2. PREDICTION OF A SMALL AREA MEAN FOR AN INFINITE POPULATION WHEN THE VARIANCE COMPONENTS ARE RANDOM

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Abstract

In this paper, we propose a new model with random variance components for estimating small area characteristics. Under the proposed model, we derive the empirical best linear unbiased estimator, an approximation to terms of order $o(1/m)$ *and an estimator whose bias is of order* $o(1/m)$ *for its mean squared error, where m is the number of small areas in the population.*

Keywords: small areas, direct and indirect estimation, infinite population, empirical best linear unbiased predictor

JEL Classification: C51, C63, C13

1. Introduction

National surveys are generally designed for estimating parameters at national level. The inference is based on the sampling distribution and the size of the sample insures an adequate level of precision for the resulting estimators.

There is a growing demand for estimations at sub-national level, region or county, for instance. In the sample survey theory these sub-populations are called domains or areas. If we base the inference on the sampling distribution, which is the distribution coming from the sampling design, the resulting estimator for some area characteristics called *direct estimator* has the same formula as the estimator for the same parameter at the national level, but it uses only the observations falling in the area. Because the national survey was designed for national estimation and not for the area under study, it happens more often than not that these observations are not enough to ensure an adequate level of precision for the direct estimator of the area parameter. This is why

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in this case the area is called small area and a new theory is needed to estimate the small area parameters.

The new theory is called small area estimation theory. A detailed account is given in Rao J.N.K. (2003). In small area estimation, the inference is model based. This means that, first, one has to find a model for the population values of the variable under study, then check the model fit and, finally, base the inference on the model to get the estimator as well as a measure for its precision. The model based estimator for the small area characteristics called *indirect estimator* uses the entire national sample and not only the small area sample, because the population model acts like a link between different areas of the population. For this reason, it is said that the indirect estimator *borrows strength* from related small areas and its precision is generally better than that of the direct estimator.

The parameter of interest is the small area mean. We consider only the infinite population case. In section 2 of our paper we propose a new model, which can be used for estimating the mean of a small area and give the formula for the best linear unbiased predictor of the mean under the proposed model. In section 3, first we derive an approximation to terms of order $o(1/m)$ for the precision of the predictor measured by its mean squared error. Then, we shall obtain an estimator of bias of order $o(1/m)$ for the precision of the predictor. In section 4, we present a Monte Carlo simulation showing the accuracy of both the approximation and the estimator of the precision. Finally, in section 5 we draw some conclusions and give a few directions for future research.

2. A new model

In our paper, we are interested in estimating the mean μ_i of small area *i* from a population composed of *m* small areas. We suppose that the sampling design is non informative, which means that the model for the population values holds true for the sample values. A wide range of models have been proposed and used in the literature for estimating μ_i. Stukel D.M. and Rao J.N.K. (1999) proposed a two-fold nested error regression model with constant variances, useful in situations when the individuals are grouped in primary units and each small area is composed of several primary units. Cleary their model for the population and the sample has the form:

$$y_{ijk} = \mathbf{x}_{ijk}^t \boldsymbol{\beta} + v_i + u_{ij} + e_{ijk}$$
$$k=1,\ldots,N_{ij}\ (n_{ij}),\ j=1,\ldots,M_i^{'}\ (m_i^{'}),\ i=1,\ldots,m \tag{1}$$

where: *m* is the number of small areas in the population; $M_i^{'}$ and $m_i^{'}$ are the population and the sample number of primary units in small area *i,* respectively; N_{ij} and n_{ij} are the population number of individuals and the sample number of individuals in primary unit *j* from small area *i*; $\mathbf{x}_{ijk}^t$ is the vector of auxiliary variable for individual *k*

in primary unit *j* from small area *i*; $\boldsymbol{\beta}$ is the vector of fixed effects, v_i is the random small area effect following $v_i \sim N(0,\sigma_v^2)$; u_{ij} is the random primary unit effect following $u_{ij} \sim N(0,\sigma_u^2)$; e_{ijk} is the model error following $e_{ijk} \sim N(0,\sigma_e^2)$; the effects v_i, u_{ij} and e_{ijk} are independent.

The variance components $\boldsymbol{\sigma}^2 = (\sigma_v^2, \sigma_u^2, \sigma_e^2)^t$ are fixed and unknown. The sample is selected as follows: from the small area *i* a sample of m_i' primary units is selected and from each selected primary units a sample of n_{ij} individuals is selected.

Model (1) supposes that the variances σ_u^2 and σ_e^2 are the same regardless of the small area or the primary unit. In practice, this hypothesis may be restrictive, especially if we deal with non homogenous small areas. For this reason, we propose a less restrictive model, with σ_u^2 and σ_e^2 depending on the small area. Clearly, our model is given by:

$$y_{ijk} = \mu + v_i + u_{ij} + e_{ijk} \qquad (2)$$
$$k=1,\ldots,N\ (n),\ j=1,\ldots,M'\ (m'),\ i=1,\ldots,m$$

where: $v_i \sim N(0,\sigma_v^2)$, $u_{ij} \sim N(0,\sigma_u^2)$, $e_{ijk} \sim N(0,\sigma_e^2)$, $\sigma_i^2 \sim law(\beta_1,\alpha_1)$ and $\tau_i^2 \sim law(\beta_2,\alpha_2)$ ($law(\beta,\alpha)$ designates an arbitrary distribution of mean β and variance α).

The effects v_i, u_{ij} and e_{ijk} are conditionally independent. The other notations in (2) have the same meaning as in (1).

Remark 1: In (2) we do not have auxiliary variables and the model supposes that we have both a balanced population and sample: *N* and *n* are the same regardless of *i* and *j*; M' and m' are the same regardless of *i*; we make these restrictive hypotheses for theoretical reasons.

Remark 2: (2) supposes that τ_i^2 depends only on *i* and not on *j*; a more general model should incorporate τ_{ij}^2 instead of τ_i^2, but the theoretical results would be more difficult to obtain under this less restrictive model.

The idea of passing from a constant to a random vector of variance components is not new. It can be found in Kleffe, J. and Rao, J.N.K. (1992), who did it for a one-fold nested error regression model, which is a model appropriate for a population without primary units. Initially, a one-fold nested error regression model with constant variances was proposed by Battese, G.E., Harter, R.M. and Fuller, W.A. (1988), but Kleffe, J. and Rao, J.N.K. (1992) transformed their model into a model with random variances.

From (2), it can be seen that the mean of the small area *i* is given by:

$$\mu_i = \frac{1}{M'N}\sum_{j=1}^{M'}\sum_{k=1}^{N} y_{ijk} = \mu + v_i + \frac{1}{M'}\sum_{j=1}^{M'} u_{ij} + \frac{1}{M'N}\sum_{j=1}^{M'}\sum_{k=1}^{N} e_{ijk} \tag{3}$$

For an infinite population M' is large enough and from (3) it can be seen that μ_i is approximately equal to a linear combination of the model effects:

$$\mu_i \approx \mu + v_i$$

As a consequence, predicting μ_i is equivalent to predicting $\mu + v_i$. We shall use the theory of the best linear unbiased predictor, denoted by $\tilde{\mu}_i$. It is linear, because it is of the form $\tilde{\mu}_i = \mathbf{l}^t\mathbf{y}$, where **y** is the vector of all the sample observations. It is unbiased, because $E(\tilde{\mu}_i) = E(\mu_i)$ and it is best in the sense that from all the linear and unbiased predictors $\tilde{\mu}_i$ has the least mean squared error. It can be verified (see Ştefan, M. (2005), chapter 4 for technical details) that under model (2) $\tilde{\mu}_i$ is given by:

$$\tilde{\mu}_i = \bar{y}_{i\cdot\cdot} - \frac{\beta}{m'\delta}(\bar{y}_{i\cdot\cdot} - \bar{y}) \tag{4}$$

where: $\beta = \beta_1 + \frac{\beta_2}{n}$, $\delta = \sigma_v^2 + \frac{\beta_1}{m'} + \frac{\beta_2}{m'n}$, $\bar{y}_{i\cdot\cdot} = \frac{1}{m'n}\sum_{j=1}^{m'}\sum_{k=1}^{n} y_{ijk}$ and $\bar{y} = \frac{1}{mm'n}\sum_{i=1}^{m}\sum_{j=1}^{m'}\sum_{k=1}^{n} y_{ijk}$.

We can also compute exactly the mean squared error of (4) (see Ştefan, M. (2005), chapter 4 for details), which will be given by:

$$MSE(\tilde{\mu}_i) = \frac{\beta\sigma_v^2}{m'\delta} + \frac{\beta^2}{mm'^2\delta} \tag{5}$$

The best linear unbiased predictor $\tilde{\mu}_i$ cannot be used to predict μ_i, because, as one may see from (4), it depends on β and δ which are unknown. In practice, we look for (if possible) unbiased estimators of β and δ, plug them into (4) and get what is called the empirical best linear unbiased predictor of μ_i, which we will denote by $\hat{\mu}_i$. Clearly, it can be proved (see Ştefan, M. (2005) chapter 4 for details) that the estimators $\hat{\beta}$ and $\hat{\delta}$ given below are unbiased:

$$\hat{\beta}=\frac{1}{m(m'-1)}\sum_{i=1}^{m}\sum_{j=1}^{m'}(\overline{y}_{ij\cdot}-\overline{y}_{i\cdot})^2\ ,\ \hat{\delta}=\frac{1}{m-1}\sum_{i=1}^{m}(\overline{y}_{i\cdot\cdot}-\overline{y})^2 \quad (6)$$

where: $\overline{y}_{ij\cdot}=\frac{1}{n}\sum_{k=1}^{n}y_{ijk}$.

As a consequence, $\hat{\sigma}_v^2=\hat{\delta}-\frac{\hat{\beta}}{m'}$ will be unbiased for σ_v^2. Then, $\hat{\mu}_i$ will be given by:

$$\hat{\mu}_i=\overline{y}_{i\cdot\cdot}-\frac{\hat{\beta}}{m'\hat{\delta}}(\overline{y}_{i\cdot\cdot}-\overline{y}) \quad (7)$$

(7) is the predictor that can be used in practice to predict μ_i.

3. Approximation and estimation of $MSE(\hat{\mu}_i)$

We now have to measure the precision of $\hat{\mu}_i$, that is to compute its mean squared error $MSE(\hat{\mu}_i)$. Contrary to $MSE(\tilde{\mu}_i)$, $MSE(\hat{\mu}_i)$ cannot be computed exactly and it would be naïve to consider that they are equal. In fact, we shall consider $MSE(\tilde{\mu}_i)$ the naïve mean squared error of $\hat{\mu}_i$, that is $MSE_N(\hat{\mu}_i)=MSE(\tilde{\mu}_i)$. $\hat{\mu}_i$ contains in its formula the estimators $\hat{\beta}$ and $\hat{\delta}$ compared to $\tilde{\mu}_i$, which contains the fixed unknown constants β and δ instead. For this reason, we expect $\hat{\mu}_i$ to have a larger variability than $\tilde{\mu}_i$, that is $MSE(\hat{\mu}_i)>MSE(\tilde{\mu}_i)$. In section 4, the numerical results will show that $MSE_N(\hat{\mu}_i)$ can represent a serious underestimation of $MSE(\hat{\mu}_i)$.

The following theorem gives an approximation to terms of order $o(1/m)$ for $MSE(\hat{\mu}_i)$ ($O(1)$ represents a quantity that remains bounded when $m\to\infty$ and $o(1/m)$ represents a quantity that, multiplied by *m,* tends to zero when $m\to\infty$):

Theorem 1. Let the model (2) and the following regularity conditions be:

a) $m'=O(1)$ and $n=O(1)$; b) σ_i^2 and τ_i^2 have finite twelfth order moments.

Then:

$$MSE(\hat{\mu}_i)=\frac{3m'-1}{mm'^2(m'-1)}\frac{\beta^2}{\delta}+\frac{2}{mm'(m'-1)}\frac{\alpha}{\delta}+\frac{1}{m'}\frac{\sigma_v^2\beta}{\delta}-\frac{3}{mm'^2}\frac{\sigma_v^4\alpha}{\delta^3}+o(1/m) \quad (8)$$

where: $\alpha = \alpha_1 + \frac{\alpha_2}{n^2}$.

Proof: See Ştefan, M. (2005), chapter 4, pp.100-156 for technical details.

(8) shows that the approximation correct to terms of order $o(1/m)$ will be given by:

$$MSE_A(\hat{\mu}_i) = \frac{3m'-1}{mm'^2(m'-1)}\frac{\beta^2}{\delta} + \frac{2}{mm'(m'-1)}\frac{\alpha}{\delta} + \frac{1}{m'}\frac{\sigma_v^2\beta}{\delta} - \frac{3}{mm'^2}\frac{\sigma_v^4\alpha}{\delta^3} \quad (9)$$

If the number *m* of small areas in the population is large enough (generally $m \geq 30$ is sufficient), then $MSE_A(\hat{\mu}_i)$ is a good approximation of $MSE(\hat{\mu}_i)$, the relative error being negligible.

$MSE_A(\hat{\mu}_i)$ is unknown because it depends on the unknown constants $\sigma_v^2, \beta, \delta$ and α. As a consequence, it cannot be used for estimating $MSE(\hat{\mu}_i)$. The naïve approach would be to replace in the formula of $MSE_N(\hat{\mu}_i)$ the unknown quantities by their estimators, resulting in the naïve estimator $mse_N(\hat{\mu}_i)$. By simulation (see section 4), we check that $mse_N(\hat{\mu}_i)$ can have large negative bias. The following theorem gives an estimator for $MSE(\hat{\mu}_i)$ of bias of order $o(1/m)$:

Theorem 2. Let the model (2) and the same regularity conditions be as above. Let:

$$\hat{\gamma}_i^2 = \frac{1}{m'-1}\sum_{j=1}^{m'}(\overline{y}_{ij\cdot} - \overline{y}_{i\cdot\cdot})^2,\ \hat{\alpha} = \frac{m'-1}{m(m'+1)}\sum_{i=1}^{m}(\hat{\gamma}_i^2)^2 - \hat{\beta}^2 \text{ and}$$
$$mse(\hat{\mu}_i) = \frac{1}{m'}(\hat{\beta} - \frac{(m-1)}{mm'}\frac{\hat{\beta}^2}{\hat{\delta}}) + \frac{4}{mm'}(\frac{1}{m'-1}\frac{\hat{\beta}^2}{\hat{\delta}} + \frac{1}{m'^2}\frac{\hat{\beta}\hat{\alpha}}{\hat{\delta}^2} + \frac{1}{m'(m'-1)}\frac{\hat{\alpha}}{\hat{\delta}}) \quad (10)$$

Then, the bias of $mse(\hat{\mu}_i)$ as an estimator of $MSE(\hat{\mu}_i)$ is of order $o(1/m)$.

Proof: See Ştefan M. (2005), chapter 4, pp.100-156 for technical details.

As above, when *m* is large enough ($m \geq 30$), $mse(\hat{\mu}_i)$ can be used as an estimator of negligible bias.

4. Monte Carlo study

In this section, we shall present the results of Monte Carlo simulations showing how $MSE_A(\hat{\mu}_i)$ and $mse(\hat{\mu}_i)$ perform as approximation and estimation of $MSE(\hat{\mu}_i)$, respectively. The results are shown for the first small area of the population. The computer program written in S-Plus is presented in the Appendix.

Without loss of generality, we fixed $\mu = 0$, $m = 30$, $m' = 2$, $n = 2$ and $\beta_2 = 300$. Then, we took several values for σ_v^2 and β_1 equal to 15, 30, 60, 150, 300 and 600, so that the ratios β_1 / β_2 and σ_v^2 / β_2 take the values 0.05, 0.1, 0.2, 0.5, 1 and 2. We then computed for the first small area of the population (*i*=1) the values of $MSE_N(\hat{\mu}_i)$ and $MSE_A(\hat{\mu}_i)$ given by (5) and (9). The Monte Carlo simulated parameters are computed from *G*=10000 samples y_{ijk}, *i*=1,…,*m*; *j*=1,…, m' and *k*=1,…,*n*. Each sample y_{ijk} was generated as follows:

- we generated σ_1^2, σ_2^2,…, σ_{30}^2 from $\chi(\beta_1)$ ($\Rightarrow \alpha_1 = 2\beta_1$) and τ_1^2, τ_2^2,…, τ_{30}^2 from $\chi(\beta_2)$ ($\Rightarrow \alpha_2 = 2\beta_2$)
- we generated e_{ijk} from $N(0, \tau_i^2)$, u_{ij} from $N(0, \sigma_i^2)$ and v_i from $N(0, \sigma_v^2)$ (which gave $\mu_{1g} = v_{1g}$)
- for *g*=1,…,*G* we computed the sample observations $y_{ijk} = v_i + u_{ij} + e_{ijk}$ and using these values we obtained: $\hat{\mu}_{1g}$, $mse(\hat{\mu}_1)_g$ and $mse_N(\hat{\mu}_1)_g$ according to their respective formulas.

Then we computed the Monte Carlo mean squared error of $\hat{\mu}_1$ given by:

$$MSE_{MC}(\hat{\mu}_1) = \frac{1}{10000}\sum_{g=1}^{10000}(\hat{\mu}_{1g} - \mu_{1g})^2 \tag{11}$$

Using (11) we computed the relative errors of $MSE_A(\hat{\mu}_1)$ and $MSE_N(\hat{\mu}_1)$ by:

$$RE = 100\frac{MSE_A(\hat{\mu}_1) - MSE_{MC}(\hat{\mu}_1)}{MSE_{MC}(\hat{\mu}_1)} \text{ and}$$
$$RE_N = 100\frac{MSE_N(\hat{\mu}_1) - MSE_{MC}(\hat{\mu}_1)}{MSE_{MC}(\hat{\mu}_1)} \tag{12}$$

The Monte Carlo values of $E(mse(\hat{\mu}_1))$ and $E(mse_N(\hat{\mu}_1))$ were simulated by:

$$E_{MC}(mse(\hat{\mu}_1)) = \frac{1}{10000}\sum_{g=1}^{10000} mse(\hat{\mu}_1)_g \text{ and}$$
$$E_{MC}(mse_N(\hat{\mu}_1)) = \frac{1}{10000}\sum_{g=1}^{10000} mse_N(\hat{\mu}_1)_g \tag{13}$$

Using (13) the relative bias of $mse(\hat{\mu}_1)$ and $mse_N(\hat{\mu}_1)$ will be given by:

$$RB = 100\frac{E_{MC}(mse(\hat{\mu}_1)) - MSE_{MC}(\hat{\mu}_1)}{MSE_{MC}(\hat{\mu}_1)},$$
$$RB_N = 100\frac{E_{MC}(mse_N(\hat{\mu}_1)) - MSE_{MC}(\hat{\mu}_1)}{MSE_{MC}(\hat{\mu}_1)} \quad (14)$$

The two tables below present the numerical results of our simulations.

Table 1

Relative errors (%) of $MSE_N(\hat{\mu}_1)/MSE_A(\hat{\mu}_1)$

(*m*=30, $m'=2$, *n*=2)

σ_v^2/β_2 \ β_1/β_2	**0.05**	**0.1**	**0.2**	**0.5**	**1**	**2**
0.05	-45.73/-6.82	-39.71/0.43	-46.22/-2.91	-51.89/-1.73	-60.30/-2.97	-68.74/-4.00
0.1	-25.24/0.01	-27.30/-0.51	-30.47/-0.87	-39.91/-3.32	-44.36/0.16	-51.52/2.65
0.2	-13.60/1.39	-14.74/1.34	-18.37/-0.22	-22.01/1.62	-32.19/-1.32	-41.21/-0.34
0.5	-8.00/-1.24	-6.58/0.72	-8.55/-0.15	-9.70/1.79	-16.80/-0.75	-25.04/-1.45
1	-2.81/0.71	-4.69/-0.86	-3.61/0.81	-5.63/0.54	-10.24/-1.31	-16.46/-2.58
2	-1.37/0.42	-2.28/-0.32	-2.33/-0.05	-3.36/-0.15	-2.78/1.96	-6.06/1.51

Table 2

Relative bias (%) of $mse_N(\hat{\mu}_1)/mse(\hat{\mu}_1)$

(*m*=30, $m'=2$, *n*=2)

σ_v^2/β_2 \ β_1/β_2	**0.05**	**0.1**	**0.2**	**0.5**	**1**	**2**
0.05	-80.50/-3.42	-84.10/2.01	-89.38/1.05	-107.02/-0.04	-117.19/1.86	-129.07/2.23
0.1	-52.35/1.85	-55.98/1.48	-60.88/1.92	-77.37/-1.16	-88.72/5.86	-112.96/6.08
0.2	-29.63/3.31	-32.05/3.39	-37.68/1.65	-47.08/4.23	-64.35/0.91	-83.28/3.77
0.5	-14.69/-0.06	-14.21/1.83	-17.31/0.94	-22.76/2.86	-33.81/0.57	-49.03/0.70
1	-6.38/1.45	-8.39/-0.11	-8.60/1.16	-12.41/1.24	-19.70/-0.45	-30.47/-1.49
2	-3.07/0.92	-4.62/-0.34	-5.09/-0.07	-6.64/0.37	-7.62/2.93	-14.39/2.30

From Tables 1 and 2 one may see that the underestimations of $MSE_N(\hat{\mu}_1)$ and $mse_N(\hat{\mu}_1)$ can be important. RE_N and RB_N decrease in absolute value when β_1/β_2 is fixed and σ_v^2/β_2 increases. On the other hand, they increase when σ_v^2/β_2 is fixed and β_1/β_2 increases. As far as RE and RB are concerned, they

are small for all the values of β_1 / β_2 and σ_v^2 / β_2 considered in our simulations, which confirms the accuracy of theoretical results in Theorems 1 and 2.

5. Conclusion

In this paper, we propose a two-fold nested error regression model with random variances. Under this model and in the infinite population case we derive the empirical best linear unbiased predictor, an approximation and an estimator for its mean squared error.

Presently, we study the finite population case. In this case, the approximation $\mu_i \approx \mu + v_i$ is no longer valid. For predicting $\mu_i = (\sum y_{ijk}) / N_i$, we split the small area population into two: the observed and the unobserved units, and the unobserved units will have to be predicted by the best (in the sense of the mean squared error) linear unbiased predictor. Then, an approximation and an estimator of bias of order $o(1/m)$ will have to be obtained. The results will be the subject of another paper.

Often in practice, statisticians deal with time series data. For instance, we have data that could be modelled using a one-fold nested error regression model, but if, moreover, the observations are done in time, then a supplementary index *t* for time is needed, and the one-fold model will transform into a two-fold model. Thus, the theory in this paper can be used to handle such data. Of course, the independence hypotheses on the effects v_i, u_{ij} and e_{ijk} will have to be dropped, because in time the observations on the same units are correlated.

In dealing with such a model, one can use a result presented in Fuller, W.A. and Battese, G.A. (1973). Clearly, they show that given the model $\mathbf{z} = \mathbf{W}\boldsymbol{\beta} + \boldsymbol{\psi}$, where $\boldsymbol{\psi}$ is a vector of mean **0** and of variance-covariance matrix **V**, it is possible to find a transformation matrix **T** so that the errors $\boldsymbol{\psi}^* = \mathbf{T}\boldsymbol{\psi}$ of the transformed model $\mathbf{Tz} = \mathbf{TW}\boldsymbol{\beta} + \mathbf{T}\boldsymbol{\psi}$ be non-correlated and with constant variances. We can use the results presented in this paper under the model $\mathbf{Tz} = \mathbf{TW}\boldsymbol{\beta} + \mathbf{T}\boldsymbol{\psi}$ and then return to the initial model via the transformation **T**.

References

Battese, G.E., Harter, R.M., Fuller, W.A. (1988), "An error components model for prediction of county crop area using survey and satellite data", *Journal of the American Statistical Association*, 83: 28-36.

Fuller, W.A., Battese, G.A., (1973), "Transformations for estimation of linear models with nested-error structure", *Journal of the American Statistical Association*, 85: 626-632.

Kleffe, J., Rao, J.N.K. (1992), "Estimation of mean squared error of empirical best linear unbiased predictors under a random error variance linear model", *Journal of Multivariate Analysis*, 43: 1-15

Rao, J.N.K. (2003), *Small area estimation*, 313 pp., John Wiley&Sons, Hoboken-New Jersey.

Ştefan, M. (2005), *Contributions à l'estimation pour petits domaines*, Ph.D. Thesis, 303 pp., Université Libre de Bruxelles.

Stukel, D.M. and Rao, J.N.K. (1999), "Small area estimation under two-fold nested error regression models", *Journal of Statistical Planning and Inference*, 78: 131-147

Appendix

The function genereazabazadate computes the relative errors and the relative bias in the case of an infinite population:

```
genereazabazadate<-function(miu,sigmav,beta1,beta2,m,mprim,n,G)
{
alpha1<-2*beta1;alpha2<-2*beta2;alpha<-alpha1+alpha2/n^2
beta<-beta1+beta2/n;delta<-sigmav+beta/mprim
EQMmiuchap1<-(3*mprim-1)*beta^2/(m*mprim^2*(mprim-1)*delta)
+2*alpha/(m*mprim*(mprim-1)*delta)+sigmav*beta/(mprim*delta)-
3*sigmav^2*alpha/(m*mprim^2*delta^3)
EQMmiuchap1N<-beta/mprim-(m-1)*beta^2/(m*mprim^2*delta)
sigmai<-rep(0,m);
listay<-array(,dim=c(n,mprim,m))
listae<-array(,dim=c(n,mprim,m))
u<-matrix(,m,mprim);v<-rep(0,m);ybari<-rep(0,m);ybarij<-matrix(,m,mprim)
EQMMmiuchap1<-0;EMeqmmiuchap1<-0;EMeqmmiuchap1N<-0
        for(g in 1:G){
        sigma<-rchisq(m,beta1)
        tau<-rchisq(m,beta2)
        v<-rnorm(m,mean=0,sd=sqrt(sigmav))
                for(i in 1:m){
                u[i,]<-rnorm(mprim,mean=0,sd=sqrt(sigma[i]))
                        for(j in 1:mprim){
                        listae[,j,i]<-rnorm(n,mean=0,sd=sqrt(tau[i]))
                        listay[,j,i]<-miu+v[i]+u[i,j]+listae[,j,i]
                        }
                ybarij[i,]<-colMeans(listay[,,i]);ybari[i]<-mean(listay[,,i])
                }
        sigmai<-rowVars(ybarij,SumSquares=TRUE)/(mprim-1)
        betachap<-sum(rowVars(ybarij,SumSquares=TRUE))/(m*(mprim-1))
        deltachap<-sommecarres(ybari)/(m-1)
        alphachap<-(mprim-1)*mean(sigmai^2)/(mprim+1)-betachap^2
        miuchap1<-ybari[1]-(ybari[1]-mean(ybari))*betachap/(mprim*deltachap)
        miu1<-miu+v[1]
        EQMMmiuchap1<-EQMMmiuchap1+(miuchap1-miu1)^2
        eqmmiuchap1<-betachap/mprim-(m-1)*betachap^2/(m*mprim^2*deltachap)+
        4*betachap^2/(m*mprim*(mprim-1)*deltachap)+
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```
        4*betachap*alphachap/(m*mprim^3*deltachap^2)+
        4*alphachap/(m*deltachap*mprim^2*(mprim-1))
        EMeqmmiuchap1<-EMeqmmiuchap1+eqmmiuchap1
        eqmmiuchap1N<-betachap/mprim-(m-1)*betachap^2/(m*mprim^2*deltachap)
        EMeqmmiuchap1N<-EMeqmmiuchap1N+eqmmiuchap1N
        }
EQMMmiuchap1<-EQMMmiuchap1/G
EMeqmmiuchap1<-EMeqmmiuchap1/G;EMeqmmiuchap1N<-EMeqmmiuchap1N/G
RB<-100*(EMeqmmiuchap1-EQMMmiuchap1)/EQMMmiuchap1
RE<-100*(EQMmiuchap1-EQMMmiuchap1)/EQMMmiuchap1
REN<-100*(EQMmiuchap1N-EQMMmiuchap1)/EQMMmiuchap1
RBN<-100*(EMeqmmiuchap1N-EQMMmiuchap1)/EQMMmiuchap1
listarezultate<list(relerror=RE,relbias=RB,EQM=EQMmiuchap1,EQMMC=EQMMmiuc
hap1,
Eeqm=EMeqmmiuchap1,naifrelerror=REN,naifrelbias=RBN,EQMN=EQMmiuchap1N,
EeqmN=EMeqmmiuchap1N)
return(listarezultate)
```