5. COMPUTATION OF OPERATIONAL RISK FOR FINANCIAL INSTITUTIONS

Ming-Tao CHUNG[1]
Ming-Hua HSIEH[2]
Yan-Ping CHI[3]

Abstract

Quantification of operational risk has led to significant concern regarding regulation in the financial industry. Basel Accord II and III for banks and Solvency II for insurers require insurance companies and banks to allocate capital for operation risk. Because the risk measure used for Basel regulatory capital purposes reflects a confidence level of 99.9% during one year and the loss distribution of operational risk has high skewness and kurtosis, it is almost infeasible to get an accurate estimate of such a risk measure if a crude Monte Carlo approach is used. Therefore, we develop a novel importance sampling method for estimating such a risk measure. Numerical results demonstrate that the proposed method is very efficient and robust. The main contribution of this method is to provide a feasible and flexible numerical approach that delivers highly accurate estimates of operational risk with a high confidence level and meets the high international regulatory standard for quantification of operational risk.

Keywords: operational risk; advanced measurement approaches; loss distribution approach; Monte Carlo simulation; variance reduction

JEL Classification: G32, G28, C63

1. Introduction

In 2004, operational risk was injected into the Basel Accord by the Basel Committee on Banking Supervision. BCBS (2004) defined Operational risk as “the risks of losses resulting from inadequate or failed internal processes, people and systems or from external events.” Furthermore, BCBS (2004) outlined eight business lines and seven event types as exposure to operational risk. Given the issues involved and the calculations required, operational risk is more complicated than credit risk and market risk. Hence, the quantification of operational risk is a continually important issue for regulations in the financial industry.

[1] *Department of Management Information Systems, National Chengchi University, Taiwan (R.O.C.), Email: 102356509@nccu.edu.tw.*

[2] *Department of Risk Management and Insurance, National Chengchi University, Taiwan (R.O.C), Email: mhsieh@nccu.edu.tw. Corresponding author.*

[3] *Department of Management Information Systems, National Chengchi University, Taiwan (R.O.C), Email: ypchi@mis.nccu.edu.tw.*

Basically, there are three methods for measurement of the capital charges for operational risk - Basic Indicator Approach (BIA), Standardized Approach (SA), and Advanced Measurement Approach (AMA). Levels of model sophistication and risk sensitivity are increased correspondingly. Banks using the BIA must hold capital for operational risk equal to the average over the previous three years of a fixed percentage (denoted alpha) of positive annual gross income. If banks choose SA, the capital charge for each business line is calculated by multiplying gross income by a factor (denoted beta) assigned to that business line. Beta serves as a proxy for the industry-wide relationship between the operational risk loss experience for a given business line and the aggregate level of gross income for that business line. It should be noted that in the SA, gross income is measured for each business line, not the whole institution. Finally, supervisors expect that AMA banking groups will continue efforts to develop increasingly risk-sensitive operational risk allocation techniques, notwithstanding initial approval of techniques based on gross income or other proxies for operational risk. Although the AMA allows banks to develop their own models for assessing their operational risk exposures, it is required to cover their yearly operational risk exposure with a confidence level of 99.9% (BCBS 2006).

Although the international regulatory standard demands a high confidence level when calculating operational risk, there is no obvious numerical approach that can achieve this goal. Crude Monte Carlo can achieve this goal in some sense because of its flexibility and easy implementation. However, crude Monte Carlo can only provide a rough estimate with low accuracy even using a lot of computing power because the operational risk loss distribution has high skewness and kurtosis; see Jorion (2007). Therefore, our goal is to provide a feasible numerical approach that can deliver highly accurate estimates of operational risk with a high confidence level. This contribution is important for financial institutions that have adopted the international regulatory standard.

In the wake of the financial crisis, BCBS (2011) proposes supervisory guidelines associated with the development and operation of key internal governance, data and modelling frameworks underlying an AMA. Furthermore, BCBS (2014, 2016) aims to propose a simpler approach called the Standardized Measurement Approach (SMA) to replace all the currently available options for computing regulatory capital (BIA, SA, AMA). But Mignola, Ugoccioni, and Cope (2016) argues that SMA does not respond appropriately to changes in the risk profile of a bank, and results generally appear to be more variable across banks than the previous AMA option of fitting the loss data.

Based on the above, AMA is still the most risk sensitive approach for calculating the Operational Capital-at-Risk (OpCaR). Therefore, we focus on the Loss Distribution Approach (LDA), which is the main method of AMA. LDA is a statistical method that has been used widely in actuarial science for computing aggregate loss distributions, so it is also known as the Actuarial Model. LDA concerns the measurement of risk for random losses generated from a matrix whose element corresponds to a combination of business line and event type in a one-year horizon. In practice, it is hard to collect the sufficiency data of all business lines; hence, the financial institutions usually separately model the number of loss events in a given year and the loss amount of a single loss event by the frequency distributions and severity distributions. Furthermore, the frequency and severity distributions are usually assumed to be independent or modeled through copulas models (Chavez-Demoulin, Embrechts, and Nešlehová, 2006). Then, the convolution of these two distributions gives rise to the loss distribution in a given year.

There are many studies that use LDA to access operational risk: Frachot, Georges, and Roncalli (2001) use the severity distribution, which follows lognormal, the frequency distribution, which follows Poisson, and the Gaussian copula, which describes the correlated aggregate loss distributions. Chapelle *et al.* (2008) chooses Pareto, Weibull lognormal distribution to model severity, negative binomial (NB) distribution to model frequency, and linear Spearman copula to model the dependence of aggregate losses. Temnov and Warnung (2008) assumes the loss severity and the loss frequency are independent, and then uses the Weibull distribution and generalized Pareto distribution (GPD) to fit loss severity and negative binomial distribution and the Poisson distribution to fit loss frequency. Fantazzini, Dalla Valle, and Giudici (2008) uses gamma, exponential, and Pareto distributions to model loss severity; negative binomial and Poisson distributions to model loss frequency; and Gaussian copula and *t* copulas to describe the dependence structure among the losses.

Traditional studies explore the estimation of the combination of different severity distributions, frequency distributions and dependence structures. They almost adopt the naïve Monte Carlo simulation method because the building blocks of the operational risk model are very diverse and complex. But the risk measure used for regulatory capital purposes reflects a holding period of one-year and a confidence level of 99.9% (BCBS, 2006); it is almost infeasible to get an accurate estimate of such risk measures if a naïve Monte Carlo approach is used (Asmussen and Glynn, 2007). Therefore, the main objective of this paper is to propose an efficient Monte Carlo simulation (a novel variance reduction) algorithm for computing such a risk measure. Except for the assumption that a common factor driving operational risk events exists, we do not impose any additional restrictions. The method can therefore be applied to a wide range of operational risk models. The rest of this paper is organized as follows. Section 2 defines the problem to be solved. Section 3 elaborates on the simulation algorithm developed in this study. Section 4 presents numerical results by a real case. Section 5 concludes the paper.

2. Problem Formulation

BCBS (2004) defined eight business lines and seven event type exposures of operational risk (Table 1). For each single risk cell (a combination of business line and event type), the total loss L_{ij} follows the standard LDA approach and is the sum of individual losses:

$$L_{ij} = \sum_{k=1}^{N_{ij}} X_{ijk},$$

where: X_{ijk} is individual loss (severities) and N_{ij} is the number of losses (frequency) in cell (*i* , *j*). The aggregate operational loss is then defined by

$$L = \sum_{i,j} L_{ij}.$$

Let the marginal distribution function of X_{ijk}, N_{ij} and L_{ij} be denoted by $S_{ij}(.)$, $C_{ij}(.)$ and $H_{ij}(.)$, respectively. Depending on the fitting of actual loss data, there are different distributions that can be used. Table 2 surveys the distribution to model loss severity and loss frequency from the past research.

According to the past research about operational risk, we can sum up the distributions that are suitable to loss severity or loss frequency. The distributions are used to model loss severity as follows: lognormal, exponential, Weibull, gamma, Pareto, generalized Pareto (GPD). In addition, the distributions are used to model loss severity as follows: Poisson and negative binomial.

Table 1

Business Lines and Event Types Matrix

Event types / Business lines	Internal fraud	External fraud	Employment Practices	Clients, Products, and Business Practice	Damage to physical assets	Business Disruption and Systems Failures	Execution, Delivery, and Process Management
Corporate finance	L_{11}	L_{12}	L_{13}	L_{14}	L_{15}	L_{16}	L_{17}
Trading and sales	L_{21}	L_{22}	L_{23}	L_{24}	L_{25}	L_{26}	L_{27}
Retail banking	L_{31}	L_{32}	L_{33}	L_{34}	L_{35}	L_{36}	L_{37}
Commercial banking	L_{41}	L_{42}	L_{43}	L_{44}	L_{45}	L_{46}	L_{47}
Payment and settlement	L_{51}	L_{52}	L_{53}	L_{54}	L_{55}	L_{56}	L_{57}
Agency services	L_{61}	L_{62}	L_{63}	L_{64}	L_{65}	L_{66}	L_{67}
Asset management	L_{71}	L_{72}	L_{73}	L_{74}	L_{75}	L_{76}	L_{77}
Retail brokerage	L_{81}	L_{82}	L_{83}	L_{84}	L_{85}	L_{86}	L_{87}

Source: BCBS (2004).

Table 2

Marginal Distribution Function Setting

Literature	$F_{ij}(.)$	$G_{ij}(.)$
Böcker and Klüppelberg (2008)	generalized Pareto	Poisson
Embrechts and Puccetti (2008)	Pareto lognormal	Poisson
Fantazzini *et al.* (2008)	gamma exponential Pareto	negative binomial Poisson
Guégan *et al.* (2011)	lognormal generalized Pareto	Poisson
Chapelle *et al.* (2008)	Pareto Weibull lognormal	negative binomial Poisson

After we fit the marginal distribution of severity and frequency, we need to model the dependence. Table 3 shows some copula setting of loss severity and loss frequency from past studies.

There are three approaches to implementing a dependent structure in LDA model (Chernobai, Rachev, and Fabozzi, 2007; Cope and Antonini, 2008):

1. The frequency distribution between cells are dependent;

2. The severities between cells are dependent;
3. The aggregated loss between cells is dependent.

Table 3

Copula Setting of Loss Severity and Loss Frequency

Literature	Copula Model
Böcker and Klüppelberg (2008)	Lévy copula to describe dependence in frequency and severity between different cells
Embrechts and Puccetti (2008)	Gumbel copula and Gaussian copula to describe dependence in severity distribution
Fantazzini *et al.* (2008)	Gaussian copula and *t* copula to describe the dependence structure among the losses

Frachot, Roncalli, and Salomon (2004) argues that the dependence considered by the Basel Committee is most likely to be the aggregate loss dependence since the form of the dependence structure becomes important primarily at the stage when capital charges from different groups are to be aggregated. Hence, we concentrate on the discussion of the aggregate loss dependence in the rest of this paper.

Assume that the joint behaviors of total loss L_{ij} within cell (*i, j*) can be described by Gaussian factor copulas that include Gaussian copulas and *t* copulas (Asmussen and Glynn, 2007). As suggested by Asmussen and Glynn (2007), copulas provide a possible approach for modeling multivariate distributions in which one has a well-defined idea of the marginal distributions but a vague one on the dependence structure.

There are several copulas that have been used in past research, including Gaussian copulas and *t* copulas, Gumbel copula, Lévy copula, to model the dependence structures of L_{ij}. In addition to the above copula settings, we can use factor copula models to fit the dependence. It is clear that H_{ij} depends on C_{ij} and S_{ij}. The function form of C_{ij} and S_{ij} can be estimated from data and are assumed to be given. If the fitted distributions of C_{ij} and S_{ij} are commonly used distributions, then it is likely that the distributions of L_{ij} are also known. If H_{ij} is unknown, we can generate independent empirical distributions of L_{ij} first. When the dependence structure can be described by Gaussian factor copulas, we may express

$$L_{ij} = H_{ij}^{-1}(\Phi(X_{ij}))$$

where: $\Phi(.)$ is the CDF of the standard Gaussian random variable and X_{ij} are the latent variables used to model the joint distributions of L_{ij}. The dependence among X_{ij} is induced through common factors M_b and M_e as follows:

$$X_{ij} = aM_b + bM_e + \sqrt{1 - a^2 - b^2}Z_{ij}$$

where: M_b, M_e, and Z_{ij} are independent standard Gaussian random variables and *a* and *b* denote constant factor loadings. The common factor M_b represents the common factor within a business line, while M_e represents the common factor within an event type. On the other hand, Z_{ij} are specific factors pertaining to each risk cell.

We can also model the dependence structure via t_v factor copulas. In particular,

$$L_{ij} = H_{ij}^{-1}(t_v(X_{ij}))$$

where: $t_v(.)$ is the CDF of the *t* distributed random variable with *v* degrees of freedom. The *t*-copula is more powerful in terms of capturing tail dependence (Klugman *et al.*, 2012). The dependence among X_i is induced through common factors M_b and M_e as follows:

$$X_{ij} = \sqrt{\frac{v}{R}}(aM_b + bM_e + \sqrt{1 - a^2 - b^2}Z_{ij})$$

where: R is an independent Chi-square random variable with v degrees of freedom.

3. The Proposed Algorithm

In this paper, we focus on the inverse function of operational VaR that is the probability of large portfolio losses (denoted by PPL or $P(L > y)$). PPL is easier to check the computation efficiency than the original measure. Without loss of generality, we assume the dependent structure follows the one-factor Gaussian copula model; then, a crude Monte Carlo (CMC) procedure for estimating the PLL can be easily implemented as follows:

Algorithm 1. The CMC Algorithm for PLL

- Draw independent *r.v.* N_{ij} from $C(\cdot)$, where $C(\cdot)$ is a selected frequency distribution.
- Draw independent *r.v.* X_{ijk} from $S(\cdot)$, where $S(\cdot)$ is a selected severity distribution.
- Compound N and X to a mixture distribution $H(\cdot)$, where $H(\cdot)$ is a hybrid distribution of $C(\cdot)$ and $S(\cdot)$.
- Generate independent *r.v.*: $M \sim \Phi(\cdot)$ and $Z_{ij} \sim \Phi(\cdot)$, where $\Phi(\cdot)$ is the CDF of Gaussian distribution.
  $\Rightarrow X_{ij} = \rho M + \sqrt{1-\rho^2}Z_{ij}$, where ρ is given.
  $\Rightarrow L_{ij} = H^{-1}(\Phi(X_{ij}))$.
- Compute $L = \sum_i \sum_j L_{ij}$
- Repeat the above procedure *K* times, and then we can calculate the $P(L > b) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{1}_{\{L^k > b\}}$ where b is given.

Algorithm 1 provides simple point estimates for PLL, denoted by

$$\hat{\alpha}_{\mathrm{PLL}} = \frac{1}{K}\sum_{k=1}^{K} \alpha_{PLL}^{k} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{1}_{\{L^k > b\}}$$

and its associated standard errors is

$$se(\hat{\alpha}_{\mathrm{PLL}}) = \frac{1}{\sqrt{K}}\sqrt{\frac{1}{K-1}\sum_{k=1}^{K} (\alpha_{\mathrm{PLL}}^{k} - \hat{\alpha}_{\mathrm{PLL}})^2},$$

K is the total number of simulation trials.

Although the CMC is suitable for solving complex problems, it is hard to efficiently estimate the rare-event quantities. This is because the CMC gives equal weight to all replications, while the Monte Carlo method hardly samples in the very low probability regions. A refinement of this method, known as importance sampling (IS), involves drawing the points randomly, but drawing more frequently where the integrand is large. However, Asmussen and Glynn (2007) argue that most IS algorithms are inefficient in high-dimensional spaces because the variance of the likelihood ratio easily blows up. Therefore, we scale down the multi-dimensional rare event to a one-dimensional question by utilizing a two-step approach. First, we must establish a method of selecting an appropriate IS distribution from among the set of possible measures to ensure default events of interest on every generated path. Second, we must ensure that the gain in variance reduction of the proposed algorithm always outweighs that of a CMC.

Let $f(\cdot)$ be the density function of L, and suppose that $g(\cdot)$ is also a density function, such that $g(\cdot) > 0$. Let $LR(\cdot) = f(\cdot)/g(\cdot)$, which is the likelihood ratio, and L^k be independent values sampled $g(\cdot)$. Then, it is well known that

$$\hat{\beta}_{\mathrm{PLL}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{1}_{\{L^k > b\}} \cdot LR(L^k) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{1}_{\{M^k < -m^*\}} \cdot LR(L^k)$$

is an alternative estimator for the PLL.

The point estimator $\hat{\beta}_{\mathrm{PLL}}$ is known as an IS estimator (Glynn and Iglehart 1989; Asmussen and Glynn 2007), and its variance depends on the choice of the IS density $g(\cdot)$. To select an appropriate $g(\cdot)$, we derive a simple alternative characterization for the operational risk event $\{L > b\}$.

Proposition 1. The loss event $\{L > b\}$ is equivalent to the event $\{M \leq -m^*\}$, where m^* is the root of the $L - b = 0$; that is, conditional on X_{ij} and Z_{ij}, the event of interest can be determined solely by the common factor M.

Proof.

Let us consider the event of interest $\{L > b\}$. Given X_{ij} and Z_{ij},

$$\mathbf{1}_{\{L>b\}} = 1 \Leftrightarrow \mathbf{1}_{\{\sum_i \sum_j L_{ij} > b\}} = 1$$

$$\Leftrightarrow \mathbf{1}_{\left\{\sum_i \sum_j H^{-1}\left(\Phi(X_{ij})\right) > b\right\}} = 1$$

$$\Leftrightarrow \mathbf{1}_{\{\sum_i \sum_j H^{-1}(\Phi(X_{ij})) > b\}} = 1$$

$$\Leftrightarrow \mathbf{1}_{\left\{\sum_i \sum_j H^{-1}\left(\Phi(\rho M + \sqrt{1-\rho^2} Z_{ij})\right) > b\right\}} = 1$$

$$\Leftrightarrow \mathbf{1}_{\left\{\sum_i \sum_j H^{-1}\left(\Phi\left(\frac{\left(X_{ij} - \sqrt{1-\rho^2} Z_{ij}\right)}{\rho}\right)\right) > b\right\}} = 1$$

$$\Leftrightarrow \mathbf{1}_{\{M \leq -m^*\}} = 1$$

By finding the root of m^* such that $\sum_i \sum_j H^{-1}\left(\Phi\left(\left(X_{ij} - \sqrt{1-\rho^2} Z_{ij}\right)/\rho\right)\right) = b$, we see that $\{L > b\}$ if and only if $\{M \leq -m^*\}$.

Proposition 1 guides us to skillfully choose the probability measure for M. It provides a simple way to ensure that, for every path generated, default events ($L > y$) always happen. With this specific truncated IS density, Algorithm 2 presents the novel IS (NIS) procedure for estimating the PLL.

Algorithm 2. The NIS Algorithm for PLL

- Draw independent *r.v.* N_{ij} from $C(\cdot)$, where $C(\cdot)$ is a selected frequency distribution.
- Draw independent *r.v.* X_{ijk} from $S(\cdot)$, where $S(\cdot)$ is a selected severity distribution.
- Compound N and X to a mixture distribution $H(\cdot)$, where $H(\cdot)$ is a hybrid distribution of $C(\cdot)$ and $S(\cdot)$.
- Generate independent *r.v.*: $X_{ij} \sim \Phi(\cdot)$ and $Z_{ij} \sim \Phi(\cdot)$, where $\Phi(\cdot)$ is the CDF of Gaussian distribution.
  $\Rightarrow X_{ij} = \rho M + \sqrt{1-\rho^2} Z_{ij}$, where ρ is given.
  $\Rightarrow L_{ij} = H^{-1}\big(\Phi(X_{ij})\big)$.
- Compute $L = \sum_i \sum_j L_{ij}$

Repeat the above procedure *K* times, and then we can calculate the $P(L > b) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{1}_{\{L^k > b\}}$ where b is given.

4. Numerical Experiments

We define the performance evaluation criterions of the estimators first. Then, we describe the details of the numerical examples and compare the simulation results for CMC and our method.

4.1. Performance Evaluation Criterions

Variance ratio (V.R.) is a common standard to measure computational efficiency (Asmussen and Glynn 2007). V.R. is defined by the ratio of the variance of the CMC estimator over that of our estimator, *i.e.*

$$\mathrm{V.R.} = \frac{var_{\mathrm{CMC}}}{var_{\mathrm{NIS}}},$$

where: var_{Method} is the variance of a specific method (CMC or NIS). It is clear that if $\mathrm{V.R.}$ is greater (or less) than one, then the computation efficiency of the NIS is higher (or lower).

In addition to using the V.R. measure, we also utilize the concept of bounded relative error (BRE) to test the robustness of our estimator. Let us consider an unbiased estimator $\hat{p}$ of *p* taken from a sample having size *K*. BRE states that the standard error of $\hat{p}$ divided by *p* is bounded as *p* tends to 0 (for a fixed sample size *K*).

Definition 1. Bounded Relative Error (BRE)

Let σ_K^2 denote the variance of $\hat{p}$ for a fixed sample size K. The coefficient of variation (C.V.) is defined by

$$\mathrm{C.V.} = \frac{\sigma_K}{p}.$$

We say that the estimator has the property of bounded relative error if $\mathrm{C.V.}$ remains bounded as $p \to 0$.

4.2. Numerical Setting and Results

We adopted the numerical examples from Fantazzini *et al.* (2008). These examples are used to compare the computation efficency and robustness between the CMC estimator and our estimator. Exponential, Gamma, Pareto, and lognormal distributions are used to fit the loss severity. Poisson, negative binomial distributions are used to fit the loss frequency. We model the dependency structure by one-factor Gaussian copula. We choose two representative values for factor loading (ρ) settings (ρ = 0.5 means the moderately correlated condition; ρ = 0.9 represents the highly-correlated condition). Furthermore, we examine two confidence levels (99% is the typical confidence level for regular statistical tests; 99.9% is the standard set by the international regulation BCBS 2006). The number of replications for CMC and NIS are 1,000,000 and 1,000, respectively. These numbers of replications are sufficient to make these estimators with the desired accuracy. These numerical examples represent a wide range of possible loss distributions of operational risk based on real loss data. The models of loss distributions can fit real loss data of each business line/event type well, and the dependence structure among business line/event type can be very flexible by using various copula models. Therefore, these numerical examples can cover almost all situations facing financial institutions.

Table 4 presents the PLL estimation results. The values of V.R. range from 4.3 to 603.4 under the different marginal distributions of business/event type and different settings of dependence structure. The values of V.R. are the speedup of our method compared to crude Monte Carlo simulation (CMC). That is, the values of V.R. represent the great computational cost saving of financial institutions. To be more precise, the computational cost is only 1/603.4 to 1/4.3 of that of CMC.

These results also show that our method is more efficient than CMC in all scenarios, especially in highly correlated conditions and a high confidence level. In addition, the pattern of V.R. results is irrelevant to the combinations of frequency distribution and severity distribution. That means our method is flexible in arbitrary combination of frequency distribution and severity distribution.

From Table 4, the values of $C.V._{NIS}$ grow slowly when the confidence level goes to 1 (or loss probability goes to zero). Such results indicate the estimator of NIS is robust and possesses the property of bounded relative error, which is the best class of estimators in rare event simulation; see Asmussen and Glynn (2007) for more details. In summary, numerical results show that our method is efficient, flexible, and robust for computing the operational risk, especially in highly correlated conditions and high confidence level.

Table 4

Simulation Results between the CMC and the NIS Method

Frequency Dist.	Severity Dist.	ρ	1-α (%)	b	$\hat{p}$	$\hat{p}_{\mathrm{NIS}}$	se	seIS	V.R.	$\mathrm{C.V.}_{\mathrm{CMC}}$	$\mathrm{C.V.}_{\mathrm{NIS}}$
Poisson	Gamma	0.5	99	5.6E+06	1.000%	0.993%	0.00010	0.00104	9.1	9.9	3.3
Poisson	Gamma	0.5	99.9	9.6E+06	0.100%	0.099%	0.00003	0.00017	35.4	31.6	5.4
Poisson	Gamma	0.9	99	7.7E+06	1.000%	0.992%	0.00010	0.00026	150.8	9.9	0.8
Poisson	Gamma	0.9	99.9	1.4E+07	0.100%	0.102%	0.00003	0.00004	603.4	31.6	1.3
Poisson	Pareto	0.5	99	2.5E+07	1.000%	0.987%	0.00010	0.00111	8.1	9.9	3.5
Poisson	Pareto	0.5	99.9	8.2E+07	0.100%	0.097%	0.00003	0.00009	113.9	31.6	3.1
Poisson	Pareto	0.9	99	3.3E+07	1.000%	1.008%	0.00010	0.00030	113.8	9.9	0.9
Poisson	Pareto	0.9	99.9	1.1E+08	0.100%	0.100%	0.00003	0.00004	599.7	31.6	1.3
NB	Gamma	0.5	99	5.6E+06	1.000%	0.998%	0.00010	0.00113	7.7	9.9	3.6
NB	Gamma	0.5	99.9	9.7E+06	0.100%	0.101%	0.00003	0.00011	80.4	31.6	3.5
NB	Gamma	0.9	99	7.9E+06	1.000%	0.997%	0.00010	0.00026	144.7	9.9	0.8
NB	Gamma	0.9	99.9	1.4E+07	0.100%	0.103%	0.00003	0.00004	562.3	31.6	1.3

Frequency Dist.	Severity Dist.	ρ	1-α (%)	b	$\hat{p}$	$\hat{p}_{\mathrm{NIS}}$	se	seIS	V.R.	$\mathrm{C.V}_{\cdot\mathrm{CMC}}$	$\mathrm{C.V}_{\cdot\mathrm{NIS}}$
NB	Pareto	0.5	99	4.7E+07	1.000%	0.986%	0.00010	0.00152	4.3	9.9	4.9
NB	Pareto	0.5	99.9	1.3E+08	0.100%	0.099%	0.00003	0.00016	39.0	31.6	5.1
NB	Pareto	0.9	99	5.3E+07	1.000%	1.003%	0.00010	0.00040	61.4	9.9	1.3
NB	Pareto	0.9	99.9	1.5E+08	0.100%	0.099%	0.00003	0.00006	310.4	31.6	1.8

Note: NB is the negative binomial distribution; ρ is the factor loading of the one-factor Gaussian copula model; b is the threshold losses; $(1-\alpha)\%$ is the confidence level; S.E. is the standard error of the estimator; C.V. is the coefficient of variation; V.R. is the variance ratio.

5. Conclusions

The empirical distribution of the aggregate operational loss *L* can be estimated by repeated sampling from the stochastic model through a crude Monte Carlo procedure. However, the converge rate of the Monte Carlo method is $n^{(-1/2)}$, which is slow if each replication is expensive to generate. Therefore, the technique of variance reduction can be used to accelerate the Monte Carlo method. Variance reduction typically involves a fair amount of both theoretical study of the problem in question and additional programming effort (Asmussen and Glynn 2007).

Since the Basel accord requires that the financial institutions quantify their operational risk measures with a high confidence level, the estimation problem becomes a rare event simulation problem and makes the computation much harder. To overcome the computational difficulties and complexity, this study develops a novel importance sampling (NIS) method to estimate the operational risk exposure. The NIS has a conceptually convincing mechanism feature for implementation. In addition, NIS is feasible for any factor copula model that is constructed by arbitrary numbers of mutually independent factors. The columns of V.R. of Table 4 show that the NIS method can estimate operational risk fast with desired accuracy. Furthermore, the columns of C.V. reveal that the NIS estimator has bounded relative errors. In summary, various numerical results show that our NIS is an efficient, flexible, and robust approach for estimating operational risk, especially in highly correlated conditions and a high confidence level. More importantly, our method can serve as a useful tool of financial institutions to meet the international regulatory standard of quantifying operational risk.

References

Asmussen, S. and Glynn P., 2007. *Stochastic Simulation: Algorithms and Analysis.* NY: Springer-Verlag.

Basel Committee on Banking Supervision, 2004. *International convergence of capital measurement and capital standard*. Available at: <http://www.bis.org/publ/bcbs107.pdf> [Accessed on June 2004].

Basel Committee on Banking Supervision, 2006. *International convergence of capital measurement and capital standard*: *A revised framework - Comprehensive version.* Available at: <http://www.bis.org/publ/bcbs128.pdf > [Accessed on June 2006].

Basel Committee on Banking Supervision, 2011. *Operational Risk – Supervisory Guidelines for the Advanced Measurement Approaches.* Available at: <http://www.bis.org/publ/bcbs196.pdf > [Accessed on May 2017].

Basel Committee on Banking Supervision, 2014. *Operational risk –Revisions to the simpler approaches.* Available at: <http://www.bis.org/publ/bcbs291.pdf> [Accessed on May 2017].

Basel Committee on Banking Supervision, 2016. *Standardised Measurement Approach for operational risk.* Available at: <http://www.bis.org/bcbs/publ/d355.pdf> [Accessed on May 2017].

Böcker, K.and Klüppelberg, C., 2008. Modeling and measuring multivariate operational risk with Lévy copulas. *Journal of Operational Risk*, 3(2), pp.3-27.

Chapelle, A., Crama, Y., Hübner, G., and Peters, J.P., 2008. Practical methods for measuring and managing operational risk in the financial sector: A clinical study. *Journal of Banking & Finance*, 32(6), pp.1049-1061.

Chavez-Demoulin, V., Embrechts, P., and Nešlehová, J., 2006. Quantitative models for operational risk: extremes, dependence and aggregation. *Journal of Banking & Finance*, 30(10), pp.2635-2658.

Chernobai, A.S., Rachev, S.T., and Fabozzi, F.J., 2007. *Operational risk: a Guide to Basel II capital requirements, models, and analysis*. NJ: Wiley.

Cope, E. and Antonini, G., 2008. Observed correlations and dependencies among operational losses in the ORX consortium database. *Journal of Operational Risk*, 3(4), pp.47-74.

Embrechts, P. and Puccetti, G., 2008. Aggregating risk across matrix structured loss data: the case of operational risk. *Journal of Operational Risk*, 3(2), pp.29-44.

Fantazzini, D., Dalla Valle, L., and Giudici, P., 2008. Copulae and operational risks. *International Journal of Risk Assessment and Management*, 9(3), pp.238-257.

Frachot, A., Georges, P. and Roncalli, T., 2001. Loss distribution approach for operational risk. Available at: <http://ssrn.com/abstract=1032523> [Accessed on November 2007].

Frachot, A., Roncalli, T., and Salomon, E., 2004. The correlation problem in operational risk. *OperationalRisk Risk's Newsletter*.

Glynn, P. and Iglehart, D., 1989. Importance sampling for stochastic simulations. *Management Science*, 35(11), pp.1367-1392.

Guégan, D., Hassani, B.K. and Naud, C., 2011. An efficient threshold choice for the computation of operational risk capital. *The Journal of Operational Risk*, 6(4), pp.3.

Jorion, P., 2007. *Value at risk - The New Benchmark for Managing Financial Risk 3rd Edition*. McGraw-Hill, New York.

Klugman, S.A., Panjer, H.H. and Willmot, G.E., 2012. *Loss models: from data to decisions*. NJ: Wiley.

Mignola, G., Ugoccioni, R., and Cope E., 2016. *Comments on the BCBS proposal for a New Standardized Approach for Operational Risk*. Place: Cornell University Library. Available at: <https://arxiv.org/ftp/arxiv/papers/1607/1607.00756.pdf> [Accessed on May 2017]

Temnov, G. and Warnung, R., 2008. A comparison of loss aggregation methods for operational risk. *The Journal of Operational Risk*, 3(1), pp.3-24.