4 TESTING MARKET EFFICIENCY VIA DECOMPOSITION OF STOCK RETURN. APPLICATION TO ROMANIAN CAPITAL MARKET

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Abstract

In this paper we are investigating the market efficiency using a model which decomposes the stock return into two components: a stochastic trend and a white noise component. This model is tested for the Romanian Capital Market, considering the time series of BET (Bucharest Exchange Trade) Index.

The conclusion is that for our data sample we cannot reject the efficient market hypothesis for Romanian Capital Market.

Keywords: efficient market hypothesis, random walk, stochastic trend, ARIMA models, Romanian Capital Market, BET.

JEL classification: C42, G14 - Information and Market Efficiency; Event studies**.**

1. Introduction

The efficiency of capital market is an important issue for both academic and non-academic specialists. Also there is a large variety of methods and techniques developed in order to test a particular form of market efficiency: random walk approach, distribution of price changes et al.(for a review of theoretical approach related to market efficiency, see Fama, 1970, 1976, 1991).

From a classical point of view (Fama, 1976), *a market in which prices always* "fully reflect" available information is called "efficient".

A usual taxonomy related to the Efficient Market Hypothesis (EMH) is to consider three main types of efficiency: weak, semi-strong and strong form of market efficiency.

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This classification arise from definition of efficiency regarding an information set (Malkiel, 1992): efficiency with respect to an information set implies that it is impossible to make economic profits by trading on the basis on that information set.

According to this taxonomy, the weak form of market efficiency deals with the information set containing past prices of a stock, the semi-strong form concerns the information set containing publicly available information, while the strong form is based on the private information regarding a certain stock(see Lo, Campbell and MacKinley,1997).

Although we can discuss about such taxonomy of market efficiency, only the weak form of efficiency can be easier to express mathematically. Also, there is a large variety of statistical tools for testing this form of efficiency. In few recent years researchers prefer to discuss rather predictability of asset return than to test a particular form of market efficiency.

From a statistical point of view the efficient market hypothesis (weak form) can be expressed using a random walk model for stock price. Indeed, as the market is efficient regarding past prices, any new information will be reflected in the new price. As the information is unpredictable, the price change will depend only on the new available information and should be uncorrelated to the past price changes; so the price changes are unpredictable and random trough time.

A random walk model (RW) for stock price is usually defined (see for instance Lo, Campbell and MacKinley, 1997) as follows:

$$P_t = \mu + P_{t-1} + \varepsilon_t \,, \qquad [1]$$

where P_t denotes stock price at moment t, the increments (innovations) $(\varepsilon_t)_t \sim \mathbf{WN}(0,\sigma^2)$ denotes a white noise, with independently and identically distributed variables:

$$\begin{cases} \mathbf{E}[\varepsilon_t] = 0, \forall t \\ \mathbf{Var}\left[\varepsilon_t\right] = \sigma^2, \forall t \\ \varepsilon_t \text{ and } \varepsilon_{t+k} \text{ are independent variables}, \forall k \neq 0 \end{cases}$$

Thus, if the last assumption holds, then:

$$\mathbf{Cov}\left[\varepsilon_t, \varepsilon_{t+k}\right] = 0 \text{ and } \mathbf{Cov}\left[\varepsilon_t^{\,2}, \varepsilon_{t+k}^{\,2}\right] = 0, \forall k \neq 0 \;.$$

In equation [1], μ is the expected price change, also called drift. It's easy to show, based on these assumptions, that the stock prices follow a nonstationary stochastic process.

In order to simplify the aspects related to inference problem, one common approach is to assume a Gaussian white noise $\varepsilon_t \sim \mathbf{N}(0,\sigma^2)$ satisfying the above conditions.

This assumption is rather unrealistic, many studies revealing the stock price do not come from a Gaussian distribution (see Mandelbrot, 1963). As a consequence, the logarithm of price instead of stock's price is used: $p_t = \log P_t$, where *log* denotes natural logarithm.

Thus, the model RW will became a lognormal model:

$$p_t = \mu + p_{t-1} + \varepsilon_t , \quad [2]$$

where ε_t is a sequence of independently and identically distributed variables and $\varepsilon_t \sim \mathbf{N}(0, \sigma^2)$.

Although the model [1] is simple, elegant and easy to formulate, it is unrealistic to assume that the price changes are identically distributed on the market. Usually the innovations are heteroskedastic, in a sense that the factors contributing to random fluctuations in prices are changing their direction and sense over time.

Moreover, the hypothesis of independent innovations is a very strong one, and usually is replaced by the weaker condition of uncorrelated innovations.

Many authors have developed statistical tools in order to test the random walk hypothesis, assuming either independent or uncorrelated innovations, homoskedasticity or heteroskedasticity of innovations.

We can mention here the well known Variance Ratio Test, developed in Lo and MacKinlay (1988). A generalised form of this test was developed by Chow and Denning (1993), the so called Multiple Variance Ratio Test, based on the SMM (Studentized Maximum Modulus) distribution.

During decades, the conclusions of the studies regarding market efficiency were more or less contradictory. For instance, in 1937 Cowles and Jones, analyzing the US capital market, come to reject the efficient market hypothesis.

Few decades later, in the 70's, Fama find more arguments for the EMH than against this hypothesis. In 2003 Malkiel states that "stock markets are far more efficient and far less predictable than some recent academic papers would have us believe".

The EMH for Romanian capital market has been analysed in several papers and the findings are rather contradictory.

Todea (2002), analyses a sample of 10 stocks covering the period 1997-2000 and concludes that cannot reject the weak form of EMH, while Dragotă and Mitrică (2004), studying a sample of 6 stocks from Bucharest Stock Exchange, for the period 1998-2000, come to conclusion of rejecting weak form of EMH.

A more complete and recent study(Dragotă et al., 2007), analyzing the daily and weekly returns for a sample of 22 stocks and market indexes, come to conclusion that stock prices follow a random walk, so we cannot reject the EMH.

2. A model for long-horizon returns

An alternative to the random walk model is the long-horizon returns model, firstly introduced by Muth in 1961.

Basically, this model proposes the breakdown of time series of prices natural logarithms into the sum of a random walk and a stationary component:

$$p_t = w_t + y_t \,, \qquad [3]$$

where:

- $w_t = \mu + w_{t-1} + \varepsilon_t, \quad (\varepsilon_t)_t \sim \mathbf{IID}(0,\sigma^2)$, meaning that $(\varepsilon_t)_t$ is a sequence of independent, identically distributed random variables with zero mean (a white noise);
- y_t is stationary process with zero mean: $\begin{cases} \mathbf{E}[y_t] = 0, \forall t, \\ \mathbf{Cov}[y_t, y_s] = 0, \ \forall t \neq s, \\ \mathbf{E}[y_t{}^2] < \infty \end{cases}$.
- $(w_t)_t$ and $(y_t)_t$ are independent processes.

In the usual interpretation of the model, the random walk $(w_t)_t$ is considered to be the fundamental component, reflecting the capital market efficiency, while $(y_t)_t$ is a stationary process reflecting the short-term deviations from the EMH[1]. The term $(y_t)_t$ synthesizes the abnormal, unpredictable behaviour of the capital market, due to certain short-term factors, which are not significant on the long run. The stationary feature of this component implies that, on the long-horizon, the expected value would be its average, whose value is zero, so the influence upon financial assets price is relevant only on the short run.

If we are using the log-return instead of log-price, we have the following expression: $r_t = \log \dfrac{P_t}{P_{t-1}} = p_t - p_{t-1}$, where $p_t = \log P_t$, P_t being the asset's price at the moment t.

According to the Muth model, the following expressions reflect the price logarithm on two successive moments in time:

$$\begin{cases} p_t = w_t + y_t \\ w_t = \mu + w_{t-1} + \varepsilon_t \end{cases} \text{ and } \begin{cases} p_{t-1} = w_{t-1} + y_{t-1} \\ w_{t-1} = \mu + w_{t-2} + \varepsilon_{t-1} \end{cases} \text{ with } (\varepsilon_t)_t \sim \mathbf{IID}(0,\sigma^2).$$

Thus, the asset's return could be written as follows:

$$r_t = p_t - p_{t-1} = w_t + y_t - w_{t-1} - y_{t-1} = w_t - w_{t-1} + y_t - y_{t-1} = W_t + Y_t \,,$$

where $W_t = w_t - w_{t-1}$ and $Y_t = y_t - y_{t-1}$.

1 According to Campbell, J.Y., Lo, W., MacKinlay, C.(1997), *The Econometrics of Financial Markets*, Princeton University, 1997, pp.55.

Based on the properties of the basic components of the Muth model, the behaviour of the new components of the model could be determined.

Thus:

- $W_t = w_t - w_{t-1} = \mu + w_{t-1} + \varepsilon_t - (\mu + w_{t-2} + \varepsilon_{t-1}) = w_{t-1} - w_{t-2} + \varepsilon_t - \varepsilon_{t-1} =$
  $= \mu + \varepsilon_t$.
- $Y_t = y_t - y_{t-1}$ is a stationary process. Moreover, the independence between W_t and Y_t is fulfilled.

Therefore, the Muth model for log-return could be written as follows:

$$r_t = W_t + Y_t,$$

where: • $W_t = \mu + \varepsilon_t$, $(\varepsilon_t)_t \sim \mathbf{IID}(0, \sigma^2)$

- Y_t is a stationary process with zero mean:

$$\mathbf{E}[Y_t] = 0, \forall t$$
$$\mathbf{Cov}\, [Y_t, Y_s] = 0, \forall t \neq s$$
$$\mathbf{E}[Y_t^2] < \infty$$

- $(W_t)_t$ and $(Y_t)_t$ are independent processes.

Although this model is simple and elegant, is difficult to test it in the real world.

3. A model with stochastic trend and random turbulences

Obviously, if in the model [3] the influence of random walk component is significantly higher than the influence of the stationary component, than we cannot make accurate predictions about the future prices.

If we invert this reasoning, then we may assume the stock price to be sum of two components: an AR(1) process and a stationary zero mean process.

$$p_t = w_t + y_t, \qquad [4]$$

where: • $w_t = \mu + \rho w_{t-1} + \varepsilon_t$, $(\varepsilon_t)_t \sim \mathbf{IID}(0, \sigma^2)$, meaning that $(\varepsilon_t)_t$ is a sequence of independent, identically distributed random variables with zero mean (a white noise);

- y_t is stationary process with zero mean: $\begin{cases} \mathbf{E}[y_t] = 0, \forall t, \\ \mathbf{Cov}[y_t, y_s] = 0, \forall t \neq s, \\ \mathbf{E}[y_t^2] < \infty \end{cases}$.

- $(w_t)_t$ and $(y_t)_t$ are independent processes.

If $\rho < 1$ then the process $(w_t)_t$ is stationary and predictable, while if $\rho = 1$ $(w_t)_t$ is a random walk.

Dealing with the model [4] in terms of returns, we obtain the following expression for the log-return:

$$r_t = p_t - p_{t-1} = w_t + y_t - w_{t-1} - y_{t-1} = w_t - w_{t-1} + y_t - y_{t-1} = W_t + Y_t$$

where $W_t = w_t - w_{t-1}$ and $Y_t = y_t - y_{t-1}$.

Moreover,

$$W_t = w_t - w_{t-1} = \mu + \rho w_{t-1} + \varepsilon_t - \mu - \rho w_{t-2} - \varepsilon_{t-1} = \rho(w_{t-1} - w_{t-2}) + \xi_t = \rho W_{t-1} + \xi_t \quad [5]$$

where $\xi_t = \varepsilon_t - \varepsilon_{t-1}$ is a white noise.

Consequently,

$$r_t = W_t + Y_t = \rho W_{t-1} + Y_t = \rho(W_{t-1} + Y_{t-1}) + Y_t - \rho Y_{t-1} = \rho r_{t-1} + \gamma_t, \quad [6]$$

where $\gamma_t = Y_t - \rho Y_{t-1}$ is a stationary process with zero mean.

The equations [5] and [6] can lead us to an interesting conclusion: if the systematic component $(w_t)_t$ in the price model [4] is an AR(1) process, then the return can also be decomposed into the sum of two components.

Based on these calculations, we can state a ***stochastic trend model*** for stock log return:

$$r_t = W_t + Y_t \quad [7]$$

where:
- r_t is the stock log return at time *t*;
- W_t is an AR(1) process, following the equation

  $W_t = \rho W_{t-1} + \xi_t$, where $(\xi_t)_t$ is a white noise;
- Y_t is stationary process with zero mean, in particular any white noise;
- W_t and Y_t are independent processes.

The autoregressive component of the model [7] can be regarded as the return due to the action of the macroeconomic environment and Y_t represents the influence of the non systematic, random factors.

If the coefficient $\rho = 1$, then W_t becomes a stochastic trend (in fact a random walk process) and we cannot reject the EMH. Instead, if $\rho < 1$, W_t is a stationary AR(1) process and due to this fact we can make predictions about the future values of log return, so we can reject the EMH.

But this conclusion is not an absolute one: even if W_t is a stationary AR(1) process, if his explained return variance is significantly low, again we cannot use past values of return in order to make accurate predictions about the future values.

Based on this model, we can describe the behaviour of the log-return, according to the following propositions[1]:

Lemma 1

Let $(X_t)_t$ *a* $ARIMA(p,d,q)$ *process and let* $(Y_t)_t$ *a* $ARMA(p',q')$ *process,* $(X_t)_t$ *and* $(Y_t)_t$ *being uncorrelated:* $Cov(X_t,Y_s)=0, \forall s,t \in \mathbf{Z}$.

Then the sum $(S_t)_t, S_t = X_t + Y_t, \forall t \in \mathbf{Z}$ *is an ARIMA (P, d, Q) process, where* $P = p + p', Q = \max(p+d+q', q+p')$.

Lemma 2

Let $(X_t)_t$ *a process* $ARIMA(p,d,q)$ *and let* $(U_t)_t$ *a white noise,* $(X_t)_t$ *and* $(U_t)_t$ *being uncorrelated:* $Cov(X_t,U_s)=0, \forall s,t \in \mathbf{Z}$.

Then the sum $(S_t)_t, S_t = X_t + U_t, \forall t \in \mathbf{Z}$ *is an ARIMA (p, d, Q) process, where Q=max (p+d, q).*

So we can formulate the following:

Proposition

If the log return can be decomposed like in model [7], then the log return is an autoregressive process of first order.

For our model, if W_t is an AR(1) process, then the return is also an AR(1) process and more, the coefficient ρ is the same as the coefficient of the autoregressive model of return.

Consequently, we can indentify a method for testing the market efficiency via testing the existence of a stochastic trend.

In fact, according to the model [7], we will estimate a first order autogressive process for log return:

$$r_t = \rho r_{t-1} + \gamma_t, \qquad [8]$$

where $(\gamma_t)_t$ is a white noise and we will test the existence of a unit root for the equation [8].

Even the model [7] is more general, we will test a particular form of this model:

$$r_t = W_t + Y_t \qquad [9]$$

where:
- r_t is the stock log return at time *t*;
- W_t is an AR(1) process, following the equation

  $W_t = \rho W_{t-1} + \xi_t$, where $(\xi_t)_t$ is a white noise;
- Y_t is white noise;
- W_t and Y_t are uncorrelated processes.

[1] *The proofs are based on time series theory and are available upon request.*

4. Application to Romanian Capital Market

After the reopening from 1995, Bucharest Stock Exchange has set up, in 1997, general index of the market, BET, quantifying the evolution of prices for the assets of the most liquid companies on the market.

In case of the Romanian capital market, the BET (Bucharest Exchange Trading) index is computed based on the formula:

$$BET_t = 1000 \cdot f \frac{\sum_{i=1}^{10} q_{i0} p_{it}}{\sum_{i=1}^{10} q_{i0} p_{i0}},$$

where: • $n=10$ is the number of shares from the index portfolio;

- p_{i0} is the prices weighted average of the share *i* at the reference moment (considered to be the moment of the last index basket updating);
- p_{it} is the weighted average price of the share *i* at the moment *t*;
- q_{i0} is the number of *i* company shares at the reference moment;
- *f* is a correction factor recalculated every time changes in component shares occur.

The criteria for choosing the 10 companies to be taken into account for the BET index basket are the following[1]:

- the companies should have the highest blue chips; likewise, the capitalisation of companies included in the BET index portfolio should account for at least 60% of total capitalisation;
- the companies' selection should ensure index portfolio diversification;
- the shares of companies included in the index should be the ones with the highest liquidity; at the same time, the total value of transactions with the shares included in the index portfolio should account for at least 70% of total value of transactions in Bucharest Stock Exchange.

The above mentioned criteria are monthly revised based on the performance analysis of the companies included in the index portfolio (the index basket) as well as of recently listed companies, in view of eventual update of the index composition.

A point of interest is to distinguish the influences determined by the macroeconomic context, supposed to last for a long period, and the incidental influences, of random nature, whose effect is not manifested on a long horizon (the time horizon of these random influences ranges between a few days and a few months, at most).

For the construction of the market pattern of the capital market return, we shall assume that the return of the financial assets is subject to the combined action of two types of influences: the influences due to the macroeconomic environment, whose action is medium and long-term and the influences due to some random factors, whose action is short-term.

[1] *According to Bucharest Stock Exchange website, www.bvb.ro*

The formula of the index log return can be written as follows:

$$r_t = W_t + Y_t \qquad [10]$$

where W_t is the return due to the action of the macroeconomic environment and Y_t represents the influence of the non systematic, random factors.

According to the model [9], we shall assume that the interest parameter, W_t (the systematic factor) is an autoregressive process AR(1), which verifies the linear equation $W_t = \rho W_{t-1} + \xi_t$, where $(\xi_t)_t$ is a white noise.

The non systematic factor Y_t is also a white noise. Estimating the parameter of the autoregressive process, we can draw a conclusion regarding the efficiency of the capital market in correlation with the economic context.

Thus: - if $\rho = 1$ then the market is efficient –the autoregressive pattern becomes a random walk pattern;

- if $\rho \neq 1$, then we can reject the efficient market characteristic.

On the other hand, we shall pay attention to the performance indicators of the pattern, too: if $\rho \neq 1$, but the pattern has a weak explicative power, we cannot reject the Efficient Market Hypothesis.

We have used daily data for BET Index, covering the period between 19/09/1997 and 09/01/2007(2305 observations for the index and 2304 observations for the return).

Descriptive statistics are presented in the table below.

Table 1

BET Index – Descriptive Statistics

	BET Index	BET Return
Mean	2369.011	0.001
Standard Error	51.436	0.000
Median	1099.950	0.001
Mode	532.330	0.000
Standard Deviation	2469.482	0.017
Sample Variance	6098339.380	0.000
Kurtosis	0.032	5.800
Skewness	1.211	-0.121
Range	8343.960	0.220
Minimum	281.090	-0.119
Maximum	8625.050	0.101
Sum	5460570.677	2.147
Count	2305	2304

In order to test the stationarity of the BET return, we have used the Augmented Dickey-Fuller test, for the presence of a unit root. In fact, we have considered the following model: $r_t - r_{t-1} = (u-1)r_{t-1} + u_t$ and we have tested the null hypothesis $H_0 : u = 1$ against the alternative $H_A : u < 1$.

By rejecting the null hypothesis we can conclude the stationarity of the analysed process, as in table below.

Table 2

Unit root test for BET return

			t-Statistic	Prob
Augmented Dickey-Fuller test statistic			-36.48	0.00
Test critical values:	1% level		-3.43	
	5% level		-2.86	
	10% level		-2.57	
Dependent Variable: D(BET)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
BET(-1)	-0.73	0.02	-36.48	0.00
C	0.00	0.00	1.95	0.05

R-squared	0.37
Adjusted R-squared	0.37
S.E. of regression	0.02
Sum squared resid	0.65
Log likelihood	6139.45
Durbin-Watson stat	1.98
F-statistic	1331.09
Prob(F-statistic)	0.00

In order to decide about the behaviour of BET return, we can study the the autocorrelation function (ACF) and the partial autocorrelation function (PACF).

Table 3

ACF and PACF for BET return

Lag	Autocorrelation	Partial Autocorrelation	Q-Stat	Prob
1	0.267	0.267	164.48	0.00
2	0.043	-0.031	168.72	0.00
3	0.009	0.005	168.89	0.00
4	0.002	0	168.9	0.00
5	0.023	0.024	170.09	0.00
6	0.037	0.027	173.27	0.00
7	0.046	0.031	178.15	0.00
8	-0.003	-0.025	178.17	0.00
9	-0.008	0	178.3	0.00
10	-0.03	-0.03	180.37	0.00
11	0.007	0.023	180.47	0.00
12	0.023	0.014	181.68	0.00

Based on the behaviour of the autocorrelation function (ACF) and the partial autocorrelation function (PACF), we can estimate the AR(1) model for BET return.

In fact, the analysis of the correlogram can suggest that we can build also an ARMA (1,1) model, but the results of the estimation were not significant (see the Table 4.)

Table 4

ARMA(1,1) model for BET return

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.00	0.00	1.98	0.05
AR(1)	0.16	0.08	2.07	0.04
MA(1)	0.12	0.08	1.55	0.12
R-squared	0.07	Mean dependent var	0.00	
Adjusted R-squared	0.07	S.D. dependent var	0.02	
S.E. of regression	0.02	Akaike info criterion	-5.33	
Sum squared resid	0.65	Schwarz criterion	-5.32	
Log likelihood	6140.56	F-statistic	89.51	
Durbin-Watson stat	2.00	Prob(F-statistic)	0.00	
Inverted MA Roots	-0.12			

The results of the estimation for model [8] are presented in the table below.

Table 5

Results for the model [8]

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.0006	0.00	1.96	0.051
AR(1)	0.27	0.02	13.29	0.00
R-squared	0.07	Mean dependent var	0.00	
Adjusted R-squared	0.07	S.D. dependent var	0.02	
S.E. of regression	0.02	Akaike info criterion	-5.33	
Sum squared resid	0.65	Schwarz criterion	-5.32	
Log likelihood	6139.45	F-statistic	176.71	
Durbin-Watson stat	1.98	Prob(F-statistic)	0.00	
Inverted AR Roots	0.27			

The autoregressive parameter is sub-unitary and the model passes the robustness tests. More, the estimation results tells that the appropriate model for BET return is a zero-drift model, as the intercept is not significant at 95%.

The fact that the non systematic factor is a white noise results from the independence tests.

This conclusion, of white noise behaviour for Y_t is confirmed by the Box-Ljung Q-Statistic, computed using EViews 5.1.

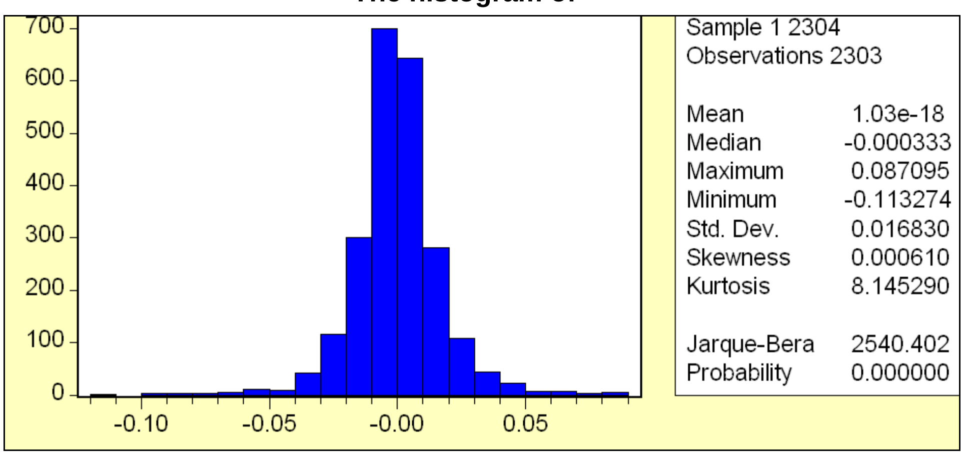
Table 6

Q-Statistic for white noise behaviour of Y_t

Lag	AC	PAC	Q-Stat	Prob
1	0.008	0.008	0.1465	0.702
2	-0.031	-0.031	2.3354	0.311
3	-0.002	-0.002	2.3483	0.503
4	-0.007	-0.008	2.4551	0.653
5	0.015	0.015	2.9689	0.705
6	0.023	0.022	4.2077	0.649
7	0.043	0.043	8.4581	0.294
8	-0.015	-0.014	8.9682	0.345
9	0	0.004	8.9686	0.44
10	-0.034	-0.035	11.662	0.308
11	0.009	0.01	11.866	0.374
12	0.02	0.016	12.822	0.382
13	0.014	0.013	13.301	0.425
14	-0.01	-0.011	13.539	0.485
15	0.014	0.018	14.017	0.524

Figure no. 1

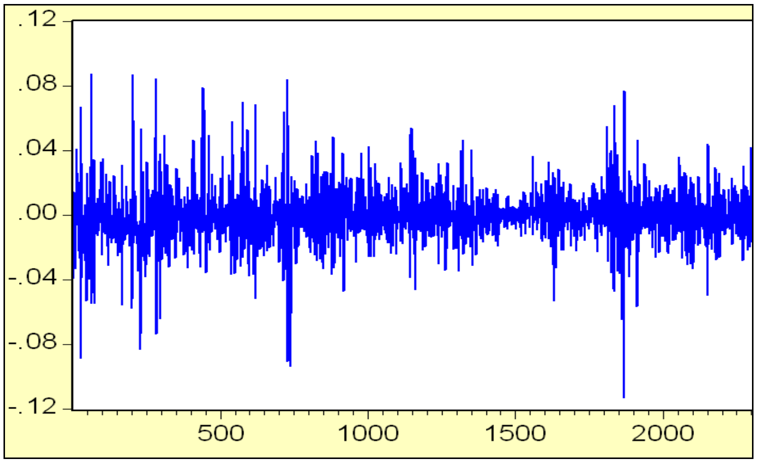
The histogram of Y_t



Moreover, from the analysis of the histogram we deduce that Y_t follows a white noise with the dispersion $\sigma_Y^2 = 0.0003$.

Figure no. 2

The non systematic factor Y_t



The pattern of the systematic factor is an AR(1) process: $W_t = \rho W_{t-1} + \xi_t$, with $\rho = 0.27$.

The dispersion of the white noise $(\varepsilon_t)_t$ can be determined applying to the AR(1) pattern the variance operator:

$$\mathbf{Var}(W_t) = \rho^2 \mathbf{Var}(W_{t-1}) + \mathbf{Var}(\xi_t)$$

or with other notations: $\sigma_W^2 = \rho^2 \sigma_W^2 + \sigma_\xi^2$, whence results $\sigma_\xi^2 = (1-\rho^2)\sigma_W^2$.

From the stochastic trend model, we also have the following relation between dispersions:

$\sigma_r^2 = \sigma_W^2 + \sigma_Y^2$, from where we can deduce the dispersion of the systematic factor: $\sigma_W^2 = \sigma_r^2 - \sigma_Y^2$ =0.00003.

We thus obtain the dispersion value of the white noise for the systematic factor: $\sigma_\xi^2 = 0.000031$.

Since for the autoregressive parameter we have $0 < \rho < 1$, we could conclude that, on long term, the capital market in Romania does not satisfy the efficiency hypothesis.

The fact that the return variation is only slightly explained with by the estimated pattern ($R^2 = 0.07$) and the non systematic factor has a significant influence, leads us towards an interesting conclusion.

In fact, what influences on long term the movement of the shares on the market, is, to a large extent, the result of the action of some punctual, short term, non general factors. Consequently, we cannot reject the hypothesis of market efficiency in the weak sense.

5. Concluding remarks

In this paper, starting from a classical model of Muth (1961), we have proposed a model for stock's return decomposition. In fact, according to Efficient Market Hypothesis, if the stock price follows a random walk model, then the stock return is unpredictable; in particular the return is a white noise.

We have proposed a return decomposition into two components, an autoregressive process and a stationary zero mean process. Then under certain assumptions, the stock return follows also an autoregressive model.

In fact, by estimating the parameter of the autoregressive process, we can draw a conclusion regarding the efficiency of the capital market in correlation with the economic context.

If the autoregressive component is in fact a stochastic trend (for example a random walk), and this component has a significant influence upon the stock return, then we cannot reject the EMH.

Moreover, even the autoregressive component is stationary but has a little influence upon stock return, again we cannot reject the EMH.

The model was tested for Romanian Capital Market, using the daily returns of BET Index, and the conclusion was that we cannot reject the hypothesis of market efficiency in the weak sense.

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Appendix

Proof of Lemma 1

From $(X_t)_t \sim ARIMA(p,d,q)$ and $(Y_t)_t \sim ARMA(p',q')$ we deduce that the process $\nabla(B)^d X_t$ is stationary and $\Phi(B)_t \nabla^d X_t = \theta(B)Z_t$, where $Z_t \sim WN(0,\sigma^2)$, $\nabla(B) = 1 - B$ is the difference operator: $\nabla(B)X_t = X_t - X_{t-1}$ and

$$\begin{cases} \Phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p \\ \theta(\mathrm{z}) = 1 + \theta_1 z + \ldots + \theta_q z^q \end{cases}.$$

Also $(Y_t)_t$ is stationary and we have $\Phi'(B)Y_t = \theta'(B)U_t$, where $(U_t)_t \sim WN(0,\tau^2)$ with $\begin{cases}\Phi'(z) = 1 - \phi'_1 z - \ldots - \phi'_{p'} z^{p'} \\ \theta'(z) = 1 + \theta'_1 z + \ldots + \theta'_{q'} z^{q'}\end{cases}$.

As the processes $\nabla(B)^d X_t$ and $(Y_t)_t$ (like $\nabla(B)^d Y_t$ also) are stationary, implies that the sum process $(S_t)_t,\ S_t = X_t + Y_t, \forall t \in \mathbf{Z}$ is integrated of order d i.e. $\nabla(B)^d S_t$ is stationary.

Further, by applying the operator $\Phi(B)\Phi'(B)\nabla^d$ to $(S_t)_t$, we have:

$$\Phi(B)\Phi'(B)\nabla^d S_t = \Phi(B)\Phi'(B)\nabla^d (X_t + Y_t) = \Phi(B)\Phi'(B)\nabla^d X_t + \Phi(B)\Phi'(B)\nabla^d Y_t =$$
$$= \Phi'(B)\theta(B)Z_t + \Phi(B)\theta'(B)\nabla^d U_t.$$

With some computations, we can derive the equality $\Phi(B)\Phi'(B)\nabla^d S_t = \Theta(B)V_t$, where $\Theta(z) = \Phi'(z)\theta(z) + \Phi(z)\theta'(z)(1-z)^d$ is a polynomial with degree $Q = \max(p + d + q', q + p')$, and $(V_t)_t$ is a white noise verifying the equation $\Theta(B)V_t = \Phi'(B)\theta(B)Z_t + \Phi(B)\theta'(B)\nabla^d U_t$.

Hence, the process $(S_t)_t,\ S_t = X_t + Y_t, \forall t \in \mathbf{Z}$ is an ARIMA(P,d,Q), with $P = p + p', Q = \max(p + d + q', q + p')$.

Proof of Lemma 2

Having $(X_t)_t$ a $ARIMA(p,d,q)$ process means that the time series $Y_t = \nabla(B)^d X_t$ is stationary and is a solution of the equation $\Phi(B)Y_t = \theta(B)Z_t$, where $Z_t \sim WN(0,\sigma^2)$, and $\nabla(B) = 1 - B$ is the difference operator $\nabla(B)X_t = X_t - X_{t-1}$, where $BX_t = X_{t-1}$. Moreover, $\begin{cases}\Phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p \\ \theta(z) = 1 + \theta_1 z + \ldots + \theta_q z^q\end{cases}$.

As $(U_t)_t \sim WN(0,\tau^2)$ one can conclude that the series $\nabla(B)^d U_t$ is stationary, for every integration degree d. Also the process $(Y_t + \nabla(B)^d U_t)_t$ is stationary and having fulfilled the identity $\nabla(B)^d (X_t + U_t) = Y_t + \nabla(B)^d U_t$, we can conclude that the process $(S_t)_t, S_t = X_t + U_t, \forall t \in \mathbf{Z}$ is integrated with order d ($(S_t)_t \sim I(d)$).

Further, we can evaluate the expression:

$$\Phi(B)\nabla^d (X_t + U_t) = \Phi(B)\nabla^d X_t + \Phi(B)\nabla^d U_t =$$
$$= \theta(B)Z_t + \Phi(B)\nabla^d U_t = \theta(B)Z_t + (1-\phi_1 B - ... - \phi_p B^p)(1-B)^d U_t.$$

Let $\Theta(z) = \theta(z) + \phi(z)(1-z)^d$ a polynomial with degree Q=max(p+d,q); then we can write $\theta(B)Z_t + (1-\phi_1 B - ... - \phi_p B^p)(1-B)^d U_t = \Theta(B)V_t$, where $(V_t)_t \sim WN(0,\sigma_V^2)$.

Finally, the process $(S_t)_t, S_t = X_t + U_t, \forall t \in \mathbf{Z}$ verifies the difference equation $\Phi(B)\nabla^d S_t = \Theta(B)V_t$, with $(V_t)_t \sim WN(0,\sigma_V^2), \deg(\Phi) = p, \deg(\Theta) = Q = \max(p+d,q)$ so we can conclude that $(S_t)_t, S_t = X_t + U_t, \forall t \in \mathbf{Z}$ is an ARIMA(p,d,Q) process.

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