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TESTING FOR HETEROSKEDASTIC MIXTURE OF ORDINARY LEAST SQUARES ERRORS

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Abstract

There is no procedure available in the existing literature to test for heteroskedastic mixture of distributions of residuals drawn from ordinary least squares regressions. This is the first paper that designs a simple test procedure for detecting heteroskedastic mixture of ordinary least squares residuals. The assumption that residuals must be drawn from a homoscedastic mixture of distributions is tested in addition to detecting heteroskedasticity. The test procedure has been designed to account for mixture of distributions properties of the regression residuals when the regressor is drawn with reference to an active market. To retain efficiency of the test, an unbiased maximum likelihood estimator for the true (population) variance was drawn from a log-normal normal family. The results show that there are significant disagreements between the heteroskedasticity detection results of the two auxiliary regression models due to the effect of heteroskedastic mixture of residual distributions. Forecasting exercise shows that there is a significant difference between the two auxiliary regression models in market level regressions than non-market level regressions that supports the new model proposed. Monte Carlo simulation results show significant improvements in the model performance for finite samples with less size distortion.

The findings of this study encourage future scholars explore possibility of testing heteroskedastic mixture effect of residuals drawn from multiple regressions and test heteroskedastic mixture in other developed and emerging markets under different market conditions (e.g. crisis) to see the generalisatbility of the model. It also encourages developing other types of tests such as F-test that also suits data generating process. Practitioners could minimize the risk of misrepresentation in advisory work by qualifying and disclaiming for possible pricing errors in cost of capital computations and valuations based on detection test results. Findings of this paper encourage stock exchanges and governments to effectively promote firm-specific trading by, for example, timely discloser of corporate announcements and investor education programs, to improve functional efficiency of stock markets.

Keywords: mixture of distributions hypothesis; heteroskedastic mixture; realized volatility; Monte carlo simulation; ordinary least squares; capital asset pricing; idiosyncratic volatility puzzle.

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1. Introduction

1.1 Background and Motivation

Since it was first put forward by Peter Clark in 1973, the mixture of distributions hypothesis of residuals drawn from Gaussian linear regression models has thus far been limited to an assumption, without any formal test. The mixture of distributions of residuals can be observed particularly in speculative markets such as stock markets, cotton futures etc. The heteroskedasticity of mixture of residuals distributions drawn from ordinary least squares (OLS) regression estimates is inevitable when the regressor is a common variable, usually a market variable (e.g. market return) drawn with reference to an active market. This is not so if one regresses, for example, gross domestic production (GDP) on per capita income (PCI) because PCI is not drawn with reference to an active market and is not subject to dynamics of market equilibrium. Therefore, in the application of stock market regressions, residuals should qualify for mixture of distributions property of stock returns in order to forecast return and volatility with precision and accuracy[3], as practitioners often use non-stochastic volatility models, such as ARCH[4] that uses OLS errors for modeling volatility in financial markets. If homoscedastic mixture of residuals distributions hypothesis[5] is invalid, given the context of application, non-stochastic volatility models may be misspecified. Scholars such as Nelson (1992), Nelson and Foster (1994, 1995) Canina and Figlewski (1993), Jorion (1995) demonstrate that these models fail to sufficiently account for the mixing properties of the ex-post squared returns.

1.2 Gap in the Existing Methodology

Levene (1960) and his successors utilize auxiliary regression based test procedures (e.g. Glejser 1969; Ramsey 1969; Breusch and Pagan 1979; White 1980). This class of test procedures does not diagnose for mixing properties of the dependent variable (e.g. return of a firm's stock), especially, when squared residuals are regressed on a common regression variable (e.g. market return) where, for example, the equilibrium price changes of a firm may have some form of association with the expectation of the market at the time of observation or transaction (Senarathne 2018). The quantum of this error may cause heteroskedastic mixture of regression residuals. Clark (1973 p 136) discuses about a class of observations that violate normality of price change distribution and demonstrate under what conditions that a distribution of price change is subordinated to that of a normal distribution. Mehmet (2008 pp. 34-35) intuitively illustrates a similar idea in OLS regression as to how the expected value of each distribution could vary depending on the type association of regression residuals with the regressor[6].

This paper contributes by filling the gap in the current literature as discussed in the preceding paragraph with regard to detection of heteroskedastic mixture of regression residuals, for

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[3] Ying (1966) points out that 'prices and volumes of sales in the stock market are joint products of a single market mechanism, and any model that attempts to isolate prices from volumes or vice versa will inevitably yield incomplete if not erroneous results'.

[4] Autoregressive Conditional Heteroskedasticity.

[5] See Clark (1973) for a complete exposition. This distributional assumption is apparent in stochastic volatility modeling (See Andersen et. al., 2001a, p 1 for a useful discussion).

[6] Dupernex (2007, p 175) demonstrates how error behavior changes over time in response to firm-specific information segments.

which, a formal test procedure is not available in the existing literature. The objective of this paper is to design a simple test procedure for detecting heteroskedastic mixture of OLS errors. This paper is organized as follows. Section two discusses the theory and specification. Section three enumerates test results including the results forecasting exercise and Monte Carlo simulation exercise and section four discusses some limitations of the study. Section five concludes the paper.

2. Theory and Specification

2.1 Theory

To explain the underlying theory regarding the heteroskedastic mixture of residuals distributions, collect δ_{st} amount of payoffs (i.e. free cash flow to equity)[7] attributable to equity and let it denote the s^{th} trade intraday equilibrium price increment[8] in day t such that;

$$\varepsilon_t = \sum_{s=1}^{n_t} \delta_{st,} \qquad (1)$$

where n_t is the number of observations at operational time t so that n and t are clearly coincided[9]. The regression residual[10] ε_t is subordinated to δ_s following Mandelbrot and Taylor (1967), Clerk (1973), Westerfield (1977), Harris (1987), Lamoureux and Lastrapes (1990). Equation 1 implies that ε_t is drawn from a log-normal normal family[11] where the variance of each distribution depends upon the operational time t and is, of course, clearly homoscedastic. The Cleak's (1973) prepositions suggest that the variance of the daily price change is a stochastic variable (assume which forms no association with a common market variable, e.g. (market return)) with a mean proportional to the mean number of daily transactions n observed at each operational time t in the market[12].

Let r_{it} be the change in price (cum dividend) p of fully equity financed firm (i.e. stock) i at time t, trading in an efficient stock market[13]. Define $r_{it} = (p_{it} - p_{it-1})/p_{it-1,}$ > 0 for

[7] Henceforth, let it be written as Free Cash Flow to Equity (FCFE) on firm-specific information segments. Unless assumed unlevered, FCF to Firm must be decomposed to identify the payoffs attributable to equity which is somewhat cumbersome. The terms 'equity holders' and 'speculators' are used interchangeably.

[8] δ_s is an uncorrelated independent increment from a stationary price process. This postulation is in line with Clark (1973), Lamoureux and Lastrapes (1990, p 222) and Bachelier (2011).

[9] Because stationary independent increments on information segments are observed by n by a flow of information at operational time $t > 0$ within the framework of Clark (1973). See also Harris (1987), Epps and Epps (1976) and Ezzat and Kirkulak-Uludag (2017).

[10] δ_s is $i.i.d.$ with mean zero and variance σ^2 so that $\varepsilon_t | n_t \sim N(0, \sigma^2 n_t)$.

[11] Clark (1973) generalizes the mixture of distributions hypothesis such that the marginal distribution (unconditional on operational time t) of realized variance $RV_t(\Delta)$ is log-normally distributed (Clark 1973, p 147) and the standardized (by realized volatility) returns, $(r_t/RV_t(\Delta)^{1/2})$ is approximately normally distributed. Also, Andersen et. al., (2001a) find that the unconditional distributions of the variances and covariances of stock return are leptokurtic and highly skewed to the right, while the logarithmic standard deviations and correlations are approximately Gaussian (See also French et. al., *1987; Zumbach et. al., 1999; Andersen et.al., 2001a, Andersen et. al., 2001b)*

[12] Tauchen and Pitts (1983) revisit and assert the Clark's prepositions within their framework.

[13] All assumptions of efficient market hypothesis and capital asset pricing do apply.

conditional[14]subordinated price increments when $p_{it} > p_{it-1}$ such that $r_{it} > \sum_{s=1}^{n_t} \delta_{st} > 0$. The price increments[15] are stationary and uncorrelated so that the random walk model, $p_{it} = \lambda p_{it-1} + \Phi$ will then have finite steps for clockwise operation, $n = t$ (see proof of theorem 3). The future change in price, for example, p_{it+1} from p_{it} is solely dependent upon idiosyncratic information. Define μ is the implied mean of an *imperfect mixture of distributions* of firm i such that $\mu = (r_{it} - \sum_{s=1}^{n_t} \delta_{st}) = \beta_m r_{mt}$

The theory suggests that the return r_{it} of capital asset i (at time t for instance), could be modeled in a linear relationship such that the return is proportional to the none-diversifiable risk (systematic risk) of the asset (See Sharp (1964))[16].

$$r_{it} = \beta_0 + \beta_m r_{mt} + \varepsilon_{it}, \tag{2}$$

where β_0 is the intercept term, β_m is the beta coefficient and r_{mt} is the return on market portfolio at time t. The error term ε_t at time t should clearly be idiosyncratic.

In the sense of Breusch and Pagan (1979) assume that the variance σ^2 is a liner function of X. Such association of common premium makes stock selection indifference *in the stock market* and are irrelevant[17] for the determination of *relevant* (conditional) payoffs[18] (i.e. the payoffs driven by idiosyncratic (firm-specific) information segments, for example, the net present value[19] of future cash flows of a new project due to equity)) attributable to equity holders. Moreover, the pure randomness associated with the stock price changes exposes speculative traders to risk, which could be minimized by appropriately diversifying the investment portfolio (Markowitz 1952). As such, there should be marginal payoffs attributable to equity holders from shifting the investment bundle, say for example, from firm i to firm k. If all firms are trading at a high systematic risk, diversification between equity classes makes indifference in the market as the marginal payoffs are eliminated and equity market becomes a single instrument (with no diversification benefits) for prospective (future) investors such as bank depositors and government bondholders. Andersen et. al., (2001a)

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[14] *Conditioning variable is the number of observations n which, of course, will be the operational time t without any counter argument as long as sampling frequency is consistent throughout (See Clark 1973 p 39).*

[15] *Firm-specific information segments are usually observed over time as and when information arrives at the market, until the equilibrium price increment in day t is determined. These may include overreaction/underreaction of stock prices (See e.g. Abarbanell and Bernard 1992; Poteshman 2001; Kadiyala and Rau 2004; Spyrou et. al., 2007). However, it is assumed that the stock prices adjust rapidly to new information (See e.g. Fama et. al., 1969; Pan and Poteshman 2006; Reboredo 2013).*

[16] *Black's (1972) version is adopted.*

[17] *This argument is only valid for the players in the stock market as these premiums are relevant, for example, for prospective (future) investors to the stock market such as bank depositors (See e.g. Senarathne and Jayasinghe 2017).*

[18] *The relevant payoffs are subordinated payoffs attributable to equity holders as accrued in equation 1. These payoffs are stochastic so that the $E[\varepsilon] = 0$. As assumed above, payoffs attributable to debt holders are ignored as firms are assumed to be fully equity financed. As such, assuming cash basis of accounting at firm i, $r_{it\,|\,n_t} = \sum_{s=1}^{n_t} \delta_{st}$ = $((w_{it}(\Delta)/Q_{it})$ + $DPS))|\, n_t$ where $w_{it}(\Delta)$ is the change in equity of firm i from time $t-1$ to t, Q_{it} is the quantity in issue of firm i at time t and DPS is the dividend per share, distributed during $t-1$ to t.*

[19] *The present value is added to equity if the project is financed by internal sources of funds (e.g. by reduction of current/future dividend and retained earnings).*

find that the reduction of these benefits accruing to equity holders is due to the presence of covariance (i.e. common premium) of individual stocks in the market[20]. In particular, this paper answers one misconception about the corporate finance and market efficiency. Scholars in traditional finance literature argue that best time to announce or make corporate financial decisions is when the stock market performs well. In fact, the equity holders most benefit from good corporate financial announcements (i.e. announcements attached with good news) made when the stock market is neutral rather than highly bullish because there is little amount of common market premium. Similarly, a bad corporate financial announcement must ideally be made in a highly bullish market because the equity holders are exposed to a high common premium under such conditions (see discussion after proof of theorem 2).

The famous work of Roll (1988) and French and Roll (1986) find that lower R^2 value from regressing return on market variable/s implies high firm-specific return variation[21]. Another set of scholars, for example, Durnev *et. al.,*. (2001, 2003), Morck *et. al.,* (2003), Durnev *et. al.,* (2004a), Durnev *et. al.,* (2004b), Jin and Myers (2006), Chen *et. al.,* (2006), DeLisle *et. al.,* (2016) shows that firm-specific information increases idiosyncratic price change variance and contributes to Tobin's (1982) form of functional efficiency (i.e. fundamental valuation efficiency). Morck *et. al.,* (2000) unearth notable facts about idiosyncratic volatility and demonstrate that it has significant implications for portfolio diversification and arbitrageurs, who seek for mispriced securities with lower level of idiosyncratic risk. The arbitrageurs could benefit from trading in mispriced securities with lower level of idiosyncratic risk, given the availability of the premium (systematic return) in the market.

A collection recent literature on idiosyncratic volatility puzzle finds that high idiosyncratic volatility results in lower expected returns of individual stocks and portfolios (See e.g. Jiang et. al., 2009; Boyer et. al., 2010; Chen et. al., 2012 and Stambaugh et. al., 2015).

Therefore, the variance σ^2 can be written as a function of X; some or all of r_m could serve as X. That is to say;

$$\sigma^2 = f(\alpha_0 + \alpha_1 X) \tag{3}$$

where α_0 is a constant and α_1 is the vector of common market premium (i.e. common expectation).

2.2 Theorems and Proofs

Theorem 1: *Under null hypothesis for perfect mixture of distribution (i.e. complete subordination),* $E(\tilde{\sigma}^2|n) = \sigma^2$

Proof of Theorem 1:

Assuming r_i as in equation two is normally and independently distributed with mean $\beta_0 + \beta_m r_{mi}$ and variance σ^2, the Maximum Likelihood (ML) estimator $\tilde{\sigma}^2$ against unbiased OLS estimator in the sense of Gujarati (2009 p 116, equation 13) can be written as,

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[20] *Although the aggregate market volatility (not idiosyncratic volatility of individual stocks) and covariances (i.e. common market premium) of individual stocks are positively related at market level, the conditional price increments driven by idiosyncratic information segments must have no or, if not, negligible correlation with common market premium at individual stock level.*

[21] *Eventually, this leads to Tobin's (1982) functional form of efficiency at individual stock level. This is further testified by Durnev et al (2001).*

$$E(\tilde{\sigma}^2) = \frac{1}{n} E \sum \hat{\varepsilon}_i^2 \quad (4)$$

If ε_i is drawn from a mixture of distributions, then $E(\hat{\varepsilon}_i^2|n) = \sigma^2 n$ (See e.g. Clark 1973 p. 140; Lamoureux and Lastrapes 1990, p. 222),

$$E(\tilde{\sigma}^2) = \frac{1}{n} \sigma^2 n \quad (5)$$

The expectation of conditional variance becomes the unconditional variance of r_i as,

$$E(\tilde{\sigma}^2|n) = \sigma^2 \quad (6)$$

and the theorem is proved with reference to Clark (1973, p 140). Heteroskedasticity in the *mixture of residuals distributions* will be addressed under theorem 2.

Equation (03) ignores central observations, if $\hat{\varepsilon}_i^2$ is regressed on X. In that respect, Breusch and Pagan (1979) obtain the maximum likelihood estimator $\tilde{\sigma}^2$, dividing sum of squared residuals[22], $\sum \hat{\varepsilon}_i^2$ by N. Obtain residuals from regression (2) and construct $g_t = \hat{\varepsilon}_t^2 / \tilde{\sigma}^2$ for each operational time t. However, the auxiliary regression omits mixing properties of returns (dependent variable) which should be factored into test procedures.

The theory suggests that the distribution of true population variance σ^2 should coincide with the distribution of realized price change variance (r^2)[23]. Let $\tilde{\sigma}^2$ now be computed as $\sum r_i^2 / N$[24] and $d_t = r_t^2 / \tilde{\sigma}^2$. Theoretically, $d_t = g_t$ when the *number of idiosyncratic information segments of firm i observed by n at each operational time becomes large and mixture of distributions of residuals (or increments) is not subject to heteroskedasticity* (see proof of theorem 3).

The test procedure against the null hypothesis (i.e. alternative hypothesis $E(\tilde{\sigma}^2) \neq \sigma^2$) would then become simple (see proof of theorem 2).

One may then regress $z_t = g_t / d_t$ on X at time t as:

$$z_t = \omega + \lambda X_t + u_t \quad (7)$$[25]

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[22] The price increments conditional on operational time t are solely attributable to equity holders as payoffs (i.e. FCFE) from idiosyncratic information segments. It is assumed that there is no transaction cost or cost of borrowing in this market (i.e. efficient market). It is also assumed that there is no agency problem (i.e. conflict of interest between firm's management and the equity holders).

[23] See footnote 11. More importantly, Andersen et. al., (2001a) contain a detailed exposition with proof and evidence as to why and under what conditions the squared return is approaching the true underlying instantaneous volatility. Squared return serves as the best proxy for idiosyncratic variance of stock returns (Renault et. al., 2016).

[24] This standardization would not omit the central observations. In the empirical application as outlined in section 3.2 below, it has been computed as $\sum (r_{it} - \bar{r}_{it})^2 / N$ given the presence of outliers in the distribution of variance, due to jumps in stock prices, although the effect of such difference on auxiliary regression is very close to zero or if not, highly negligible.

[25] For sample regression function, r_{mt} is used for X_t.

for the significance[26] of λ under alternative hypothesis for the presence of heteroskedastic mixture (i.e. imperfect mixture, $\lambda \neq 0$[27]). Label this regression as *model (a)* for comparison purpose at the empirical application stage. The case of more than one regressors could however be explained by a chi square distribution with q degrees of freedom (i.e. LM test) as NR^2 from auxiliary regression, where q is the number of regressors and N is the number of observations. The standard version of Breusch and Pagan (1979) is given as:

$$g_t = \omega + \Pi X_t + u_t \tag{8}$$

for the significance of Π under alternative hypothesis against null hypothesis (i.e. homoscedasticity) for the presence of *heteroskedasticity* (i.e. $\Pi \neq 0$). Label this regression as *model (b)* for further comparison in the succeeding paragraphs.

***Theorem 2*:** *If ε_i is drawn from a homoscedastic mixture of distributions (from a well specified regression), the successive price increments should be independent and generated from a homoscedastic mixture of distributions. Then, $E(\beta_m r_{mi}|n)$ =0 and $E(r_i\hat{\varepsilon}_i)|n = E(\hat{\varepsilon}_i^2)$. Under alternative hypothesis for violation of null hypothesis, $E(\tilde{\sigma}^2) \neq \sigma^2$, may occur in the presence of common market premium,X.*

Proof of Theorem 2*[28]*:

Assume r_i as in equation two is normally and independently distributed with mean, $r_{mi}^T\beta_m$ and variance σ^2 and the proof of *theorem one prevails*, then, $E(r_i\hat{\varepsilon}_i)|n = E(\hat{\varepsilon}_i^2) = \sigma^2 \sim r^2$

Suppressing the constant term, let r_i be written as,

$$r_i = r_{mi}^T\beta_m + \varepsilon_i \tag{9}$$

where $r_{mi}^T\beta_m$ is the expected outcome indexed by time and T denotes the transpose. One would then write the expectation of unconditional variance of ε_i as,

$$E(r_i\hat{\varepsilon}_i) = E\left[r_i\left(r_i - r_{mi}^T\hat{\beta}_m\right)\right] \tag{10}$$

and if $E(r_{mi}^T\, r_{mi})$ is non-singular, then the OLS estimator $\hat{\beta}_m$ for r_i in the sense of Hoaglin and Welsch (1978, p. 17) becomes,

$$\hat{\beta}_{mi} = (r_{mi}^T r_{mi})^{-1}\, r_{mi}^T\, r_i \tag{11}$$

and the collection of variables up to n steps gives,

$$\hat{\beta}_m = \left(\sum_{i=1}^{n} r_{mi}^T r_{mi}\right)^{-1} \sum\nolimits_{i=1} r_{mi}^T\, r_i \tag{12}$$

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[26] *t-test is superior over F-test or X^2 test as it reveals the causes of the heteroskedasticity and is more appropriate when the behavior of errors changes over time. Moreover, if one attempts detect the heteroskedastic mixture of this nature with an F-test, given the fact that the nature of data generating process, causes only two types of variability in the variance and the variance group associated with heteroskedasticity g_t and the mixing property d_t, the corresponding test statistics are coincided as $F = t^2$.*

[27] *u is a well-behaved error term of the regression.*

[28] *Letter i has been used in the conceptual framework chapter to denote ith firm in the market. However, standard notations for rows and columns should be read in the matrix operation as there is no English letter is left to be used.*

In the sense of Hoaglin and Welsch (1978), draw term $r_{mi}\left(\sum_{j=1}^{n} r_{mj}^{T} r_{mj}\right)^{-1} r_{mi}^{T}$ from a hat matrix obeying the properties, symmetry, idempotence and positive definite. Then, $H = X(X^T X)^{-1} X^T$ where X= $(r_{mi}, \dots\dots, r_{miN})$ such that[29];

$$E(r_i \hat{\varepsilon}_i) = E(\varepsilon_i^2)\left\{1 - E\left[r_{mi}\left(\sum_{j=1}^{n} r_{mj}^{T} r_{mj}\right)^{-1} r_{mi}^{T}\right]\right\} \quad (13)$$

accordingly,

$$Trace\ (H) = \sum_{i=1}^{n} h_{ii} = Rank(H) \quad (14)$$

where h_{ii} is the diagonal term of H that can be verified[30] by,

$$h_{ii} = \sum_{j=1}^{n} h_{ij}^2 = h_{ii}^2 + \sum_{j \neq i} h_{ij}^2 \quad (15)$$

of which the number of nonzero eigenvalues[31] is equal to the rank of the matrix. Hence,

$$Rank(H) = Rank(X) = \sum_{i=1}^{n} h_i = p \quad (16)$$

The average value of a diagonal element of the hat matrix is p/N and the sample size, $N \geq p$. Particularly, how the observation points in the regression are affected by speculative conditions of the market determines the behaviour of errors. Consider the following simple explanation. There is little amount of market activities (i.e. little amount of transactions by the listed firms) in a dull market (i.e. natural market) with little macroeconomic information. If a firm makes a good corporate announcement in this period, firm's stock price may deviate from the market trend with high frequent or high amount of firm's stock transactions (active firm's stock trading than overall market trading status). Unless there is firm-specific information such as above, speculative investors could not benefit (as measured by payoffs) from the market by picking any stock randomly as efficient market hypothesis suggests. Homoscedastic mixture is more likely to be present under these conditions, if the firm's stock price is driven by firm-specific trading observations (i.e. on corporate announcements for example). In an active market with large macroeconomic information events (e.g. end of a long-standing civil war which impress a promising economic future of the country), high amount of transactions can be observed in the market with a uniform direction of index movements. An investor is likely to benefit from trading by picking any stock without firm-specific information because there is a common premium in the market. However, they are not the payoffs mentioned in equation 1. Speculators do not benefit from firm-specific information events or announcements in highly active market on macroeconomic information because their payoffs do not come from a homoscedastic mixture of distributions. These

[29] All columns of X are linearly independent.

[30] Whenever $h_{ii} = 0$ or $h_{ii} = 1, h_{ij} = 0$ for all $j \neq i$, $h_{ij} = \frac{1}{2} + \left[(r_{mi} - \bar{r}_m)(r_{mj} - \bar{r}_m)\right] / [\sum_{k=1}^{n}(r_{mk} - \bar{r}_m)^2]$.

[31] It is clear that eigenvalues of a projection matrix are either zero or one and $1 \geq h_{ii} \geq 0$. (See Hoaglin and Welsch 1978, p. 18).

scenarios explain the influence of firm-specific return observations for the evolution in residuals towards homoscedastic mixture of distributions under two market conditions. The proxy for firm-specific return variance is r^2 which was used as the ML estimator $\tilde{\sigma}^2$ in the standardization of residual variance and the variance of residuals, $Var(\varepsilon)|n = \sigma^2 n = \sigma^2 = \sigma^2(1-h_i) \sim r^2$.

Theorem 3: *The expectation of conditional subordinated price increments at each operational time are clearly zero for all steps leading to finite variance, in the presence of heteroskedasticity.*

Proof of Theorem 3

Assume each step of increments in the price change process at operational time t-intraday falls within a unit circle. Let C_0 denote the circle of radius R centered at δ_{st-1}, $|\delta_{st} - \delta_{st-1}| = R$ for clockwise operation with finite steps, $n = t$[32]. Then, the counter clockwise *direction* for conditional expectation in the presence of heteroskedasticity (i.e. common premium X) would yield[33];

$$\delta_{st} = \delta_{st-1} + R\mathrm{e}^{X\theta}(-\pi \le \theta \le \pi) \tag{17}$$

where X is the imaginary unit that turns out to $\int_{C_0}(\delta_{st} - \delta_{st-1})^{n-1}\, d\delta_{st} = 0 \ (n = \pm 1, \pm 2, \dots)$. It is known for the fact that $\delta_{st} - \delta_{st-1} = R\mathrm{e}^{X\theta} and\ d\delta_{st} = RX\mathrm{e}^{X\theta} d\theta$ so,

$$\int_{C_0}(\delta_{st} - \delta_{st-1})^{n-1}\, d\delta_{st} = \int_{-\pi}^{\pi} R^{n-1}\, e^{X(n-1)\theta}. X\, Re^{X\theta} d\theta = \int_{-\pi}^{\pi} R^n\, Xe^{Xn\theta} d\theta$$

$$= R^n X.\frac{e^{Xn\theta}}{nX}\Big|_{-\pi}^{\pi} = \frac{R^n}{n}(\mathrm{e}^{n\pi X} - \mathrm{e}^{-n\pi X}) = 0 \tag{18}$$

Therefore, at end of each trading day t, $\mathrm{E}(\mu \mid n_t) = 0$

3. Data and Empirical Findings

The statistical population of this study includes all firms listed in the Colombo Stock Exchange (CSE) as of 31st December 2011. Fifteen firms were selected from the sampling period1st June 2010 to 31st December 2011. Daily data are obtained from the CSE publications (https://www.cse.lk) and the sampling period reflects the most active period of trading in the history of CSE (see Figure 1). Fully equity financed and low levered firms[34] were given priority in the sample selection in line with the conceptual model. The data were first sorted in Microsoft Excel documents before they were entered into the estimation process on Eviews statistical software. Individual stock price data are matched with the market price and properly sorted. Same method is adopted for non-market level regressions

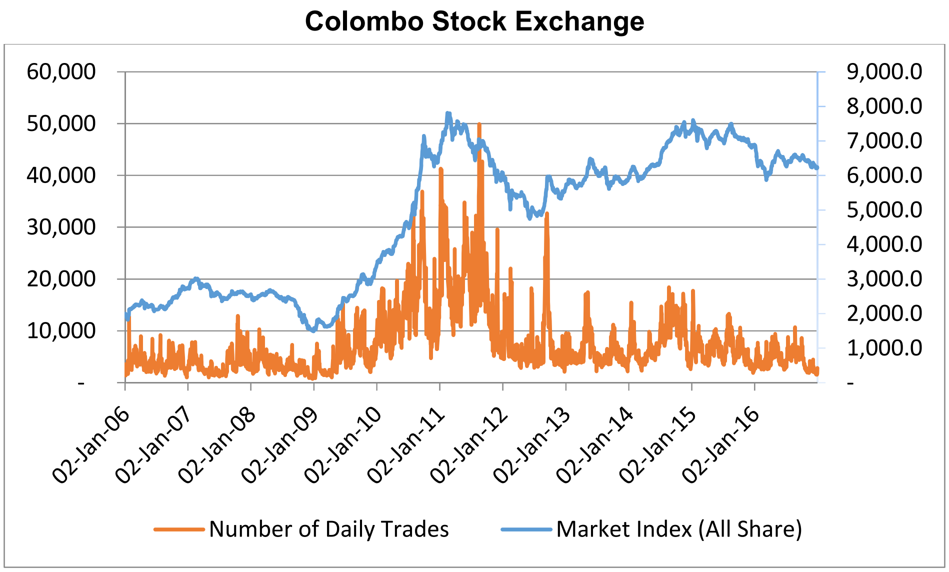
32 See proof of theorem 1.

33 Operational clock is always ticking clockwise where the direction is counterclockwise with a positive angle.

34 This is particularly because there is a positive association between financial leverage and equity return volatility (see e.g. Christie, 1982). However, the impact of this asymmetry on expected return is minimal at individual stock level rather than aggregative market level (Andersen et. al., 2001a). Also, Dennis and Strickland 2009 find that firm-specific price variance is positively related to leverage.

in the forecasting stage. Estimator variables were generated by '*generate*' function of Eviews according to the specific formula given in the text and the regressions are run on Eviews according to the respective equations. Economic data are obtained from economic data library webpage (https://www.cbsl.lk/eresearch/) of Central Bank of Sri Lanka.

Figure 1



Graphic 1. Performance of Colombo Stock Exchange

3.1 Descriptive Statistics Stock Return Data

Figure 1 outlines the change in all share price index (ASPI) and number of trades of the Colombo Stock Exchange during the sampling period. As it can be seen, the most active period of trading is from January 2010 to January 2012 (approximately). This period is chosen because sufficient regularity conditions for central limit theorem (CLT) must held throughout the process of estimation as the mixing variable (i.e. directing variable) of this research is the number of transaction (n). On the other hand, this period reflects the major microeconomic events taken place in Sri Lanka such as conclusion of 30-year long civil war. As such, trading in this period also reflects the effect systematic risk. This period serves as the best period for the purpose of this research.

As far as the descriptive statistics of returns are concerned, nonnormality of the distribution of returns is clearly observed as kurtosis exceeds 3 in the unconditional distributions of all firms and the skewness exists. The test statistic of Augmented Dickey–Fuller exceeds the critical value of -2.86 in all firms, confirming the stationarity of return series. Ljung-Box Q statistic exceeds the critical value 31.41 in five firms, displaying the serial correlation in returns in their unconditional distributions and the null hypothesis for no serial correlation returns is accepted for ten firms in the sample. Under a reasonable rule of thumb, null hypothesis for no autocorrelation in returns could be accepted (Wang and Jain 2003, p 254) as d statistic of thirteen firms are in the range 1 to 2.5, except for firm 8 (d = 1.45) and 9 (d = 2.71).

Table 1

Empirical Properties of Return Data

Firm's Stock Code	No.	N	Skew.	Kurt.	ADF	Q(20)	DW
MGT-N	1	361	1.083	7.025	-12.489	24.715	2.347
OSEA-N	2	383	1.957	13.084	-19.658	15.303	2.016
RHTL-N	3	382	1.286	8.298	-20.004	22.623	2.052
UML-N	4	375	-4.035	69.807	-20.781	9.1097	1.929
YORK-N	5	380	1.869	11.234	-20.464	39.112	2.099
CLND-N	6	383	4.958	46.062	-17.927	28.519	1.832
CLPL-N	7	381	-1.194	25.535	-18.162	19.020	1.863
DIPD-N	8	350	1.352	11.941	-27.171	75.116	2.715
DPL-N	9	381	5.521	59.712	-14.744	74.091	1.457
KDL-N	10	364	1.050	5.746	-18.008	92.372	2.378
CARS-N	11	372	-6.935	116.49	-19.633	11.645	2.043
CHOU-N	12	367	1.040	5.673	-24.405	39.616	2.479
KGAL-N	13	374	0.425	6.338	-19.752	17.798	2.047
LMF-N	14	372	0.451	5.157	-19.444	33.1	2.023
LVEN-N	15	369	0.814	5.158	-21.370	22.325	2.209

Note:

1. ADF is the Augmented Dickey–Fuller test statistic for stationarity. Under null hypothesis for return having unit root, the critical value at 5% significance level is -2.86
2. Q (20) is the Ljung-Box Q statistic for serial correlation upto 20 lags, in the returns. Under null hypothesis for no serial correlation, critical value of χ^2 (20) distribution at 5% significance level is 31.41.
3. DW is Durbin–Watson d statistic for detecting autocorrelation in the return series.

3.2 Detection of Heteroskedastic Mixture of OLS Residuals

Table 2 outlines the test results of heteroskedastic mixture. As per the results, residuals of most of the firms are homoscedastic, except for YORK, CARS, KGAL, LMF and LVEN in the model (b). In respect of model (a), the distributions of residuals of all most all firms are homoscedastic as coefficient λ is statistically insignificant, except for RHTL, YORK, CLPL and KDL that are heteroskedastic. It is observed that there is a significant disagreement between the t-statistics of the two models. More specifically, residues of RHTL, CLPL and KDL are heteroskedastic as per model (a) but homoscedastic as per model (b) displaying a clear disagreement. Similarly, error terms of CARS, KGAL, LMF and LVEN are heteroskedastic according to model (b) but homoscedastic as per model (a). Even, there is a significant difference between the absolute value of t-statistics of *agreeing coefficients* in addition to disagreeing coefficients. These results clearly show that there is a presence of the effect of heteroskedastic mixture of residuals distributions on OLS regression outcome which can be traced by the auxiliary regression proposed. However, there is a possibility that this heteroskedastic mixture may have been affected by the financial leverage of firms because there is a positive association between financial leverage and equity return volatility (Christie, 1982). In particular, firm-specific price variance is positively related to leverage (Dennis and Strickland 2009) so that it may have some implications for each return observation (i.e. idiosyncratic observations or transactions of investors). Since the *asymptotic superiority* of detection of heteroskedastic mixture is not directly observable from the empirical test results of the two models at this level, just by comparing the t-statistics, Monte Carlo simulation exercise is carried out under section 3.4 and the results are outlined as an appendix.

Table 2

Test Results of Heteroskedastic Mixture of OLS Residuals

Firm	Model (b)			Model (a)			D/E
	Π	t-stat	*p*-value	λ	t-stat	*p*-value	
1	34.50**	3.056	0.0024	329960**	5.472	0.0000	7.38%
2	7.256	0.437	0.6621	3931.62	0.572	0.5673	1.61%
3	15.267	1.185	0.2364	2280.92**	3.000	0.0029	0.00%
4	56.684	1.434	0.1524	-195492.2	-0.948	0.3434	0.00%
5	33.714**	2.227	0.0265	239.70*	1.808	0.0713	0.00%
6	-13.655	-0.426	0.6702	-200175.7	-1.006	0.315	0.00%
7	30.390	1.285	0.1994	-4973.8**	-1.979	0.0485	1.39%
8	-13.750	-0.874	0.3822	4900.42	0.603	0.5467	1.75%
9	16.039	0.437	0.6617	-1857.37	-1.502	0.1339	1.16%
10	6.285	0.611	0.5412	332.79**	2.845	0.0047	0.00%
11	175.87**	3.513	0.0005	-35766.7	-0.359	0.7193	0.00%
12	10.51	1.020	0.3083	1640.81	1.557	0.1202	10.55%
13	22.316**	2.014	0.0447	-4919.90	-0.712	0.4768	21.00%
14	20.45**	2.114	0.0351	387.002	0.645	0.5189	5.49%
15	16.90*	1.744	0.0819	-47577.9	-0.991	0.3222	13.72%

*Note: 1. ** Statistically significant at 5% assuming conditional normality. *Statistically significant at 10%. D/E is the Debt to Equity ratio which is computed using data from the quarterly financial statements of each firm for the sampling period.*

3.3 Forecasting Exercise

Suppose that one needs to forecast national remittance on Sri Lanka's merchandise exports to revise the tax policy on profits retained abroad by businesses (*equation 1*). At the same time, he also wants to understand how the returns of county's blue chip stock portfolio will perform into the future by regressing S&P Sri Lanka 20 index portfolio return on all share price index return (ASPI) (*equation 2*). In above regressions, both variables in the regression (regressor and dependent) are market variables in market level regression and non-market variables in non-market level regression. Consider another example of regressor being highly market driven and dependent variable is derived from labour (wage) market and regress wage rate index on average weighted lending rate (AWLR) (*equation 3*) which is determined by market-wide factors in the banking and financial industry operation. Also, consider regressing tourist earnings on total tourist arrivals which is solely a non-market level regression (*equation 4*).

Data are obtained from Central Bank of Sri Lanka. Regress (OLS) national remittance on merchandise exports (i.e. non-market regression) and S&P SL 20 on ASPI (market-level regression). Also, regress wage rate index on AWLR and tourist earnings on total tourist arrivals (all four regressions with a constant term). Sample period covers monthly data from January 2010 to December 2016. In this sample, regressions with explanatory variables ASPI and AWLR are subject to the effect of *heteroskedastic mixture* as assumed above because they are market variables, while merchandise exports and tourist arrivals are non-market level explanatory variables. They are not subject to any dynamic market conditions (e.g. equilibrium price formation process). In other words, they are not subject to the common expectation of a market because they are not market variables.

Table 3 outlines the test results of the *detection of heteroskedastic mixture* based on the two models considered in this study (i.e. (a) and (b)). The results of OLS regression forecasts are not reported due to limitation on space. In fact, the most important results of this forecasting exercise are the implications of the detection of *heteroskedastic mixture* as this study is about testing for *heteroskedastic mixture*.

Table 3

Results of Forecasting Exercise

Regression	N	Model (b)			Model (a)		
		Π	t-stat	*p*-value	λ	t-stat	*p*-value
Non-market (1)	84	-0.002	-1.175	0.243	-0.075	-0.902	0.370
Market (2)	84	9.562	1.241	0.218	18.912	0.047	0.962
Market (3)	84	0.213	3.326	0.001	388.596	1.379	0.172
Non-market (4)	84	-1.5E-06	-0.561	0.576	8.6E-05	0.334	0.739

As per the results outlined in Table 3, it is clear that there is a significant difference between estimates from model (a) and (b) with regard to market level regression. The regression residuals from the regression of S&P SL 20 on ASPI (*equation 2)* are *homoscedastic* as *p*-value is greater than 5 percent in both model (b) and (a). However, there is a significant difference between the absolute value of t-statistics (i.e. 2540%) from model (a) to (b). This difference can be clearly attributable to the effect of *heteroskedastic mixture* that contemporary heteroskedasticity detection models fail to account. A careful observation reveals that the significance levels and the absolute t-statistics of the non-market level regressions are not significantly different as they are not subject to the effect of *heteroskedastic mixture* because the two variables are not market variables and are not subject to speculative price formation process. The difference between the absolute value of t-statistics (i.e. 30%) from model (a) to (b) in *regression (1)* is not significant when compared with the market level *regression (2).* More importantly, residuals from *regression (3)* are heteroskedastic as per model (b) but drawn from a homoscedastic mixture as per model (a) The percentage change for *regression (3)* and *(4)* are 141% and 67% respectively. As such, model (b) has little power of detecting the *heteroskedastic mixture of distributions* property associated with the residuals, when the regression variables are drawn with reference to an active market.

3.4 Monte Carlo Simulation Exercise

The statistical sensitivity of the test to provide more prudent results must be demonstrated by Monte Carlo simulation exercise. Table 4 outlines the power of test, simulated for 5000 experiments at different significance levels. Sample sizes 5, 10, 20, 50 and 100 have been set in each experiment and the effect of sample size on the model performance of two tests has been presented for the following expressions together with corresponding hypothesis of the experiment.

(a) $z_t = \omega + \lambda X_t + u_t$ for ε_t as in equation 2 is distributed with $\varepsilon_t|n_t \sim N(0, \sigma^2 n_t)$. Under null hypothesis, $\lambda = 0$ and $E(z_t) = \omega$

(b) $g_t = \omega + \Pi X_t + u_t$ for ε_t as in equation 2 is distributed with $N(0, \sigma^2)$. Under null hypothesis $\Pi = 0$ and $E(g_t) = \omega$

Monte Carlo simulation test results are outlined in Table 4 as annexed. Model (a) improves the power of the test even when sample size is substantially small and triggers around the cut-off sample size of Central Limit Theorem (CLT)[35] whereas model (b) shows marginal improvements in the performance for small sample sizes. The departure from regularity conditions as measured by the difference between simulated critical values and the respective t-statistics is higher in model (b) than model (a). In particular, the power of the test (a) is improved substantially for small and large samples. Also, the test (a) is more powerful for large samples with lower significance levels. Likelihood ratio tests, for example, Lagrange Multiplier (LM) test has shown to provide a poor statistical power for small samples (See Breusch and Pagan 1979, p. 1293). Moreover, when small sample sizes have larger variances (significant changes in error behaviour), the t-test is more powerful than other tests (See especially Ruxton 2006; De Winter and Dodou, 2010; De Winter 2013). As per the simulation results, model (a) is shown to provide a more asymptotic superiority over model (b).

4. Limitations of the Study

The sample of this study consists of low-leveraged firms (see Table 2) and the correlation between heteroskedasticity and firm leverage was assumed to be zero as per the conceptual model. Any practical implications of such association on the conclusions were thus ignored. While there are functional differences among stock exchanges such as number of listing and listing rules, automated trading technology, trading rules and regulations, market size etc. that may impact the findings of this study, it is plausible to surmise that similar results could be found in other emerging markets. The empirical findings of this study are strictly based on the sample of fifteen actively traded firms during the most active period of the Colombo Stock Exchange. Different results could sometimes be found under other market conditions (e.g. dull market). Practically, tax on firm-specific cash flows accruing to equity stock holders (i.e. in the form of dividend) may affect the number of transactions of a particular firm's stock. However, the impact of dividend tax induced trading around ex-dividend and dividend payment dates was assumed to be zero.

5. Conclusions and Implications

5.1 Conclusions

There is no test procedure available in the current literature to detect heteroskedastic mixture of regression residuals and the residuals are usually assumed to be drawn from a homoscedastic mixture of distributions in the interpretation of OLS regression results. Heteroskedastic mixture is more likely to present in speculative markets as speculators lean from market sources, for example, corporate announcements, the behaviour of errors including its product, (i.e.$\varepsilon_i, \varepsilon_j$) changes over time. This should be considered in designing test procedures for detecting heteroskedasticity because errors are used to forecast the variance of dependent variable and variance is included in the coefficient estimates[36]. As

[35] *This approach seems to be encouraging given the limitation associated with CLT on its application on equity return distributions as noted by Andersen et al (2001b).*

[36] *Andersen* et al *(2001a) conclude with a discussion on this problem.*

such, omission of error behaviour may substantially reduce one's confidence about the precision of coefficient estimates.

The empirical findings show that there are significant disagreements between the heteroskedasticity detection results of the model (a) and (b) and it suggests that there is a presence of the effect of heteroskedastic mixture of distributions on auxiliary regression results, subject to the effect of financial leverage on equilibrium price formation process of firm's stocks (i.e. expectations). In addition to heteroskedasticity detection, there are significant differences between reported t-statistics. These differences are further testified by the results of Monte Carlo simulation. The power of the test (a) as measured by the quantum of deviation from the regularity conditions of asymptotic student-t distribution is greater than model (b) even for substantially small samples (less size distortion). The results of forecasting exercise show that two market level regressions are subject to the effect of heteroskedastic mixture as there are significant differences between the absolute values of t-statistics of market and non-market level regressions. In every respect, the proposed test is shown to provide a good fit for detecting ***heteroskedastic mixture*** of ordinary least squares errors.

5.2 Implications for Economic Forecasting

5.2.1 Theoretical Implications for Forecasting and Avenues for Future Research

The implication of this research is twofold (i.e. theoretical implications and practical implications for future research). The test procedures proposed in this study are based on two-variable regression whereas the residuals from multiple regressions also suffer from the heteroskedastic mixture. The findings of this study therefore encourage future scholars to explore possibility of designing a test procedure for detecting heteroskedastic mixture of residuals drawn from multiple regressions. The conclusions of this study particularly encourage scholars to test heteroskedastic mixture in other developed and emerging markets to see the generalisability of the findings. It would also be interesting to test heteroskedastic mixture under other market conditions (e.g. dull, crisis). On the other hand, this study proposes a t-test (auxiliary regression) to test the heteroskedastic mixture of regression residuals. Although a X^2 test can be derived from the auxiliary regression, the findings encourage to develop a F test given the fact that second order conditions of stock price change (i.e. r^2) together with residual variance also follow a F distribution (see page 6-7).

5.2.2 Practical Implications for Policymaking and Business Application

Statisticians, government policymakers, investment analysts and advisors often use ordinary least squares regressions for forecasting. Decisions are made based on the regression outcome, assuming that the regression residuals are generated from a homoscedastic mixture of distributions in the absence of a formal test. If this assumption is invalid, given the context of application, their forecasts may provide inaccurate directions such as pricing errors and inaccurate firm valuations in the forecasts (e.g., cost of capital computations). The test procedures proposed in this paper is therefore useful for government policymakers, market analysts, fund managers and investment advisors to put forward their advisory work with justifiable reasons as to the accuracy and reasonableness of the reported coefficients. They could minimize the risk of errors (e.g. risk of misrepresentation in advisory work) in their advice on forecast by qualifying and disclaiming the conclusions of reports where appropriate based on this test results. From the perspective of stock market efficiency, the governments must effectively promote firm-specific trading by designing regularity framework (e.g., timely discloser of corporate announcements) and investor education programs to improve *functional efficiency* of stock markets.

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Annex: Tables

Table 4

Monte Carlo Simulation Test Results (Simulated Asymptotic Critical Values of t-distribution)

Level of Significance (two-tailed)	Model (a)					Model (b)				
	Sample size					Sample size				
	5	10	20	50	100	5	10	20	50	100
1%	-4.4934 (-4.604)	-2.8241 (-3.250)	-2.5904 (-2.861)	-2.3426 (-2.679)	-2.4246 (- 2.626)	-4.3426 (-4.604)	-2.7606 (-3.250)	-2.3797 (-2.861)	-2.3256 (-2.679)	-2.2645 (- 2.626)
5%	-2.5575 (-2.776)	-2.0243 (-2.262)	-1.8420 (-2.093)	-1.7284 (-2.009)	-1.6066 (- 1.984)	-1.9916 (-2.776)	-1.7089 (-2.262)	-1.6554 (-2.093)	-1.6521 (-2.009)	-1.5585 (- 1.984)
10%	-1.8958 (-2.132)	-1.5771 (-1.833)	-1.3754 (-1.729)	-1.2230 (-1.676)	-1.2553 (- 1.660)	-1.4872 (-2.132)	-1.2731 (-1.833)	-1.2391 (-1.729)	-1.2854 (-1.676)	-1.2546 (- 1.660)

Note: Asymptotic t-statistics (two-tailed) which assist identifying the departure from regularity conditions appear in parenthesis.