7. PORTFOLIO DECISION USING TIME SERIES PREDICTION AND MULTI-OBJECTIVE OPTIMIZATION

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Abstract

Randomness, volatility, and nonlinearity displayed by the stock market lead to the uncertainty of the stock market index and stock prices. The purpose of the study is to find a straightforward method for portfolio decision applicable to strong-form and weak-form efficient markets. Thus, a methodology for porfololio decision base on the Nonlinear Autoregressive Exogenous Model (NARX) and multi-objective optimization (MO) was proposed. First, two of eight quarters from 2018 to 2019 were chosen to buy S&P 500 stocks on the basis of the predicted stock market trend using the NARX with a single exogenous variable. The variable was selected from 67 macroeconomic factors by Shannon entropy or relevance. Then, the stocks were selected for a portfolio on the basis of the predicted stock returns from the NARX with a mean relative error as the criteria. Next, a reverse conditional probability indicator was imported as a risk indicator for the objective function of MO, and the stock weights of the portfolio were allocated by MO following the principle of maximizing predicted portfolio return and minimizing portfolio risk. The final findings demonstrate that the portfolio return is 8%–14% below the S&P 500 return and is increased to approximately 5% above the S&P 500 return after the stock weights were allocated by MO. The final investment return for eight quarters is 60% above the S&P 500 return if the proposed investment strategy was adopted. Therefore, the proposed method in the study combining the NARX and MO with certain criteria can guide investors to make a rational portoloio decision and give a reference for scholars to establish effective method for the prediction of stock prices and assets allocation.

Keywords: portfolio; NARX; multi-objective optimization; prediction of stock prices

JEL Codes: G1

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1. Introduction

Stock investment can be highly profitable but risky. Investors have to balance risk and return to obtain excessive profit with low risk. Therefore, investors must accurately predict stock prices and stock market index and consider diversity to avoid unsystematic risk (Markowitz, 1952). The efficient market hypothesis of Eugene Fama (1970) doubted the predictability of stock prices. For instance, the S&P 500 is a strong-form efficient market, and fundamental and technical analyses are ineffective. Contemporarily, many prediction methods have been developed for investors, and time series models are the most popular (Renouard and Ezvan, 2018; Villar-Rodriguez et al., 2018). Traditional time series models mainly include MA, AR, ARMA, ARIMA, and seasonal model. Many harsh assumptions for the models are not consistent with the fact that financial market data present huge noise, non-stationarity, and nonlinearity. Therefore, the traditional econometric equation is not ideal in the analysis of high dimension, complexity, and noise (Shlafman et al., 2018). Recently, the quantitative investment strategy and black-box models have gradually become the focus of scholars' study because of the development of an artificial neural network and search algorithm (Planuch-Prats and Salvador-Valles 2018; Ormiston, 2019). For example, nonlinear autoregressive exogenous models (NARXs) are applied to predict stock prices over a long period with accuracy limited to 100 days (Xiu and Chen, 2017). However, many difficulties exist for those models such as independent variables selecting, parameters setting, and data preprocessing. Nonetheless, the inevitable error of the forecast value brings risks to the portfolio decision. Therefore, risk aversion strategies should be applied to reduce the risks, but those strategies cannot provide specific operational guidelines. Therefore, how to change those strategies to objective functions for optimization is crucial to portfolio decision. For this purpose, the present study proposes a method for the decision, including stocks selection using a NARX and weight allocation using multi-objective optimization (MO). Moreover, this study intends to find an easy-to-use method for portfolio decision applicable to strong-form and weak-form efficient market.

2 State of the art

NARX is popular for the prediction of stock prices and stock market index because of its accuracy compared with the conventional prediction methods for time series (Narang, 2014). Grigoryan (2015) adopted principal component analysis on 30 technical analysis indicators and five price-related variables from March 12, 2012 to December 30, 2014 to obtain exogenous variables for the NARX and predict the closing price of a NASDAQ stock in two periods. The predicted prices demonstrate the effectiveness of NARX prediction. Gandhmal and Kumar (2020) successfully predicted two stock prices with the NARX using 12 technical analysis indicators extracted from the historical data from January 2000 to March 2019. They utilized many technical analyses, which are highly subjective and time-consuming. Xiu and Chen (2017) applied the NARX for the prediction of the Shanghai Securities Composite Index in 100 days to prove that the long-term prediction is possible. However, the effectiveness of the NARX for the long-term prediction of a strong-form efficient market is doubtful because the Shanghai Stock Exchange is a weak-form efficient market. Labde et al. (2017) found the validity of the NARX for stock prices prediction in the Indian stock market. However, three exogenous variables for the NARX are all microeconomic indicators. Montenegro and Molina (2020) employed a NARX with the historical data of 1,259 trading days to predict the opening index of the S&P 500. The results verify the validity of the short-term prediction, but 10 exogenous variables for the NARX are all microeconomic indicators.

Yassin et al. (2017) collected 1533 historical data from September 1995 to August 2013 for the NARX to predict the weekly stock prices of Apple Inc., and the predicted prices were acceptable. However, the five exogenous variables for the NARX were still microeconomic indicators. Mahendran et al. (2020) built a NARX with the historical data from January 2, 2007 to March 22, 2010 to predict the next five-day closing prices of five stocks in the Indian stock market. However, the study has the same deficiencies as the aforementioned.

Assets allocation using optimization to reduce unsystematic risks is key to a successful portfolio decision due to inevitable prediction errors. Miryekemami et al. (2017) introduced a time-consuming genetic algorithm to gain an optimal solution for a portfolio on the premise of the risk analysis of the Tehran stock market. However, the beta coefficient and liquidity for the optimization is historical and not predicted. Ding et al. (2017) considered five financial ratios as risk indicators and conducted optimization with the Lagrange multiplier. However, the five indicators are not suitable for all stocks because of differences of companies in size and industry background. Meghwani and Thakur (2017) optimized portfolios with heuristic algorithms, but the data of rebalanced transaction costs for the study are historical and not predicted. Hu et al. (2019) chose multi-swarm algorithm for MO with p-optimality criteria called p-MSMOEAs, but the study mainly focused on p-MSMOEAs. Chen et al. (2019) imported genetic algorithm to optimized portfolios established with radial basis function neural network. However, the portfolio built by clustering analysis violated the risk aversion principle of diversification, and the internal correlation between indicators cannot be defined accurately. Chen and Wei (2019) confirmed that the robust effective solution based on the set order relationship has high applicability for investors, but the data employed for optimization were historical. Zhao et al. (2020) adopted a multi-objective evolutionary algorithm with a decomposition strategy to solve the problem of conflicts between many objective functions in MO, but the statistical indicators for MO remain historical.

The above studies were chiefly aimed at the NARX or MO. Thus, the present study combines two approaches for portfolio decision. Particularly, the NARX and MO can be improved to obtain more satisfying portfolios. In this study, macroeconomic indicators were introduced as a single exogenous variable for the NARX to improve forecasting accuracy. Moreover, this study aims to solve the problem of long-term prediction and strong-form efficient market prediction for NARX. Thus, quarterly stock prices and the S&P 500 index were predicted in the study. As for MO, predicted data as a substitute for historical data were served for MO to increase the practicability of the proposed method. Moreover, a risk indicator, reverse condition probability, was introduced for MO. The final portfolio return was compared with the returns of the S&P 500 and a treasury bill to test the effectiveness of the method.

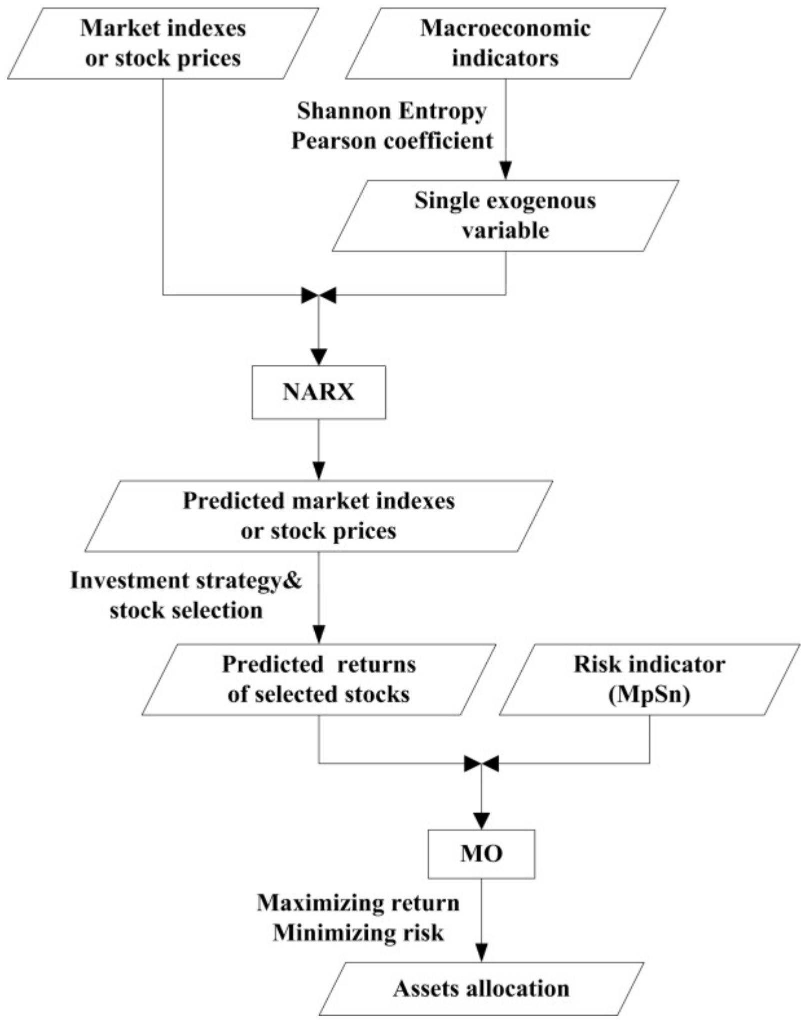
The rest of the study is organized as follows. Section 3 presents the samples and assumptions of the study, the parameters setting of the NARX, and the objective functions of MO. The selection of a single exogenous variable for the NARX and stocks for MO is also specified in the section. Section 4 gives the results of the case study and discusses the effectiveness of the proposed method. The last section summarized the study and provided the conclusions.

3 Methodology

This study has three assumptions: market friction and background risk are disregarded, short selling is forbidden, and portfolio decision is not subject to capital position. The portfolio decision can be divided into three steps, namely, sample selection, market index and stock prices prediction, and assets allocation. All tasks were accomplished through MATLAB2016. The program chart is shown in Figure 1. The specific method is stated as follows.

Figure 1

Program chart



Source: Authors' calculations.

3.1 Samples selection

In a strong-form efficient market represented by the S&P 500, a weathervane of the U.S. economy, most scholars regard that the stock prices are unpredictable. Thus, the S&P 500 and its component stocks were chosen as the study objects to find a universal method for portfolio decision applicable to strong-form and weak-form efficient markets. Moreover, the raw data of the S&P 500 is accessible and free to obtain. All raw data were downloaded from finance.yahoo.com and fred.stlouisfed.org and preprocessed to meet the requirements of the NARX and MO. The closing stock prices and the S&P 500 index of the last trading day in the quarter were defined as quarterly ones. Quarterly macroeconomic indicators were acquired through raw data or its simple handling. Quarterly data were regularized to improve the generalization ability of the NARX and MO and accelerate the convergence speed of the algorithm. The quarterly S&P 500 index and the closing prices of stocks were predicted in this study, and the period is from December 31, 2019 to January 1, 2000. The eight quarters in 2018 and 2019 (Table 1) of eighty quarters were set as the predictive periods. Only 382 component stocks that meet the time requirement were selected. A total of 67 primary macroeconomic indicators in the same period were alternatives to a single exogenous variable for the NARX. The regularization formula of quarterly data is as follows:

$$x_i' = \frac{x_i}{\max\limits_{1 \le i \le n} |x_i|}, \tag{1}$$

where: n is the length of a time series, and x_i is the ith element of the time series.

Table 1

Predictive periods

Quarter	Time span
73	2018/01/01-2018/03/31
74	2018/04/01-2018/06/30
75	2018/07/01-2018/09/30
76	2018/10/01-2018/12/31
77	2019/01/01-2019/03/31
78	2019/04/01-2019/06/30
79	2019/07/01-2019/09/30
80	2019/10/01-2019/12/31

Source: Authors' calculations.

3.2 Parameters setting of the NARX

For the NARX (see Appendix 1), exogenous variable x(t) is the time series of a macroeconomic indicator, y(t) is the time series of a stock price or the S&P 500 index, and y(t+1) is the prediction of y(t). The number of hidden layers is 40, and the number of input delays and feedback delays is 20. Moreover, 70% of the data are used for training, 15% for validation, and 15% for testing for each prediction. The Levenberg-Marquardt algorithm was selected as the training algorithm because of time efficiency. In each prediction, a macroeconomic indicator was chosen as a single exogenous variable for the NARX using Shannon entropy (SE) and correlation coefficient (CC) as criteria. The computational formula of SE is as follows:

$$\mathrm{H(X)} = \sum_{i=1}^{n} p_i log p_i, \tag{2}$$

where: n is the number of possible values for a macroeconomic indicator, and p_i is the corresponding probability of x_i.

The computational formula of CC is as follows:

$$\rho_{x,y} = \frac{cov(x,y)}{\sigma_x \sigma_y} \tag{3}$$

where: $\mathrm{cov}(x, y)$ is the covariance for x and y, and σ_x and σ_y are standard deviations for x and y.

3.3 Investment strategy and stock selection

After the predicted S&P index and stock prices were obtained using the NARX, the predicted return of the S&P 500 or a stock had to be calculated for investment strategy and stock selection. The computational formula is as follows:

$$\mathrm{r} = \frac{p_{t+1} - p_t}{p_t} \tag{4}$$

where: p_{t+1} is the predicted S&P index or the stock price of the next quarter, and p_t is the S&P 500 index or a stock price of the present quarter.

The choice between stocks and bonds in a quarter is the investment strategy of this study. Stocks were bought in the present quarter and sold in the next quarter if the predicted S&P 500 return is positive. Otherwise, the three-month treasury bill becomes the substitution for stocks. The predicted stock returns and the mean relative error (MRE) were imported as the criteria in selecting stocks to build a portfolio. The computational formula of the MRE is as follows:

$$\mathrm{MRE} = \frac{1}{\mu}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2} \tag{5}$$

where: n is the quarter number from the 21st quarter to the present quarter, μ is the mean of the real S&P 500 index or the stock price from the 21st quarter to the present quarter, Y_i is the real S&P 500 index or stock price, and $\hat{Y}_i$ is the predicted S&P 500 index or stock price.

3.4 Assets allocation

MO was employed for assets allocation in the study, and the objective functions reflected the basic principle of maximizing the return and minimizing the risk. MO is expressed as follows:

$$\text{Maximize } R_p = \sum_{i=1}^{n} x_i r_i, \tag{6}$$

$$\text{Minimize } MpSn = \sum_{i=1}^{n} x_i p_i, \tag{7}$$

$$\text{subject to } \sum_{i=1}^{n} x_i = 1 \;\&\; 0 \le x_i \le \frac{1}{mininumber},$$

where: n is the number of selected stocks, R_p is the portfolio return, MpSn is the reverse conditional probability of a portfolio, x_i is the weight of an individual stock, r_i is the return of an individual stock, p_i is the reverse conditional probability of an individual stock, and $\mathrm{mininumber}$ is the minimum number of stocks whose weight is above 0.

The reverse conditional probability of a stock p_i was introduced as a risk indicator. The indicator represents the falling probability of a stock price when the S&P 500 index is rising in a quarter, and the probability was based on the analysis of historical data. The computational formula of the indicator is as follows:

$$p_i = \frac{p(MpSn)_i}{p(Mp)}, \tag{8}$$

where: $p(MpSn)_i$ is the joint probability for a negative stock return with the positive S&P 500 return, and $p(Mp)$ is the probability for the positive S&P 500 return.

The variable $\mathrm{mininumber}$ was employed for the upper bound of x_i to implement the risk aversion principle of diversification. The weight allocation strategy in the study was to set a lower bound for the number of stocks whose weight is above 0, as shown in Table 2.

Table 2

Weight allocation strategy

Number of stocks	mininumber	Upper bound of weight
$\mathrm{n} \leq 20$	n/2	2/n
$20 < \mathrm{n} \leq 50$	n/3	3/n
$50 < \mathrm{n} \leq 150$	n/4	4/n
$150 < \mathrm{n}$	n/5	5/n

Source: Author calculations

4. Results analysis and discussion

4.1 Procedure of NARX prediction

For the prediction of a stock price or the S&P 500 index, SE (<0.9) and CC (>0.8) calculated from Equations (2) and (3) were applied as the criteria in choosing a single exogenous variable for the NARX from 67 primary macroeconomic indicators. The predicted results were eliminated if the MRE is greater than 0.1. The prediction of the S&P 500 index for the 73rd quarter (Table 3) was given as an example to illustrate the procedure. Eighteen macroeconomic indicators are eligible because their entropies are lower than 0.9. Seven predicted indexes were eliminated because their MREs calculated from Eq. (5) are higher than 0.1. Finally, the mean of the rest of the predicted returns calculated from Eq. (4) is the predicted S&P 500 return for the 73rd quarter.

Table 3

Prediction of market return for 73rd quarter

Macroeconomic indicators	Shannon entropy	MRE	Predicted Return (%)
IPG2211A2NQ	0.781	0.068	0.095
UNDCONTNSA	0.873	0.069	0.016
M2NS	0.882	0.111	
M1NS	0.870	0.040	-0.078
TWEXBMTH	0.861	0.048	-0.004
NA000334Q	0.728	0.147	
USSTHPI	0.804	0.082	0.093
BC0ACBQ158SBOG	0.896	0.048	-0.061
TCU	0.874	0.121	
CILACBQ158SBOG	0.880	0.063	0.037
CPIAUCSL	0.878	0.108	
A191RP1Q027SBEA	0.791	0.147	
GPDI	0.853	0.217	
HOUST	0.871	0.063	-0.108
INDPRO	0.864	0.055	-0.225

Macroeconomic indicators	Shannon entropy	MRE	Predicted Return (%)
IPG211111CSQ	0.875	0.064	-0.101
M2SL	0.893	0.055	-0.038
W070RC1Q027SBEA	0.880	0.172	
Mean			-0.034

Source: Authors' calculations.

The following codes were noted: IPG2211A2NQ is the electric and gas utilities of industrial production. UNDCONTNSA is the total new privately-owned housing units under construction. M2NS is the M2 money stock. M1NS is the M1 money stock. TWEXBMTH is the trade-weighted U.S. dollar index (Broad, Goods). NA000334Q is the gross domestic product. USSTHPI is the all-transactions house price index for the United States. BC0ACBQ158SBOG is the bank credit of all commercial banks. TCU is the capacity utilization of the total industry. CILACBQ158SBOG is the commercial and industrial loans of all commercial banks. CPIAUCSL is the consumer price index for all urban consumers (all items in U.S. city average). A191RP1Q027SBEA is the gross domestic product (seasonally adjusted). GPDI is the gross private domestic investment, HOUST is the housing starts of the total new privately-owned housing in units started. INDPRO is the industrial production index. IPG211111CSQ is the mining crude oil of industrial production. M2SL is the M2 money stock (seasonally adjusted). W070RC1Q027SBEA is the state and local government current tax receipts.

4.2 Investment strategy decided by the stock market trend

The investment strategy was constructed on the ground of the predicted S&P 500 returns of eight quarters (Table 4). SE and CC were employed as the criteria. The NARX was run 10 times to eliminate the randomness of the prediction. At least seven times, the predicted S&P 500 returns of the 77th and 80th quarter are positive on both conditions. Thus, stocks were bought in the two quarters, and the three-month treasury bill was bought in the rest of the six quarters. SE criterion may be better than the CC criterion because stocks will be bought in the 76th quarter if the CC criterion is followed. However, the S&P 500 index of the quarter increasingly fell.

Table 4

Predicted market returns of eight quarters

Quarter	Original Return (%)	Number of times (predicted return>0)		Mean return for ten times (%)	
		SE<0.9	CC>0.8	SE<0.9	CC>0.8
73rd	-1.225	4	2	-0.522	-1.961
74th	2.935	5	6	0.246	0.184
75th	7.196	1	2	-2.406	-1.376
76th	-13.972	5	8	0.355	1.154
77th	13.066	8	10	4.195	5.220
78th	3.788	0	0	-6.232	-5.268
79th	1.189	0	0	-6.956	-7.602
80th	8.534	7	7	2.387	0.972

Source: Authors' calculations.

4.3 Stock selection for portfolio establishment

Stocks were selected to build the portfolio for the 77th and 80th quarter on the ground of the predicted returns. The criteria of SE, CC, and MRE were employed for the NARX. Moreover, the return of the selected stocks should be more than 0.05. The results illustrated in Table 5 demonstrate that the mean return of selected stocks for the portfolio is approximately 15% below the market return. Therefore, optimization is necessary to raise portfolio return.

Table 5

Selected stocks for quarter 77 and 80

Quarter	Number of stocks		Market return (%)	Original mean return (%)		Predicted mean return (%)	
	SE<0.9	CC>0.8		SE<0.9	CC>0.8	SE<0.9	CC>0.8
77th	104	95	13.066	12.630	12.899	17.258	14.818
80th	118	98	8.534	5.923	6.817	14.594	13.930
total			21.600	18.553	19.716	31.852	28.748

Source: Authors' calculations.

4.4 MO of portfolio

Objective functions were obtained using Equations. (6), (7), and (8) for MO. Then, the conventional algorithm (CA, function fgoalattain in MATLAB2016) and genetic algorithm (GA, function gamultiobj in MATLAB2016) were compared from the optimization results, as shown in Table 6 and Appendix 2). Real portfolio return using CA is approximately 20% above that using GA. Therefore, MO is more appropriate than GA because of optimization effectiveness and time efficiency. The final portfolio return using CA for SE and CC criteria are shown in Table 7.

Table 6

Comparison of two optimization algorithms

Quarter	Market return (%)	Original mean return (%)		Time consumption (h)	
		CA	GA	CA	GA
77th	13.066	15.767	13.143	0.05	36
80th	8.534	6.949	5.896	0.05	41
	21.600	22.715	19.040	0.1	87

Source: Authors' calculations.

Table 7

Final CA optimization results

Quarter	Number of stocks		Original portfolio return (%)		Predicted portfolio return (%)	
	SE<0.9	CC>0.8	SE<0.9	CC>0.8	SE<0.9	CC>0.8
77th	104	95	15.7665	15.1414	13.1504	10.8993
80th	118	98	6.9489	7.4821	12.4526	10.3946
total			22.7154	22.6235	25.6030	21.2939

Source: Authors' calculations.

4.5 Final investment return

Following the above-mentioned investment strategy, the final return of eight quarters is shown in Table 8. The final investment return is 60% above the S&P 500 return and 110% above the treasury bill return for eight quarters.

Table 8

Final investment return for eight quarters

Quarter	Return (%)			
	S&P 500	Treasury Bill	SE<0.9	CC>0.8
73rd	-1.225	1.559	1.559	1.559
74th	2.935	1.841	1.841	1.841
75th	7.196	2.036	2.036	2.036
76th	-13.972	2.311	2.311	2.311
77th	13.066	2.388	15.767	15.141
78th	3.788	2.305	2.305	2.305
79th	1.189	1.984	1.984	1.984
80th	8.534	1.577	6.949	7.482
total	21.512	16.000	34.751	34.659

Source: Authors' calculations.

5. Conclusions

This study aims to search for an effective method for portfolio decision to gain an investment return above the stock market return. Thus, the NARX and MO were built using MATLAB2016 for the stocks' selection and assets allocation in the study. The NARX and MO with the criteria of SE, CC, MRE, and return limit were combined for the decision, including stock selection and weight allocation. Investment strategy based on the predicted market trend is crucial to portfolio decision. Results prove that the SE and CC criteria combined for the prediction presented a rational judgment on the market trend. For the 77th and 80th quarters, the mean return of the portfolio including stocks selcected according pridection result is lower than the market return. However, the portfolio return was raised to be more than the market return after assets were allocated using MO. The final portfolio return demonstrates that neither SE nor CC is better or worse. Following the investment strategy, stocks were bought only in the 77th and 80th quarters, but the two quarters have the highest market return. Therefore, the final investment return for eight quarters is higher than that of the S&P 500 index. The innovative method used in this study can effectively improve the accuracy of stock selection and present some references for the prediction of stock prices and assets allocation. However, the portfolio includes more than 100 stocks. Thus, this approach only applies to institutional investors. As a result, chaotic or wavelet models can be introduced to improve the accuracy of predictions. Thus, the number of stocks in the portfolio will be significantly reduced.

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References

Chen, B. L., Zhong, J. D. and Chen, Y. Y., 2019. A Hybrid Approach for Portfolio Selection with Higher-order Moments: Empirical Evidence from Shanghai Stock Exchange. *Expert System with Applications*, 145(2), pp.2-26.

Chen, C. and Wei, Y., 2019. Robust Multiobjective Portfolio Optimization: A Set Order Relations Approach. *Journal of Combinatorial Optimization,* 38(6), pp.21-49.

Ding, Y., Liu, O., Yao, Y. W. and Chan, Ch. O., 2017. Multi-Objective Portfolio Optimization in Stock Market. *International Journal of Design. Analysis and Tools for Integrated Circuits and Systems*, 1(6), pp.63-67.

Gandhmal, D. P. and Kumar, K., 2020. Wrapper-Enabled Feature Selection and CPLM-Based NARX Model for Stock Market Prediction. *The Computer Journal*, 00(00), pp.1-16.

Grigoryan, H., 2015. Stock Market Prediction using Artificial Neural Networks. Case Study of TAL1T, Nasdaq OMX Baltic Stock. *Database Systems Journal*, 2(6), pp.14-23.

Hu, Y. B., Chen, H. N., He, M. W., Sun, L. L., Liu, R. and Shen, H., 2019. Multi-Swarm Multi-Objective Optimizer Based on p-Optimality Criteria for Multi-Objective Portfolio Management. *Mathematical Problems in Engineering*, 2019, pp.1-22.

Labde, S., Patel, S. and Shukla. M., 2017. Time Series Regression Model for Prediction Of Closing Values of the Stock using an Adaptive NARX Neural Network. *International Journal of Computer Applications*, 1(158), pp.29-34.

Mahendran, A., Vasavada, K.A., Tuteja, R., Sharma, Y., and Vijayarajan, V., 2020. Stock Market Analysis and Prediction Using Artificial Neural Network Toolbox. In: Bhateja V., Satapathy S., Satori H., eds. *Embedded Systems and Artificial Intelligence. Advances in Intelligent Systems and Computing,* (vol 1076), Singapore: Springer Press, pp.559-569.

Malkiel, B. G. and Fama, E. F., 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, 25(2), pp.383-417.

Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance*, 7(1), pp.77-91.

Meghwani, S. S. and Thakur, M., 2017. Multi-objective Heuristic Algorithms for Practical Portfolio Optimization and Rebalancing with Transaction Cost. *Applied Soft Computing Journal*, 67(3), pp.865-894.

Miryekemami, S. A., Ehsan, S. and Sabegh, Z. A., 2017. Using Genetic Algorithm in Solving Stochastic Programming for Multi-Objective Portfolio Selection in Tehran Stock Exchange. *Advances in Mathematical Finance & Applications*, 2(4), pp.107-120.

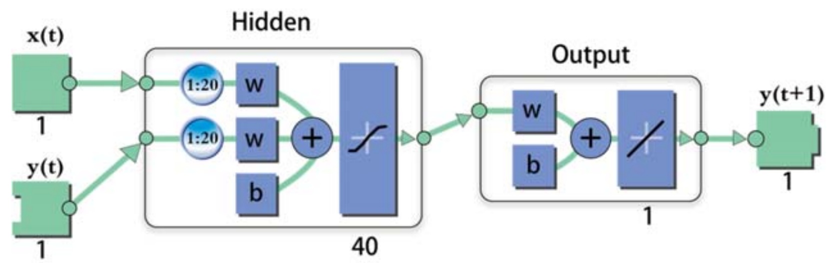
Montenegro, C. and Molina, M., 2020. Using Deep Neural Networks for Stock Market Data Forecasting: An Effectiveness Comparative Study. In: Botto-Tobar M., León-Acurio J., Díaz Cadena A., Montiel Díaz P., eds. *Advances in Emerging Trends and Technologies. ICAETT 2019. Advances in Intelligent Systems and Computing,* (vol 1066), Switzerland: Springer, Cham Press, pp.399-408.

Narang, R. K., 2014. *Inside the Black Box*. NJ: China Machine Press.
Ormiston, J., 2019. Blending Practice Worlds: Impact Assessment as A Transdisciplinary Practice. *Business Ethics: A European Review*. 28(4), pp.423-440.
Planuch-Prats, C. and Salvador-Valles, R., 2018. Digitalization and Knowledge Management in Financial Services. *DYNA*, 93(4), pp.356-371.
Renouard, C. and Ezvan, C., 2018. Corporate Social Responsibility Towards Human Development: A Capabilities Framework. *Business Ethics: A European Review*. 27(2),pp.144-155.
Shlafman, N., Frolina, K. and Gotal, D. L., 2018. Modeling for the Analysis of the Investment Potential of the Construction Sector. Tehnički Glasnik, 12(4), pp.236-243.
Villar-Rodriguez, E., Osaba-Icedo, E. and Ser-Lorente, D. J., 2018. Quantum Computing Six Key Factors to Understand the Future of Computation. *DYNA*, 93(3). 238-245.
Xiu, Y. and Chen, X. Y., 2017. Study on Prediction of the Shanghai Composite Index Based on EMD and NARX Neural Network. *Information Technology and Intelligent Transportation Systems*, 296(5), pp.590-596.
Yassin, I. M., Abdul-Khalid, M. F., Herman, S. H., Pasya, I., Wahab, N. A. and Awang, Z., 2017. Multi-Layer Perceptron (MLP)-Based Nonlinear Auto-Regressive with Exogenous Inputs (NARX) Stock Forecasting Model. *International Journal on Advanced Science Engineering Information Technology*, 3(7), pp.1098-1103.
Zhao, P., Gao, S. and Yang, N., 2020. Solving Multi-Objective Portfolio Optimization Problem Based on MOEA/D. *In 2020 IEEE 12th International Conference on Advanced Computational Intelligence*, Dali, China, pp.30-37.

Appendix 1

Figure 2

Structure of the NARX

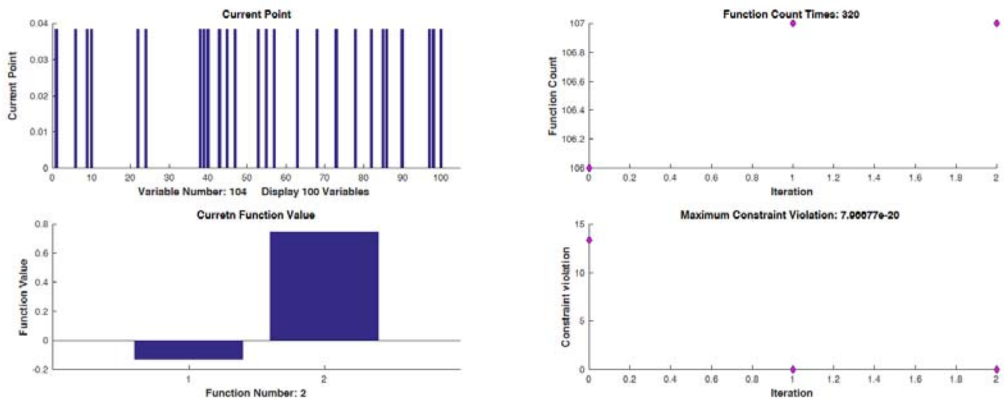


Source: Authors' calculations.

Appendix 2

Figure 3

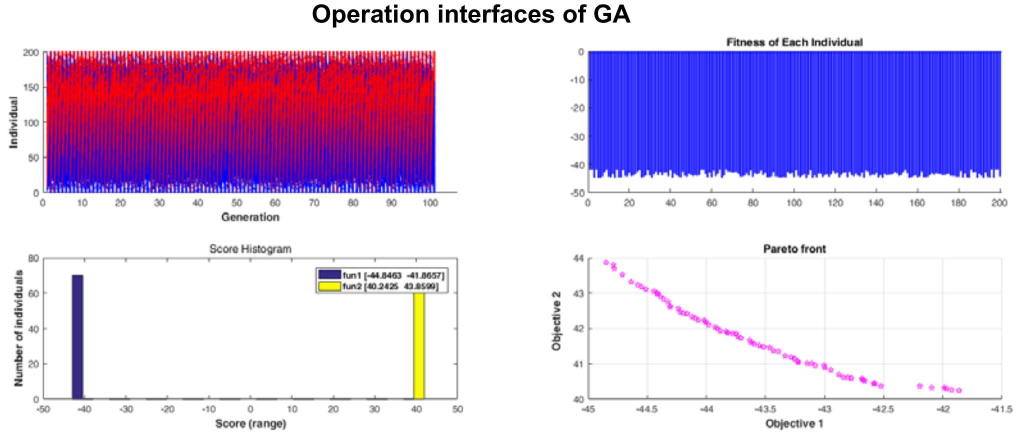
Operation interfaces of CA



Source: Author calculations

Figure 4

Operation interfaces of GA



Source: Authors' calculations.