8. OSCILLATORY DYNAMICS OF INDUSTRIAL PRODUCTION

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Abstract

GDP and its components' evolution show an oscillatory behavior. As an alternative approach to classical cyclical behavior producing models the paper analyses this behavior both by Fourier transforms of the data series and by a discussion of the roots configuration of the associated second order differential equations describing the process. Specific cycles that associate with the economic sectors contributing to the generation of GDP are identified. This oscillatory system-based approach is providing a complementary way to describe economic behavior dynamics.

Keywords: nonlinear models, oscillatory behavior, GDP cycles

JEL Classification: C3, C61, C62, D7, D87

Introduction

Oscillatory processes are imbedded in economic systems' behavior due to the intrinsic nature of human activities and natural phenomena with which they interact. Various names are coining this behavior such as cyclical, seasonal, yearly or quarterly periodic, etc. Moreover the scale of time constants of various activities range from seconds, e.g. the ticks of the stock exchange, to tens of years, e.g. T bonds of the US Treasury.

There are various authors that have identified nonlinear behavior and described it with various models that show features such as bifurcation, discontinuity, periodicity, etc. What we intend to present here is a systematic analysis of the oscillatory behavior of the GDP and its components that applies well known mathematical instruments, e.g. Fourier transform, differential equations, flows, etc.

By applying this type of analysis to the real GDP data from a given economy, Romania. we obtain a system of second order differential equations. Each of these equations has its specific solutions. Actually the resulting amplitudes of the Fourier analysis could be considered eigenvalues of these equations and the flow of trajectories may be discussed in terms of convergence/divergence, stability and underlying potential.

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We will do here a basic analysis of this sort for the industrial production.

First, let us review how cyclical behavior came to be acknowledged with a few examples of the observations of important scientists related to such behavior.

We present below, in chronological order, a few relevant quotes related to cyclic behavior:

The general character and agreement in the periodic turn in movements of factors of circulation -- these are the specific problems of business cycle theory which have to be solved within the closed interdependent system....If a business cycle theory which is *system-conforming* cannot be built, then general overproduction will not only drive the economy but also economic *theory* into a crisis., Adolph Lowe, How is Business Cyle Theory Possible at All?, *WWA*, 1926: pp. 175..

Since we claim to have shown in the preceding chapters what determines the volume of employment at any time, it follows, if we are right, that our theory must be capable of explaining the phenomena of the Trade Cycle.

(John Maynard Keynes, *General Theory*, 1936: pp. 313.

Keynesian economics, in spite of all that it has done for our understanding of business fluctuations, has beyond all doubt left at least one major thing quite unexplained; and that thing is nothing less than the business cycle itself....For Keynes did not show us, and did not attempt to show us, save by a few hints, why it is that in the past the level of activity has fluctuated according to so definite a pattern.

(John Hicks, *Contribution to the Theory of the Trade Cycle*, 1950: pp. 1.

Further on, one should mention Samuelson, Minsky and Gandolfo, as well as S. Keen, who contributed substantially to the formulation of models that present oscillatory dynamics and to the discussion of their various basins of behavior. As an example, Keen says;

The economic fixation upon equilibrium appears quaint to these mathematically literate economists, and this alone may significantly undermine the hold which static thinking has on economics

(Steve Keen, *Debunking Economics*, 2007: p309.

We will leave for the references the multiple papers at the basis of our survey of models imbedding cyclical behavior done by Hicks, Dussenbery, Kalecki, Kaldor, Goodwin, etc. which either introduce limitation such as ceilings and floors or consider nonlinear components that allow the occurrence of cycles.

Process behavior considerations

We will go on now to propose a different approach based on the fact that the data series of GDP and its components show oscillations. Our approach is based on the fact that the determination of the solutions of the associated differential equations to the oscillatory behavior mentioned above may be determined from the Fourier transform generated characteristic amplitudes and frequencies of the oscillations and to discuss the behavior of the system in the associated complex space representation of the solutions that indicate the attractors and the evolution of the system trajectories.

This approach allows the description of a dynamic system, for the moment of linear differential equations. which later on we will analyze in terms of nonlinearities and potential complex behavior.

The specific conditions of human economic activity involve a cyclical behavior that is also evident in the evolution of GDPP. The various component sectors that generate value added are each having their own cycles, e.g. agriculture is obviously different from industry or from constructions but, they all combine to generate GDPP.

In Purica, 2010. and Purica and Caraiani, 2009., we show that the shocks in the economy are triggering a typical exponentially amortized oscillatory response, e.g. in industrial activity, which allowed the determination of the specific coefficients of the second order differential equation describing the process.

The basic formulae for the above are given below:

A	ϕ	T_d	1/a	- parameters from fitted data above
0.18	1.52	45.3	130	

ω_d= 0.138701662 T_d = 45.3 months (years d = 3.775.

ϕ = 1.515393705

EXP(-ax.*A*SIN(2*π/T_d*x+ϕ. - characteristic function

where:

ω_n=A*ω_d	undumped natural frequency
ϕ =atan(ω_d/a.	phase
ω_d=2*π/T_d	dumped natural frequency
ζ_i=a/ω_n	dumping ratio

Deduction of the values of interest is given below:

a=ζ_i*ω_n
ω_d=ω_n*SQRT(1-ζ_i^2.

a=ω_d(ζ_i/(1-ξ_i^2..
ζ_i=SQRT(1/(1+(ω_d/a.^2..

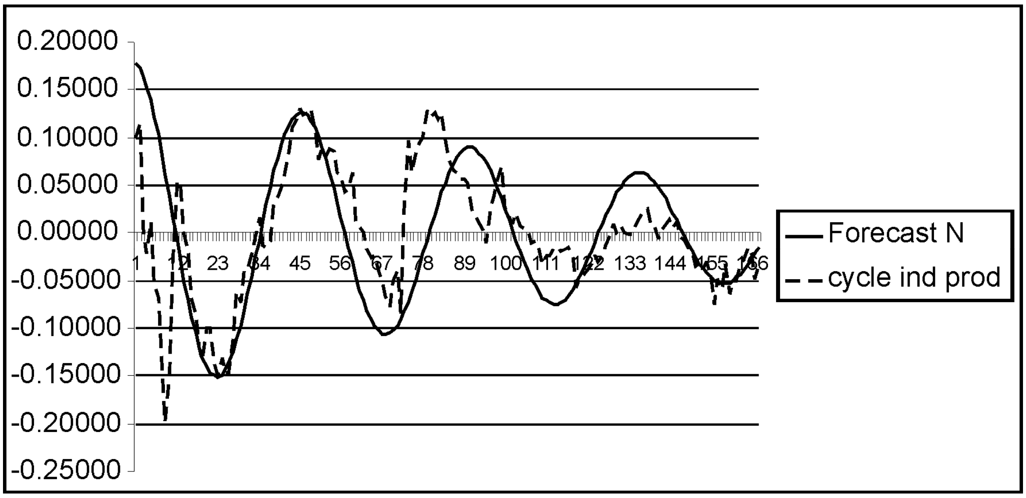
ζ_i= 0.055374283
ω_n=a/ξ_i = 0.138914804 T_n= 45.23049473 months (years n = 3.769207894.

And the differential equation results as:

$$\frac{d^2y}{dt^2} - 0.015\frac{dy}{dt} + 0.019y = 0.019u$$

Figure 1

Industrial production response after the 1990 shock

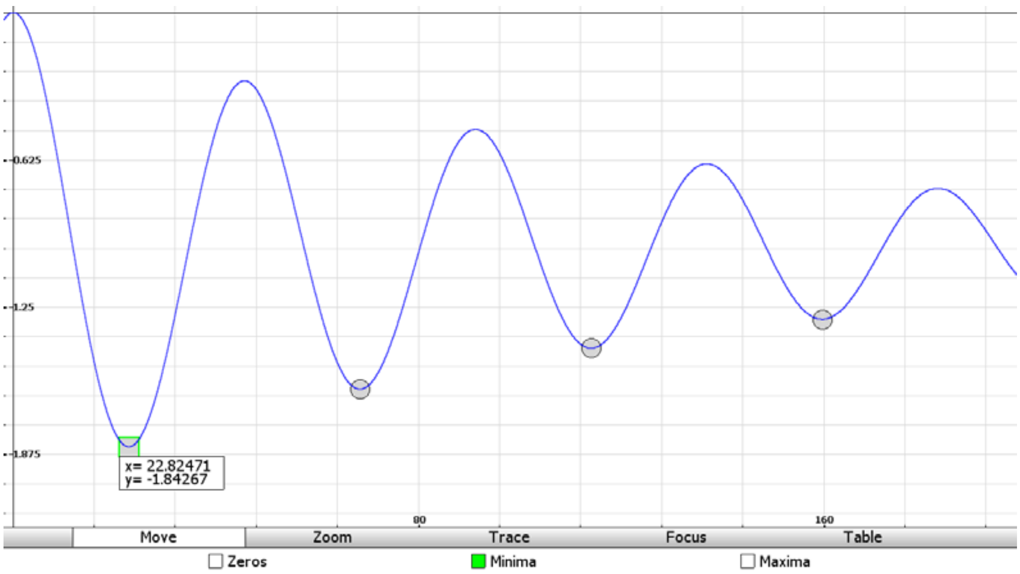


Source: Purica 2010.

The fact that the specific equation for the industrial activity was determined enables us to apply this behavior to a later shock, such as the economic crisis at the end of 2008 in Romania and to assess duration and amplitude of the economic response, industrial activity.. Such response is given in the figure below.

Figure 2

Evolution of industrial production from Jan.2009 - estimated



Source: Author calculations.

One may see that the decreasing trend is reaching a first minimum after 22.8 months meaning almost the end of 2010, after which the industrial production starts growing again. Considering the amplitude and knowing that the decrease in GDP, we assume the same in industrial production. in 2009 was 7.1% and that this represents -1 in normalized value of y above, then the decrease in 2010 will be of aprox. 0.8 x 7.1% = 5.7% related to the value of 2008, i.e. 6.11% if related to the previous year, 2009.. An increase is expected in 2011 and it should be noticed that other oscillations will impact the economy later on. Obviously the values calculated above are indicative and refer to industrial production only. Any decision taken and implemented that changes the basic conditions is liable to change the above trends either upward or downward.

The basic result here is the fact that there is predictive power in this approach and it is worth trying to extend it to other components of the GDPP.

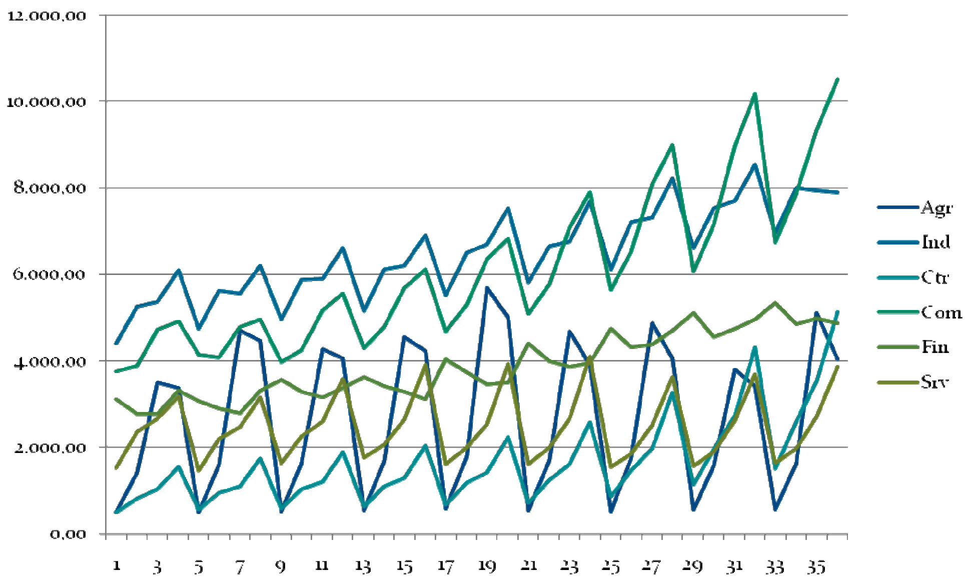
Oscillatory behavior of the GDP and its components

To analyze this we have considered the evolution of GDP on a Quarter disaggregation for 2000-2008. Figure 3 shows this trend for Agr(iculture., Ind(ustry., C(ons.tr(uction., Fin(ance. and Serv(ices..

These components combine into the GDP and the analysis, we are starting with this paper, is due to take into consideration each component and the way they combine into the GDPP. For the moment we remain with the industrial production.

Figure 3

Oscillatory behavior of GDP components



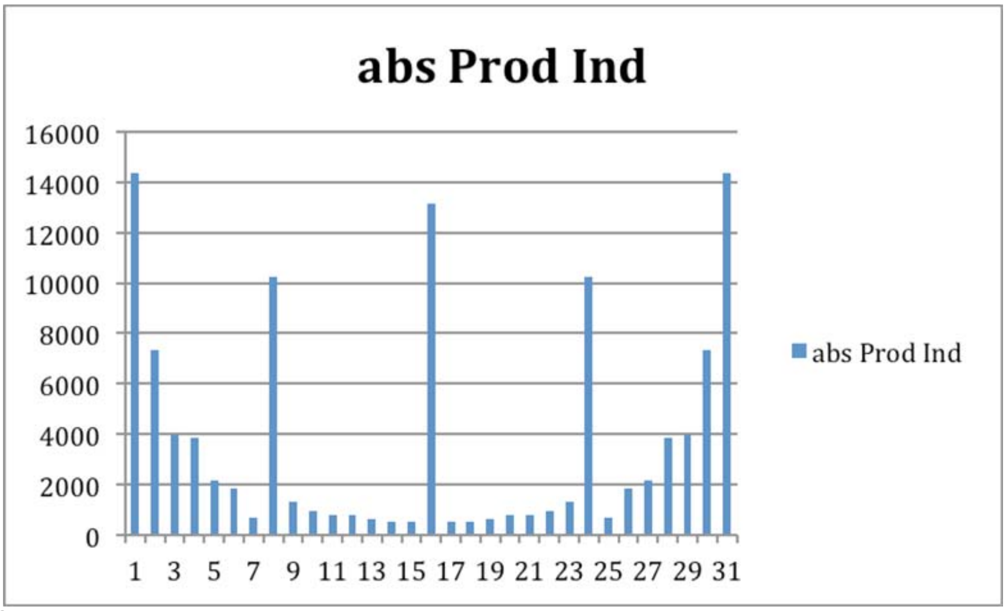
Source: INS, National Institute of Statistic data plotted by the author..

Let us give now a full example of the procedure we use to analyze the cyclic process.

Considering the case of the industrial production we are determining the Fourier transform of the cyclic process. This requires to only consider 32 out of the 36 data points representing the quarterly values of the GDP and its components. Based on these data we compute the Fourier transforms for the resulting 32 frequencies and represent the amplitudes, in absolute value. in a frequency-amplitude space. The result is given in Figure 4.

Figure 4

Industrial Production Frequency – Amplitude representation of the Fourier transform.



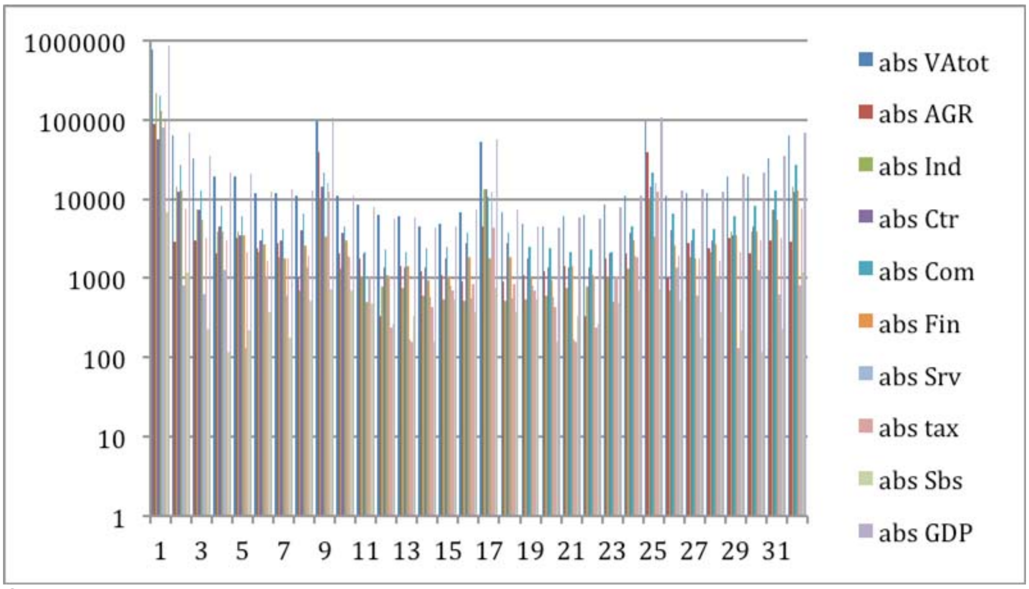
Source: author's calculation.

Repeating the procedure for other GDP components we get their specific representations in the frequency–amplitude space. We have grouped all the spectra into one graph but one may see the variations in the spectra for various GDP component, see Figure 5.

Having done the above we are now in the position to go one step forward and discuss the type of behavior described by the differential equation associated with the components. We consider that the spectra above contain the eigen values of the associated process behavior. Since this paper is opening the way to a comprehensive analysis we will present only the example of the industrial production, the rest of the components will follow in further work.

Figure 5

GDP components' spectra



Source: author's calculation.

Stability and complex space analysis

In order to assess the behavior of the given component we first apply the Bode plot such as to detect the existence of a characteristic frequency resulting from the phase frequency and amplitude frequency plots, see also Schaum, 2003..

We consider the industrial production system described by the differential equation presented above and write it in the time domain, s., then determine the transfer function of the system. This results in:

((-0.019./(s*(s^2+0.015*s+0.019...

The Bode plot for the above, author's calculation using Math Studio. is given below Figure 6.a

One may see that the maximum in amplitude is occurring for a given frequency which is the normal frequency of the industrial production cycles, $0.8632/2/\pi=0.1389=\omega_n$.. While the phase is also having a change at about the same frequency range.

Looking at the complex plane containing the solution represented by the Fourier transform representation, Elmore and Heald, 1985., the roots of the equation, associated to the characteristic, eigen. frequencies, Figure 7., show small values close to zero that we may neglect for now and the remaining ones are one real and two complex conjugates all in the negative real axis. We will call them λ_1, λ_2 and λ_3 with

$$\lambda_3<\mathrm{Re}\ \lambda_{1,2}<0$$

Figure 6.a

Amplitude Frequency plot

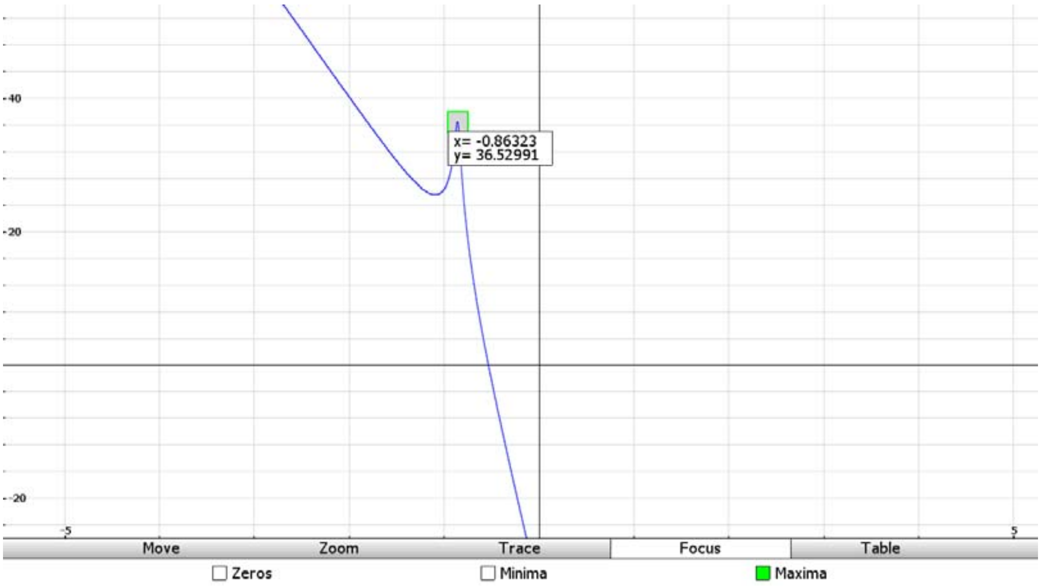
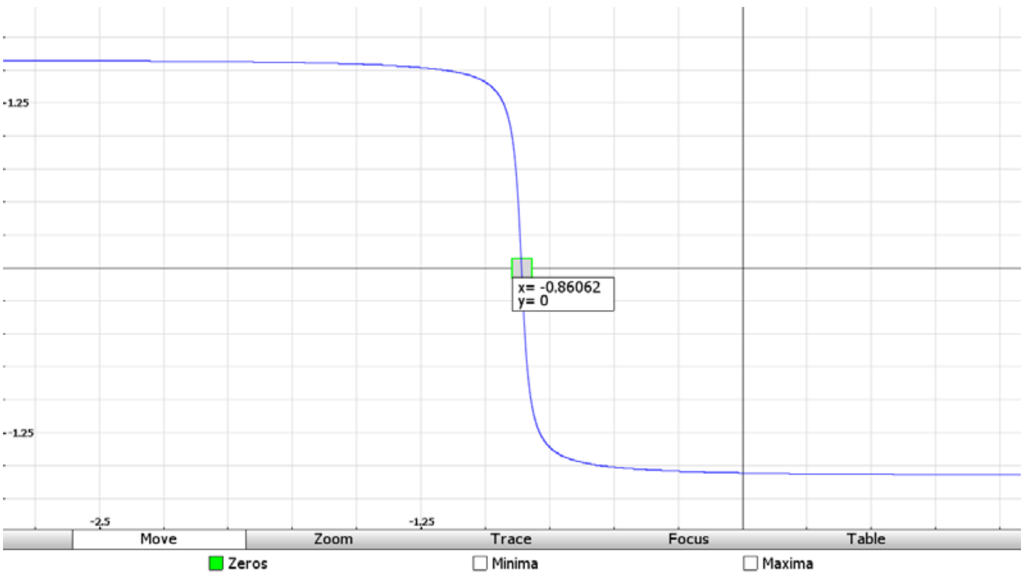


Figure 6.b

Phase Frequency plot



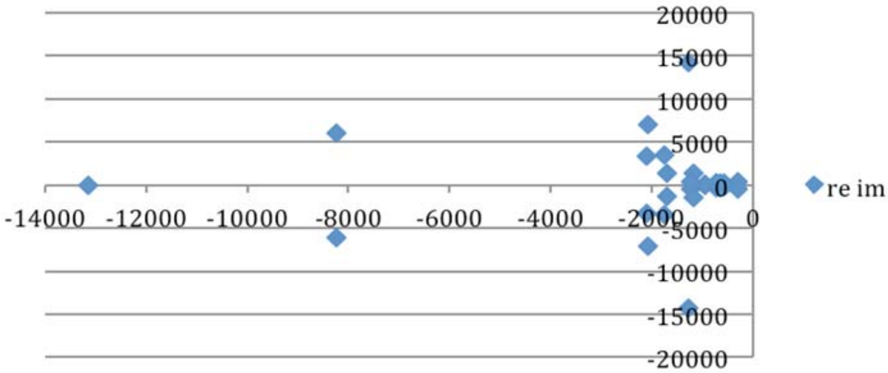
Source: author's calculations.

This configuration is specific to a rotation followed by a slower contraction toward the origin.

Let's analyze now the complex space representation of the roots.

Figure 7.a

Complex plane for the industrial production

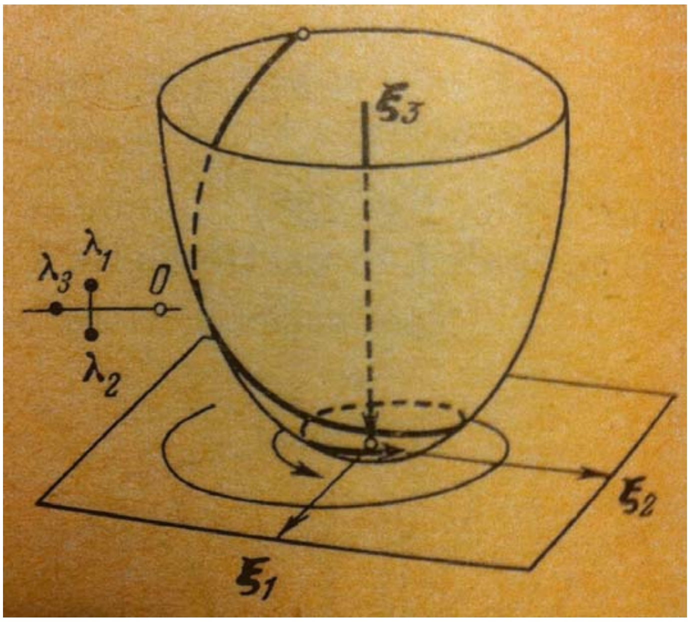


Source: author's calculation

We have chosen in the above the data series of the industrial production to give an example of the way the analysis can be made both in the 'phase space' and in the time domain of the component. The complex plane representation showed that the evolution trajectory is rotating with a slow convergence toward the origin, which suggests a dynamically stable trend, see also Arnold, 1974..

Figure 7.b

Trajectories with oscillatory behavior in the phase space, see also Arnold, 1974



We will not discuss here the other small points close to zero but leave their potential influence for a later paper. We mention, though, that the trajectories associated with the points considered, further away from zero. will decay the first in the transient oscillations leaving the trajectories associated to the closer to zero, negative and positive. points. These ones are the ones that persist on long term and will potentially determine small amplitude potentially nonlinear behavior.

Conclusions

Considering the data series of the industrial production we have made some forecasting of its evolution after the shock at the end of 2008 based on the second order differential equation determined from the previous shock of 1990. The basic result was the upturn of the trend at the end of 2010 leading to an expected increase in 2011. Further on we have extended the analysis based on the data of the GDP and its components on a quarterly basis. We have calculated the Fourier transform and the associated spectra for the industrial production as well as the other GDP components.

Moreover, based on the complex space representation we have used the Bode plot and the roots in the complex space to assess the level of dynamical stability of the industrial production evolution trajectory. The results show a rotational movement with a contraction toward the origin that suggests the existence of a dynamically stable behavior. This enhances the amortized behavior described by the differential equation determined previously.

The paper opens the way toward a thorough analysis of each GDP component and the GDP itself, in the framework of the oscillatory dynamics presented here. The spectra above are definitely suggesting different behaviors and combining them into the GDP may result in oscillatory behavior of this last one that has not been modeled in this way till now. This contributes to increasing the variety of models available to make predictions on economic evolution.

Finally, the time interval till the first up swing in industrial production oscillation, of about two years, considered as an indication of the crisis duration, is confirmed by the evolution of the GDP that had decreased more in 2009 and still decreased less in 2010 following a slight increase in 2011. One should beware of the second decrease period, hoping the growth will overcome it.

We are continuing these ideas with an extension to all GDP components to be presented in further papers.

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