4. MODELING STOCK INDEX RETURNS USING SEMI-PARAMETRIC APPROACH WITH MULTIPLICATIVE ADJUSTMENT

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Abstract

In this paper we utilize a semi-parametric approach with multiplicative adjustment to estimate the distributions for a series of stock index returns including developed and emerging economies.

The semi-parametric approach has potential improvements over both pure parametric and non-parametric estimators. Firstly, in the case where the parametric model is misspecified so that the parametric estimator for the true density is usually inconsistent, the semi-parametric estimator can still be consistent with the true density. Secondly, in comparison with the kernel density estimator, the semi-parametric estimator can result in bias reduction as long as the parametric model can capture some roughness feature of the true density function, whereas the two estimators have the same asymptotic variance.

The simulation results show that the proposed approach has good finite sample performance compared with non-parametric approach. We apply the approach to the empirical data of a series of stock index returns and find support for it in each of the markets under consideration.

Keywords: semi-parametric density estimation, multiplicative adjustment, heavy-tailed returns

JEL Classification: C13, C14, C16, G10

I. Introduction

The change of stock returns is of particular interest to investors, analysts and financial regulators, which can significantly affect the performance of an investment over a long time period and even threaten the stability of the financial system. For example, the global financial crisis began in 2007 when the subprime mortgage crisis originated in the US spread rapidly to most financial markets around the globe and has resulted in global stock markets experiecing substantial fall in asset prices and entered a period

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of high volatility (Chevapatrakul and Tee, 2014). Mishkin (2011) discussed both the crisis and global recession.

A popular assumption usually made is that financial logarithmic returns follow a normal distribution. However, this assumption is not supported by empirical evidence. The existing literatures showed that distribution of stock returns exhibits negative skewess and heavy tails (Fama, 1965; Gray and French, 1990; Bekaert *et al*., 1998). This property is very important in risk management. For example, a good description of the returns series is important for risk measures like VaR and ES (Expected Shortfall).

To explain heavy tail and asymmetry phenomena, many authors used parametric distributions such as skewed-t distribution (Hansen, 1994), skewed generalized t distribution (Theodossiou, 1998), skewed exponential power distribution (Fernandez, *et al*., 1995; Ayebo and Kozubowski, 2004; Komunjer, 2007), generalized hyperbolic distribution (Necula, 2009), two-parameter Weibull distribution (Gebizlioglu *et al*., 2011) and so on. Instead of the parametric models, Butler and Schachter (1998) employed a non-parametric kernel quantile estimators of the pdf of the returns on a portfolio.

The parametric approach is attractive for a number of reasons. First of all, the parameters of a model often have important interpretations to a subject matter specialist. Another attractive aspect of parametric approach is its statistical simplicity, i.e., estimation of the entire function boils down to inferring a few parameter values. The third reason is that it can provide an excellent estimator if the class of parametric functions happens to be correctly chosen. However, it is model dependent and is subject to errors of model misspecification.

The non-parametric approach, has in general a slower rate of convergence, but has attractive flexibility that can be used without the structural assumption that underlying structure is controlled by a finite dimensional parameter. So, the non-parametric estimator has the advantage of being free of distributional assumptions on returns, while being able to capture fat-tail and asymmetry distribution of returns automatically.

In recent years, there have been increasing interests and activities in the general area of semi-parametric approaches. A semi-parametric approach with multiplicative adjustment has been used to improve the density estimation. The approach can be viewed as semi-parametric in such a case that it combines parametric and non-parametric methods. In the proposed approach, a parametric estimator is used as a crude guess of true density function. This initial parametric approximation is adjusted via multiplication by a non-parametric factor. It was shown that the semi-parametric estimator has the very same asymptotic variance as the standard non-parametric method, while there is substantial room for reducing the bias if the chosen parametric initial function belongs to a wide neighborhood around the true density function. Hjort and Glad (1995) proposed a density estimator based on the naive estimator of the non-parametric factor. Hjort and Jones (1996) suggested and investigated two versions of multiplicative density estimator. Naito (2004) proposed a local L_2-fitting criterion with index α. Wang and Lin (2008) showed that the multiplicative adjustment method can be applied to density estimation for time series. Wang (2012) utilized this method to propose an alternative way to implement the historical simulation approach to VaR estimation. Similar ideas has been used to improve the regression estimation

(Glad, 1998, Wang *et al*., 2009), time series conditional variance estimation (Mishra et al., 2010) and quantile regression (Ghouch and Genton, 2009).

In this paper we utilize the above semi-parametric approach to estimate the distributions for a series of stock index returns including developed and emerging economies. We presents a Monte Carlo simulation for different distribution shapes and the results show that the proposed approach has good finite sample performance compared with non-parametric approach. We then apply the approach to the empirical data of a series of stock index returns and find support for it in each of the markets under consideration.

The paper is organized as follows. In the next section we introduce the semi-parametric approach with multiplicative adjustment and present its advantages over parametric and non-parametric ones. Section III presents a Monte Carlo simulation to examine the finite sample performance of the proposed approach. In section IV we present the empirical results for a series of stock index returns and perform goodness-of-fit tests. Section V includes conclusions.

II. The Semi-Parametric Approach with Multiplicative Adjustment

Let $\{X_t\}_{t=1}^{T}$ be a realization from a stationary process with marginal density f and distribution function $F(\cdot)$. In the proposed approach, a parametric density estimator is utilized, but it is seen as a crude guess of the true density f. This initial parametric approximation is adjusted via multiplication by an adjustment factor ξ which can be determined by non-parametric approaches using some criteria.

Suppose $g(x,\theta)$ be a given parametric family of densities, where the possibly multidimensional parameter $\theta = (\theta_1, \cdots \theta_p)'$ belongs to some open and connected region in p-space. Let the parametric-start estimate be $g(x,\hat{\theta})$, where $\hat{\theta}$ is an estimator of the least false value θ_0 according to a certain distance measure between f and $g(\cdot,\theta)$. For concreteness we here chose $\hat{\theta}$ as the maximum likelihood estimator and define θ_0 as the minimizer of the kullback-Leibler distance on θ.

The next problem is the determination of the adjustment factor ξ. Hjort and Glad (1995) proposed a density estimator based on the naive estimator of ξ. Hjort and Jones (1996) suggested and investigated two versions of multiplicative density estimator. Naito (2004) proposed a local L_2-fitting criterion with index α, including the above estimators proposed by Hjort and Glad (1995) and Hjort and Jones (1996) as special cases. However, the focus of all of the above mentioned papers is in i.i.d. observations. Wang and Lin (2008) showed that the multiplicative adjustment method can be extended to density estimation for time series.

II.1 The Local L_2-fitting Criterion with Index α

Let $K_h(z) = h^{-1}K(h^{-1}z)$ and $K(z)$ is a kernel function, which is taken to be a symmetric probability density, and h is the bandwidth. Following Naito (2004) and Wang and Lin (2008), the adjustment factor ξ is determined by minimization of the empirical version of the function

$$Q(x,\xi \mid \alpha) = \int K_h(u-x)\frac{[f(u) - g(u,\hat{\theta})\xi]^2}{g(u,\hat{\theta})^{\alpha}}du \qquad \text{[II.1]}$$

for a fixed target point x, where α is a real number called the index. This method is called the local L_2-fitting criterion. After omitting the irrelevant term, the empirical version of [II.1] can be expressed as

$$Q_T(x,\xi \mid \alpha) = \xi^2 \int K_h(u-x)g(u,\hat{\theta})^{2-\alpha}du - \frac{2\xi}{T}\sum_{t=1}^{T} K_h(X_t - x)g(X_t,\hat{\theta})^{1-\alpha} .$$

The minimizer can be easily determined as

$$\hat{\xi}(x) = \arg\min_{\xi} Q_T(x,\xi \mid \alpha) = \frac{T^{-1}\sum_{t=1}^{T} K_h(X_t - x)g(X_t,\hat{\theta})^{1-\alpha}}{\int K_h(u-x)g(u,\hat{\theta})^{2-\alpha}du} .$$

Using this $\hat{\xi}$, a class of semi-parametric density estimators is obtained by

$$\hat{f}_{\alpha}(x) = g(x,\hat{\theta})\hat{\xi} = g(x,\hat{\theta})\frac{T^{-1}\sum_{t=1}^{T} K_h(X_t - x)g(X_t,\hat{\theta})^{1-\alpha}}{\int K_h(u-x)g(u,\hat{\theta})^{2-\alpha}du} . \qquad \text{[II.2]}$$

When $\alpha = 0$, 1 and 2, we have the following relationships:

$$\hat{f}_0(x) = \hat{f}_{HJ}(x),\ \hat{f}_1(x) = \hat{f}_{LL}(x),\ \hat{f}_2(x) = \hat{f}_{HG}(x),$$

where $\hat{f}_{HJ}(x)$ and $\hat{f}_{LL}(x)$ are two estimators proposed by Hjort and Jones (1996), and $\hat{f}_{HG}(x)$ is the density estimator proposed by Hjort and Glad (1995).

The corresponding estimate for the distribution function $F(\cdot)$ is

$$\hat{F}_{\alpha}(x) = \int_{-\infty}^{x} \hat{f}_{\alpha}(y)dy = \frac{1}{n}\sum_{i=1}^{n} \tilde{K}_h(x - X_i),$$

where $\tilde{K}_h(t) = \int_{-\infty}^{t} \frac{g(t,\hat{\theta})k_h(X_i - t)g(X_i,\hat{\theta})^{1-\alpha}}{\int k_h(u-t)g(u,\hat{\theta})^{2-\alpha}du}dt$.

II.2 Advantages of the Semi-Parametric Approach with Multiplicative Adjustment

According to Wang and Lin (2008), under some mixing and smoothness conditions, let $g_0(x) = g(x, \theta_0)$, with θ_0 be the best parametric approximation to f, the asymptotic bias and variance of $\hat{f}_\alpha(x)$ are respectively,

$$Bias\{\hat{f}_\alpha(x)\} = \frac{h^2}{2}\sigma_K^2\left[\frac{(g_0(x)^{1-\alpha} f(x))''}{g_0(x)^{1-\alpha}} - \frac{f(x)(g_0(x)^{2-\alpha})''}{g_0(x)^{2-\alpha}}\right] + O\left(\frac{h^2}{T} + h^4 + T^{-2}\right),$$

$$Var\{\hat{f}_\alpha(x)\} = \frac{1}{Th}R(K) f(x) + o\left(\frac{1}{Th}\right),$$

Where $\sigma_K^2 = \int z^2 K(z)dz$ and $R(K) = \int K(z)^2 dz$.

Then the asymptotic MISE (AMISE) of $\hat{f}_\alpha(x)$ is

$$AMISE\{\hat{f}_\alpha(x)\} = \frac{h^4}{4}(\sigma_K^2)^2 \Re(\hat{f}_\alpha) + \frac{R(K)}{Th}, \qquad \text{[II.3]}$$

Where $\Re\{\hat{f}_\alpha\} = \int\left[\frac{(g_0(x)^{1-\alpha} f(x))''}{g_0(x)^{1-\alpha}} - \frac{f(x)(g_0(x)^{2-\alpha})''}{g_0(x)^{2-\alpha}}\right]^2 dx$.

The kernel density estimator $\tilde{f}(x)$ of f is defined by

$$\tilde{f}(x) = T^{-1}\sum_{t=1}^{T} K_h(X_t - x),$$

and its AMISE is (Fan and Yao, 2003, p.206)

$$AMISE\{\tilde{f}(x)\} = \frac{h^4}{4}(\sigma_K^2)^2 \Re(\tilde{f}) + \frac{R(K)}{Th}, \qquad \text{[II.4]}$$

Where $\Re\{\tilde{f}\} = \int [f''(x)]^2 dx$.

Then the kernel estimator $\tilde{F}_n(x)$ of distribution function is

$$\tilde{F}_n(x) = \int_{-\infty}^{x} \tilde{f}_n(y)dy = \frac{1}{n}\sum_{i=1}^{n} K_h(x - X_i),$$

where $K_h(t) = \int_{-\infty}^{t} k_h(t)dt$.

Several properties of $\tilde{F}_n(x)$ have been investigated. Moreover, it has been shown by several authors that the asymptotic performance of $\tilde{F}_n(x)$ is better than that of the empirical distribution function $F_n(x)$. For example, Reiss (1981) proved that the relative deficiency of the E.D.F. $F_n(x)$ with respect to $\tilde{F}_n(x)$ quickly tends to infinity as the sample size increases. Swanepoel (1988) derived optimal smoothing parameter in the sense of minimizing the mean integrated squared error (MISE) and asserted that the kernel-type estimator $\tilde{F}_n(x)$ is asymptotically more efficient than the E.D.F. $F_n(x)$.

From [II.3] and [II.4], the new semi-parametric approach is better than the traditional kernel estimator in all cases where $\Re\{\hat{f}_\alpha\}$ is smaller in size than $\Re\{\tilde{f}\}$. Furthermore, if f is in the model $\{g(x,\theta):\ \theta \in \Theta\}$, that is, $g_0(x) = \mathrm{f}(x)$, then the $O(h^2)$ term of the bias vanishes and, consequently, the new estimators achieve a parametric convergence rate. Naito (2004) compared $\Re\{\hat{f}_\alpha\}$ with $\Re\{\tilde{f}\}$ for the case in which f belongs to a class of normal mixture densities used in Marron and Wand (1992). The class of normal mixture densities is a very broad one because any density can be approximated arbitrarily closely in various senses by a normal mixture (Marron and Wand, 1992). Naito confirmed that $\hat{f}_\alpha$ is better than, or at least competitive with, $\tilde{f}$ for all cases in that comparison.

III Simulation Study

Finite sample performance of the proposed distribution estimators was investigated by Monte Carlo simulation. The distribution functions studied in this section cover three different density shapes. The Gaussian, skewed unimodal and kurtotic unimodal distributions were considered as the distribution function (see Marron and Wand (1992) for the definitions about the last three densities) because they typify some different challenges to distribution estimation. In each model, 500 samples of size $n = 100$, 500, 1000 are generated respectively. The MISE(α, h) value for a given combination of (α, h) was estimated by the average of these 500 realizations of ISE(α, h). To obtain a precise approximation to the minimum MISE, a grid search of the combination (α, h) was implemented. The Gaussian kernel was used throughout. The estimators compared in this study were $\tilde{F}_n(x)$ and $\hat{F}_\alpha(x)$ for $\alpha = 0$, 1 and 2.

We utilized $g(x,\hat{\theta}) = \phi_{\hat{\sigma}^{-1}}(x - \overline{\mu})$ for all cases, i.e., we started with a Gaussian approximation, where $(\overline{\mu}, \hat{\sigma}^2)$ is the MLE of (μ, σ^2).

Values of $10^8 \times \min$ MISE are tabulated in Table 1 - Table 3, where the minimum is taken over h. Also tabulated in parentheses are 10^6 times the standard error (SE) for each case.

Table 1

The Value of Estimated $\min_h MISE(h) \times 10^8$ for Samples of Size $n = 100$.

	Gaussian	Skewed unimodal	Kurtotic unimodal
$\tilde{F}_n(x)$	106.66 (24)	149.77 (33)	78.43 (18)
$\hat{F}_0(x)$	114.32 (25)	149.77 (33)	77.89 (17)
$\hat{F}_1(x)$	48.75 (11)	148.58 (33)	78.78 (17)
$\hat{F}_2(x)$	21.58 (5)	147.99 (33)	79.71 (18)

Table 2

The Value of Estimated $\min_h MISE(h) \times 10^8$ for Samples of Size $n = 500$.

	Gaussian	Skewed unimodal	Kurtotic unimodal
$\tilde{F}_n(x)$	28.41 (6)	50.84 (11)	44.32 (10)
$\hat{F}_0(x)$	21.12 (5)	49.81 (11)	44.09 (10)
$\hat{F}_1(x)$	20.41 (5)	49.07 (11)	44.20 (10)
$\hat{F}_2(x)$	2.06 (0.5)	48.50 (11)	44.30 (10)

Table 3

The Value of Estimated $\min_h MISE(h) \times 10^8$ for Samples of Size $n = 1000$.

	#1 Gaussian	#2 Skewed unimodal	#3 Kurtotic unimodal
$\tilde{F}_n(x)$	2.72 (6)	59.30 (13)	52.11 (12)
$\hat{F}_0(x)$	1.83 (0.4)	57.71 (13)	51.59 (12)
$\hat{F}_1(x)$	1.44 (0.3)	58.15 (13)	51.83 (12)
$\hat{F}_2(x)$	1.16 (0.3)	58.60 (13)	52.07 (12)

Firstly, case 1 is that f is in the parametric model so that the $O(h^2)$ term of the bias of $\hat{F}_\alpha(x)$ vanishes, as mentioned in section II. For #1 all of $\hat{F}_\alpha(x)$ (except $\hat{F}_0(x)$ in

Table 1) are significantly better than $\tilde{F}_n(x)$. Secondly, for cases #2, all $\hat{F}_\alpha(x)$ are better than $\tilde{F}_n(x)$ and the best α is 2 ($n=$100 and 500) or 0 ($n=$1000). Finally, for case #3, the estimator $\hat{F}_0(x)$ is the best.

The simulation results show that the proposed approach has good finite sample performance.

IV. Empirical Results

IV.1 Data and Tests of Normality

The data under consideration consists of daily returns for four stock indexes: USA (S&P 500), Japan (Nikkei 225), Germany (DAX), China (SSEC) in two different periods: a pre-crisis period (for the period January 1998 to June 2006) and a post-crisis period (for the period July 2006 to June 2011). Returns are defined as the first difference of the natural logarithm of each index, i.e.,

$$R_t = 100 \cdot [\log(I_t) - \log(I_{t-1})],$$

where: R_t and I_t are the return and the index in day t, respectively.

The summary information about the empirical distributions of stock returns, together with Jarque-Bera nomality test statistics are presented in Table 4.

Table 4

The Distributional Characteristics of the Returns

Index		Skewness	Kurtosis	JB statistic
S&P 500	pre-crisis	0.0021	5.4994	555.4701
	post-crisis	-0.2451	11.4479	3.7624e+003
Nikkei 225	pre-crisis	-0.0349	4.5982	222.5390
	post-crisis	-0.5415	11.3400	3.6189e+003
DAX	pre-crisis	-0.1062	5.1807	431.2442
	post-crisis	0.1902	10.5722	3.0418e+003
SSEC	pre-crisis	0.4444	8.4062	2.5540e+003
	post-crisis	-0.4129	5.3389	311.9625

Table 4 shows that all the distributions are skewed and leptokurtic, thus exhibiting heavy tails (and high peaks), and Jarque-Bera test rejects the null hypothesis that the returns are normally distributed.

IV.2 Estimation Results

For the semi-parametric approach, we utilize normal distribution as an initial parametric approximation of the true density, i.e.,

$$g(x, \hat{\theta}) = \phi_{\hat{\sigma}}(x - \hat{\mu}),$$

where: $(\hat{\mu},\hat{\sigma}^2)$ is the MLE of (μ,σ^2). For the adjustment factor, the Gaussian kernel was used, the index α was selected by data-based method (Naito, 2004), and the bandwidth is choosed by the unbiased least squares cross-validation (Hjort and Glad, 1995).

In order to compare the relative fit of the semi-parametric approach and the normal fit, we performed goodness-of–fit tests. We divide the range of returns into 20 equal, non-overlapping intervals contained in [-10%, 10%]. The results of these tests are shown in Table 5. The goodness-of-fit test follows a chi-square distribution with $p-k-1$ degree of freedom, where p is the number of intervals and k is the number of parameters estimated for each distribution.

The results show that, the normal distribution provides bad fit, it is cleared rejected in all markets. The semi-parametric approach, on the other hand, cannot be rejected in any market.

Table 5

Goodness-of-fit Results

Index		Normal	Semi-parametric
S&P 500	pre-crisis	3.9522e+005	8.3471
	post-crisis	1.2844e+006	12.5154
Nikkei 225	pre-crisis	1.4042e+003	6.4118
	post-crisis	4.6355e+003	9.2260
DAX	pre-crisis	535.2262	5.6983
	post-crisis	2.3901e+005	9.5908
SSEC	pre-crisis	5.9828e+006	10.2417
	post-crisis	676.9179	8.4619

V. Concluding Remarks

In this paper, we proposed a semi-parametric approach with multiplicative adjustment to estimate the distributions of stock index returns.

The semi-parametric approach presents several potential improvements over both pure parametric and non-parametric estimators. Firstly, in the case where the parametric model is misspecified so that the parametric estimator for the true distribution is usually inconsistent, our semi-parametric estimator can still be consistent with the distribution. Secondly, in comparison with the nonparametric kernel estimator, our estimator can result in bias reduction as long as the parametric model can capture some roughness feature of the true distribution function, whereas the two estimators have the same asymptotic variance.

The simulation and empirical results show that the proposed approach has good performance.

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References

Ayebo, A. and Kozubowski, T.J., 2004. An Asymmetric Generalization of Gaussian and Laplace Laws. *Journal of Probability and Statistical Science*, 1, pp. 187-210.

Bekaert, G. Erb, C. Harvey, C. and Viskanta, T., 1998. Distributional Characteristics of Emerging market Returns and Asset Allocation. *Journal of Portfolio Management*, 24, pp. 102-15.

Butler, J.S. and Schachter, B., 1998. Estimating Value at Risk with a Precision Measure by Combining Kernel Estimation with Historical Simulation. *Review of Derivatives Research*, 1, pp. 371-390.

Chevapatrakul, T. and Tee, K.H., 2014. The Effects of News Events on Market Contagion: Evidence from the 2007-2009 Financial Crisis. *Research in International Business and Finance*, 32, pp. 83-105.

Fama, E.F., 1965. The Behavior of Stock Market Prices. *Journal of Business*, 38, pp. 34-105.

Fan, J.Q. and Yao, Q.W., 2003. *Nonlinear Time Series: Non-parametric and Parametric Methods*, Springer-Verlag.

Fernandez, C. Osiewalski, J. and Steel, M.F.J., 1995. Modeling and Inference with v-distributions. *Journal of the American Statistical Association*, 90, pp. 1331-1340.

Gebizlioglu, O.L. Şenoğlu, B. and Kantar, Y.M., 2011. Comparison of Certain Value-at-risk Estimation Methods for the Two-parameter Weibull Loss Distribution. *Journal of Computational and Applied Mathematics*, 235, pp. 3304-3314.

Ghouch, A.E. and Genton, M.G., 2009. Local Polynomial Quantile Regression with Parametric Features. *Journal of the American Statistical Association*,104:1416-1429.

Glad, I.K., 1998. Parametrically Guided Non-parametric Regression. *Scandinavian Journal of Statistics*, 25, pp. 649-668.

Gray, B. and French, D., 1990. Empirical Comparisons of Distributional Models for Stock Index Returns. *Journal of Business, Finance and Accounting*, 17, pp. 451-459.

Hansen, B.E., 1994. Autoregressive Conditional Density Estimation. *International Economic Review*, 35, pp. 705-730.

Hjort, N.L. and Glad, I.K., 1995. Non-parametric Density Estimation with a Parametric Start. *Annals of Statistics*, 23, pp. 882-904.

Hjort, N.L. and Jones, M.C., 1996. Locally Parametric Non-parametric Density Estimation. *Annals of Statistics*, 24, pp. 1619-1647.

Komunjer, I., 2007. Asymmetric Power Distribution: Theory and Applications to Risk Measurement. *Journal of Applied Econometrics*, 22, pp. 891-921.

Marron, J.S. and Wand, M.P., 1992. Exact Mean Integrated Squared Error. *Annals of Statistics*, 20, pp. 712–736.

Mishkin, F.S., 2011. Over the Cliff: from the Subprime to the Global Financial Crisis. *Journal of Economic Perspectives*, 25 (1), 49–70.

Mishra, S. Su, L. and Ullah, A., 2010. Semi-parametric Estimator of Time Series Conditional Variance. *Journal of Business Economics & Statistics*, 28, pp. 256-274.

Naito, K., 2004. Semi-parametric Density Estimation by Local L_2-fitting. *Annals of Statistics*, 32, pp. 1162–1191.

Necula C., 2009. Modelling Heavy-tailed Stock Index Returns Using the Generalized Hyperbolic Distribution. *Romanian Journal of Economic Forecasting*, 2, pp. 118-131.

Reiss, R.D., 1981. Nonparametric Estimation of Smooth Distribution Functions. *Scandinavian Journal of Statistics*, 8, pp. 116-119.

Swanepoel, J.W.H., 1988. Mean Integrated Squared Error Properties and Optimal Kernels When Estimating a Distribution Function. *Communication in Statistics - Theory and Methods*, 17:3785-3799.

Theodossiou, P., 1998. Financial Data and the Skewed Generalized T Distribution. *Management Science*, 44, pp. 1650-1661.

Wang, K.P., 2012. Value at Risk Estimation by Combining Semi-parametric Density Estimation with Historical Simulation". *Economic Computation and Economic Cybernetics Studies and Research*, 4, pp. 163-178.

Wang, K.P. and Lin, L., 2008. Semi-parametric Density Estimation for Time Series with Multiplicative Adjustment. *Communications in Statistics - Theory and Methods*, 37, pp. 1274-1283.

Wang, K.P. Lin, L. and Qi, R.H., 2009. Multiplicative Adjustment Method for Semi-parametric Regression with Mixing Dependent Data. *Communications in Statistics-Theory and Methods*, 38, pp. 3654-3665.