

# 4. DIFFERENTIAL ELASTICITY OF SUBSTITUTION IN THE INDIAN INDUSTRIES

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## Abstract

*The paper specifically focuses on the impact of reforms to see whether there has been any shift in the differential elasticity of substitution between labour and capital in Indian industry. The main conclusion of the paper is that there are differential elasticities of substitution between labour and capital, both pre and post economic reform period in India, hence substitution possibilities are relatively skewed in favour of labour during post economic reform in the Indian industry.*

**Keywords:** India, CES Production Function, Differential Elasticity, Economic Reforms

**JEL Clasificación:** E23, J3, J24, C32, Q48

## 1. Introduction

The empirical value of the estimate of differential elasticity of substitution between the capital and labour in the Indian industry is very imperative to comprehend the extent of substitution possibilities between labour and capital during post economic reform period. There are many empirical studies on the elasticity of substitution between labour and capital (Asif Banarji [1975], Goldar [186], Gujarati [1966], Isher Judge Ahulwalia [1981], Sinha and Sawhney [1970], Upender [1996], Diwan and Gujarati [1968], Mehta [1980], Swamy [1984], Shankar [1970], Umar Kazi [1980], Vijay Basin and Vijay Seth [1977], Sanjib Pohit, Rajesh Chadha, Bina and Sangeeta [1996], and Inderpal Kaur [1997]) in India. The empirical information on differential elasticity of substitution between labour and capital in the Indian Industry during the post economic reform period is required to understand the extent of substitution

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possibilities during the post economic reform period. Keeping this in view, an attempt has been made to provide empirical content on the estimate of differential elasticity of substitution between labour and capital in the present paper by using the time series observations for the period from 1980-81 to 2004-05. The results of the present paper will be an addition to the existing stock of empirical literature on the elasticity of substitution in the Indian industry, as the differential elasticity of substitution during the post economic reform period has been examined having ensured that the constant elasticity of substitution production function has a structural shift by chow break point test.

## **2. Empirical Methodology**

### **Data Source**

The required annual data on production, employment and wages for the period from 1980-81 to 2004-05 has been collected from various issues of Annual Survey of Industries (Factory Sector). The period after 1991 is considered as post economic reform period to examine the prognostications of economic reforms, initiated by the central government in India, on elasticity of substitution between labour and capital in the Indian Industry.

### **Variables Used**

[i] Gross Value Added [GVA] : It comprises total ex-factory value of products and by-products manufactured, as well as other receipts such as receipts from non-industrial services rendered to others, work done for others on material supplied by them, value of electricity produced and sold, sale value of goods sold in the same condition as purchased, addition in stock of semi-finished goods and own construction] and Net Value Added [NVA] : It is arrived by deducting total input and depreciation from total output] are considered as surrogate for Production [P]

[ii] Wages to workers [W]: Wages and Salaries are defined to include all remuneration in monetary terms and also payable more or less regularly in each pay period to workers as compensation for work done during the accounting year. It includes (a) direct wages and salary (i.e., basic wages/salaries, payment of overtime, dearness, compensatory allowance, house rent and other allowances), (b) remuneration for the period not worked (i.e., basic wages, salaries and allowances payable for leave period, paid holiday, lay-off payments and compensation for unemployment, if not paid from sources other than employers), (c) bonuses and ex-gratia payment paid both at regular and less frequent intervals (i.e., incentive bonuses, good attendance bonuses, productive bonuses, profit sharing bonuses, festival or year-end bonuses, etc.). It excludes lay off payments which are made from trust or other special funds set up exclusively for this purpose i.e., payments not made by the employer. It also excludes imputed value of benefits in kind, employer's contribution to old age benefits and other social security charges, direct expenditure on maternity benefits and crèches and other group benefits. Traveling and other expenditure incurred for business purposes and reimbursed by the employer are excluded. The wages are expressed in terms of gross value.

[iii] Total number of workers [L]: Workers are defined to include all persons employed directly or through any agency whether for wages or not and engaged in any manufacturing process or in cleaning any part of the machinery or premises used for manufacturing process or in any other kind of work incidental to or connected with the manufacturing process or the subject of the manufacturing process. Labour engaged in the repair & maintenance, or production of fixed assets for factory's own use, or employed for generating electricity, or producing coal, gas etc. are included.] are considered as labour in the present study.

[iv] Labour Productivity [P/L] is measured in terms of gross value added per unit of labour employed and net value added per unit of labour employed.

[v] Wage Rate [W/L] is wage per unit of labour.

### **Empirical Model for Differential Elasticity of Substitution**

The following constant elasticity of substitution production function has been considered in the present paper to know the extent of substitution possibilities between labour and capital in the Indian Industry. The specification of the constant elasticity of substitution production function is

$$P = A [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-1/\rho} \quad [1]$$

where: P = Production [GVA or NVA]

K and L = Capital and Labour inputs respectively.

A = Efficiency parameter

$\rho$  = Extent of substitution between labour and capital related to  $\sigma = 1/(1+\rho)$

$\delta$  = Distribution parameter

The above equation has been estimated empirically under the marginal productivity conditions. [Marginal productivity of labour = wage rate]. The marginal productivity of labour is obtained from the above function [equation -1] as follows:

$$\partial P / \partial L = A [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-1/\rho} [\delta K^{-\rho} (1-\delta) L^{-\rho}]^{-1} (1-\delta) L^{-(1+\rho)} = [P \times P^\rho] / A^\rho [1-\delta] (1/L^{1+\rho}) = [(1-\delta) / A^\rho] \times P^{1+\rho} \times (1/L^{1+\rho})$$

Equilibrium condition between  $\partial P / \partial L$  (Marginal Product of labour [MPL] and W / L [Wage Rate]) is

$$\partial P / \partial L = W / L$$

$$[(1-\delta) / A^\rho] [P / L]^{1+\rho} = W / L$$

Solving for [P / L]

$$[P / L]^{1+\rho} = [A^\rho / (1-\delta)] \times W / L$$

$$[P / L] = [A^\rho / (1-\delta)]^{1/(1+\rho)} \times W / L$$

Taking logarithms both sides

$$\log [P / L] = [1 / (1+\rho)] \log [A^\rho / (1-\delta)] + [1 / (1+\rho)] \log [W / L]$$

$$[1 / (1+\rho)] \log [A^\rho / (1-\delta)] = \text{Constant}$$

$1 / (1+\rho) = \sigma$  = Elasticity of Substitution or Elasticity of labour productivity with respect to money wage rate.

$$\log [P / L] = \text{Constant} + \sigma \log [W / L] \quad [2]$$

The coefficient on  $\log W/L$  in the above regression of  $\log P/L$  on  $\log W/L$ ,  $\sigma$ , is the estimate of constant elasticity of substitution between labour and capital.

With a view to estimate the magnitude of differential elasticity of substitution between labour and capital during the post economic reform period the model [2] has been extended by taking dummy [D] and interaction variables [ $D \times \log (W / L)_t$ ] as shown below

$$\log [P/L]_t = \log \beta_0 + \beta_1 \log [W / L]_t + \beta_2 D + \beta_3 (D \times \log [W / L]_t) + \text{error} \quad [3]$$

Where  $(P / L)_t$  = Labour productivity

$(W / L)_t$  = Money Wage rate

D = Dummy variable that takes value 0 for the years 1980-81 to 1990-91 [Pre economic reform period] and 1 for the years 1991-92 to 2004-05 [Post economic reform period]

$\beta_0$  = Intercept during the pre economic reform period (1980-81 to 1990-91), D = 0

$\beta_2$  = Differential intercept during the post economic reform period (1991-92 to 2004-05). D=1

$\beta_1$  = Magnitude of elasticity of substitution during the pre economic reform period (D = 0)

$\beta_3$  = Magnitude of differential elasticity of substitution during the post economic reform period (D = 1)

As the interaction variable [ $D \times \log W/L$ ]<sub>t</sub> enters the equation in dichotomous form [i.e. = 0 in pre economic reform period and D = 1 in post economic reform period] the derivative of  $\log [P/L]_t$  with respect to [ $D \times \log W/L$ ]<sub>t</sub> does not exist. Instead, the coefficient of [ $D \times \log W/L$ ]<sub>t</sub> subject to statistical significance, measures the discontinuous effect of the changes in wage rate and economic reforms [policy decisions] [D = 1] represented by the interaction variable on the labour productivity. The variable [ $D \times \log W / L$ ]<sub>t</sub> has been introduced in model to capture the interaction effect of economic reforms and changes in wage rates [cost of labour] on labour productivity. The interaction variable takes a value equal to  $\log [W / L]_t$  during post tax reform period and 0 during pre tax reform period.

The above equation [3] has been estimated by the Ordinary Least Squares method under the equilibrium [marginal product of labour = wage rate] conditions with one way causation between labour productivity and money wage rate.

### **Structural Shift - Chow test**

The magnitude of differential elasticity of substitution between labour and capital during the post economic reform period has been examined having ensured that the constant elasticity of substitution production function has a structural shift by chow break point test (Gregory C Chow [1960]). The differential coefficient of elasticity of substitution refers to the analysis of, how much the magnitude of elasticity of substitution during the post economic reform period differs from that of pre economic reform period. The estimates of F ratio [chow test] are reported in table -1. The results of chow test [F - Statistic] furnished in Table – 1 show that there is evidence of structural shift in the constant elasticity of substitution production function

Table 1

**Search for Structural Shift in CES Production Function**

Constant Elasticity of Substitution Production Function $\log [P / L ] = \text{Constant} + \sigma \log [W/L] + \text{error}$	Chow [ F ] test Statistic	No shift in CES Production function during post reform period
Dependent Variable: $\log (\text{Gross Value Added} / \text{Labour})$	12.19	Rejected
Dependent Variable: $\log (\text{Net Value Added/Labour})$	7.52	Rejected

**3. Analysis of the Empirical Results**

**Degree of variability in labour productivity and wage rates**

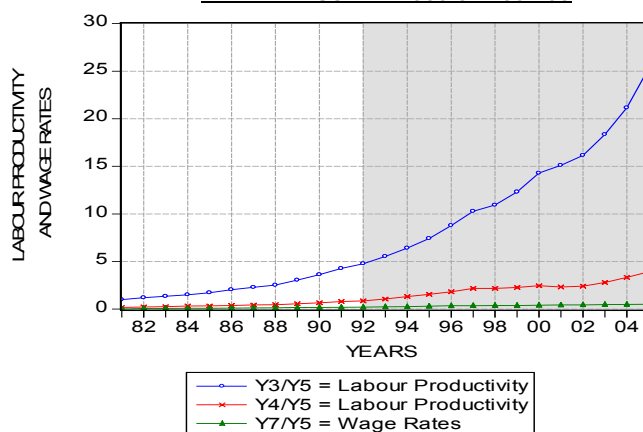
The coefficient of variation has been estimated to see the degree of variability in labour productivity and wage rates during pre and post economic reform periods. The summary statistics together with the coefficient of variation are presented in Table 2. The degree of variability in labour productivity in case of gross value added is somewhat high as compared to the net value added and wage rates during post tax reform period. The variability is relatively high in labour productivity during post economic reform period as compared to pre economic reform period. The variability in wage rate is very high during pre economic reform period as compared to post economic reform period showing the presence of consistency in wage rate during post economic reform period. The movements in labour productivity and wage rates during pre and post economic reform periods during pre and post tax reform [shaded area] periods are shown in the graph below.

Table 2

**Degree of variability in labour productivity and wage rates in the Indian industries**

Statistic	Pre economic reform period [1951-1991]			Post economic reform period [1992-2005]		
	Gross Value Added/ Labour	Net Value Added/ Labour	Wage Rate	Gross Value Added/ Labour	Net Value Added/ Labour	Wage Rate
Mean	2.245727	0.430005	0.122459	12.63503	2.186182	0.384841
Median	2.052820	0.387824	0.121875	11.62204	2.234295	0.389634
Maximum	4.289727	0.816753	0.209157	25.34454	3.938402	0.509676
Minimum	1.010222	0.197281	0.015624	4.772594	0.874520	0.216662
Standard Deviation.	1.052203	0.192449	0.055850	6.133743	0.838768	0.094056
Coefficient of Variation (%)	46.85	44.76	45.61	48.55	38.40	24.74

MOVEMENTS IN LABOUR PRODUCTIVITY AND WAGE RATES IN THE INDIAN INDUSTRY -1980-81-2004-05



**Growth rates of the labour productivity and wage rates**

The growth rates of labour productivity and wage rates during the pre economic reform period and differential growth rates of labour productivity and wage rates during the post economic reform period have been worked out having ensured that there is a structural shift in the growth rates by Chow break point test statistics (F - Statistic in case of [1] GVA/L= 12.19 : [2] NVA/L =11.29 and [3] W/L=7.52). The growth rates are set out in Table 3.

Table 3

**Differential growth rate of labour productivity [GVA/L and NVA/L] and Wage Rate**

Growth rate of the Variable	Constant $\beta_0$	Coefficient Time $\beta_1$	Coefficient Dummy $\beta_2$	Coefficient of Time x Dummy $\beta_3$	R <sup>2</sup>	Adj. R <sup>2</sup>
log(GVA/L)	0.012483	0.139737*	0.288113*	-0.018553*	0.9982	0.997973
t-values	0.496282	32.86782	4.904322	-3.582896		
log (NVA/L)	-1.594905*	0.132465*	0.629736*	-0.036928*	0.9892	0.987726
t-values	-27.42520	13.47563	4.636238	-3.084328		
log(W / L)	-3.172221*	0.182262*	1.106818*	-0.120597*	0.9060	0.892625
t-values	-20.99711	7.137187	3.136641	-3.877224		

Notes:

Estimates are based on the following equations:

$$\log [GVA / L] = \beta_0 + \beta_1 \text{ time} + \beta_2 D + \beta_3 ( D \times \text{time} ) + \text{error}$$

$$\log [NVA / L] = \beta_0 + \beta_1 \text{ time} + \beta_2 D + \beta_3 ( D \times \text{time} ) + \text{error}$$

$$\log [W / L] = \beta_0 + \beta_1 \text{ time} + \beta_2 D + \beta_3 ( D \times \text{time} ) + \text{error}$$

Annual average growth rate during the pre economic reform period =  $\beta_1$ .

## Differential Elasticity of Substitution in the Indian Industries

Annual average growth rate during the post economic reform period =  $(\beta_1 \pm \beta_3)$ .

\* Significant at one percent level.

The growth rate of the labour productivity [ratio of gross value added to labour employed] is 13.97% per annum during pre economic reform period. The differential growth rate is significantly negative, showing a downward shift in the growth rate during the post economic reform period. The growth rate of the labour productivity [ratio of net value added to labour employed] is 13.24% per annum during pre economic reform period. The differential growth rate is significantly negative showing a downward shift in the growth rate of the labour productivity during the post economic reform period. The growth rate of the wage rate is 18.22 % per annum during pre economic reform period. The differential growth rate is significantly negative showing a downward shift in the growth rate of wage rate during the post economic reform period.

### Estimates of Differential Elasticity of Substitution

The regression results of the Constant Elasticity of Substitution Production Function fitted to the time series for the Indian Economy have been set out in Table 4.

Table 4

**CES Production Function: Differential Elasticity of Substitution**

Dependent Variable	Constant	Coefficient of Dummy	Coefficient of $\log(W/L)$	Coefficient of Dummy* $\log(W/L)$	Elasticity of substitution during post economic reform period	R <sup>2</sup>	Adj. R <sup>2</sup>
	$\log \beta_0$	$\beta_2$	$\beta_1$	$\beta_3$	$\beta_1 \pm \beta_3^{**}$		
$\log(GVA/L)$	1.955873*	2.289851*	0.550532*	1.299290*	1.87	0.966781	0.962036
t-values	9.682914	7.990231	6.434936	5.988609			
$\log(NVA/L)$	0.265517	1.939732*	0.529919*	0.989485*	1.52	0.969314	0.964930
t-values	1.455724	7.495740	6.859485	5.050675			

Note:

[1] \* Significant at one percent level.

[2] \*\* = Since  $\beta_1$  and  $\beta_3$  are statistically significant, the sum  $[\beta_1 + (3)]$  or difference  $[(1) - (3)]$  is deemed to be statistically significant as well.

The regression results of the constant elasticity of substitution production function with the ratio of gross value added to labour [GVA/L] as the dependent variable presented in Table 4 illustrate that the coefficient of money wage rate [labour cost], known as the elasticity of labour productivity with respect to wage rate, is positive but less than unit during pre economic reform period in the Indian economy, illuminating that labour productivity is relatively inelastic to the changes in money wage rates. This shows that a one percent increase in money wage rate leads to increase the labour productivity by 0.55 percent. The differential elasticity of substitution is significantly positive and more than unit more than unity during the post economic reform period. The constant elasticity of substitution during the post

economic reform period is also more than unit, emphasizing the fact that the substitution possibilities are more in favour of labour in the Indian industries [factory sector] during post economic reform period. The estimate of the elasticity of substitution during the post economic reform period implies that a one percent increase in wage rate will lead to increase the labour productivity by 1.8 percent in the Indian industries.

The regression results of the constant elasticity of substitution production function with the ratio of net value added to labour [NVA/L] as the dependent variable presented in Table 3 illustrate that the coefficient of money wage rate, the elasticity of labour productivity with respect to wage rate, is also significantly positive and less than unit during pre economic reform period in the Indian industry. The differential elasticity of substitution is more or less equal to unit. The sum of elasticity of substitution during the pre economic reform period and differential elasticity of substitution during the post economic reform period is 1.5, showing that a one percent increase in money wage rate leads to increase in the labour productivity more than one percent. This leads to deduce the fact that the substitution possibilities are more in favour of labour in the Indian industries [factory sector].

#### **4. Conclusion**

This paper conducts an empirical test on the differential elasticity of substitution between labour and capital in the Indian industry during the post economic reform period by employing a logarithmic form of constant elasticity of substitution production function with two different dependent variables, namely the ratio of gross value added to labour and net value added to labour, having ensured that constant elasticity of substitution production function has a structural shift by Chow break test. The differential elasticity of substitution is significantly positive and more than unit during the post economic reform period when the ratio of gross value added to labour is considered as dependent variable and close to unit when the ratio of net value added to labour is considered as dependent variable. The sum of the elasticity of substitution during pre economic reform period and differential elasticity of substitution during post economic reform period exceeds unit, evincing the fact that substitution possibilities are relatively more in favour of labour in the Indian industry during the post economic reform period.

#### **References**

- Arrow, HB, Chenery BS, Minhas and RM Solow, (1961), "Capital-Labour Substitution and Economic Efficiency," *The Review of Economics and Statistics*, XLIII(3):225-250.
- Asif Banarji (1975), *Capital Intensity and Productivity in Indian Industry*, MacMillan Co., New Delhi.
- Cobb, CW and Douglas PH, (1928), "Theory of production", *American Economic Review*, XVIII:139-165.



- Diwan R.K. and D.N. Gujarati (1968), "Employment and Productivity in Indian Industries – Some Questions of the Theory and Policy", *Artha Vijnana*, 10(1): 29-67.
- Goldar, B.N, (1986), *Productivity Growth in Indian Industry*, Allied, New Delhi.
- Gregory C. Chow, (1960), "Tests of Equality between sets of Coefficients in two linear Regressions", *Econometrica*, 28(3): 501-605.
- Gujarati, D.N, (1966), "A Test of ACMS Production Function: Indian Industries" 1958, *Indian Journal of Industrial Relations*, 2(1): 95-97.
- Inderpal Kaur, (1997) , *Impact of Liberalisation on Indian Industry*, Deep Publications, New Delhi.
- Isher Judge Ahulwalia, (1981), *Productivity Growth in Indian Manufacturing*, Oxford University Press.
- Krishna KL (Ed), (1997), *Econometric Applications in India*, Oxford University Press.
- Mehta, S.S, (1980), *Productivity, Production Function and Technical Change – A Survey of Some Indian Industries*, Concept Publishing Co., New Delhi.
- Panchamukhi, V. R, (1978) , "A Survey of Econometric Studies in the fields of Production, International Trade and Money and Banking" in *A Survey of Research in Economics*, Vol.VII, Econometrics (ICSSR) Allied.
- Sanjib Pohit, Rajesh Chadha, PL Bina and N Sangeeta, (1996), *Estimation of Substitution Parameter in Indian Industries*, National Council of Applied Economic Research, New Delhi.
- Shankar, U. (1970) , "Elasticities of Substitution and Returns to Scale in Indian Manufacturing Industries", *International Economic Review*, Vol. II, No. 3, October, pp. 399-411.
- Sinha J.N. and P.K. Sawhney, (1970), *Wages and Productivity in Selected Indian Industries*, Vikas Publications.
- Swamy, T.L.N. (1984), "Factor Substitution in Indian Industries: Some Cross-Section Evidence", *Margin*, January, pp. 31-38.
- Umar, A. Kazi, (1980), "The VES Production Function: A Case Study for Indian Manufacturing Industries", *Oxford Economic Papers*, 32(1): 163-175.
- Upender, M, (1996), "Elasticity of Labour Productivity in Indian Manufacturing Sector", *Economic and Political Weekly*, XXXI(21): M7-M10.
- Vijay, K. Basin and Vijay K. Seth, (1977), "Estimation of Production Functions for Indian Manufacturing Sector", *Artha Vignana*, 19(2): 113-128.