarrow (1-Q)p_n$

$z_i \rightarrow (1-Q)p_i$

$z_{m+3} \rightarrow (1-Q)p_{m+3}$

$z_{m+2} \rightarrow (1-Q)p_{m+2}$

$z_{m+1} \rightarrow (1-Q)p_{m+1}$

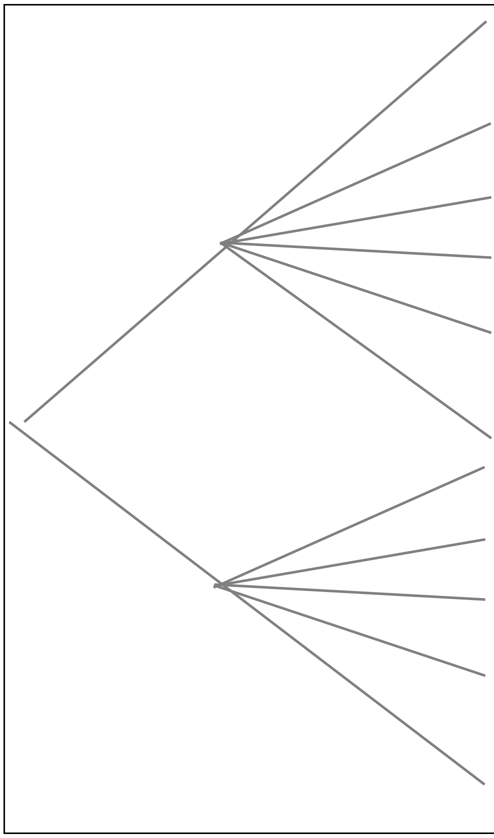
$z_j \rightarrow (1-Q)p_j$

$z_1 \rightarrow (1-Q)p_1$

$z_1 \rightarrow (1-Q)p_1$

Figure 5

The lower bound



z_{j^*+1}	$\rightarrow (1\text{-}P)\,\overline{p}_{j^*+1}$
z_i	$\rightarrow (1\text{-}P)\,\overline{p}_i$
z_{m+3}	$\rightarrow (1\text{-}P)\,\overline{p}_{m+3}$
z_{m+2}	$\rightarrow (1\text{-}P)\,\overline{p}_{m+2}$
z_{m+1}	$\rightarrow (1\text{-}P)\,\overline{p}_{m+1}$
z_j	$\rightarrow (1\text{-}P)\,\overline{p}_j$
z_1	$\rightarrow (1\text{-}P)\,\overline{p}_1$
z_{j^*}	$\rightarrow P\,\hat{p}_{j^*}$
z_i	$\rightarrow P\,\hat{p}_i$
z_{m+3}	$\rightarrow P\,\hat{p}_{m+3}$
z_{m+2}	$\rightarrow P\,\hat{p}_{m+2}$
z_{m+1}	$\rightarrow P\,\hat{p}_{m+1}$
z_j	$\rightarrow P\,\hat{p}_j$
z_1	$\rightarrow P\,\hat{p}_1$

where $\overline{p}_j = \dfrac{p_j}{\sum_{i=1}^{j^*+1} p_i}$ and $\hat{p}_j = \dfrac{p_j}{\sum_{i=1}^{j^*} p_i}$.

Thus, the probabilities of the two distributions must be risk-neutral and must satisfy the provision that any European option's payoff must yield a higher price when it is valued under the upper bound probabilities than under the lower bound probabilities. Masson and Perrakis (2000) proved that the expectation of any continuous, non-decreasing and convex function C(z) defined over the assumed stock returns after a certain period of time is larger under the upper bound cumulative probability function than under the lower cumulative probability function. They derived a theorem for the necessary and sufficient conditions that this provision be satisfied:

i) the two probability distributions are risk-neutral (i.e. their means are R);

ii) the probability of z_1 in the lower bound distribution is larger than the probability of z_1 in the upper bound distribution

iii) the cumulative probability of all the states up to n-1 is lower under the upper bound distribution than under the lower bound distribution;

$$\sum_{j}^{n-1} p_{ju} \le \sum_{j}^{n-1} p_{jl}$$

iv) $$\sum_{k}^{j}\left(\sum_{h}^{k} p_{hu}\right)\Delta S_k \ge \sum_{k}^{j}\left(\sum_{h}^{k} p_{hl}\right)\Delta S_k$$, where j=1,…,n, and $\Delta S_k = S_{k+1} - S_k$.

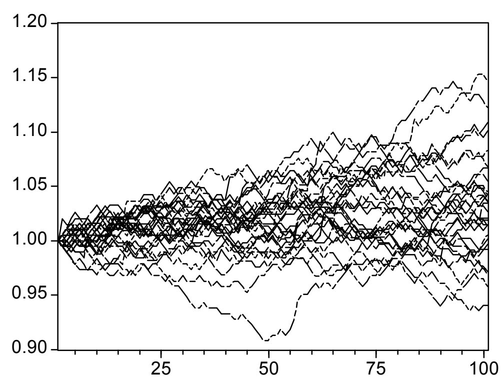
Numerical results

In this section we present some numerical comparisons of the option bounds computed for the three state diffusion, the diffusion with 2 jumps (up and down), diffusion with 10 jumps (5 ups and 5 downs), diffusion with 20 jumps (10 ups and 10 downs), diffusion with 200 jumps (100 ups and 100 downs) and diffusion with 2000 jumps (1000 ups and 1000 downs). In each of these situations the bounds will be estimated for 5, 10, 20, 50 periods until maturity (a period has the length ∆T).

For the computation of the bounds we use Monte Carlo simulations for different periods until expiration and for different number of jumps. The evolution of the stock returns are simulated for both the upper and lower bound distributions for 100 000 paths. The 100 000 final values will be used to compute the expected payoff of the European option. These two values (for the upper and lower bounds) will be discounted to obtain the present value of the option payoff under the two risk-neutral distributions for the two bounds. The values of these discounted payoffs are more thoroughly computed as the number of paths is larger.

Figure 6

Monte Carlo Simulation



We expect that after K periods the option prices should lie between the values $R^{-K} E_l \left[Max\left(SZ^{(K)} - X,0\right)\right]$ and $R^{-K} E_u \left[Max\left(SZ^{(K)} - X,0\right)\right]$, where E_l and E_u denotes the expectations over the lower and upper bound risk-neutral distributions respectively.

The computations of both the probabilities and the sizes of returns for all the states are realized using the up-mentioned formulae. For all the situations considered the conditions for the probabilities are also verified. The drawings from the computed probabilities are realized by dividing the [0,1] interval into n intervals (n=2m+3 states of the world) with sizes equal to the probabilities of the states under the two distributions. A random drawing from the uniform distribution will indicate the realization of state i if its value is in the i'th interval. The value of the uniform random drawing is used for randomly choosing the states under both of the distributions. More than 100 000 sequences of such random numbers are generated and each sequence represents a possible evolution of the stock under the two risk-neutral distributions.

The numerical analysis presented in this paper used the parameters from the numerical example in Perrakis (1993): μ = .0001, σ = .01, λ = .3, r = .0002 and S = X = 1. We also set γ = .9. Δt = 0.1 and the period to maturity is consider t = {0.5, 1, 2.5, 5} representing 5, 10, 25 and 50 periods until maturity respectively.

In order to compute the lower bound distribution the computation of j* (the state for which the return is the highest return smaller than the riskless rate of return) is necessary. For these parameters, the bounds b^n_{d1} and b^n_{um} are chosen such that j* is as large as possible, to let the difference between the average of Z under the real distribution be very close to the riskless rate of return. For all the cases b^n_{d1} = 0 b^n_{um} = 1.706.

Thus, for all the 5 cases considered we found the following values for the sizes of the jumps and for the j*'s:

Case	J*	z1	zn
Diffusion (3 states)	2	0.967216100482	1.03389511351
Diffusion and 2 jumps (5 states)	4	0.679267295975	2.30850596367
Diffusion and 10 jumps (13 states)	12	0.384619570916	1.96909934177
Diffusion and 20 jumps (23 states)	22	0.395097603007	2.01471841487
Diffusion and 200 jumps (203 states)	202	0.370489432262	2.01753751002

The conditions presented in Masson and Perrakis (2000) are satisfied for all the 5 cases. We present here the situations for two of these conditions for each of the 5 cases. We may add that these two conditions verify if the probabilities for the upper bound distribution are larger in their extreme values (the highest and the lowest states) when compared with the probabilities of the lower bound distribution.

Case	$\sum_{j}^{n-1} p_{ju}$	$\sum_{j}^{n-1} p_{jl}$	p_{1u}	p_{1l}

Diffusion (3 states)	0.554056489024	0.554941760549	0.454958261356	0.454042983452
Diffusion and 2 jumps (5 states)	0.987286670323	0.991840453311	0.0261855629166	0.0172502040518
Diffusion and 10 jumps (13 states)	0.9977224535	0.998114025252	0.00330103334488	0.00268442717409
Diffusion and 20 jumps (23 states)	0.998884751368	0.998896742741	0.00130329724586	0.00128317419784
Diffusion and 200 jumps (203 states)	0.999890645748	0.99991824791	0.000167368156403	0.00012277443391

The Monte Carlo bounds for the **one** option are:

3-state Diffusion – 100 000 Monte Carlo repetitions

Periods to maturity t	Upper bound	Lower Bound
0.5	0.000899359	0.000889635
1	0.0009.92802	0.000965809
2.5	0.0011.69781	0.001150502
5	0.001241671	0.001240906

1 Jump and Diffusion – 200 000 Monte Carlo repetitions

Periods to maturity t (Δt=0.1)	Upper bound	Lower Bound
0.5	0.000271086	0
1	0.003733558	0.003491863
2.5	0.004594	0.004255
5	0.012221384	0.01207356

5 Jumps and Diffusion - 200 000 Monte Carlo repetitions

Periods to maturity t (Δt=0.1)	Upper bound	Lower Bound
0.5	0.00107923	0.001067417
1	0.002596009	0.002524686
2.5	0.0035603	0.003341797
5	0.010239831	0.010033483

10 Jumps and Diffusion - 200 000 Monte Carlo repetitions

Periods to maturity t (Δt=0.1)	Upper bound	Lower Bound
0.5	0.000359807	0.000358662
1	0.001158375	0.001157863
2.5	0.003670091	0.00365782
5	0.00142562	0.001419356

100 Jumps and Diffusion – 200 000 Monte Carlo repetitions

Periods to maturity	Upper bound	Lower Bound

t (Δt=0.1)		
0.5	0.000539929	0.000534873
1	0.000693	0.000679
2.5	0.0063398	0.0063214
5	0.009961168	0.009955671

By varying the parameters and the number of the jumps we can obtain interesting results about the values of the bounds. The simulations are developed in E-Views and the number of paths for the Monte Carlo should be as high as possible. The results of the Monte Carlo are asymptotically valid. The 200 000 repetitions here are not sufficient in order to obtain the best results.

The use of 100 jumps (203 states) provides a finer representation of the domain for the sizes of the jumps, and as such, we may conclude that the higher the number of jumps, the jump-diffusion process of the stock returns is better represented.

Remarks

Econometric models which test for the existence of jump components in stock returns analyse the conditional density for each jump and use assumptions that the arrival intensity is a stochastic process too, which means that the probability of occurrence is different in various time intervals. Maheu and McCurdy (2003) realize a period-by-period comparison between a jump-diffusion process which allows for heteroskedasticity and a stochastic volatility model. In our example this would mean that λ could take the form of an AR(1) which could also be tested by Monte Carlo simulation.

Another approach to the construction for the risk-neutral distribution of the bounds could allow the n upper different jumps to be represented by a single state and the n lower jumps to be represented by another state. In this case we could consider that we have five different states (z_i, i = 1…5) where z_1 is the average of the n lower jumps and z_5 is the average of the n upper jumps. The construction of the risk-neutral distribution would assign different probabilities to the 2m + 3 states (as compared to the model presented in the numerical example) – for the upper bound all the up-states and down-states would receive much higher probabilities than the respective states for the lower bound. However the model used for the numerical computation here should provide tighter bounds.

A test of the jump-diffusion model for the evolution of the stock returns should provide the necessary parameters for the implementation of the bounds computation method. These results could be backtested in order to obtain a measure of its representation on the real data.

The Monte Carlo simulation is also useful when the bounds are computed for an American option as it allows for dividends to be taken into account after each period of time. The numerical analysis in this paper may be used for the computation of bounds of currency options and the model of Maheu and McCurdy (2003) can be used for the derivation of the necessary parameters.

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