UNDERGROUND ECONOMY AND FISCAL POLICIES MODELING*

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1995

Abstract

Over the last decades a growing concern over the phenomenon of the underground economy has increased attention among officials, politicians, and economists. There are several important reasons why officials and the general public should be concerned in post-communist countries about the real size of the underground economy, as following: an impressive development of underground economy occurred after the collapse of former communist regimes and general liberalization of economic activity; under a growing underground economy, macroeconomics policy is based on mistaken official indicators (such as: income, consumption, unemployment, etc.). In such situation, an extended underground sector may cause severe difficulties to politicians, because it "provides" unreliable official indicators; an accelerated increase in the size of the underground economy, usually caused by a rise in the overall burden of taxes and regulations, may lead to an erosion of the tax base, a decrease in tax receipts and thus to a further rise of the budget deficit (in case of Eastern countries, these are perhaps accentuated due to a weak government bonds market and of a high inflation). First part of this study deals with a critical survey of main approaches of underground (informal) sector in specialized literature; second part focuses on models existing in literature, on their comparative estimating results of the size of underground economy and evaluation of fiscal policy effects. Also, some our results obtained by using specific models to investigate problems of underground economy in East European countries are presented.

Keywords: underground economy, Laffer curve, informal activity, fiscal policy, transition
JEL classification: E26, P26, H26, O17

* This research was undertaken with support from the European Commission's Phare ACE Programme 1994, Contract Number: 94-0139-F.

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Acknowledgments

The author gratefully acknowledges the financial support of ACE-Phare Programme for a six months stay at CEPREMAP. In the same time, he would like to thank Pierre-Yves Hénin for his support and encouragement without this work could not have been written and for the favourable conditions to work at CEPREMAP. Also many thanks to Philippe Adair for many detailed comments and suggestions and for the possibility offered to author to participate in the GRATICE's Workshop on the Informal Economy. Special thanks to Robin de Vilder for his stimulating ideas and suggestions concerning the emergence of chaotic regime within some economic models.

1. INTRODUCTION

Over the last two decades a growing concern over the phenomenon of the underground economy has increased attention among officials, politicians, and economists. For Romania, as for many other Eastern countries, there are several important reasons why officials and the general public should be concerned about the real size of the underground economy. Among the most important are the following:

1 - An impressive development of underground economy occurred after the collapse of former communist regimes and general liberalization of economic activity. This was favoured by the abolition of regulatory laws including ambiguous texts and the concrete way of translating them and of the quasi-comprehensive state own sector forms. But, an uncontrolled development of underground economy may lead to a slowness of the transition process itself.

2 - Under a growing underground economy, macroeconomics policy is based on mistaken official indicators (such as: income, consumption, unemployment, etc.). In such situation, an extended underground sector may cause severe difficulties to politicians, because it "provides" unreliable official indicators.

3 - An accelerated increase in the size of the underground economy, usually caused by a rise in the overall burden of taxes and regulations, may lead to an erosion of the tax base (the Laffer's curve principle), a decrease in tax receipts and thus to a further rise of the budget deficit. Moreover, in case of Eastern countries, these are accentuated because of a weak government bonds market and of a high inflation.

These growing concerns (among which the last two are also true for the Western countries) have led many authors to challenging tasks of measuring the size or growth rate of the underground economy, to trace back the main causes of it and to analyse the interactions of official and unofficial sectors of the economy. In this work we present the main opinions existing in literature regarding the underground economy approach. There are two divergent mainstreams: a first group of authors that consider unofficial or informal sector as being marginal and parasite, having insignificant role within the global economic system, its dynamics being unrelated to the changes of fiscal policy, and consequently it must be strong controlled and even repressed; the second mainstream, on the other hand, attributes to the unofficial sector an important compensatory role, its dynamics being strong related to the fiscal policy change. Generally, the partisans of first mainstream think direct and detailed methods (such as: periodic surveys, frequent controls of the potential tax payers by fiscal or other specialized organisms) as being adequate to estimate the size of each compose the underground economy and then to include in the next planned budget. Opposite to this viewpoint, the partisans of second mainstream consider that the main instrument to restrain underground sector is to operate changes in fiscal policy, which must be oriented toward stimulation of supply in free market conditions, and consequently to increase the allocation effects (these opinions are concentrated within the so-called supply-side economics). The first focused on regulation, control, and structure of income tax, but the second focused on budgetary policy and structure of covering governmental expenditure by sources. Our analysis shows that the mentioned divergence is often artificial, it occurring only when the ideological aspects prevail in disputes. In our opinion, the two mainstreams are more complementary. The two mainstreams are important for the transition period of Eastern economies, they being confronted with two simultaneous processes: de-regulation process relating to the former economic system and regulation process relating to the free market system. Regarding the methods used to estimate the size of underground economy in these countries, clearly indirect methods are more adequately at least for the present day period. This study has two parts: a first part which deals with main approaches of underground (informal) sector in specialized literature, its main components and methods used to estimate its size and dynamics (a distinct point represents the approach of underground sector in East European countries, its quick expansion during the transition period); the second part deals in detail with main classes of models existing in literature, presents their comparative estimating results of the size of underground economy and evaluation of fiscal policy effects, and some of our results regarding problems of underground economy modelling in East European countries.

PART ONE: **CONCEPTIONS AND METHODS**

2. COMPONENTS OF UNDERGROUND ECONOMY

In this chapter, after a general presentation of the underground economy structure within national economy, we analyse its main components in case of each type of economic system.

2.1 Architecture of Underground Economy

In a manner large, the underground economy covers very various activities: illegal (traffic, corruption, etc.) but as legal, that self is not counted by the national accountancy (domestic work, voluntary service, convivial activities or mutual aid of vicinity), or are not declared to social and fiscal administrations (fiscal fraud, clandestine work). In addition, the forms and dimension of the underground economy vary by economic system, legislation and others local particularities.

Function of criteria adopted for the approach, there are various forms of underground economy. The most frequently, the approach begins with the investigation of relationships between underground economy and national accountancy. In this context, taking account the character of reports with the State, often some authors oppose underground activities or generally the underground economy to the official economy (private and public). So, the underground exchanges therefore undertake in margin of controls of the State and under two distinct modes: the concealment (occult economy) and the autonomy (autonomous economy) (Rosanvallon, 1980). However, in framework of national accounts, the notion of production is heard as "the economic activity socially organized consisting in create goods and services exchanging on the market and/or obtained from productions changing on the market". Such definition of the production allows, at least from a theoretical viewpoint, the evaluation, within national accounts, of productive clandestine activities, but it excludes, by convention, the non-merchant underground activities that are exerted in the domestic framework. Elsewhere, in the recent years the national accounts strive to take in consideration the underground economy, to leave from inquiries and rectification. The international definition (adopted notably by the European Community) of the underground economy understands two components: the fiscal fraud and the work to the black or "black economy".

The first reefers to the legal productive activities exerted by legally recorded units that make the object of declarations under - evaluated. The second understands the legal productive activities exerted by units non declared (stowaways) (these activities are designed as "economy to the black" by Eurostat, the statistical office of the European Community) and also the illicit productive activities or "criminal economy" (production and trade of drugs, illicit trading, etc.).

There are some authors that have concentrated on the identification one by one of the components of the underground economy and on classification of motivations that they are underlying. Such taxonomy has their permits to evaluate the extent of the various activities, to discuss on causes and implications and to conclude, by unifying the particular estimations, on a global appreciation (Heertje and Barthelemy, 1984; Barthe, 1988; Pestieau, 1989; Debare, 1992). This is the so-calls analytic method.

Contrary, other authors, on the rule those who are the partisans of the global or composite methods adopt an approach on an inverse way. They obtain, by aggregated methods and models, global estimations for the entire underground economy and then they seek explanations on the obtained results and to establish the place of various components within the underground economy. This represents the so-called synthetic method.

We are interested equally in the both types of approach, but concerning the application of quantitative methods on the Eastern economies during the period of transition to a free market system, we are forced by objective reasons (mainly because of lack of data from samples and statistical surveys) to be more concentrated on indirect or global methods.

The first approach considers a detailed structure by activities of the underground economy as following: 1 - penal activities that comprise transfers (flights, defrauds to insurance, scythe - currency, swindles, etc.), production and distribution of goods (drugs, pornography, etc.), production and distribution of services (whoremongering, illegal games, etc.) and crimes against people; 2 - fraudulent activities understanding fiscal fraud and work to the black (including the clandestine immigration); 3 - non-marketable activities understanding domestic work (cooking, housekeeping, gardening, serves of vicinity, etc.) and voluntary service or the convivial economy (to the associations or person benefit).

All classifications of this gender call immediately reserves (1). They are more empirical than systematic and do not avoid the crosschecking. For example, in the fraud, as in the work to the black, there are some penal aspects as also in the case of clandestine labour migrations. Also, in other examples, income of an independent concealing only a part of its income or a wage earner not declaring income of a second job can be resumed in official statistics by usual rectification (clandestine labourer fee can be reflected in statistics by the form of expenses of the beneficiaries of their services). Other example, the refund undertaken by a companion of insurance to compensate a proprietor of a building to which it has voluntarily put the fire is taken in official statistics. There are also frequently transfers of activities from the marketable sphere to the non-marketable sphere. To fasten the domestic work and voluntary services to the underground economy generated many discussions among the economists. On the one hand, they resemble activities that would have to be added to the national production if one wanted to measure it correctly. On the other hand, in many cases, they are motivated reasonably consciously by an economic calculation in which the intention to escape all taxation and all regulation plays a considerable role.

Therefore, overlapping being numerous, there is convention to place each activity in an alone category taking into account the major motivation that is underlying the concerned activity (Pestieau, 1989).

Some that concerns the stallion to appreciate the relative importance of the underground economy we use more frequently in this work the GDP (Gross Domestic Product). Thus, excepting all debates reported till now, we consider the GDP recorded in official statistics as reflecting the activity in the visible sector of a national economy. To evaluate all activity unfolded during a year in a country, we consider a greater level of GDP (global or total GDP) including so the activity in the visible sector that the activity in the invisible sector of the underground economy. Making the difference between the two levels of the GDP can obtain a global estimation of the dimension of the invisible underground economy at macroeconomic level (the so-called invisible GDP). To obtain the dimension of the whole underground economy it is necessary to add the visible share of the underground economy, it is to tell that already recorded in national accounts (because of the discoveries realized by the fiscal authorities or by estimations coming from the extrapolations made with occasion of polls, inquiries, controls, etc.). The problem of what share of the underground economy is already enclosed in the national accounts will be approached in the next chapter. In Figure 2.1 we present a structural scheme of the national economy, including all activities, formal and underground, production of goods and services generating total GDP obtained during a year in a country (2).

According to several authors, the structure by components of the underground economy varies by category to which is affiliated a country or other.

In the countries of the Organization for Economic Cooperation and Development (OECD), the underground economy presents some sameness, so some that concerns practice them "black" that in their motivations. Underground activity appears as parasitic in the industrialized countries of the West, regulatory of shortages in Eastern countries, and factor of development in Southern countries.

Until the collapse of the communist regimes, to leave from 1989, the Eastern countries had their "second economy", a particular parallel economy. Today, it seems to survive and adapt to under way economic mutations in Central and East European countries. Developing countries, in spite economic situations very heterogeneous, characterize by a large traditional sector that escapes the State. This "informal economy" constitutes in number of case the mode of dominant production and competes, otherwise to the development, the less to the surviving of population.

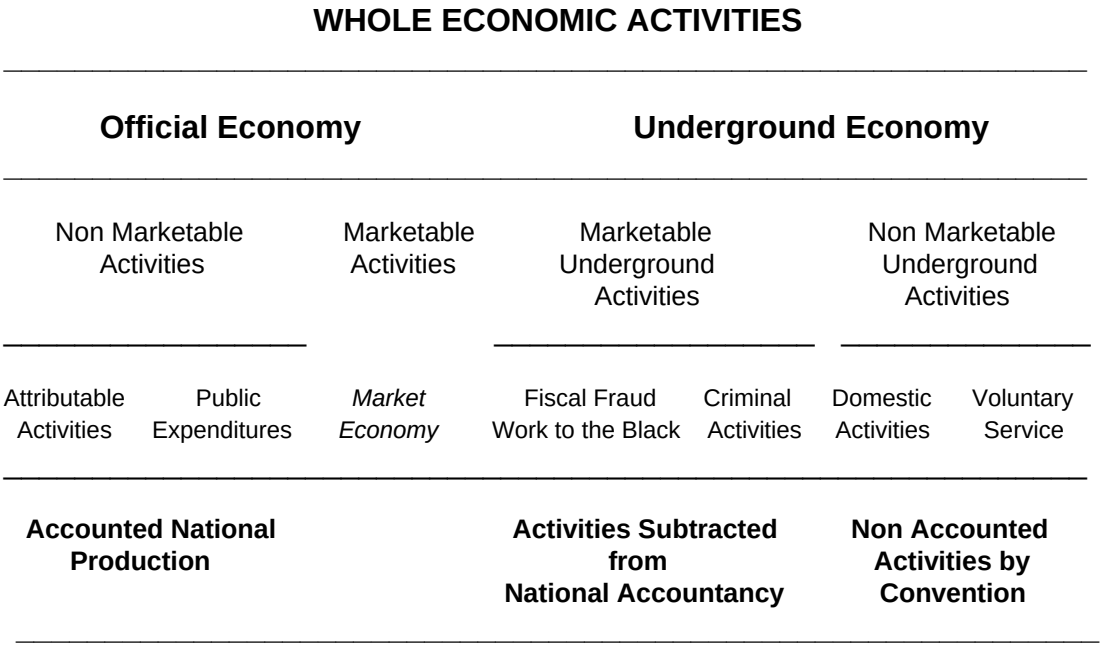


Figure 2.1

Generally, one can assert that in Western countries there is a tendency to count and control underground activities, but in developing countries there is a tendency to tolerate them. In post-communist countries, at least until the present, there is an ambiguous attitude regarding the underground activity.

By many authors, the underground economy (including the domestic economy and voluntary service) represents only a fraction of the official economy in the case of Western countries, but it is sometimes greater than official activities in the case of developing countries. In the case of Eastern countries at least in present period there are no clear estimations at global level (3).

From current system of national accounts one can notice the exclusion by convention of underground activities non-marketable, those notably exerted in the domestic framework. This share of "forgotten accountancy" has consequences limited in industrialized countries and especially in Europe where the non-declared legal activities represent really the most important part of the underground economy. However, this is not the case in developing countries, where the self-consumption remains important and where the non-official production is often the most important part of the marketable economy. Probably, in Eastern countries there is an intermediate situation.

In the resentment that the problem to what basis it is necessary to bring the figures on a component or another of the underground economies remains opened, we present synthetically the estimations obtained by various authors relatively to the dimension of the underground economy and its components.

2.2 Underground Economy in Industrialized Countries

In Western countries, a certain part of each component of underground economy, except for the fraudulent and criminal activities, there is already included in national accounts. After some authors, alone in Italy and USA it seems that it was enclosed a share of fraudulent and criminal activities in the official GDP (4).

2.2.1 Non-Merchant Activities

The domestic work occupies a preponderant place not only among components of the underground economy, but also in the whole production of the society. On the other hand, the convivial activity is a growing component of the underground economy in Western countries.

After some authors, in the last decades, there is a transition of some productive sectors from the formal framework to a domestic framework that constitutes the most important transformation that has assigned the totality of the economy. Thus, Gershuny refuses to consider as base of economic development what he appoints the growth in unique sense - it is to tell the passage of the primary sector to the secondary, and of the secondary sector to the third sector. His position consists on the contrary in defend the idea of a swing between formal economies and informal economies - it is to tell marginal, domestic, and community (Gershuny, 1979).

In the same framework, we can place the central idea of the theory of Gary Becker, that asserts that the time is an economic resource whose optimal allocation between different usage, leisure, sleep, domestic activity, trip and profession, allows to household to reach its level maximal well-being (Becker, 1965). Other studies have insisted on the possibility of substitution between domestic work and merchant work (Gronau, 1973). In several countries, one has been able to verify the predictions of the model on empirical data. These predictions have generally been validated. Also one can go more far and to measure with a certain precision the effect that a modification in social or demographic variables can have on this offer of work (Pastieu, 1989). In the next chapter we will insist on methodological aspects of the underground economy evaluation, but here we expose alone the results of estimations.

From numerous studies, the value of the domestic production is estimated generally in Western countries between 1/3 and 1/2 of the national product. In Annex 2.1 we present a table containing these estimations. One can observe an important variation according to the method used and to the year of reference. Generally convergent as absolute values in case of the same country, however the estimations on the dynamics must be considered with prudence. It is because, generally based on surveys and samples of households, in time occur many changes and influences of various factors, that is a weak structural stability in time.

Apart from numerous discussed on the role of the voluntary service within a modern society and on the structure of the population employed in this gender of activity, there is unanimity to consider a true explosion during the last years. Generally, it is considered traditionally a greater extension in USA than in Europe and greater in Nordic countries than in Mediterranean countries. Also, it is considered that the extension of voluntary service is greater to the richer population, also that there are some differences between countries concerning the structure of population participating in this type of activities by age, sex, occupation, education, religion, etc. To estimate the importance of the voluntary service at national level exist only some subjective estimations that seem to converge to a value of about 5% of the official national product.

2.2.2 Merchant Activities

In their totality, the marketable underground activities or "black economy" represent about 15% of GDP. According to the method and to the country but there are great differences between estimations walking from 2% of GDP to more a quarter. Thus, Manasian has registered in 1987 those that were disposable for Europe. They vary evidently from a country to other, but also in case of the same country. In percentage of Gross National Product (GNP), "black economy" situates between 0.5% and 17.2% in Sweden, between 3.8% and 12.7% in Belgium, between 2.4% and 16% in the United Kingdom, between 25% and 30% in Greece, and between 15.4% and 30% in Italy.

2.2.2.1 Fiscal Fraud and Work to the Black

Fiscal fraud and work to the black go often peer; there are however cases when they go one without the other. A surreptitious worker can fulfil most of his fiscal obligations

because of the fact that the taxes on income and social contributions are collected at the source. On the other hand, an independent who works on street and conceals to the fiscal authorities a part of his income cannot be qualified as labour to the black. There are some other differences between these two underground practices. The fiscal fraud covers a larger terrain since it does not concern solely income of the work. Motivations are different. The labour to the black seeks an income; if this income avoids the taxation will be better. The defrauder has an income that he wants to preserve in integrity. The labour to the black does not respect firstly social laws while the defrauder skirts the Code of the taxation.

Generally, it seems that the labour to the black is found mainly in weak income classes but the defrauder on average belongs to the richer classes. For social and professional categories, one finds defrauders in the groups of the independent workers or renters and in the liberal professions. Labourers to the black are mainly the inactive persons, the stowaway-immigrants, and the employees having a second occupation.

The main impact of the fiscal fraud is on the side of public receipt losses, but the work to the black is important for its impact on the labour market. The fiscal fraud idea gathers the totality of illegal practices that allow avoiding partly or totally the taxation. These practices are susceptible to penalties, perhaps penal. According to countries and periods, the social tolerance regarding the fiscal fraud varies, and with it, the frequency and severity of the controls. The fiscal fraud is not the alone manner to evade the taxation. There are many other efficient methods to evade the payment of fiscal duties, such as fiscal evasion and entire panoply of deductions, exemptions and credits on taxes that understands all fiscal system. Excepting the judicial aspect, the distinction between these techniques of non- payment of the taxes does not justify truly. The effects of fiscal evasion are analogous to that provoked by the fraud. It provokes less for the budget and inequity between taxpayers. In some cases, the public opinion is also against the one or the other of these methods of tax-payment avoidance. Thus some times, the public opinion has learnt that because of the technique to evade the fiscal duties, the rich politicians often avoid the payment of the entire amount of their fiscal duties. Moreover, aside of the fiscal evasion that exploits the gaps and contradictions of the fiscal regulations, there is other method of avoidance that feeds from the break in the profit of some taxpayers' categories. All time, the most powerful professional organizations use their influence on parliament to grant exemptions for their members (5).

One sees it on the channel from the legal to the fraud; there is no rupture but continuity. Between behaviours clearly legally and behaviours that are without ambiguity illegally, there is a zone curl. It is gradually that the taxpayer slips from the error to use fiscal option to the simple abstention and to the ability and the abuse of legal rights to the qualified fraud.

Concerning the evaluation of the fiscal fraud, the method the surest is considered that that leans on the fiscal administration data in matter of verifications. Then, these data are extrapolated to the totality of the taxable population and completed by punctual studies for income that escape from all verifications. This method is applied only in some countries, like in United States. In other countries, as in France, the periodic reports of designed organisms (Council of taxes in France) yield statistics on the verifications and fiscal outputs. In France, for example, for the former years the contribution of the controls to fiscal receipts represents approximately 5% of emissions of tax on the income and 8% of the product of the tax on the enterprises. This type of information does not teach us but anything on the actual importance of the fraud. From time to time, ministers of Finances, the Budget or the Justice give their clean evaluation but without quoting some the source or the method of calculation. Generally, figures that circulate are relatively low - approximately 3%. To the total, according to the INSEE, the economy "to the black" represents 4% of the GDP whose 3% for the fraud and fiscal evasion and 1% for the work to the black.

In revenge, non-official sources give greater estimations. According to Godefroy and Laffargue (1984), in 1979, the fraud represented 17% of the French fiscal receipts. According to Frank (1977), this figure was in Belgium of 18% for 1975. The rate of fraud varies according to the nature of the taxes.

2.2.2.2 Delinquent Activities

The crime is a violation of the penal law. So the content of the law, but that the manner how it is interpreted and applied may vary at one time and in the space. To lean on the criminal phenomenon to surround some economic impacts is without null doubting a delicate and common bit. By Kellens, "the data on the cost of the crime are not purely and simply transposable in terms of profit: the cost is not the simple reverse of the profit" (Kellens, 1987). Pestieau (1989), after an analysis of this phenomenon, distinguishes three categories of criminality, that those destroy the human or physical resources, that that produces goods and services and finally that that constitutes a simple transfer. In the national accountancy viewpoint the first category would correspond a subtraction, the second - an addition and the third would have a neutral effect.

Certain prudence is required nevertheless in the manner to post the criminal productions, a similar prudence that in the case when the national accountancy ignores the degradation of the environment that keep up some productive activities. Also, there is little estimation from an economic viewpoint of the properly criminal activities.

We are interesting more in economic crime in a broad sense because it can be better evaluated that the criminality whose motivation is not before all economic and whose impact is not directly economic (rape, political assassination, etc.). In a such study that present an monetary estimation for USA, besides the fiscal fraud that includes the work to the black and the clandestine immigration, the authors distinguish two great categories: illegal transfers of goods and services (flight, swindling, corruption, fraud) and the illegal production and distribution of goods and services (drugs, whoremongering, usury) (Simon and Witte, 1981). To the total, it gives, for the year 1974, some of more of 12% for the entire illegal activities, from which about a half for what is not fiscal fraud and work to the black. It is a specific feature of USA by report with many other countries. The properly criminal activities occupy there an important place. Unfortunately, it does not exist evaluations more recently. By some appreciation, it would seem that the trade of drugs, mainly that cocaine, has strongly increased since. The fiscal American administration gives for 1981 the figure of 23 thousand million dollars (IRS, 1983) comparing with fifteen billions as evaluation for 1974. Estimations more recent go until more hundred thousand million dollars.

In revenge, in a study provided by the Center of Sociological Studies, the criminal activities in France have been evaluated in 1982 to the total as some of 4.1% of the GDP (2.4% fiscal fraud and 1.7% activities properly criminal). In this study it has been evaluated also the cost of the control of the criminal phenomenon. To the total it (including repression, prevention and protection) represents approximately 1.2% of the GDP (Godefroy and Laffarque, 1984).

In a global estimate for West - European countries the properly criminal activities are appreciated to approximately 2% of the official national product (Pastieu, 1989).

In the last decade, some authors consider that there is a diversification as well as an internationalisation of underground activities. Among new developing forms of underground economy are considered: extension of speculative activities on the financial markets and especially on financial bourses; internationalisation of the networks of production, transport, and distribution of drugs; occurrence of new zones from the so-called category "fiscal paradise"; extension of illicit international trade and the so-called "organized crime". For instance, during the period 1980 - 1990 the difference between the international exports and imports increased from 20 thousand million dollars to about 100 thousand million dollars. In these conditions more international organizations become high interested in research of the "hidden face of the world economy" (Debare, 1992).

2.3 Underground Economy in Developing Countries

The general situation of developing countries is very heterogeneous. However, these countries are characterized often by the subsistence of a large traditional sector whose State is absent. In these countries, non-official economy constitutes generally the mode of dominant production, and determines for part the level of development. This situation explains that under - developed economies are sometimes qualified as "duality".

2.3.1 Components of the informal sector

The importance of the informal or non structured sector, notion created by international institutions - World Bank, International Labour Bureau (BIT) - to the beginning of the years 1970s, in developing countries can be partially and paradoxically explained by the strong intervention of the State in number of these economies. Even if an evolution to a very fort economic liberalization is made day, States of the third-world are long attached to stimulate the development, notably by plans and the implantation of industries that they expected to be "industrial-incentive". This state interventionism did not integrate the traditional modes of activity. This idea of the development condemned some leaves the traditional activities; non-to vanish, but to become informal.

Forts of this distinction between an official economy planned by the State and an informal sector gathering the activities which escape from all controls, some observers assert that the informal sector allows alone the surviving and even the development of these societies (BIT, 1972; Adair, 1985, 1995; Lautier, 1994; Cherif and Nafii, 1995, etc.).

One distinguishes traditionally in developing countries two fundamental forms of the informal economy. Such there is a "primitive" form that covers mainly the agricultural auto-production and the auto-equipment of rural zones. It concerns the activities that undertake in the proximate continuation the domestic work, the frontier being well difficult to trace. The other part of this informal sector gathers the merchant activities, those that make the object of change. It concerns artisan or commercial activities and services, generally at small scale. They are multiplied with the rural exodus and the demographic explosion that provokes the increase of cities and urban outlying districts. The proliferation of these activities undertakes in margin of all legal obligations and escapes from the state regulation. However, the local public authorities tolerated them, because they allow attenuating social tensions by absorbing a part of the underemployment and by diminishing some shortages.

To illustrate this heterogeneity, one can examine the degree of marketization of a country. In an economy of barter, goods are generally changed against goods with the losses well known of a system: it limits the number of transactions; it breaks a rational allocation of resources; it makes necessary an ineffective risky stocking of goods; finally, it breaks the specialization and the division of the work. By some gross evaluations, in the years 1970s in some developing countries, especially African countries, the fraction of non-monetary economy within the national economy is very important: 49% in Rwanda, 45% in Ethiopia, 42% in Niger, 39% in Malawi, 38% in Burkina Faso, 33% in Mali, 28% in Tanzania, 20% in India and in Malaysia, 13% in Dominican Republic, etc. (Chandavarkar, 1985).

The analysis of the non-structured sector has taken its genuine flight due to the launching of the BIT's world program on the labour. Seven criteria there were considered to define the informal sector. They concern all directly or indirectly the job. They are: 1 - The facility to entry that means that each person can provide its basis needs by exerting this type of activity (6); 2 - The utilization of local resources: economy of proximity, job of family assistance and auto-financing; 3 - The family property of enterprises; 4 - The small scale of enterprises: number of their staffs are less than 10, according to the statement of Sethuraman (1976); 5 - A labour-intensive technology adapted to the local conditions; 6 - Of acquired training outside of the scholastic system; 7 - Of the markets of competition non-regulated to the relative disposition look to the salary, to the security, to the conditions of work, etc. (Lubell, 1991).

Other authors propose different criteria, as: absence of institutional credits, a production destined to the final consumer, absence of fix work schedules, etc. (Sethuraman, 1976). On the look methodological, the informal sector be characterize by escape to empirical investigation ways: producers working without permanent job, exemption of the licence or import payment, exclude the social regulations, having no accountancy or by sources of income, on the one hand formal - salaries and various allocations - and, on the other hand informal, according to the criterion of the salary and the auto - job (Hart, 1973). However, the informal job regroups generally family jobs in the artisan sector and the small profession as well as the occasional or temporary activities within the modern sector (construction of the buildings, daily of stocks, wheel of work hand, etc.). The informal sector also distinguishes by a judicial and social organization different from that in formal sector. Thus, Mazumdar (1976) defined a labour market non-protected without the system of social securities or a market of competitive products and non-regulated that delimit the field of the non-structured activities. Several other works taking some counts the heterogeneity of the informal sector, distinguish a

modern traditional sector (Nihan, 1980), or subdivide the non-structured sector in two sub-sectors (Steel and Yasuoki, 1978; Steel, 1980). The first sub-sector, unregulated or residual, encloses a large menu of legal activities - marginal (gardening, washing of cars, begging, etc.) and small prestige - and number of illegal activities; the second sub-sector, regular or intermediary (7), understands the economic activities at small scale by family enterprises and generally non-salaried.

2.3.2 Economic Policy and Informal Sector

In developing countries, can be probably stronger than in the other countries two opposite theses, concerning effects that the informal sector induces, the decisions of economic policy that it calls, are put in discussion. For some, losses registered by the State are considerable, the lack to earn are recorded at all levels. The informal economy would carry wrong on the totality of an national economy, such as: the non perception by the rights and tax State being able to reach several billions, the loss registered on the stage of the marketing (indirect taxes) and the diminution of the stocks of strong currencies. The national accountancy tends to under evaluate the GDP and the national wealth by inhabitant. The contraband has internationalised, and there exists of parallel circuits having important financial resources and ways of communication, fraud and modern and sophisticated transportation that endanger the security of the State. In this context, the objective no longer is to maximize indirect or direct fiscal receipts. It is doubled by an objective concerning the security of the "gendarme" State or "providence" State.

For the partisans of the second more liberal mainstream, the informal economy represents a spontaneous response to the incapacity of the State to satisfy aspirations of the poorest people. In now a legal device a lot too constrains, if one believes some the Laffer curve, it corners agents to operate in margin of the law. By escaping such system, agents of the informal economy would be far more productive that the State. Thus, in these conditions, the crisis and the recessions seem to be only a statistical illusion? More again, the economic policies are not they led to failure, since the majority of the activities and agents escape completely from the public decisions? Moreover, these all are not integrated by the economists in their plan, and whose main virtue is to profit the totality of the population, farmers, carriers, traders, etc. Today, some authors from developing countries consider that the developed countries tent to formalize the informal economy and restrain the field of action of the informal activities, with often of perverse effects such that the development of the exclusion. To the reverse, in some developing countries the informal economy is often considered as being a main factor of development, it is the case notably of economies of Maghreb (Cherif and Nafii, 1995).

Some of such problems are current also for the post-communist economies of the Central and East-Europe so that others of them are specifically.

As an order of magnitude of this informal economy in developing countries, by some estimations, it represented in 1980: 40% of the GDP and half of the active population in Peru; 46% of the active population in Brazil, 40% in Mexico, 29% in Chile, 26% in Argentina, etc. Estimations more recently, for countries of Maghreb that have adopted of policies sustaining the informal sector, give: 50% of the GDP and 57% of the active population in Morocco, 36% and respectively 26% of the active population in Tunisia and respectively in Algeria (Charmes, 1990).

2.4 Underground Economy in East European Countries

Before even to undertake the reforms destined to establish market economy, Eastern country had their private sector: the underground economy. Until the collapse, to leave from 1989, the communist regimes and their planned economies, these countries knew a parallel economy, fundamentally illegal, but that had everywhere straight to existence. Here we can mention Grossman (a pioneer concerning the study of underground economy within socialist countries) who qualifies it as "second economy". The upheavals intervened in Europe at the end of the 1980s do not threaten this "second economy". Null doubts that it will know to adapt to economic mutations under way at Warsaw, Prague or Bucharest. But "shadow economy" will lose without doubt its specificity.

The socialist economies of Eastern Europe, including Romania, were also qualified as "economies of shortage" (Kornai, 1980), the production and services being always smaller than demand. During the 1980s, the disparity between increased waits and the system's deficient performance emphasized (8).

In official economy, the systems of cooperation and the networks of supply degraded, causing the stops of work that one sought to avoid, to the exterior by informal exchange with other enterprises, to the interior by the substitution of supply sources and by the improvisation. A growing share of the production as well as of the time of work was turned aside to the households and their social networks, because it was in the distribution of consumption goods that the shortage was made feel the most vigorously to see it the abundance in West - European countries that demonstrates "to doors". More, the households produced and exchanged themselves a goods and services share that made absence. The transition from countries of Europe Central and Oriental to the political pluralism and market economy favours the temporary development of new underground activities, due to the fact notably of the disorganization of traditional distribution circuits. Economic mutations will limit without doubt some forms of parallel activities specifically to countries of planned economy (9). The disappearance of term of "shortage economy" would have to render void number networks, circuits and illegal markets whose one can paradoxically underline the role in the apprenticeship of market economy. It remains that the ex - socialist countries would not have to see disappear underground economy and that they seem to registered, due to the favour of the disorganization of the ancient system and the too slow installation of the free market, a development worrying of the Mafioso activity.

Under the former communist regimes the underground economy development had some particularities. That is why firstly we expose briefly these particularities and then we analyse the present problems of underground economy development during the transition period.

2.4.1 Underground Economy in Communist Planned Economies

Before 1989, the omnipresent State and the scarcity of goods carried a particular underground economy, product of the planning and its associated shortage, factor and of the depersonalising of the property due to the collectivisation. This "second economy", by opposition with the official system of planned economy, covers "all what relieves of the research of private gain, more all what is in fine with the law" (Grossman, 1977; Duchêne, 1981).

This heterogeneous system of prosperous but illegal activities is often tolerated under the shade of planned economy. The degree of repression depends of the appreciation of public authorities and especially of the economic situation, to the extent in which practice them underground play often a role of regulation of shortage. The repression depends often of the degree of corruption of the responsible premises, and the visibility more or less great of underground economy. Many repressed activities by the law, at least from a theoretic viewpoint, benefit of the social consent. For as much, the tolerance of the State has limits, that Pestieau underlines when he evokes condemnations for "economic crimes" in the ex - USSR: about a million per year, what gives elsewhere an idea of the generality of this gender of practices.

This second economy feeds transfers of productive factors from the planned sector to the non-official sector: work and goods are turned aside. The disguised unemployment, characteristic for Eastern countries, and the "work double" - the official time of work may be shortened to allow an external activity, it can also may be put in profit to undertake instead of the official production a clandestine production - provide its labour to the second economy. To the activity exercised in an enterprise of State or a cooperative is added or combined often a work of the shade. In the same manner, products are frequently "deducted" on stocks of the State enterprises. Underground economy was developed therefore as parasitic of official economy. It brings however the responses to defaults and rigidities of planned economy and notably to the shortages, to the standardization, to the bad quality of products or to the problems of supply of cities.

In a general approach, one can consider that the informal activities of the households cover four distinct forms: the self-production of goods and services consummated by the

households themselves; the production for and exchange within the social networks; the work to the black and exchange of materials on the black market; semi - formal economy, in the framework of which official institutions buy informal products or remunerate of "volunteers" works. In the same time, in the enterprises, there were three forms of informal work: the diversion of stocks, the diversion of equipment, and the diversion of time of work.

If second economy is globally destined to produce goods that will be self-consummated or changed on the parallel market, the underground activities in planned economies know forms also various that in industrialized countries. To aside of official economy official there were constituted "private - family" and "informal" economies. These clandestine economic activities carry essentially on the production and exchange of rare goods or rationed (gasoline, meat, vodka, etc.), and on services of all sort (repairing, medical benefits, works of construction, transportation, etc.), other activities are very diffused: flight for domestic purposes), exchange of services, trades, and work to the black. Pestieau distinguishes between illegal private economy (clandestine private enterprises, black market), legal private economy (notably in the agricultural sector, where some patches give place to a private activity far more productive that in the State cooperatives and farms), and public illegal economy (deduction of produced goods or those destined to the State enterprises, diversion of the official work time to the private purposes).

In enterprises, the material was turn aside. This was not only of the various pieces of replacement that made always absence, and that all the enterprises stock great quantities, because all the sector was under - developed. It was of the quantities more or less great of the production or current distribution - especially those that survive only periodically: exceptional or rare foods, the clothes product exclusively for the export to the West, from woods or the taps or costs of stones utilized for the small buildings constructed often without permission, cure or place prescriptions of holidays, etc. One used in an informal manner the workroom and services of enterprise, is for the repairs, is for transportation in cars of service.

The time of work was turn aside, been to accomplish orders "under the table" in the enterprise, is to make commissions outside (often to the service of the entire collective of work). Friday in the afternoon (or in Romania, Saturday in the afternoon), the supplementary or "volunteer" work, night shifts were the preferred times of informal activities in the enterprises. Nevertheless, the former were currently practiced only in some branches and in some situations of job; heavy industry, personal and social services, the administration of routine were propitious bit, as well as the work at piece or to the little window for public services. Some enterprises searched for to channel these activities by offering contingents of their products or second hand pieces of change. Most propose elsewhere of the materials and works "out statement" to their employees or their retreats; the field of the informal activities becomes blurred (10).

Generally, the self-production of the households had a best reputation and was encouraged by the State. Most important areas concern the construction and the maintenance of the accommodation, the gardening, the mutual aid, etc. (11). The social and charitable commitment was almost exclusively feminine: it profits especially to relatives and to close friends. There was also a remarkable voluntary assistance to poor children or to the handicapped persons, the activities of protection of the nature, etc. The State sought to encourage and to channel these activities in the framework of official organizations. The qualified force of work was often associated within a particular professional association: the great repairs, the more refined wardrobes, the judicial advice was found more easily and at a smaller cost in the social networks (12).

The economic importance of these activities is very difficult to be evaluated. The research in Poland or in Hungary line of the increasing role of the domestic work, estimated a value of about 1/4 of the GDP, and for the informal work estimated a value of 1/10 of the GDP; both have known an expansion in the course of the 1980s (Sik, 1994).

Apart from the household, there was a narrow circle of relatives and close friends that benefited first by the informal work. It concerns less a logic of the utility or maximizing of the "social capital", than motives of moral obligation and social commitment that prevailed in the distribution of informal productions in the "narrow circles" in East-European economies (13).

Parallel economy of planned economies covers also hatches a bond informal, the networks, circuits and markets parallel. The markets that organize to palliate defaults of the system and notably the absence of plug in account by the population demand plan. Contrarily

to the planned economy, the underground economy is oriented by this demand. It tenders to satisfy it is by local productions, is in restoring a certain international product circulation. Eastern countries, indeed, were struck by restrictions on imports, and the non - convertibility of their currencies. Finally, the logic of second economy is merchant, and it is the confrontation on the black market between the offer and demand that allow the fixing of prices.

If the totality of Eastern economies was characterized by state control, the extreme centralization and the authoritative mobilization of the material and human resources for the realization of plans, there were, despite the alignment on the Soviet "model", some differences from a socialist country to another. The differences could affect importantly on forms and conditions of exercise of the underground activities. Romania during the Ceaucescu's regime has known, in the 1980s, a terrible hardening of the repression against underground economy - some explain even the destruction of villages and the roundup of the population in collective real estate by the will to diminish the possibilities of embezzlement of the agricultural production. But this repression has not been able to come to end from informal practices and mechanisms rendered inevitably by the width of the shortages. In Hungary, second economy was on the contrary limited by measures enlarging the field of recognized action for the small private enterprise. In Poland, public enterprises themselves participated in the "second circulation" and in the development of underground economy by organizing distribution of missing goods.

2.4.2 Underground Economy in Transition Period

In a short period, Central and East-European countries has undergone deep transformations that may be conceived on three plans: the marketization of economy and development of the private sector, the machine of State on which institutions and the national regulations were grafted, the economic framework and institutional of the daily life.

From the viewpoint of the informal sector, some of more important transformations were the abrupt extension of market economy and especially the marketization of the supply of the households. Previously, these were sustained for an important share by subsidies to products and services of basis; number of cultural and social services was almost "gratuitous". More of a total income quarter of the households had consisted in "indirect income". Today, this is monetary income of the households that determined largely the standard of living. Public assistance is made to complain as individual rights. Since transformations of the regimes, a relative product abundance and private services supplied mainly by Western assistance, by the foreign enterprises and an opening of goods imports of consumption subsidized has submerged a national market once deficient. On rule, during the first year, after the changes of communist regimes, the households consume some largely. Industry of consumption goods that in same time had lost Oriental markets, as well as Western market (by elimination of the some ancient facilities of subsidy), undergoes a second hard knock. A great number of gratuitous social services, almost many of the consumption subsidies, and a share of assistance with families were demolished since, that drugged a rapid rise of prices of basis consumption, fares and rents.

But income records a rise slower than the productivity, because unions and the State were forced to maintain a switchboard of life higher than the past. Effects of these modifications were distributed unequally, originated the two appearance categories: the "winners" and the "losers". Also, the other continuations from transition to an economy of market in the East having impacts on the underground economy were unemployment, elimination of restrictions on labour market and on the time of the people, passage to a more liberal legislation concerning the withdrawal from legal job and multiple-job, a lot of successive transformations of fiscal laws and the inefficiency of activities destined to controls and penalty on the non-declared income.

There are two hypotheses concerning the social transformation and the impact on underground economy that would seem plausible. The first is that of social polarization. Odds of job diversifying socially, and the standard of living depending almost entirely of the situation of job, there will be a polarization between, on the one hand, the households with several jobs that have thus an informal potential more important (in terms of property, opportunities, social relationships, etc.) and, on the other hand, the households partially or wholly in

underemployment that, by lack of ways, realize less of informal activities (Pahl, 1988). The second hypothesis is the marketization of social relationships. The standard of living depending largely of their monetary resources and social rights, the active persons see exposed to fluctuations of the labour market, but the non - active persons depend of their individual capacity to impose their rights in a system of justification. The development of monetary economy tenders to render anonymous and to make objective social relationships: institutional relationships will be depersonalised; among personal relationships, those that are oriented to material purposes will be diminished, relationships in the "large circles" will be made objective, the "narrow circles" covering exclusively an affective character. At present, there are many debates on the availability of one or other hypothesis (Neff, 1995). Surely, today, some of objective conditions of the economy of shortage have disappeared. On the other hand, the deepen liberalization of the social and economic life as rapid legislative transformations, sometimes non - correlated and sometimes non-coherently, have opened, at least during the present period of transition and in same time of crisis, a large manifestation field for the underground activities exerted by a population that was forced to practice them several years during the former communist regimes but in conditions incomparably harder.

In enterprises, it is expected that in some years the informal work will diminish to a comparable level to that usual in Western economies. But, in countries where the process of privatisation advances too slowly or where the intervention of the State in economy continues to be great this is not place. On the other hand, an atmosphere of distrust and competition seems to forbid the informal activities. In the public life, in revenge, trends of sociability, factor determinant of the informal work for the social networks, diversify.

A minority, rather youths and these of average age, uses fully the new possibilities of expression and free organization and the opportunities of consumption. The oldest, the persons without job and those having low income have tendency to pull in a narrow circle of relatives and ancient. This has contributed to diminish social relationships "large", which characterized the former socialist society. In the closed circle of relatives and friends, contacts have become non-frequently, but in the same time more intense than previously (Neff, 1995).

Since 1989, by some authors, at least in the case of Eastern Germany, exchanges between colleagues and contacts of leisure were diminished strongly, the debates and consultations within family and between friends increased, the mutual aid diversified (diminution in case of active persons and increase in case of non-employees).

Also, since 1989, the households have reduced essentially works of sewing as well as the production and the repairing of utensils; fewer forts being the diminution of works destined to conserving of food and repairing of car (the electronic pieces in the new cars constricting the repairing). The increased offer on the market is designed as essential cause. Also, on the pressure of the marketization of economy, it seems that many of the ancient domestic activities diminished.

Concerning the estimation of underground economy size in Eastern countries, after 1989, the literature is too spare. Although there was some preoccupation (14), the estimations remain most divergent, various figures being emitted by different institutions without an explanation of the calculus mode (15).

NOTES

(1) There are many other criteria and possibilities to present the structure of underground economy. In this subject, we recommend without the works already mentioned those of INSEE (1976), Gershuny (1979), Blades (1982), Feige (1982), Archambault and Greffe (1984), Gaudin and Schiray (1984), Greffe (1984), Porter and Bayer (1984), Adair (1985).
(2) The structure of national economy presented in Figure 2.1 is based on that proposed by Smith (1986).
(3) This situation explains especially by lacking data relative to the dimension of the underground economy during the communist regimes and, on the other hand, by multiples reform during the recent transition period so of the statistical data system and national accountancy that the legislation.

(4) According to Blades, who has questioned the statistical institutes of all countries of the OECD, it seems that Italy and United States are the alone two countries where illegal goods and services record in national accounts. In Italy, one mentions cigarettes of contraband. In the United States, one estimate a share about 10% of all production of illegal goods how appears in declarations of income under the particular rubric "other income" (Blades, 1982).
(5) See, in this matter, the book of Bennett and Dilorenzo (1984), that treats in large the practice, by various methods, of tax-payment avoidance in case of public enterprises.
(6) This criterion is a cause that the non-structured sector is considered as a main factor of the absorption of rural exodus (Charmes, 1987).
(7) Contrary to modern sector using intensively fixed capital that represents an obstacle to admittance of individual entrepreneurs, the intermediate sector disposes of small quantity of fixed capital, criterion being in this case a positive marginal productivity (Steel and Yasuoki, 1978).
(8) In literature, there are many studies on this subject. Here we mention only some of that realized after 1989 within The Leuven Institute for Central and East European Studies or the Ace-Phare programs (Linotte, 1992; Jackson, 1995). Also, in Romania's case we mention only some studies on this subject (Ionete, 1994; Albu and Ungureanu, 1994; Albu, 1995).
(9) To understand better this subject, it is useful that we remember some conclusions from some comparative studies (Grossman, 1977; Katsenelinboigen, 1977; Kornai, 1980; Simis, 1982; Dallago, 1987). So, besides the legal "regular" economy an illegal "irregular" or "underground" economy exists in all capitalist regulated market economies. Such an underground economy existed even in the Eastern countries during the former communist regimes. In certain respects underground economy was similar in Eastern countries and Western countries. For instance, in both systems underground enterprises tend to specialize in labour-intensive activities without significant economies of scales. Also, enterprises in underground sector may get inputs from, or supply outputs to, firms in the official or visible sector. In both systems, by omitting underground activity, official statistics understate employment, production, consumption, and other macroeconomic indicators. But in the same time there were many differences between the two economic systems. So, whereas the main reason for underground activity in Western economies is evasion of taxes and government regulations, in Eastern economies the chief reason was to produce goods and services in shortage at official prices set below market-clearing levels. Thus, in the first case customers usually pay lower prices for underground goods and services than for the output of regular firms, whereas in the second case they pay higher prices for underground products than for the visible output. Also, with open unemployment common in Western countries but not in the former central planned economies, underground work in the first case often is a person's primary job, while in the second it usually was supplementary to regular employment. In former Eastern socialist economic system there were two major components that together form the so-called underground entrepreneurial sector: underground private enterprises and underground private activities (Grossman, 1977; Katsenelinboigen, 1977; Simis, 1982). The first was real enterprises run by private owners and entrepreneurs; these enterprises hire workers to gain profit. They were underground because law prohibited this type of enterprise or because an underground existence was more profitable. Underground private enterprises generally supplied consumption goods and personal services because of their simpler and less risky marketability. Less frequent was the case of the supply of production goods and services, including to socialist enterprises. The vast range of production items included garments, footwear, household articles, houses, home brew alcohol, and various services (car and home repair, sewing and tailoring of garments, transportation of persons and goods, and the like). Underground private activities, on the other hand, reach large dimensions and are not independent, but are in symbiosis with a socialist enterprise. The latter unofficially produced goods and services, using inputs paid for by private persons, owners of the products and services supplied. Another case of this symbiosis between visible and invisible activities was typified by socialist enterprises managed and run as virtually private enterprises. Here, production taken place according to the plan but yields a higher quantity. The surplus remained underground and was sold on the irregular market; and organizers of the underground activity took the revenue. Evidently without widespread corruption, this type of mechanism could not function. Activities are said to be underground because they break the law, escape regulation, or transgress social agreements. They either are prohibited (such

as private enterprises in certain sectors of central planned-type economies) or are performed underground to circumvent prohibitions, limitations, and controls. The existence of these activities is explained by the institutional structure of the economic system. In communist system, underground activities were all pervasive. They seem to be prosperous mainly because there was a wide demand for goods and services not satisfied by the official economy, and because underground activities grant the organizers considerable rewards. The institutional organization of the economy causes shortage in the regular economy, thereby fostering demand for goods and services supplied by underground activities (Kornai, 1980). There existed three major groups of goods. The first consisted of goods and services used by enterprises. When enterprises had no inner constraints against input increases (as in the former socialist-type system), only an administrative policy was effective. In such circumstances, input shortage was unavoidable, creating ample room for irregular activity. However, owing to technological factors, and control and repression activity, the underground economy was confined mostly to irregular exchange of production goods and services among socialist enterprises and much less to underground production. A second group is made up of consumption goods and services used by families and the private sector of the economy. Both consumers and private sector have hard inner constraints against demand increases. Underground activity permits an increase in both income and consumption. This group of goods constitutes the main area of underground production; controls are difficult, and production technology is often simple. The third group is made up from goods and services used by enterprises, on the one hand, and families and private sector, on the other hand. This is a very wide category and the most difficult to control through economic policy. In fact, there is an imbalance between socialist enterprises and families, since only the latter have an inner constraint against demand increases. For this reason, economic policy produces a displacement effect to the disadvantage of families. Utilization of policy instruments to limit demand will be effective on the consumers' side but will have little effect on discouraging enterprise demand. The consequence for aggregate demand may be negligible (Dallago, 1987).

(10) See, for more details, Neef (1995).

(11) In mentioned study, about the informal economy in East Germany, Neff affirms that 1/5 of urban households and 1/2 of rural households live in individual buildings that were constructed mainly by personal work.

(12) This except for Romania, where the communist regime of Ceaucescu would forbid the most of civil associations and organizations.

(13) Regarding the analysis of social networks in the communist regime, a large list of authors there is in a recent study of Neef (1995).

(14) In case of Romania, it can be mentioned the Workshop on "Parallel Economy", organized within the National Institute of Economic Research under the coordination of Academician Emilian Dobrescu, among 1992-1993.

(15) Till the present, for Romania there is only some global appreciation about the size of underground economy. So, the President of the National Commission for Statistics showed that, based on national accounts, the underground economy in Romania represented 9-10% of GDP in 1993. Also, by the Director of the Romanian Intelligence Service, the size of parallel economy represents 38% of GDP (Capital, 1994). This value seems nearly of some our ancient estimations, for the period 1990-1992, obtained in a global manner based on a generalized model of Laffer curve (Albu, 1994). Recently, the Prime Minister of Romanian Government has been shown that the fiscal evasion represents about 60% of the State Budget, but the Minister of Internal Affairs showed that the corruption "costs" two thousand million dollars by year in Romania (InfoMatinal, 1995).

Annex 2.1

Value of Domestic Production
(percentage of national product)

Country	Year	% of GDP	Method *	Source
Belgium	1983	35	EGN **	Driessens (1987)
	1983	44	ESN **	
Canada	1961	39	ESN	Alder-Hawrylyshyn (1978)
	1971	41		
	1961	44	GPN	
	1971	40		
Finland	1980	32	EGN	Suviranta (1982)
	1980	42	EGB	
France	1975	31	EGN	Chadeau-Fouquet (1981)
	1975	44	EGB	
	1975	44	GPN	
	1983	28	EGN	Sauvy (1984)
	1983	36	ESN	
	1983	43	GPN	
W. Germany	1964	34	EGB	Schettkat (1985)
	1974	33		
	1980	32		
	1964	47	GPB	
	1974	43		
	1980	42		
UK	1956	44	EGN	Clark (1958)
USA	1929	42	GPN	Nordhaus-Tobin (1972)
	1965	48		
	1960	34	GPB	Weinrobe (1974)
	1970	31		
	1960	38	GPN	Murphy (1978, 1982)
	1970	37		
	1976	51		
	1960	31	ESN	
	1970	34		
	1976	44		
	1966	24	EGB	Kendrick (1979)
	1973	25		

* EGN = net marketable global value (equivalent); EGB = brut marketable global value;
ESN = net marketable specific value; ESB = brut marketable specific value;
GPN = net potential gain; GPB = brut potential gain.

** Only women.

Source: Pestieau (1989).

3. METHODS TO APPROACH UNDERGROUND ECONOMY

There is not a single term that allows qualifying unanimous the underground activities. Thus, the International Bureau of Employment counts 15 designations whose usage differs (Thomas, 1992). Several times it seems to be adequately to qualify this particular economy by the terms "informal economy" and "underground economy". From an author to other but on the even appoint it is covered more varied definitions. This large diversity of approaches of the underground economy there is also regarding to the methods of evaluation.

In an extensive viewpoint, at macroeconomic level, the underground economy term covers the totality of merchant and non-merchant goods and services. There are three modes to participate to production and exchange: non-declared remunerated activities, domestic production, and other domestic activities (Adair, 1985). Stricto sensu, on the rule, it is to distinguish merchant goods and services, that design the non-declared remunerated activities and the non-merchant goods and services, that come from the household activities (1). On the other hand, as we have already presented, among merchant activities, it takes place there to distinguish two components: the fiscal fraud and the "work to the black" and respectively the properly criminal activities. In analytic or statistical viewpoints, to estimate the underground economy there are two types of approaches: direct and indirect; the former cover several methods, generally non - comparable, and end to very divergent evaluations by reason of the field more or less extensive that is considered.

Considering a recent study (Adair, 1995), we present in table of Annex 3 some evaluations. Generally, these are based on an interpretation stricto sensu of the underground economy, but several times they do not include the non-merchant activities and the criminal component of the underground economy. As we are concerned in this work, the underground economy being considered in its larger meaning we consider these evaluations the inferior limit of the underground economy.

3.1 Direct Methods

The micro-approaches employ either well-designed surveys and samples based one voluntary reply or tax auditing and other compliance methods (2). In most cases they lead only to point estimates (that is, one estimate at a specific time). It is unlikely that they capture all "underground" activities, so they can be seen as providing low estimates. Moreover, they are unable (at least at present) to provide estimates of the growth of the underground economy over time. Therefore, they have at least one considerable advantage: they can provide detailed information about the structure and composition of the labour force in the underground economy.

Several times, direct estimations result from extrapolations realized from inquiries on representative household samples. It concerns, on the one hand, inquiries aiming to estimate the deduced fiscal fraud of incomplete declarations or leave: it is the case of the inquiry on a sample of some 50000 households that has been realized to USA in 1976 (Carson, 1984). It concerns, on the other hand, inquiries aiming to evaluate relative expenses to the purchase of goods and undeclared services (3), or the offer of "work to the black" of the households which was the subject of several researches in Belgium and in Norway (Carson, 1984), as well as in Canada (Lacroix and Fortin, 1992; Lemieux et al., 1994). If their extrapolations are disputable, these various inquiries - completed by some monographs - allow to apprehend characteristics of actors and sectors of the underground economy and to sketch some element analysis. In the last years there is an important preoccupation to include the results of direct methods (it is to tell the approach at microeconomic level) in national accounts (it is to tell the approach at macroeconomic level) by an operation of rectification and by successive iterations. Thus one can obtain several levels for some macroeconomic indicators, function of steps covered in the process of data integration, obtained by direct methods, in the national accounts and

sometimes one bit to continue even to revise the official GDP. It is the case of Italy that has included the results obtained by inquiries on the job in national accounts. This method told the implicit supply of labour can be also as well as considered among the indirect methods (Adair, 1995).

3.2 Indirect Methods

The other indirect approaches, aside the implicit labour supply, are: national accountancy, monetary approach, and the so-called composite method.

3.2.1 Implicit Labour Supply

This method focused on the discrepancy between the official and actual labour force. If total labour force participation is assumed to be constant, a decreasing official rate of participation can be seen as an indicator of the increase in activity in the underground economy (4). The weakness of this method is that differences in the rate of participation may also have other causes. Moreover, people can work in the underground economy and have a job in the official economy. In practice, the implicit labour supply is considered as representing the difference between the official activity rate and the actual rate of activity measured by various inquires. The active population is thus increased by a coefficient of full-time jobs resulting from the conversion of multiple-activity and of non-declared activity in full-time jobs. These jobs, under the strong disputable hypothesis that the productivity of the work is identically in both sectors (official and underground), are assigned of an income that can be incorporated in the GDP (5).

Alone Italy has proceeded to the calculation of the implicit supply of labour and has reintegrated it in the GDP, which thus has been officially re-evaluated of some 17.7% in 1987. The other European countries challenge this procedure.

3.2.2 National Accounts

The estimations obtained by the methods based on the national accountancy are systematically smaller than those obtained by other methods (Barthelemy, 1982). Among causes, there are at least the following two: 1 - in the production viewpoint, as related base is considered only the GDP resulted from the economic activities included by convention in the national accountancy sphere; 2 - one considers that a part of underground economy is already enclosed in the official GDP and, moreover, that an other part of underground economy could be also included by successive corrections provided by direct methods of investigations (controls, voluntary replies, tax auditing, etc.).

Many authors, as well as international or national statistic institutions, consider that the national accounts record number of hidden transactions. But, in the same time, the Blades's analysis showed that, on the one hand, an important part of the visible economy is not registered and that, on the other hand, a fraction of invisible economy make object to an official measure (Blades, 1982; Berthelemy, 1986). There is here a notional problem caused by the criteria used to appoint a part of economy in report with another part of economy. That is why in this work we consider, on the one hand, that the national economy has initially two economies: official and underground (hidden). On the other hand, we consider also two sectors: visible and invisible. Finally, the visible sector encloses the initial official economy, in a first stage, to which one adds afterwards, in a second stage, the part discovered by authorities (due to controls, tax auditing, etc.). So, this part of underground economy, initially hidden, becomes visible to the end of a year, for instance. Thus, to the end of a year (or other considered period, but including at least two stages) the invisible sector will result from the initial hidden economy less the activities discovered by authorities, which have been transferred into the visible sector (even many times this transfer led to a disappearance of those activities or to their reconstruction on official bases).

A first modality concerns to estimate size of underground economy on base of the discrepancy between national expenditure and income statistics. In most countries the size of GDP is computed both from the expenditure and income side of national accounts; this often reveals that expenditure is higher than income. This initial discrepancy can be seen as a

result of underground activities (6). The weakness of this "fiscal" method is that the differences may arise not only because of activities in the underground economy, but also because of other errors in measurement statistics (De Leeuw, 1985 and 1986). These estimates may therefore be very crude and of questionable reliability.

A second modality, most elaborated, represents that that tries to enclose in national accounts all components of underground economy, obtained by various direct methods. This supposes an iterative process of their integration within national accountancy system and consequently re-adjustments of national accounts. Function of integration stage of data, there are computed some different levels of GDP and certainly of other macroeconomic indicators.

In the last years, this approach made subject of many debates on the definition and implementation of European System of Integrated Economic Accounts. So, to elaborate national accounts it has been adopted three methods - production method, expanding method, and income method - to which corresponds three levels of GDP. Also, at least from a theoretical viewpoint, it was accepted that a rigorous analysis of the dynamics of market economy supposes the integration of non-marketable activities in the marketable production (Archambault, 1980). On the other hand, in a practical viewpoint, it was admitted that, because of heterogeneity, it is not possible to make an aggregation between domestic activities and the other goods and services already enclosed in the national accounts (Durand and Kartchevsky, 1993). In these conditions, within the European System of National Accounts, the underground economy must enclose only the following two components: fiscal fraud (under-evaluated activities exerted by legal economic units) and work to the black (activities exerted by illegal economic units). For each component it was proposed three distinct methods (Coin, 1994).

To capture data about the fiscal fraud in order to adjust national accounts, there are used the following methods: 1 - utilization of results of fiscal controls operated by fiscal authorities (France, Netherlands); 2 - "Franz's Method", which yields estimations by comparing the income declared by the owner of a small company with the average salary of employees from analogous enterprises (Italy); 3 - "implicit" method, which yields estimations of fraud computed on demand-side (expenditures for buying goods and services may be correlated with the production non-declared of goods and services, that generates non-declared incomes). Also, to capture data about the "work to the black", there are used following methods: 1 - comparison between labour demand, evaluated by inquiries in the production units, and labour supply, evaluated by inquiries in households (Greece, Ireland, Italy, Portugal); 2 - selection of some representative activities and economic units followed by detailed economic and social researches or by administrative investigations (France, Denmark); 3 - "implicit" method, which produces estimations on labour demand.

Generally, in case of the OECD's countries, the estimations of underground economy obtained by this category of methods vary between 2% and 6% of GDP (7).

3.2.3 Monetary and Composite Methods

This category includes two approaches: transactions' approach and respectively currency demand approach.

First method (included also among the monetary aggregates' approach) developed by Feige (1979, 1982) assumes that there is a constant relation over time between the volume of transaction and official GDP. Also, to estimate the underground economy, it was assumed a base year in which there is no underground economy, and therefore the ratio of prices/total transactions to official GDP was "normal" and would have been constant over time without underground activities. The weakness of such hypotheses generated many critical replies in literature (Thomas, 1992).

The currency demand approach assumes that underground transactions are undertaken as cash payments, to leave no observable traces for the authorities (8). An increase in the size of underground economy will therefore increase the demand for currency. To isolate the resulting "excess" demand for currency, an equation for currency demand is econometrically estimated over time, with controls for all possible conventional factors, such as the development of income, payment habits, interest rates, etc.

Adding some other variables (such as: tax rate, public employment, unemployment rate, average time work, etc.) to that properly monetary leads to the so-called composite

method. So, for instance, additionally, such variables like the tax burden and government regulations, which are assumed to be important major factors that cause people to work in the underground economy, are included in the estimation equation. The "excess" increase in currency, which is the amount unexplained by the conventional or normal factors, is then attributed to the rising tax burden. Figures for the size and development of the underground economy can be calculated by comparison of the difference between development of currency when the tax burden and government regulations are held at its lowest value, and the development of currency with the current (much higher) burden of taxation and government regulations. This method is one of the most commonly used. It has been applied to the OECD countries (Boeschoten and Fase, 1984; Lundager and Schneider, 1986), but has nevertheless been criticized on various grounds (Garcia, 1978; Blades, 1982; Frey and Pommerehne, 1984; Klovland, 1984; Schneider, 1986).

An interesting approach, which tries to capture the impact of tax system "complexity" on the development of underground economy, is that developed of Schneider and Neck (1992). We consider that this method may be used to analyse the impact of various alternatives of fiscal policy on the size of underground economy, and consequently to choose adequately reforms.

NOTES

(1) In USA's case, the domestic work was object to 12 estimations among 1909 and 1973. Function of various methods used to estimate (cost of replacement or cost of opportunity), the work within households represents between 25% and 50% of GDP (Thomas, 1992). In Canada, the disposable data show that the domestic activities represented, in 1981, more than 1/3 of GDP: 37.1% in case of replacement cost and respectively 38.3% in that of opportunity cost (Chicha-Pontbriand, 1988). In a study of INSEE achieved in 1975, the domestic activity in France is evaluated at a level between 50% and 77% of the official GDP (Albertini, 1988). Also, Bonke (1992) shows that in most Western countries the value of households' production represents between 40% and 50% of GDP.

(2) Direct method of voluntary sample surveys has been extensively used for Norway (Isachsen et al., 1982; Isachsen and Strom, 1985) and for Denmark (Mogensen, 1985). They report, on rule, estimations around 5%, but in any case above 10%.

(3) It is the case of "Family Expenditure Survey" that analyses every budget of about 7000 households in UK.

(4) Such studies have been made for Italy (Contini, 1981; Del Boca, 1981) and for United States (O'Neill, 1983).

(5) Despite mentioned inconveniences, we to estimate the limits between can vary the underground economy in Romania have used a developed variant of this method. We considered some differences between the two sectors regarding productivity, salary, and profit rate (see Chapter 6.3).

(6) In this question, the most quoted study is that of Macaffe (1980). He obtained by this method an estimation of about 3.5% of official GDP for the underground economy in Great Britain. Similar approaches are those of Park (1979) for the United States and Petersen (1982) for Germany.

(7) For instance, in a more recent study (Leal, 1994), that presents minutely the application of methods of national accounts adjustment on Portugal's economy, the share of underground economy is estimated, in 1988, at 6.6% of GDP.

(8) This method was first used by Cagan (1958), who calculated a correlation of the currency demand and the tax pressure as one cause of the underground economy for the United States over the period 1919-1955. Then, Gutmann (1979) used the same approach, but done not use any statistical procedures; instead he "only" looked at the ratio between currency and demand deposits over the period 1937-1976 (Schneider and Neck, 1992). Cagan's approach was further developed by Tanzi (1980, 1983) who estimated a currency demand function for the United States for the period 1929-1980 to measure the underground economy.

Annex 3

Estimations of Underground Economy
(percentage of GNP)

Country	National Accounts	Monetary Aggregates	Currency Demand	Composite Method	Direct Estimates
Belgium	11% (1978) (Frank)		15.2% (1980) (Mont) 19.4% (1980) (Geeroms) 17.5% (1985) (Geeroms, Mont)	12.1% (1978) (Frey, Wreck)	
Canada	16% (1981) (Ethier)	11.1% (1977) (Haas)* 10.5-12.8% (1982) (Mirus, Smith)*	8.4% (1981) (Ethier)	8.7% (1978) (Frey, Wreck)	
Denmark	6% (1975) (Frey, Pommerehne)		6.9% (1980) (Schneider, Lundager)	11.8% (1978) (Frey, Wreck)	5.5% (1980) (Mogensen)
France	4.3%* (1985)[c] (Willard)		6.3-6.7% (1979)* (Barthelemy)	9.4% (1978) (Frey, Wreck)	
W. Germany	4.8% (1974) (Petersen)	27% (1980) (Langfelt)[b]	3.7% (1980) (Langfelt) 12% (1980) (Kirchgassner)	8.6% (1978) (Frey, Wreck)	1.1% (1984) (Hellberger, Schwarze)
Greece	28.6% (1984)[c] (Pavlopoulos) 30.2% (1984)[c] (Dermenakis)				
Ireland			8% (1980) (Boyle)		
Italy		30% (1978) (Saba)[a]		11.4% (1978) (Frey, Wreck)	
Netherlands	5% (1979)*	7.5% (1982)		9.6% (1978)	

	(Broesterhuizen)	(Boechoten, Fass)		(Frey, Wreck)	
		12.7% (1982) (Renooy)			

continued

Country	National Accounts	Monetary Aggregates	Currency Demand	Composite Method	Direct Estimates
Norway			6.3% (1978) (Isachsen, Klovland, Strom)	9.2% (1978) (Frey, Wreck)	4-6% (1983) (Isachsen, Strom)
Portugal	6.6% (1988) (Leal)	22% (1981)* (Santos A.)	11.2% (1981)* (Coco, Santos E.)		
Spain					6.5% (1978) (Frey, Wreck)
Sweden	3.8-6.4% (1979)* (Hansson)	5% (1975)* (Hansson)[b]	7-17% (1978) (Klovland)	13.2% (1978) (Frey, Wreck)	
UK	3.5% (1978)* (Macafee)	15.5% (1979)* (Feige)[b]	8% (1984) (Bhattacha-ryya et al.)	8% (1978) (Frey, Wreck)	2.3% (1977) (Dilnot, Morris)
		12% (1979)* (Mathews)[b]			
USA	3.8-6.4% (1977) (BEA)	14.6% (1982) (Porter, Bayer)[a]	4.5-6.1% (1980) (Tanzi)	8.7% (1978) (Frey, Wreck)	1.5% (1981) (Smith et al.)
	7.1% (1978) (Molefsky)	14.6% (1979) (Feige)[b]	15% (1984) (Bhattacharyya et al.)		

* GDP
a - ratio chash/bank deposit (Gutmann's method)
b - monetary transactions (Feige's method)
c - fiscal fraud, non-declared work, etc. (Blades' method)

Sources: Adair (1995), Barthelemy (1988, 1990); Carson (1984); Feige, ed. (1989); Gaertner and Wenig (1985), Tanzi, ed. (1982), Thomas (1992), Willard (1989).

4. FISCAL POLICY AND UNDERGROUND ECONOMY

Many authors consider that budgetary policy and especially fiscal policy have a major impact on the development of underground economy. However in literature this subject is treated more implicitly within the general models of fiscal policies.

Till the present, only few studies analysed explicitly, at macroeconomic level, the consequences of ignoring underground economy within economic policies. One of these showed that, in the case of the USA's economy, the growth of invisible sector would offer explanations for 35% of forecast errors in case of production volume, for 46% of forecast errors in case of inflation rate, and for 40% of forecast errors in case of unemployment rate (Feige, 1982).

In this chapter we present the general context of the theory of fiscal policy and then some comparative statistics as well as some methodological aspects of evaluating different tax rates.

4.1 General Context

In the West-European countries, till the First World War, the share of governmental expenditure represented below of 10% of GDP. There was the time of "invisible hand" or "Gendarme-State", when the role of state is only to assure the security of people and the respect of lows. From that time till the present, but more accentuated after the Second World War, the intervention of state in the economy increased continuously. Today the state became "Providence-State" and the share of total public expenses (including general government taxes, local government taxes, and social security expenses) increases up to about a half of GDP (comparing with about 15% in 1920s and 30-35% in 1950s). In the Scandinavian countries this share is even more than 50% of GDP.

Some authors make responsively to this evolution the impact of the Keynes' theory and his multiplier of budgetary expenditure. This although Keynes himself showed that the maximal level of the share must be around 25% of GDP. However in the last two decades more and more the Keynes' theory and consequently the existence of a multiplier effect of governmental expenses were contested. Many authors coming from some extensive econometrically studies tried to capture the multiplier effect budgetary expenditure on the economic growth. Their conclusion was that such effect does not exist in reality but governmental expenses are only complementary to the private expenses - expenditure of enterprises and respectively of households (Roubini & Sachs, 1989; McCallum, 1990; Mundell, 1990; etc.).

In these conditions, a new theory emerged. It is the so-called supply-side economics, which focused on the efficiency-incentive considerations relating to the taxation; the rising dead-weight welfare cost as the tax share of GDP rose; the changing elasticity of government revenue with respect to tax rates (the Laffer curve); the high transfer cost of transfers (including interest payments on the public debt); the inefficiency of double taxation (on both corporate profits dividends received); the inefficiency of taxing capital gains at the same rate as personal incomes (especially in an inflationary economy); and the importance of entrepreneurial profits in the allocation of investment. The supply-siders stressed also that total taxes plus the budget deficit - in other words, government spending - was the correct measure of the burden of government in a full-employment economy. If economic transfers could be made without friction, the public debt involves a transfer of income from taxpayers to bondholders; the debt itself has a dead-weight efficiency cost (Meade, 1945; Modigliani, 1961). A further disadvantage of a high public debt in a monetary economy is that it invites inflation and currency depreciation (Mundell, 1990).

An important tool of analysis, and a strong selling point, of supply-side analysis was the Laffer curve, relating tax rates and tax revenues. This schedule illustrates the important

fact that tax revenues, as a function of tax rates, is a two-valued function in which the same tax revenue can be achieved by both high and low tax rates, except at the maximum revenue point where the points coincide. In principle, there is a Laffer curve corresponding to each individual tax. As Mundell showed, the aggregate Laffer curve, however, is relevant for macroeconomic policy. It would be a mistake to consider the change in tax revenue from a single source when a particular tax rate is changed. Particular taxes have a general equilibrium effect on other tax revenues, and a correct analysis requires consideration of the effect of a change in tax rates on revenues from all taxes in the system. For instance, a cut in the corporation income tax that increases investment, employment and output might raise income tax revenues from all sources of tax even if the proceeds from corporate profits taxes alone decline. Finding the maximum revenue point for each tax in isolation does not maximize tax revenues. There is a maximum revenue position corresponding to both money-financed and bond-financed government spending. As Bailey (1956) has shown, real government revenue from inflationary finance is maximized at that inflation rate at which the decline in real money balances (the base of the tax) is equal to the increase in the inflation rate (the rate of the tax). Government revenue from bond finance reaches a maximum when bond prices fall (interest rates rise) at the same rate that the quantity of bonds is increasing.

The proposition that tax rates increase revenues up to a point of maximum revenue after which they fall was not at all new (1). Nevertheless, if not new, it was certainly forgotten in the half century among 1932 and 1981 when marginal tax rates were never less than 60% and hovered at times near 100%. The importance of it looms larger when it is generalized to include added tax revenues that arise from increases in output and employment (2).

However, in the last years, even international organizations, as International Monetary Fund and World Bank, adopted a new classification of economic policies. So, there are demand-side policies, concentrated on restraint of domestic demand by monetary and fiscal measures, and respectively structural or supply-side policies, focused on incentives to saving and investment (Khan, 1987). Also, Mundell (1990b) showed that stabilization program to have success in long run; the measures of economic policy must provoke favourable responses on the supply side of economy.

Now, we present a simple global model that may eventually to capture the impact of tax rate changing on some important macroeconomic indicators. Being essentially static, however it may be developed to capture some dynamic implications in short-run. In this sense, we introduced a feedback connection between the level of GDP produced in a year and that of GDP in the next year.

Coming from the works of McCullum and Mundell, mentioned already above, we considered initially the two next relationships to express the GDP and, respectively, the aggregate demand:

$$Y = C + E + T \tag{4.1}$$

$$AD = C + I + G + Z - M \tag{4.2}$$

where C is consumption; E - savings; T - taxes; I - investments; G - government expenditures; Z - exports; M - imports.

Also, we considered the injections (J) and, respectively, withdrawals (W), expressed by the following relations:

$$J = I + G + Z \tag{4.3}$$

$$W = E + T + M \tag{4.4}$$

According to the macroeconomic theory, the equilibrium condition supposes the equality between the GDP and AD, what implies the equality between J and W.

Considering that the available income (Yd) is, by definition, equal to the sum of consumption and savings

$$Yd = C + E \tag{4.5}$$

and the model represented by Figure 4.1, we can write the relationship between the efficiency of investments (b) and the interest rate (i) as following:

$$b = (DYd / I) = i / [1 - (I / E)] \quad (4.6)$$

where DYd represents the yearly absolute increase of the available income.

On this basis, now we can express the national net savings (En) by the following relations:

$$En = E - I = E - [E \times (b - i) / b] = E \times i / b \quad (4.7)$$

Introducing the following notations:

$$c = C / Yd ; \; t = T / Y ; \; q = E / Yd ; \; g = G / Y ; \; m = M / Y ; \; z = Z / Y$$

the diagram of the global model shows as in Figure 4.2 and the relationship of equilibrium will be:

$$(E - I) + (T - G) + (M - Z) = 0 \quad (4.8)$$

or

$$(\; En \;) + (\; - Di \;) + (\; \; De \; \;) = 0 \quad (4.9)$$

where Di is the internal public debt and De - the external public debt.

Dividing by Y and taking into account of what we have exposed above, the last relationship can be written under the next form:

$$t - g + \{ a \times i / [(1 - t) \times g - i] \} + m - x = 0 \quad (4.10)$$

where $a = I / Y$; $g = DY / I = b / (1 - t)$; and DY is the yearly absolute increase of GDP.

Now, grouping terms in the equilibrium equation by the parameter t, the relation (4.10) can be written as following:

$$(- g) \times t^2 + (g - i + g \times g - m \times g + z \times g) \times t +$$

$$+ (g \times i - g \times g + a \times i + m \times g - m \times i - z \times g + z \times i) = 0 \quad (4.11)$$

Considering the left member of the equation as a function of variable t, the equilibrium equation will be written under the next form:

$$F (t) = - g \times t^2 + k1 \times t + k2 = 0 \quad (4.12)$$

where k1 and k2 are coefficients.

The analysis of the behaviour of the function F has permitted to separate the inflation zones and respectively the decreasing inflation zones, as is shown in the table of Annex 4.1.

The model can be developed by the introduction of some other quasi-exogenous functions to express the behaviour of investments, consumption, etc. But, in this case, new complicated problems will emerge (Albu, 1994).

Generally, the function F may be considered in terms of Laffer curve. So, after some algebraic transformations, we can express the amount of taxes (T) as a function of tax rate (t), as following:

$$T (t) = - b^* \times Y \times t^2 + Y \times (1 + b^* - b^* \times c) \times t \quad (4.13)$$

where $b^* = DY / E$.

Regarding the analyses of economic dynamics in medium-run, a large variety of alternatives of fiscal or budgetary policies can be analysed coming from the Hénin's approach, based on a calibrated model of endogenous growth (Hénin and Ralle, 1994). Also, Day (1994) developed a model of business cycles by including the tax rate as an essential behavioural parameter. Function of some threshold-values of the tax rate, he separated

various regimes on the map of GDP and on that of T. Also, the model may produce chaos in some special conditions of tax rate dynamics.

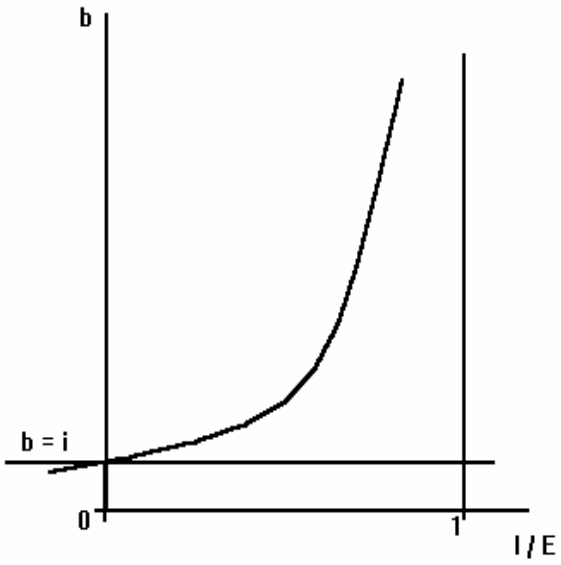


Figure 4.1

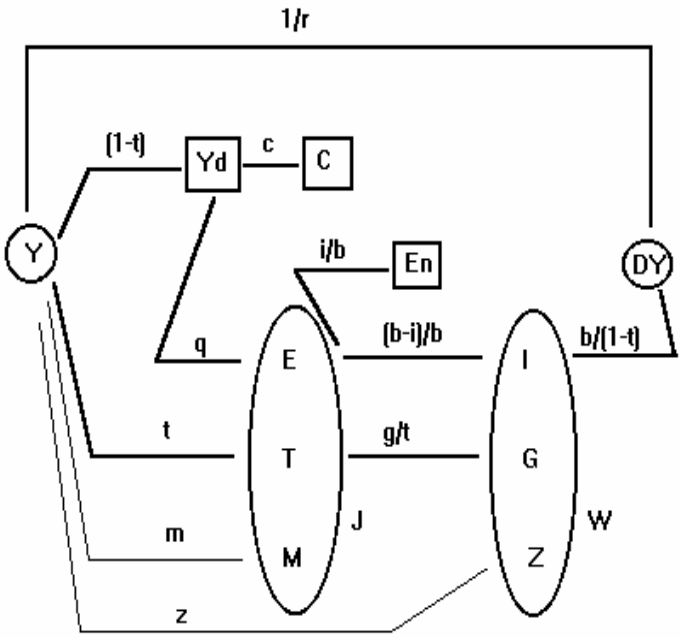


Figure 4.2

4.2 Trends in Government Spending and Taxation

In case of Western countries, throughout the past half century, there has been a steady increase in the share of government spending, G, in the total national product, Y, as how is shown in the Table 4.1. What is notable, however, is the sharp rate of increase in G/Y beginning in the mid-1960s. During the period 1973-82, the share of government experienced its most rapid jump for any sub-period during the past 50 years. After 1982, government spending as a share of GDP seems to be stabilized, and in some countries has even fallen.

Table 4.1. Public expenditure in selected OECD countries (% of GDP)

Year	1938	1950	1960	1965	1973	1982	1985	1990
France	21.8	27.6	34.6	38.4	38.5	51.1	52.4	49.9
Germany	42.4	30.4	32.4	36.6	41.5	49.4	47.2	46.0
Japan	30.3	19.8	17.5	19.0	22.4	33.7	32.7	32.3
Netherlands	21.7	26.8	33.7	38.7	45.8	61.6	60.2	55.6
UK	28.8	34.2	32.2	36.1	40.6	48.2	47.7	42.1
US	18.5	22.5	27.0	27.4	36.6	30.5	36.7	36.7
Italy	29.2	30.3	30.1	34.3	37.8	47.6	50.8	53.0

Sources: Lybeck and Henreckson (1988) for 1938, 1950, 1985; OECD Economic Outlook for 1973 and 1965; "La dépense publique en France: évolution sur trente ans et comparaison internationale", Journal Officiel de la République Française, N° 26, 1994 for 1960 and 1990.

In the same time, until 1973, government deficits were sufficiently low in most countries to lead to a falling ratio of net public debt to GDP, which we denote as D/Y. This is in the line with Barro's prediction of a falling debt-GDP ratio during periods of peacetime (Barro, 1979 and 1987). But after 1973, the trend was reversed: almost every OECD economy experienced a significant rise in the debt-GDP ratio (Roubini & Sachs, 1989).

There is a huge literature concerning the causes and possible remedies by budgetary policies, but it is not our main preoccupation in this study. Here, we note only that the estimation of the impact of budgetary policy on the size and dynamics of underground economy represents a very complicated problem due to the existence of many factors of influence varying by country and by period. Some of these are: the structure of public expenditure and of receipts of taxes, differences between incomes (from work, property, capital, etc.) from the viewpoint of taxation degree; the behaviour and reaction of tax-payers to the measures of fiscal policy adopted, etc. Also, another obstacle against to capture the impact of tax policy on underground economy, at national level, represents the difficulty to evaluate accurately the effective tax rates in case of each category of taxpayers. This problem is important for to assure correctly some international comparisons. In this subject we mention a very useful study elaborated recently by Mendoza et al. (1994) within the IMF.

Regarding the budgetary policy, and fiscal policy especially, in Eastern countries after 1989, we can separate two distinct periods. In the first year of transition, it was a massive liberalization of fiscal policy following to the collapse of the former planned-socialist system. But this liberalization, which in normal conditions must be followed by a transfer of activity from invisible sector to that visible, had not the expected effect. There are multiple causes of this unexpected effect, which are related to the general process of economic transition from a central-planned system to the one based on the free market. Then, in the second phase of transition, due to the accentuated economic crisis, followed by a deterioration of the trade balance, and by a huge growth of public debt, external debt, and budgetary deficit, the fiscal policy was reinforced and fiscal pressure on incomes increased.

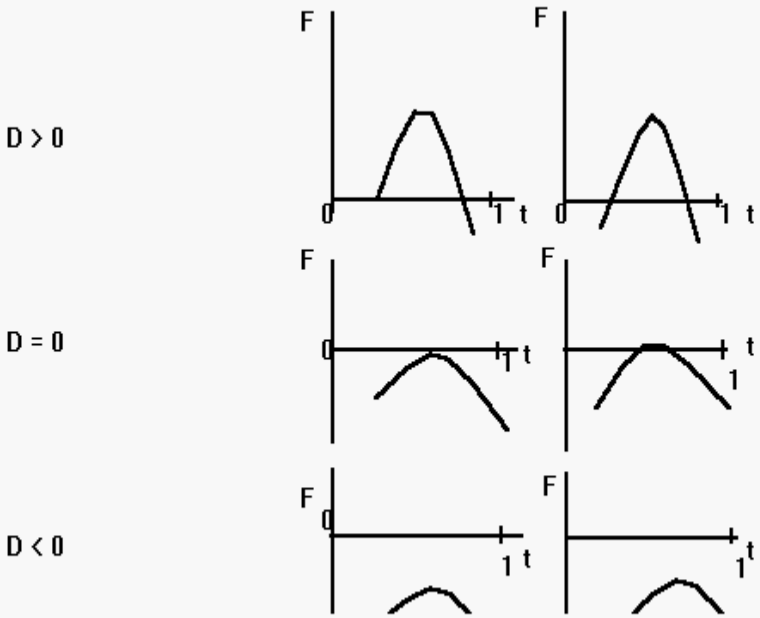
For instance, in Romania, the share of public expenditure in total GDP, expressed in the current prices of each year, has the following evolution: 47.1% in 1989, 35.8% in 1990, 35.1% in 1991, 33.7% in 1992, and 27.8% in 1993. But, after our estimations, using a proper method, in constant prices (prices of the year 1989), it was registered the following levels: 47.1% in 1989, 37.9% in 1990, 41.4% in 1991, 48.2% in 1992, and 45.5% in 1993. However, it can remark in the last time a relative stabilization and even a diminution in the public expenditure ratio.

NOTES

(1) It must have been known to wise chancellors of the ancient world devising tax systems, as David Hume (Rotwein, 1955) or Jules Dupuit (1844).
(2) There is a separate "Laffer curve" corresponding to each level of employment. It is easy to see how a tax cut, even in a Keynesian model, can raise aggregate tax revenue. According to conventional Keynesian theory, a tax cut results in a multiplied increase in income and therefore in a smaller reduction in tax revenue than if income had stayed constant. If, however, the tax reduction stimulates investment, the budget deficit may actually be reduced, or the budget surplus be increased by the tax cut (Mundell, 1990).

Annex 4.1

	$\gamma > 0$	
	$(1 - t) \times \gamma - i > 0$	$(1 - t) \times \gamma - i < 0$
$k1^2 + 4 \times k2 \times \gamma = D$	$F < 0$ - inflation $F > 0$ - deflation	$F > 0$ - inflation $F < 0$ - deflation



PART TWO: **THEORETIC AND QUANTITATIVE MODELS AND APPLICATIONS**

5. THEORETIC MODELS

The last crisis of the 1980s has promoted the underground economy to prominence in the literature. Also, many works focused on underground or informal economy in developing countries and recently, since 1989 in East-European countries. But, in opinion of some authors, while the underground sector is now well studied from an empirical or historical viewpoint, the theoretical literature continues to be sparse. However, some recent contributions (Stark, 1982; Adair, 1985; Chauduhuri, 1989; Schneider and Neck, 1992; Gibson and Kelley, 1994) have begun to provide the foundations of a theory of the underground or informal economy (1).

In this chapter, after a general presentation of the trends in the specialized literature, we shall concentrate on the main results of the models, regarding possible theoretical developments as well as some implications on the macroeconomic policy plan. We will use many of these in the next chapter to construct some quantitative models designed to evaluate the size and dynamics of underground sector, especially in the case of Romania's economy.

5.1 General Context

As it is known, the fiscal policy is considered many times main instrument within the economic theory and the tax rate as an essential parameter.

There is a vast literature concerning the impact of fiscal policy and tax rate on macroeconomic stabilization programs. New developments are registered regarding this subject from the emergence of supply-side economics. Non-ignoring the regulatory role of government budget, the main idea is that the expansion of tax rate level over some thresholds, continuing to have positive effects on the demand-side and consequently on stabilization process in short-run, can have negative impact on the supply-side part of economy. In the long run, the stabilization process itself will be affected. The general explanation is that an increasing tax rate may limit the stimulation to invest of the private (capitalistic) firms, which is the main factor of development in a free market economic system.

Despite the fact that there are many complications, at macroeconomic level, induced by the structural changes, significant differences between the responses to various taxes and as well as by the business cycles and many other factors, however, the mentioned impact was demonstrated, at least as a general tendency in the long-run. Whether, the subject of how tax rate and fiscal policy influence the evolution of economy is extensively treated in literature, the impact on underground sector is more spire till the present.

Generally accepted the idea that an increase of tax rate provokes a migration of activity from visible sector to invisible sector, the remained problem is what is the intensity of the correlation between the tax rate level and proportion of this migration. Moreover, there is the problem of how should change this intensity and by what quantity under the impact of other economic or/and non-economic factors. To evaluate some essential parameters of this transfer of activity (migration) and to produce explanations accepted by standard economics are the main goals of the underground economy modelling. However, the main problem is that studying underground economy implies many times to evade from the cannons and rules of standard economics. So, some times, the underground economy approach makes necessary a general re-discussion of the fundamentals and taboos of standard economics. That is why the approach of underground economy provokes frequently disputes between the economists coming from different mainstreams. For instance, we can mention the limits of the National Accounts System (based on standard economics conception) occurred when it was

desired to estimate the size of underground sector, non-taking about the case when it was desired to quantify its dynamics or the disputes held by the problem of implementation of reforms within this system.

5.2 Types of Approaches

There are many works that treat the psychological effect of the modern State development that has been to dig an important gap between the government and the citizen. Expressions as "government", "State", "Authority", "Downing Street", "Hotel Matignon", or all simply "they", throw the image of an anonymous power, a tyrant that gradually dominates the life itself and imposes all sorts of disagreeable homework; the fact to pay its tax is simply one of these. It seems perfectly natural that nobody pays its good tax taste to such body. Some do not hesitate to see in the underground economy a refuge against the will of the State to organize the society. A denunciation illustrated by the development of "second economy" in Eastern countries (2).

Duchêne affirms that the main "production" of a state is the law. Sometimes this production becomes an inside goal. Than it is a vicious cycle: the state produces many laws of which some are not respected in practice and then the state produces more laws to counteract this inefficiency in practice and then more laws are not respected in practice and so on. By his opinion, to which we also are partisans, to be optimal a state must produce only those laws that will have a chance to be respected in practice. Each over-output relating to this optimal level will generate inefficiency and perverse effects as well as an increasing underground economy (3).

By describing attitudes of taxpayers, four reasons can be evoked - coercive nature of fiscal policy, lack of identification with the administration, aspects relying to the behaviour face to risk and the inefficiency of administrative management - that have been processed in detail in many works. It is possible that such arguments do not belong to the area of the economic science, but we mention already that the study of the underground economy transcends the standard economic theory. We think that there is no pain to mention these subjects, for the purpose to provide some psychological and sociological foundations to some aspects of the underground economy. There is a problem of more, and the economists are certainly qualified to process it, it is that the threshold of taxation and the other rises of taxes by the State. This is not to properly speak a fifth repugnance reason to pay its taxes; this new argument gives a new dimension to each of the four proceeding causes. More again, this aspect of the problem emphasizes the influence of the four repugnance reasons to pay its taxes. To describe it in a sentence, more the fiscal cost is strong, greater will be the desire to avoid paying for each proceeding reason. The Laffer curve according to whether receipts of the State begin by increasing, and then decrease beyond one certain threshold, as the average taxation rate increases, has put this phenomenon in obviousness. Gutmann that thinks that the increase of the rate of taxation incites increasingly individuals to escape the fiscal controls by penetrating in the sphere of underground activities to the point to deform the curve of Laffer has amended this curve of Laffer. According to Gutmann the Laffer curve must be in fact deformed toward the right part of graph. These problems appear them in detail in the next section of chapter. The weight of the fiscal payments and social contributions, it is to tell the obligatory payments, has increased constantly during this century and its rate of growth is now such that the acceleration is perceptible in an alone generation. These aspects we have already approached in the former chapter.

Generally speaking, high taxation rates, few controllers of taxes and relatively weak amend for fiscal fraud contribute to persuade peoples to take their chance not to be discovered. On a theoretic plan, the model of Allingham and Sandmo (1972) strengthen this position. The problem of taxpayer consists indeed in maximize the hoped usefulness of the obtained income if he develops a fiscal fraud strategy.

Now, we present this model where a risk-averse taxpayer is allowed to declare less than his actual income, W. Declared income, $X > 0$, is taxed by a constant rate, $\theta > 0$, whereas undeclared income, $W - X$, is taxed, if detected, by a higher rate, π. The taxpayer chooses X^* to maximize his expected utility:

$$E[U] = (1 - p) \cdot U(Y) + p \cdot U(Z) \qquad (5.1)$$

where p is the (exogenously given) probability of detection, and

$$Y = W - \theta \cdot X \qquad (5.2)$$

$$Z = W - \theta \cdot X - \pi \cdot (W - X) \qquad (5.3)$$

represent his income in case of detection and non-detection, respectively. The first-order condition for the maximization of (5.1) is

$$dE [U] / dX = - \theta \cdot (1 - p) \cdot U' (Y) + (\pi - \theta) \cdot p \cdot U' (Z) = 0 \qquad (5.4)$$

from which the taxpayer's response to a change in θ may be derived. This is given by

$$dX^* / d\theta = - D^{-1} \cdot (1 - p) \cdot U' (Y) \cdot \{ \theta \cdot X \cdot [R_A (Z) - R_A (Y)] - [\pi / (\pi - \theta)] \} \qquad (5.5)$$

where $D = \theta^2 \cdot (1 - p) \cdot U'' (Y) + (\pi - \theta)^2 \cdot p \cdot U'' (Z) < 0$ is the second-order condition for the maximization of (5.1) and $R_A (I) = - U'' (I) / U' (I) > 0$ is the Arrow-Pratt absolute risk-aversion measure, evaluated at I = Y, Z.

Under decreasing absolute risk-aversion [$R_A (Z) > R_A (Y)$], the sign of (5.5) is ambiguous, as asserted by Allingham and Sandmo.

Since this work, a sizable literature applying economic analysis to tax evasion has appeared (Yitzhaki, 1974; Isachsen and Strom, 1980; Clotfelter, 1983; Feldstein, 1983; Cowell, 1985; Pestieau and Possen, 1991; Yaniv, 1994; Jung et al., 1994).

Many of the theoretical papers on this topic have used models in which a representative taxpayer receives income from a single source and then decides how much of that income to declare to the tax authorities. While such studies reveal much about the behaviour of a single underreporting agent, they do not incorporate one important empirical fact about tax evasion, namely that the extent to which one evades taxes is strongly correlated with the source of one's income (Clotfelter, 1983).

In the recent years, several theoretical papers have recognized that opportunities for evasion differ among occupations. Such papers have also emphasized that these differences may affect an agent's labour market behaviour. An agent may, in other words, base his labour supply decisions in part on the ability to evade taxes. Theoretical models that recognize tax evasion to be easier when income is received from certain sources (self-employment, for instance) rather than other sources (corporate employment, more frequently) typically assume that the economy has two sectors: one in which evasion is impossible, presumably because of tax holding and information reporting, and another in which evasion is possible. Such papers are often referred to as studies of the underground economy.

One early example of two-sector model of tax evasion is that examined by Watson (1985). Then several other authors have investigated related two-sector evasion models, which added some realistic features by including as variable labour supply and various demand-side considerations (Fortin and Hung, 1987; Kesselman, 1989). They, however, also remove a feature by assuming that agents active in the avoidable sector declare none of their income to the tax authorities. Furthermore, these papers do not consider how the relationship between the tax rate and the size of the avoidable sector is affected by attitudes toward risk bearing. Such problems, on the other hand, are approached by other authors (Jung et al., 1994).

Generally, the models focused on the situation of underground economy in Western countries. Coming from the fundamental tension in the applied literature, which refers to the interpretation of the labour force in underground sector (4), Gibson and Kelley (1994) developed a theoretical model, which, aside of general problems of underground economy focused on the case of informal sector in developing countries. An important conclusion derived from their model is that the costs in the informal sector must be greater than in formal sector. Then, if the informal sector uses more resources per unit of output, the social surplus will expand as the formal sector replaces the informal sector. The outcome, however, depends upon the existing distribution of income, which further complicates matters.

For evaluate the impact of underground sector on the national economy in the case of a central-planned system as in former Eastern countries, by disposable information to us, the alone rigorous model is that elaborated by Professor Hénin (1986). Covering a wide number of "regimes" of disequilibrium, the model can be used as an important instrument of analysis in the centralized economies.

Some studies seem to confirm the hypothesis of Laffer's curve model (Lemieux et.all, 1994), but other authors contest the availability of this model at least at macroeconomic level (Pestieau, 1989).

5.3 Generalized Laffer's Curve Model

The main goal of our investigation is to find if the simple Laffer curve model is able to produce some relevant results for fiscal policy or it remains restricted to the already known results. Because many other models used by economists seem to be close by the Laffer curve type, possibly, some conclusions of our investigation should be extended.

5.3.1 A General Presentation of the Model

Many times, in the existing literature the Laffer curve model is represented by a simple concave square curve. This expresses graphically the amount of tax income (T) as a function of tax rate (x), having two intersections with the abscise axis and a maximum value for x = 50% of total income or GDP.

In fact, this figure represents only a part of the general graph of Laffer curve model, which derives from a fundamental hypothesis, also intensively used in many other economic models. The hypothesis states that within a given system having two complementary elements, if one of the two constituents grows by an amount, then the other element decreases automatically by an identical quantity. In terms of the underground economy mechanism, the hypothesis states that within the economic system, having only two constitutive sectors - official or visible sector and invisible or underground sector - when the visible sector decreases, the invisible sector increases in a similar manner and vice-versa. In a static analysis the amount of all quantitative changes must be zero, but in a dynamic analysis the amount of all relative changes must be zero.

Generally, it is introduced an additional hypothesis that gives the sense of modification when the other variable, considered exogenous, changes. Now, the problem is that in fact this variable will be never completely exogenous in the case of economic models. As a rule, it is conceived as a relative indicator or as a share, being even indirectly related to the two elements of the system (5). The mentioned situation can generate a feed back relation (more exactly, cascades of feed back relations), which may alter our initial model. The common solution is to ignore this feed back relation, that is equivalent to be interested only in a very closed region of the entire dynamics of the system, namely around the static or fixed equilibrium. Other solution, more productive regarding the progress of economics is to consider the static equilibrium only as a particular case. In terms of Laffer curve model, the additional hypothesis states: when the variable tax rate increases the base of tax rate decreases, that is the income available to taxation decreases. The cause can reside in a migration of economic activity from visible sector to invisible sector.

Concerning the analysis of Laffer curve model, we present in Figure 5.1 the complete map, where both functions of tax rate - the income produced in the visible sector (Yv) and respectively the income produced in the underground sector (Yu) - are supposed to be linear curves. Also, it is shown the resulting curve of the total available income (Yd). The model presented in Figure 5.1 can be expressed by the following system of equations:

$$T(x) = x \cdot Yv \qquad (5.6)$$

$$Yv(x) = c1 \cdot x + c2 \qquad (5.7)$$

Taking into account that Yv passes the points (0;k) and (1;0) results -c1=c2=k, where k represents the total income produced in the economy. Now, substituting (5.6) by (5.7) it results the new expression for T:

$$T(x) = - k \cdot x^2 + k \cdot x \qquad (5.8)$$

which also can be written as:

$$T(x) = x \cdot (k - k \cdot x) \qquad (5.8')$$

And writing

$$Yu(x) = k \cdot x \qquad (5.9)$$

it results that

$$T = x \cdot (k - Yu) = x \cdot Yv \qquad (5.10)$$

Now, the deduced equation of the total available income (Yd) will be:

$$Yd = k - T = Yu + (Yv - T) \qquad (5.11)$$

which also can be written as:

$$Yd (x) = k \cdot (x^2 - x + 1) \qquad (5.12)$$

It results that when the governmental income from taxes is maximized, at x = 50% of Yv, the available income of agents (firms and households) is minimal. When x = 0 and T = 0, Yd equals the total income (k) and the entire activity becomes visible (Yd = k = Yv). On the other hand, when x = 1 and T = 0, Yd equals also total income, but now the entire activity becomes invisible (Yd = k = Yu). A deeper analysis of the model permits the identification of other important points on the map: Tmax = k/4 and Ydmin = 3k/4 (both for x = 1/2 of Yv).

Although in this chapter we shall continue to investigate the presented version of Laffer curve model, now we consider usefully presenting briefly other possible versions of this model. So, if we replace the two linear functions Yv(x) and Yu(x), by non-linear functions it can obtain other possible graphical representations. In cases presented in Figures 5.2 and 5.3, the income tax curve is deformed to left side and respectively to right side. In the last case, income tax curve is so-called Gutmann curve. Also, by some authors this case would seem to be the most adequately to represent the economic situations. In the case of discrete version of Laffer curve, we shall show that stability of the system depends not only on the tax rate level, but also on both the tax rate value and its rate of change. We can see that these Laffer curve versions cover many models frequently used in the economic literature (6). However, to produce other interesting points on the tax rate scale the analysis must be deepened.

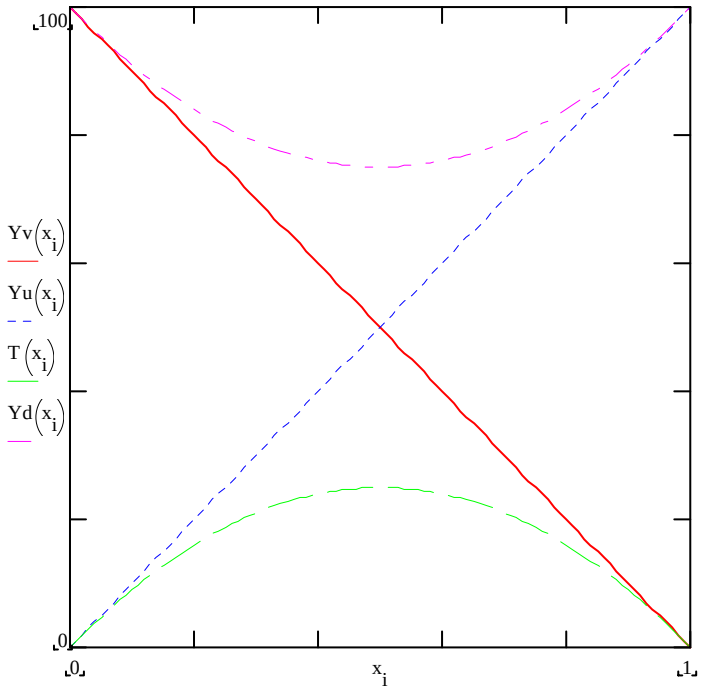


Figure 5.1

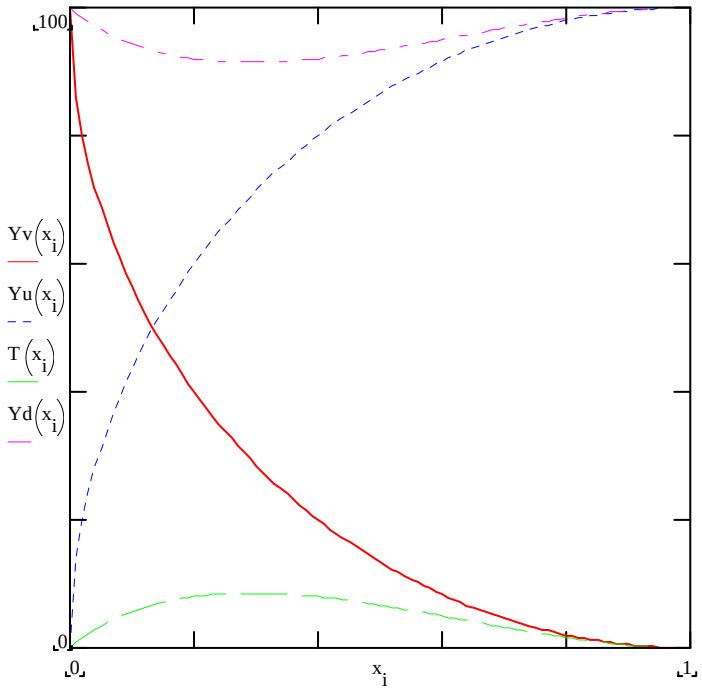


Figure 5.2

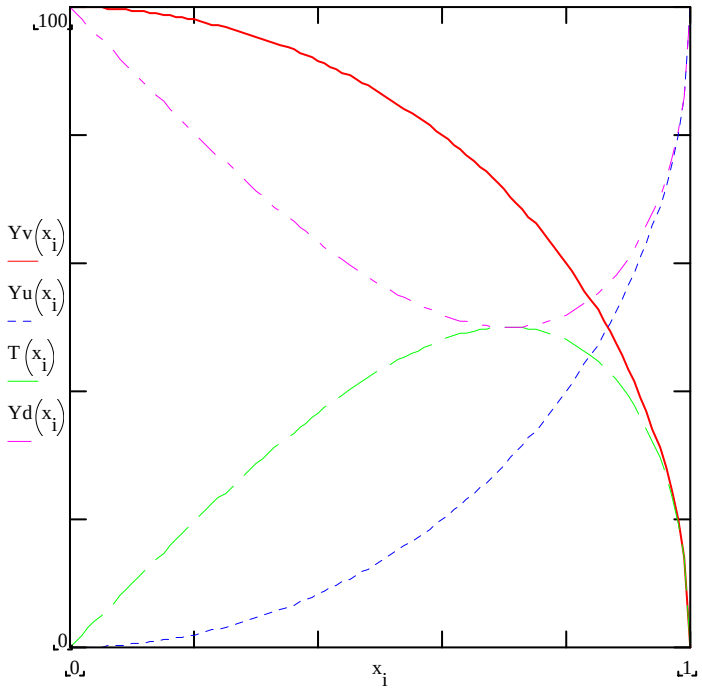


Figure 5.3

5.3.2 Developing Continuous Version of the Model

In this section we called some results derived from the Catastrophe Theory or, more generally, from the Structural Stability Theory (7). Now, to elaborate a continuous-time model we consider that the main equation of Laffer curve model can be derived from an existing potential function V (x;m):

$$dV / dx = f (x ; m) = 0 \qquad (5.13)$$

where x is the rapid variable of the system and m - the slow or control variable (both are implicitly functions of time). In our case, we chose the following potential function:

$$V (x ; m) = (- x^3) / 3 + (x^2) / 2 - m \cdot x \qquad (5.14)$$

to which it corresponds the following equation of potential surface:

$$- x^2 + x - m = 0 \qquad (5.15)$$

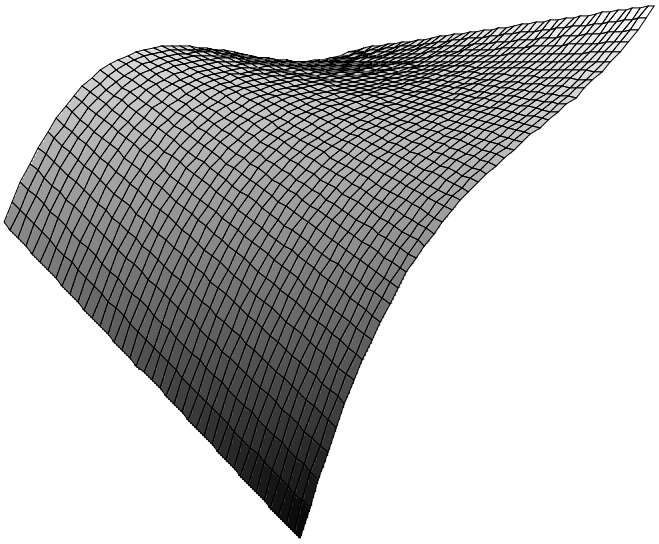
Comparing this with the relation of T already obtained in the first section of paper, it results that in terms of Laffer curve model the slow variable has the following expression:

$$m = T / k \qquad (5.16)$$

Considering the analysis of the graph of function V, it results some threshold values of m. So, for x having values among 0 and 1, there are the following cases:

when $0 < m < 3 / 16$, V has 3 real roots (0 and other two separated);
when $m = 3 / 16$, V has 2 real roots (0 and an other double root);
when $3 / 16 < m < 1$, V has only 0 as real root .

The representation of function V is shown in Figure 5.4.



M

Figure 5.4

A very important threshold value of parameter m is 1/4, where the maximum, minimum, and inflexion points are confounded. This remarkable value was also obtained in the simple version of the Laffer curve model. Other important conclusions concerning evolution and stability of the system are followings:

- there are two equilibrium curves - a stable equilibrium curve (C1) and an unstable equilibrium curve (C2);
- for values of x smaller than (C2) the trajectories are attracted to (C1) (the long-run effect);
- for values of x smaller then m the attraction is stronger than in case of greater values of x than m;
- for value of x greater than C2 the system is strong attracted in an intense troubled zone (hole) as well as in the zone beyond m = 1/ 4;
- a rich menu of alternatives can be deduced by investigation of the function V map moreover this can offer some larger possibilities of statistical data interpretation.

A decisive problem represents the estimation of k. If this would be quantified we should adopt many desired alternatives knowing their consequences. The resulting decisive importance of quantifying k strongly contradicts the authors who consider underground economy having insignificant or neutral influence on the entire economic system and consequently it must be ignored or strongly repressed. The analysis demonstrates that its existence assures that m < x. If it would be eliminated, then m equals x, which by rule is greater than 1/ 4 of Yv, and for this point of graph as it was shown the system is strongly attracted in a so-called hole-region (8). However, the remaining question is how we could solve or at least to avoid the problem of the size of k and implicitly of underground sector. For this we shall try to adopt a discrete model as an alternative.

5.3.3 A Discrete Version of the Model

Now, we recall the equation (5.8) of T, which, dividing by Yv, can be rewritten as:

$$x = a\,x \cdot (1 - x) \qquad (5.17)$$

where

$$a = k / Yv \qquad (5.18)$$

Considering parameter a as an essential parameter that assures the dynamics of the system within the discrete model, we can write the equation (5.17) as following:

$$x_t = a \cdot x_{t-1} \cdot (1 - x_{t-1}) \qquad (5.19)$$

This equation (which represents a canonical form of May's equation) possesses a wide range of dynamic behaviour, which is well known in the specialized literature. Limiting our attention to the initial conditions of the dynamics of x included in the interval [0 ; 1], the following "windows" of a were identified: $0 < a < 1$, x moves monotonously towards the stationary solution $x = 0$; $1 < a < 2$, x moves monotonously towards the stationary solution $x = (a - 1) / a$; $2 < a < 3$, x converges with a flattened oscillatory movement to the stationary solution $x = (a - 1) / a$; $3 < a < 4$, x demonstrates a complex of permanent oscillations. For $a = 3.57$, one can observe an infinite number of fixed points with different periodicity and an infinite number of different periodical cycles; there also exist innumerable combinations of initial conditions from which completely a-periodical, although bounded, trajectories begin; it is from this threshold that the chaotically region begins. For $a > 4$, the model explodes, that confirms the result obtained in the continuous version of model.

In order to estimate the value of parameter or its trend many methods could be used. Some of them are shown in Annex 5.1, where we presented an application on available data in case of French economy, including calculus of fractal dimension (Hurst exponent). Here, we present some conclusions: (a) the model that possesses the properties of chaos is verified and economically consistent, since a transformation that arrives at the canonical form is possible; (b) the values of parameters, however, are quite far from their chaotic bifurcation. Our application on the time series of tax rate in the case of France among 1973 and 1993 demonstrated value of parameter a around 1.8. Other studies obtained similar results. For instance, applying a similar model to the time series of the growth rate of GDP in case of Italy among 1960 and 1982, Candela and Gardini (1986) obtained the following estimation for parameter a: 1.34 for the period 1960-1974 and 0.68 for the period 1975-1982. To note that in our model the values of parameter a smaller than 1 value are excluded. In short, according to our observations, the chaotic model was verified, but it did not produce chaos in this case. It remains to verify in cases of other countries and other periods. In other economic fields, such as financial markets, it was already demonstrated that the chaotic model produces chaos.

5.3.4 Developments in Continuous Version

Introducing the new values of parameter as derived from the discrete-time analysis, we obtained the following system of relations in case of the continuous version:

$$m (x) = - x^2 + x \qquad (5.20)$$

$$a (x) = 1 / (1 - x) \qquad (5.21)$$

Now, the relation between the two parameters can be written as follows:

$$m = (a - 1) / a^2 \qquad (5.22)$$

and the graphical representation is shown in Figure 5.5. On this graph, we can see three remarkable points of parameter a: for a smaller than 1, the parameter m becomes negative (this means that the value 1 represents the lower limit of parameter a in case of our model, corresponding to the situation in which k = Yv; for parameter a equal to value 2, parameter m has a maximum value (this is the case of the well-known value 1/4); for parameter a equal to value 3, the function of m has an inflexion point. However, from the previous section we know that for parameter a having a value greater than 3 the way to chaos is already open. Thus,

now we can affirm that for our Laffer curve model solely the condition m<1/4 is not sufficient. So, this condition must be doubled by the last restriction demonstrated on the scale of parameter a (this is in case of considering the discrete version of the model).

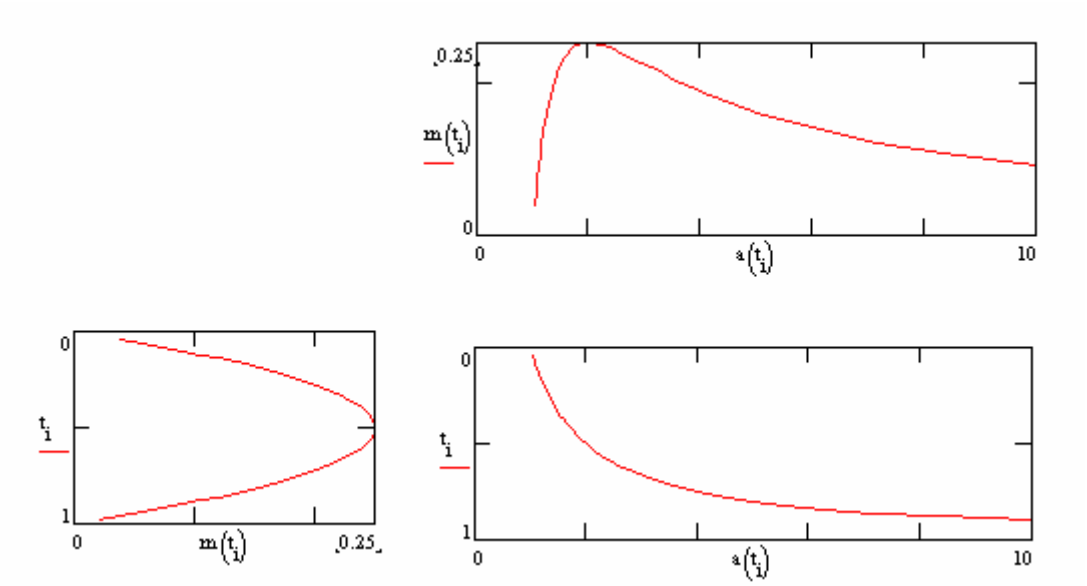


Figure 5.5

Taking into account new information, some deeper analysis in case of the continuous map of potential function V demonstrates that for the value m=2/9 the maximum's branch (curve C2) has an inflexion point on the potential surface. This is the image of inflexion point of function m(a) in the potential function representation (in our graphical representation this inflexion point on V surface does not appear too clear). Only by introducing parameter a in analysis permitted us to discover it. In economic terms, the model shows, under the conditions of presented analysis, that a rise in the share of underground sector to values in the proximity or over 2/3 of k provokes firstly multi-cycles and then emergence of a chaotic behaviour of the system. In contrast with the authors considering that the adoption of the Laffer curve model will led to a drastically reduction in tax pressure and an unlimited increase of underground sector, from our analysis resulted that this model (and its possible generalizations) includes in fact a limitation of underground sector and moreover it can show, at least from a theoretic viewpoint, when this limit was reached.

Surely, the analysis based on Laffer curve model can be refined further more by considering new parameters, such as: structure by sources - taxes and bonds - of governmental expenditure, structure by sources - capital, salaries, dividends, wealth, etc. - of governmental income from taxes, etc. Unfortunately, this development introduces new complicated deformations in Laffer curve model, new non-linearity, and surely the model must be fundamentally changed. But the solution will be also to analyse separately grouped the relations and variables, considering simultaneously only two or, under special conditions, three of them. In terms of the models derived from our existing economic theory this is the manner of restriction (9). Our results may be considered as an interlocution in the chaos/ chance debate, on one hand, and in the standard economics/supply-side economics debate, on the other hand. In fact, the non-linear estimate is one that is confirmed but the irregularities are explained by the stochastic, not the deterministic component, inasmuch as the control parameter never reaches either the chaotic or the oscillatory bifurcation. A similar relation seems to exist in the second case. Our present results in case of the tax rate policy and its close supply-side analysis are promising at least for three reasons: 1 - France does not represent the extreme case in the taxation field; 2 - the estimated value of parameter a is greater than other estimations (as it was shown above) and moreover it is situated in the

neighbourhood of the oscillatory bifurcation zone; 3 - some important factors such as distribution of sources used to finance the governmental expenditure (which could lead to an increase in the value of parameter a, especially in case of Eastern countries) was not yet considered.

NOTES

(1) An extended review of the existing literature on informal economy can be found in Porters et al. (1989).
(2) In opinion of some authors, there exists an indisputable relationship between the development of the underground economy and the role of the State in the social and economic life. So, the strong development of the parallel economy in Eastern countries is, to a certain extent, a retort of the society to the imposed and ineffective economic system (Debare, 1992).
(3) Professor Duchêne exposed his opinions within the GRATICE's Workshop, organized at University of Paris XII - Val de Marne in June 1995 (Duchêne, 1995). Also, he presented an interesting paper on the underground economy in the transition period in the East-European countries (Duchêne, 1995).
(4) For some, the informal sector represents a vibrant band of small-scale entrepreneurs who will eventually emerge as the foundation for capitalistic development. This viewpoint is opposed to the "reserve army of unemployment" that sees the informal sector as surplus labour to be absorbed in the process of growth and accumulation.
(5) This represents one of the most important differences between economics and the experimental scientific disciplines, such as physics, for instance.
(6) Here, we emphasize that whatever model based on the mentioned complementary hypothesis and additional hypothesis, in the shown conditions, is qualitatively identical with the Laffer curve model (so, in this class it can be included models as: investment - consumption; governmental sector - private sector; enterprise sector - household sector, etc.). Under these conditions, we can affirm that it was wrong to criticize only the Laffer curve model as being unreasonably or psychologically and not the other models from the same class. For instance, the Laffer curve model may cover the so-called currency demand methods, where is supposed that in a base year there is no underground economy. This assumption is equivalent, in terms of Laffer curve model presented in Figure 5.1, to a movement of the Yu line from the origin of coordinate axes (0; 0) to an other point situated on the abscise axis, that corresponding to the initial value of tax rate in the considered base year.
(7) In this version of paper we omitted the part of theoretical argumentation. However, here we present only a brief incursion in the recent literature and some possible connections with the subject of underground economy development. So, in the last years there was an increasing preoccupation of scientists from mathematical economics field for studying the dynamic economic systems in non-linearity and discontinuity conditions. Although the disputes between the theorists representing the two great theories (the catastrophe theory, as a special case of bifurcation theory, whose emphasis is on discontinuity in the large; and the chaos theory, and its close relative fractal geometry, whose emphasis is on discontinuity in the small) continued, there are signs of a reconciliation within a meta-theoretical sketch of the General Theory of Economic Discontinuities at least concerning elaboration of some works of syntheses (Lorenz, 1989; Rosser,1991; Brock et al., 1991; Zhang, 1991; Medio and Gallo, 1992; Puu, 1993; Day, 1994) or including in the same book authors belonging to different mainstreams (Grandmont, ed., 1988; Barnett et al., eds., 1989; van der Ploeg, ed., 1990; Pesaran and Potter, eds, 1993; Semmler, 1994; Creedy and Martin, 1994). Based on new mathematical developments (Grandmont, 1988), an increasing number of studies investigate non-linearity in models of economic dynamics. Some of the most important problems from the available literature which have been approached recently are: endogenous business cycles and competitive market dynamics; overlapping generations models; speculative bubbles and crashes; optimal accumulation in two-sector models; imperfect financial intermediation; multiplier-accelerator models; cobweb model; dynamics of the perfect foresight and rational expectations; dynamics of the adaptive expectations; discrete time model of monetary

dynamics. An interesting approach relatively to our research is the Chiarella's "analysis of the effect of time lags and nonlinearities in macroeconomic models including the government budget restraint", where taxation of disposable income and bond financing regime are considered (Chiarella, 1990). Weddepohl who analyses types of chaotic equilibrium within the overlapping generation models makes another approach. He demonstrated that a backward perfect foresight equilibrium path can be: stationary; converging to a stationary path; periodic; converging to a periodic path; chaotic, that is, without any structure; diverging (Weddepohl, 1990). Also, Woodford, studying the complex dynamics of a macroeconomic model under the imperfect financial intermediation hypothesis, demonstrates the existence of many types of the system behaviour and the possibility for the equilibrium dynamics to be strongly chaotic. This might occur when some conditions are fulfilled (Woodford, 1989). Also, important results in the same way have been obtained by other authors (Brock and Hommes, 1995; Hommes and de Vilder, 1995; de Vilder, 1995). At the same time, but in parallel, a growing concern for the phenomenon of the hidden economy has increased attention among officials, politicians, and social scientists. As we showed, the methods adopted to estimate the size of underground economy can be grouped in two parts: direct approach (at microeconomic level) and indirect approach (at macroeconomic level); but there are still many controversies concerning the notion and sphere of comprising the underground economy (at present, there are over 100 names that are attributed to it, such as hidden, shadow, second, informal, etc.). Regarding the problem of underground (or parallel) economy in former communist countries, there are only few studies. One of the most useful works in this field is the Hénin's model, which offers a various menu of analysing the paths of emergence and development on underground side of Eastern economies (Hénin, 1986). There is extensive Western literature that deals with the fiscal policy impact on main macroeconomic indicators' changes or on individual agents' behaviour, but as a rule the underground sector is only implicit considered or it is ignored. Even in these conditions some recent studies demonstrate that chaotic regimes may occur from some threshold values of tax rate (Day, 1994). Also, the so-called "complexity" of tax rate system is questionable regarding the possibility of some unstable or even chaotic regimes' emergence. Some authors consider this complexity has an important impact on underground sector extension (Schneider et al., 1989; Lemieux et al., 1994). However, a remained major question is which is the mechanism of emergence and expanding the underground economy? Generally, the economic literature accepts that the rise of fiscal pressure leads to an erosion of the base of tax. This erosion is due to the limitation of the amount available for saving and investment and therefore for the income subject to tax in the next period. Therefore it will be necessary to consider a time lag and a discrete choice of economic modelling. But it is known that a discrete model of a logistic equation type (as the Laffer curve) leads, even in a single dimension space, to a chaotic behaviour for some values of an essential state-parameter. This opens the way for an approach under the terms of the general theory of discontinuities. Also, even within a continuous choice of economic modelling, the problem will be complicate by distribution of the taxpayers varying with level of income and wealth. Other complications emerge when a spatial distribution within a territory is considered. A map of this distribution, where the different points are in most of the cases placed on continuous curves and areas having smooth slopes, can be conceived. But on this map there will be also some critical points in the neighbourhood of which the system behaviour will register great and abrupt changes (jumps or falls) at slight deviations in value of some essential parameters (control parameters). In the "neighbourhood" zones the system behaviour and structure of the map are fundamentally different from the smooth zones. This type of approach corresponds to one of the catastrophe theory approach. Another interpretation of the tax base erosion is that it is caused by the migration of economic activity from the official sector to hidden economy sector, where the chaotic behaviour is supposed to prevail. When the regulations and fiscal pressure raise the share of underground sector increases, the "amount" of chaos of the entire economic system rises too. It is possible that at a certain level of this share, corresponding in its turn to a certain tax rate level (critical level A) an "explosion" of the system occurs, that is the entire economic system is subject to a chaos regime. The question is how large is this threshold level? On the other hand, when the fiscal pressure decrease the share of the underground sector decreases too, and the corresponding "amount" of chaos infused in the economic system by hidden economy declines; but in the official sector may occur a rising instability (as a rising budget deficit, due

to the decrease of tax amount input). It is possible that this increasing instability to attract the official sector in a chaotic zone, and consequently the entire system enters a chaotic regime. In this case, the question arises: how low is this tax rate level (critical level B) beyond the systems explodes? A simplified model, as a Thom-type, a fold catastrophe with hysteresis effects, or a cusp catastrophe, can be imagined for representing this sort of an entire economic system evolution. Note that the two critical levels seem as two attractors, in their neighbourhood the system behaviour being fundamentally different from the one of stable zone. So, it is important to estimate the bounds of an assurance zone. Still note that the two types of chaos are fundamentally different. The first one (high level of tax rate) is due to the predominance of the "amount" of chaos from the underground sector, but the second one (low level of tax rate) is due to the predominance of the "amount" of chaos from the official sector, whose source is in an increasing instability in the regular sector. One of the recent developments of literature concerning the underground economy is the introduction of the term "complexity", but often in a standard economics context. It is used in two ways: as a general complexity of income tax (that is a complex or less visible income tax schedule) or as a complex income tax distribution according to criteria as age, sex, income level, wealth level, etc. For modelling, there is the problem of when its degree or level of other parameter may provoke the "explosion" of the initial model used and another must be the choice (also, it is possible that this situation occurs in reality). Another kind of complexity, important by the impact on the general plan of stabilization policy in the context of supply-side theory, is that relating to the methods of financing government budgetary deficit and government expenditure. Also, must note that the present day hard fiscal policy in Eastern European economies, which led to an explosive expansion of the underground sector, is probably due to the existence of an only incipient market of bonds. This situation practically restricts the financing of government deficit mainly by income taxation, in conditions of strong crises of investment and consequently of a weak economic efficiency. Concerning the time series obtained by using different methods for estimating the size of underground sector or generally those of tax rate registered, mention should be made that often there is large irregularity. Using a treatment in terms of fractal dimension methods may solve this situation at least partially.

(8) To impute to Laffer curve model that it encourages to tolerate and to stimulate underground economy development is a wrong affirmation in our opinion. The mentioned conclusions may result from our limited possibilities to model, from the limits of our present science and our existing mathematical apparatus, from our own power of understanding the reality (the scientists from other disciplines as physics, for instance, had already the power to affirm frankly their own limits in investigating and understanding, but some economists have yet reticence to affirm this). Maybe, the main preoccupation of research would be to register the evident facts, even when they refuse to come into our present models.

(9) On the other hand, the investigation of some economic time series or the constructing some models including until 4 or even 5 slow parameters, can be developed by using the recent results of Chaos Theory and its close Fractal Geometry and respectively Catastrophe Theory. But many times the explanations will be beyond those accepted by standard economic theory. Consequently, the partisans of classical economics mainstream will contest many of these studies.

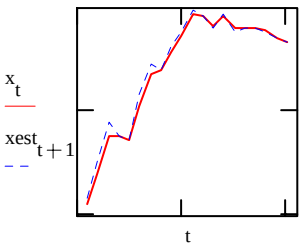
$$a := \frac{\sum_t x_t + \sum_t \left(x_{t-1} \cdot x_t\right) + \sum_t \left[\left(x_{t-1}\right)^2 \cdot x_t\right]}{\sum_t x_{t-1} - \sum_t \left(x_{t-1}\right)^4}$$

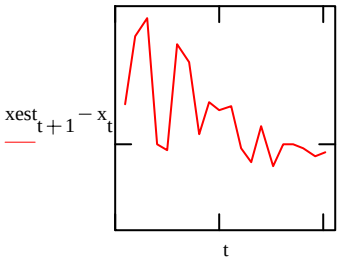
$a = 1.736$

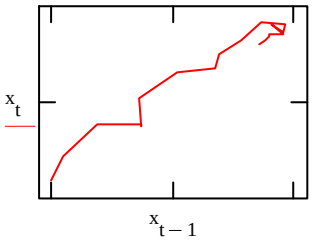
$$ad_t := \frac{x_t}{x_{t-1} \cdot \left(1 - x_{t-1}\right)}$$

$$xest_{t+1} := ad_t \cdot x_t \cdot \left(1 - x_t\right)$$

t	x_t	x_{t-1}	ad_t	$xest_{t+1}$
1	0.355	0.35	1.56	0.357
2	0.369	0.355	1.612	0.375
3	0.387	0.369	1.662	0.394
4	0.387	0.387	1.631	0.387
5	0.386	0.387	1.627	0.386
6	0.402	0.386	1.696	0.408
7	0.417	0.402	1.735	0.422
8	0.419	0.417	1.723	0.42
9	0.428	0.419	1.758	0.43
10	0.436	0.428	1.781	0.438
11	0.446	0.436	1.814	0.448
12	0.445	0.446	1.801	0.445
13	0.44	0.445	1.782	0.439
14	0.445	0.44	1.806	0.446
15	0.439	0.445	1.778	0.438
16	0.439	0.439	1.783	0.439
17	0.439	0.439	1.783	0.439
18	0.438	0.439	1.778	0.438
19	0.435	0.438	1.767	0.434
20	0.433	0.435	1.762	0.433







$z_2 := 1$

$z_3 := 1.3678940093$ $z_4 := 1.775861181$ $z_5 := 2.2433412106$

$z_6 := 2.7879268638$ $z_7 := 3.1558521557$ $z_8 := 3.3355826753$

$z_9 := 3.5884756707$ $z_{10} := 4.1711599598$ $z_{11} := 4.7490255953$

$z_{12} := 5.2229461305$ $z_{13} := 5.6811880633$ $z_{14} := 6.2069669396$

$z_{15} := 6.6882190924$ $z_{16} := 7.139098945$ $z_{17} := 7.563236572$

$z_{18} := 7.9648813877$ $z_{19} := 8.3463962272$ $z_{20} := 8.7063894461$

$z_{21} := 9.0471238731$

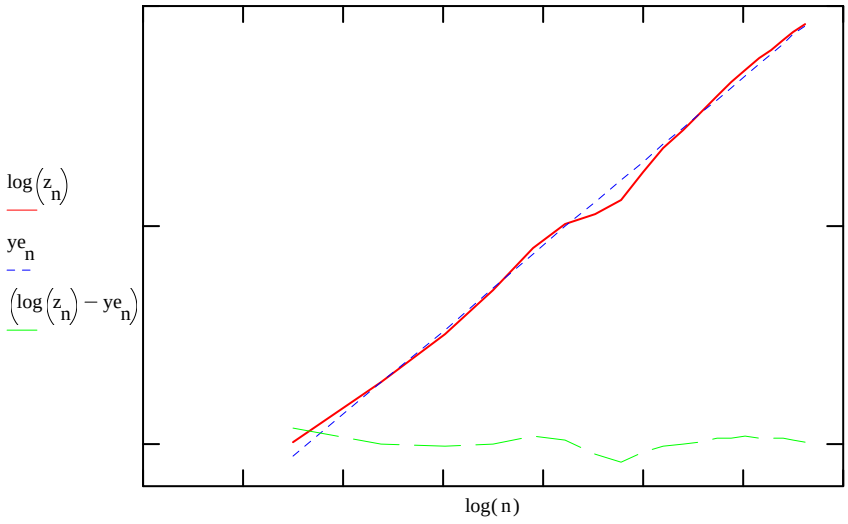
$$H := \frac{20 \cdot \sum_n \left(\log(n) \cdot \log\left(z_n\right)\right) - \left(\sum_n \log(n) \cdot \sum_n \log\left(z_n\right)\right)}{20 \cdot \sum_n \log(n)^2 - \left[\sum_n (\log(n))\right]^2}$$

$$b := \frac{\left(\sum_n \log(n)^2\right) \cdot \left(\sum_n \log\left(z_n\right)\right) - \left[\sum_n \log(n) \cdot \sum_n \left(\log(n) \cdot \log\left(z_n\right)\right)\right]}{20 \cdot \sum_n \log(n)^2 - \left[\sum_n (\log(n))\right]^2}$$

$ye_n := H \cdot \log(n) + b$

$H = 0.966$

$b = -0.322$



6. QUANTITATIVE MODELS AND APPLICATIONS

In this chapter, we present main trends concerning underground economy modelling coming from the existing literature as well as some of our results, including applications to the Romania's economy in the transition period.

Our incursion in the specialized literature is from objective reasons very selectively. It focused on the main conclusions derived from quantitative approaches. Also, some of these resulting ideas will use as hypotheses in our in our following more or less quantitative models. Unfortunately, due mainly to the lack of some accurate large time series, the resulting estimations regarding the dimension and dynamics of underground economy in Romania are not very robust. Consequently, they must consider with prudence. The extension of time series to incorporate the period behind 1989 was not many times possibly by reason of incompatibility between the central-planned period and transition period.

A reasonable solution, to capture the degree of underground economy extension, may be to intersect the estimations coming from different models, as we some times proceeded. Other times, the models produced only simulation results, remaining for the future to apply them on empirical data.

6.1 Models Based on Direct Approaches

Many times, the estimations of underground economy are directly obtained by extrapolation of data collected from a limited number of households by surveys and samples. However, in recent years, the modern theoretical models on tax evasion, which started with the work of Allingham and Sandmo (1972) and continued by Cowell (1990) and other studies already mentioned in previous chapter, there has been developed in a quantifying way. The support provides by some rigorous organized empirical studies. The main impediment in this case is the difficulty to collect information on number of hours worked by persons who illegally evade taxes, which makes it impossible to measure the effect of taxes on the allocation of time. To remedy these impediments, some studies analysed empirically labour-supply decisions in the underground economy using micro data from rigorous organized surveys such as that conducted in Quebec by Fortin and Frechette (1).

Coming from surveys they identified some key empirical regularities about the work in the untaxed sector and then they build adequate quantifying models. However, their survey seems to be less accurate for tax evaders operating at the margin of being detected. In this case data based on extensive audits (2) is more revealing for this tax evades (3).

The main conclusions deduced by Lemieux, Fortin and Frechette from these empirical findings are the following three: 1 - labour earnings in the underground sector are concentrated among workers with low earnings in the regular sector, while expenditures on goods and services produced in the underground sector are typically undertaken by people with high earnings in the regular sector; 2 - the wage rate in the regular sector and the wage rate in the underground sector are positively correlated with hours worked in the regular sector but negatively correlated with hours worked in the underground sector; 3 - earnings in the regular sector are a linear or slightly convex function of regular-sector hours, while earnings in the underground sector are a concave function of underground-sector hours (Lemieux et al., 1994).

Then, they developed a model based on the idea that labour earnings in the underground sector are a concave function of hours of work, while in the regular sector labour earnings are a linear function of hours of work. The concavity of earnings' function in the underground sector implies that the marginal revenues of underground producers decrease as producers reach the limits of the informal markets in which they operate. By contrast, the wage rate of a worker in the regular sector does not vary with the number of hours worked (4).

The results of Lemieux, Fortin and Frechette's study suggest that hours worked in the underground sector are quite responsive to changes of the net wage in the regular sector. Most important, relating to our interest in this paper, their model also provides a natural link

between the slope of the relationship between tax revenues and tax rates (the "Laffer curve") and a more conventional measure of the marginal excess burden of taxes due to the misallocation of productive resources from the regular to the underground sector (5).

The Lemieux's model is based on a concave Cobb-Douglas earnings function in the underground sector

$$Y_1 = A_1 \cdot h_1^{\theta} \qquad (6.1)$$

where $\theta < 1$,

and on a linear earnings function in the regular sector

$$Y_0 = W_0 \cdot h_0 \qquad (6.2)$$

The variable h_0 represents hours of work in the regular sector; h_1 represents hours of work in the underground sector; W_0 is the wage in the regular sector; A_1 is a revenue-shifter in the underground sector.

To simplify the representation, it considered the quasi-linear utility function:

$$U (l, C) = \gamma \cdot C + v (l) \qquad (6.3)$$

where the function v (.) is a strictly concave utility function (v' > 0, v'' < 0). C represents a composite consumption good (the numeraire), while *l* is the number of hours of leisure that satisfies the time constraint

$$T = h_0 + h_1 + l \qquad (6.4)$$

The budget constraint is given by

$$C = I + W_0 \cdot h_0 + A_1 \cdot h_1^{\theta} \qquad (6.5)$$

where I represents non-labour income, including any lump-sum transfer.

In these conditions, the worker's problem is

$$\max_{(C,\, h_0,\, h_1)} \; [\gamma \cdot C + v \cdot (T - h_0 - h_1)] \qquad (6.6)$$

subject to equation (6.5), $h_0 > 0$, and $h_1 > 0$. For workers who supply positive hours in both sectors (interior solution), the first order conditions for hours yield:

$$\delta U / \delta h_0 = - v' \cdot (T - h_0 - h_1) = - \gamma \cdot W_0 \qquad (6.7)$$

$$\delta U / \delta h_1 = - v' \cdot (T - h_0 - h_1) = - \gamma \cdot \theta \cdot A_1 \times h_1^{\theta - 1} \qquad (6.8)$$

These two conditions imply the following equality for the marginal revenue of an hour of work in the two sectors:

$$W_0 = \theta \times A_1 \cdot h_1^{\theta - 1} \qquad (6.9)$$

Hours of work in the underground sector are thus determined by setting the marginal revenue in the underground sector equal to the regular-sector wage. This is a very important result, which implies that, conditional on the regular-sector wage, underground-sector hours do not depend on preferences.

As Lemieux affirms, this "separation" between the determination of hours in the underground sector and preferences is similar to the separation result in the development literature on farm households that states that the labour demand of a farm is determined

independently of the preferences of household members (6). This separation result has several implications that are consistent with the empirical regularities. The main considers the average wage rate in the underground sector:

$$W_1 = Y_1 / h_1 = A_1 \cdot h_1^{\theta - 1} \qquad (6.10)$$

Equations (6.9) and (6.10) imply that $W_0 = \theta \cdot A_1$. Since $\theta < 1$, it follows that the regular-sector wage, W_0, is smaller than the underground-sector wage W_0. This prediction holds, on average, when the measure of W_0 used is the regular wage net of the tax rate (7). This result will be also incorporated within the hypothesis of our global model used to evaluate the dimension of underground economy in Romania.

Following the basic model of tax evasion and considering that net earnings in regular sector, WN_0, are given by

$$WN_0 = (1 - t) \cdot W_0 \cdot h_0 \qquad (6.11)$$

where t is flat tax rate, Lemieux postulates that workers who supply positive hours to the underground sector face a probability P of being detected by authorities. Becoming caught entails a penalty proportional to the amount of tax evaded (8). The penalty rate (tp) on underground income can write as:

$$tp = n \cdot t \qquad (6.12)$$

n being the penalty rate on evaded tax ($n > 1$). The expected rate of tax penalty on evaded income is thus given by $P \cdot n \cdot t$.

Again, in case of this model of tax evasion the result was that underground-sector hours do not depend on preferences conditional on W_0 and t. This is because of fact that the utility function is linear in consumption, which implies risk-neutrality in consumption. The case in which workers are risk-averse but face a parametric wage in both the regular and the underground sector has been analysed by several authors (Sandmo, 1981; Cowell, 1985; Fluet, 1987). This case leads to few interesting comparative-static results, however, even with strong restrictions on preferences for consumption and leisure. As noted by Cowell (1984), the basic reason for these ambiguities is that "in reaction to any perturbation, the individual can substitute across two margins (risk/no-risk and labour/leisure), so that in principle all sorts of behaviour could be consistent with rational expected utility maximization" (9). One main result of the Lemieux's model developed in this way is that an increase in tax rate has a positive effect on hours of work in the underground sector, but a negative effect on hours of work in the regular sector.

To analyse the implications of tax policy, Lemieux considered an experiment in which total hours worked in the two sectors does not depend on the marginal tax rate. In response to a change in the tax rate, people therefore reallocate a given hour of work from the regular to the underground sector, so that:

$$dh_0 / dt = - dh_1 / dt \qquad (6.13)$$

The excess burden (EB) caused by the misallocation of hours between the two sectors is given by:

$$EB = W_0 \cdot [h_1 (t) - h_1 (0)] - A_1 \cdot \{ [h_1 (t)]^{\theta} - [h_1 (0)]^{\theta} \} \qquad (6.14)$$

where $h_1 (t)$ and $h_1 (0)$ are the hours worked in the underground sector with and without a tax rate t respectively. The marginal excess burden (MEB) measures how much the excess burden has to increase in order to raise an additional currency unit (dollar) of taxes:

$$MEB = \delta EB / \delta T = (\delta EB / dt) \times (\delta t / \delta T) =$$

$$= [W_0 - \theta \cdot A_1 \cdot h_1^{\theta - 1}] \cdot (dh_1 / dt) \cdot (\delta t / \delta T) \quad (6.15)$$

where T represents total tax revenues ($T = t \cdot W_0 \cdot h_0$) and $\delta T/\delta t$ is the slope of the Laffer curve. This slope depends on how hours of work respond to a change in the tax rate.

After some transformations, and supposing

$$\theta \cdot A_1 \cdot h_1^{\theta - 1} = (1 - t) \cdot W_0 \quad (6.16)$$

MEB can be written as:

$$MEB = t \cdot W_0 \cdot (dh_1 / dt) \cdot (\delta t / \delta T) = (1 - \eta) / \eta \quad (6.17)$$

where η represents the elasticity of the Laffer curve or $\delta \ln (T) / \delta \ln (t)$.

Some results of application of the model on empirical data collected in Quebec City, Canada, by samples reported by the Lemieux's study are:

- the estimate value of θ is 0.67;
- since both η and MEB depend on the ratio of underground to regular income Y_1/Y_0 and on the marginal tax rate t, their respective values vary considerably across the different groups of the population;
- the estimates of the elasticity of the Laffer curve (η) range from 0.407 for workers who received some welfare payments during the year to 0.994 for workers aged 40 and more;
- these two extreme values of η yield values of the marginal excess burden in production (MEB) of 1.457 and 0.006, respectively;
- on average, an increase in the tax rate does not substantially distort labour-market activities from the regular sector to the underground sector;
- on the other hand, the same increase in the tax rate, or in the tax-back rate embodied in social-welfare programs, has a very substantial effect on the allocation of time of social-welfare claimants.

These conclusions only hold, however, when the probabilities of detection by the authorities and penalty rate are negligible ($P \cdot n = 0$). Although this might be true for some underground activities, the results of Lemieux's study suggest that is not true in general. Government enforcement policies might thus be offsetting some of the distortion due to the presence of the tax and transfer system (Lemieux et al., 1994).

Other model, calibrated to replicate the observed labour supply in a random sample conducted in the metropolitan area of Quebec City for 1985, is that of Fortin and Lacroix (1994). So, they used a simultaneous model of labour supply in the regular and irregular sectors that facilitated to compute the marginal cost of public funds associated with tax and enforcement instruments.

Also, there are some other interesting quantitative models in the existing literature which treat the behaviour of households relaying to the work in underground sector and that are based on micro-data samples, especially in case of Northern European countries (Isachsen and Strom, 1981 and 1985), but they being over the goal of our study that focuses on the macro-data approach of the underground sector.

6.2 Models Based on Monetary Approach

On the rule, there are two categories of models, function of the methods of approach: the transaction approach and the currency demand approach respectively.

The first category of models, developed firstly by Feige (1979), comes from Fisher's quantity equation:

$$M / v = p / Tr \quad (6.18)$$

where M represents money; v - velocity; p - prices; Tr - total transactions.

This approach assumes that there is a constant relation over time between the volume of transaction and official GDP. Assumptions have to be made about the velocity of money and about the relationship between the value of total transactions (p/Tr) and nominal

GDP. Relating total nominal GDP to total transactions, GDP in the shadow economy can be derived by subtracting the official GDP from total nominal GDP. To derive figures for the shadow economy, Feige has had to assume a base year in which there is no shadow economy, and therefore the ratio of p/Tr to total nominal (official = total) GDP was "normal" and would have been constant over time if there were no underground economy. This method too has several weaknesses: for instance, the assumption of a base year with no shadow economy, and the assumption of a "normal" ratio of transactions, constant over time. Moreover, to obtain reliable estimates, precise figures for the total volume of transactions should be available. This availability might be especially difficult to achieve for cash transactions, because they depend, among other factors, on the quality of paper used in the currency. Generally, although this approach is theoretically attractive, the empirical requirements, necessary to obtain reliable estimates, are so difficult to fulfil, that its application may lead to doubtful results.

The second category of models based on currency demand approach was developed firstly by Cagan (1958) and Gutmann (1977). Then, important contributions based on econometric applications had done by Klovland (1980,1984), Tanzi (1982) and Isachsen and Strom (1985).

In the Klovland's approach, for instance, to model behaviour of economic system the following variables are needed: C = currency held by the public; P = price index; Y = real GDP; i = rate of return on time deposits; π = rate of inflation; c = private consumption as a share of GDP; t = total taxes as a share of GDP; θ = stock adjustment parameter ($0 < \theta < 1$).

Klovland specified his model by the following two relations:

$$\ln C - \ln C_{-1} = \theta \cdot (\ln C^* - \ln C_{-1}) \qquad (6.19)$$

$$\ln (C^* / P) = \alpha_0 + \alpha_1 \cdot \ln Y + \alpha_2 \cdot i + \alpha_3 \cdot \pi + \alpha_4 \cdot c + \alpha_5 \cdot t \qquad (6.20)$$
$$\qquad\qquad\qquad\qquad + \qquad - \qquad - \qquad + \qquad +$$

where C* is the long run demand for currency and α_0, ..., α_0 are coefficients.

The signs below the last equation give the expected signs of the coefficients. Isachsen and Strom (1985) used this model to estimate the size of underground economy in Norway. Annual data were used, covering the period 1952-78.

The estimation gave result for ln(C/P). All the estimated coefficients, except for the coefficient attached to the inflation rate, have the expected signs. Then, to proceed with the estimation of the size of the underground economy they rewrite (6.20) as

$$\ln C = \ln P + Z\alpha^* + \alpha_5 t \qquad (6.21)$$

where the term $Z\alpha^*$ includes the α's variables except for the tax rate. α^* and $\alpha 5^*$ are estimated values. Then predicted currency holdings at time t is

$$C_\tau^* = \exp(\ln P_\tau + Z_\tau \alpha^* + \alpha_5 t_\tau) \qquad (6.22)$$

In order to obtain an estimate of the size of the underground economy they selected 1952 as a base year. Thus, if the tax rate had remained at the 1952 level, the predicted value of currency holdings would have been

$$C_{52,\alpha}^* = \exp(\ln P_\tau + Z_\tau \alpha^* + \alpha_5 t_{52}) \qquad (6.23)$$

The difference, $\Delta C_\tau = C_\tau^* - C_{52,\tau}^*$, would then give the increase in the amount of currency needed to fuel the tax evasion part of the economy compared to the currency needs if tax rates had remained at the 1952 levels. They next assume that no tax evasion took place in the base year or in years before that. In these conditions, ΔC_τ would yield an estimate of all currency circulating in the underground economy at time τ. Then, according to Tanzi (1980) and others they assume that the income velocity of currency in the underground economy equals the velocity of M1 money in the official parts of the economy. Under a such

hypothesis, for each year of the analysed period the velocity rate, V_τ, and respectively the GDP contribution from the underground economy, yu_τ, is given by the following relations:

$$V_\tau = Y_\tau / (M1_\tau - \Delta C_\tau) \quad (6.24)$$

$$yu_\tau = (Yu_\tau / Y_\tau) = \Delta C_\tau / (M1_\tau - \Delta C_\tau) \quad (6.25)$$

where Yu_τ ($Yu_\tau = V_\tau \times \Delta C_\tau$) and Yt are the GDP in underground sector and respectively the official GDP.

Using this model, Isachsen and Strom obtained the following values of the share of underground economy in Norway: 1.3% in 1971; 0.6% in 1973; 4.3% in 1976; 6.3% in 1978. Also, Klovland (1980) employed a narrow definition of the money stock and arrived at a higher velocity for the year 1978, that is 6.7. As part of observed GDP, the underground economy then becomes 9.2%.

The first objection relates to the fact that not all transactions in the underground economy are paid in cash. Also, Isachsen and Strom (1981) used the survey method to discover that in Norway in 1980 roughly 80% of all transactions in the underground sector were paid in cash. The size of the total underground economy (including barter) may thus be even larger than previously estimated. Most studies consider only one particular factor, the tax burden, as a cause of the underground economy. Other reasons such as the impact of regulation, the complexity or visibility of the tax system, taxpayers' attitudes to the state, "tax morality", and so on are not considered because data for most countries is not available. If, as seems likely, these other factors also have an impact on the extent of the underground economy, it might be larger than reported in most studies (10).

A further weakness of this approach, at least when applied to the United States by Tanzi (1980, 1983), is discussed by Garcia (1979) and Park (1979). They point out that increases in currency demand deposits are due largely to a slow-down in demand deposits rather than to an increase in currency caused by activity in the underground economy. Blades criticizes Tanzi's studies because the US-Dollar it is used as an international currency, so that Tanzi should have considered (and compensated for) the amount of US-Dollars held in cash abroad. Finally, Frey and Pommerehne (1984) claim that Tanzi's parameter estimates are not very stable (11).

Another weak point of this approach, as applied in most studies, is the assumption that the velocity in both types of economy is the same. As Klovland (1984) argues for the Scandinavian countries, there is already considerable uncertainty about the velocity of circulation of currency in the official economy; the velocity of currency in the underground sector is even more difficult to estimate. Without knowledge about the velocity of currency in the underground economy, one has to accept the assumption of an "equal" velocity of money in both sectors. Finally, the assumption of no underground economy in a base year is open to criticism. Relaxing this assumption would again imply an upward adjustment of the figures attained in the bulk of the studies already undertaken.

We also applied this model in the case of Romania's economy. Unfortunately, because of lack of comparable data concerning the tax rate for to create large time series, the results obtained were ambiguously. In these conditions we are forced to replace tax rate by final government consumption rate. In this case, the estimated size of underground economy is among 1.5% in 1989 and 4.8% in 1994 (Annex 6.1). We consider again these figures as representing only a fraction of the marketable part of the national economy (12).

6.3 Fiscal Pressure and Penalty Models

In this category we included models that take into account the impact of penalty tax. This type of tax applied when the fiscal authorities register a subtraction from tax pay duties or an illegality. It is on rule greater than normal tax rate but smaller than value 1 (case that corresponds to confiscation of the entire registered income that did not declared to the fiscal authorities).

To capture the impact of penalty tax rate on the yearly average tax rate we develop the Laffer's curve model in the following way. Therefore, we consider two conventional stages

of tax process gathering. Firstly, the normal paid taxes gather from taxpayers. To express the total amount of income taxes received by fiscal authorities in this stage we recall the simple expression of Laffer's curve model:

$$Tn = tn \cdot Yn = tn \cdot (Y - Yh) = tn \cdot (1 - tn) \quad (6.26)$$

where Tn is the total amount of taxes received by normal procedures; tn - average normal tax rate; Yh - hidden GDP at the end of normal tax procedures; Y - total GDP (including Yn and Yh).

In the second stage the fiscal authorities register a part of hidden activity, which became visible, and by applying penalty procedures they gather:

$$Tp = tp \cdot Yp = tp \cdot (Yh - Yi) = tp \cdot Yh \cdot (1 - tp) \quad (6.27)$$

where Tp is the total income from penalty procedures; tp - average penalty tax rate (1>tp>tn); Yi - invisible GDP (remaining hidden after the application of penalty procedures).

Unifying the two types of taxes we can write:

$$T = tm \cdot Yv = tn \cdot Y \cdot (1 - tn) + tp \cdot Yh \cdot (1 - tp) \quad (6.28)$$

where T is the total yearly amount of paid and received taxes; tm - the resulting average tax rate calculated at the end of year; Yv - the visible GDP registered at the level of year.

Similar to generalized Laffer's curve model we considered the following two hypotheses:

$$Yh = Y \cdot tn \quad (6.29)$$

and respectively

$$Yi = Yh \cdot tp \quad (6.30)$$

They suppose that in both stages of gathering of taxes the proportion of hidden GDP equals to the respective applied tax level.

After some algebraic operations, we can write the yearly average tax rate as following:

$$tm = [T / (Yn + Yp)] = tn \cdot [(1 - tn) + tp \cdot (1 - tp)] / [(1 - tn) + tn \cdot (1 - tp)] \quad (6.31)$$

Supposing tm as a given parameter, the relation between normal tax and penalty tax will be:

$$- tn^2 + tn \cdot [1 + tp \cdot (1 + tm) - tp^2] - tm = 0 \quad (6.32)$$

under the condition:

$$Dn = [1 + tp \cdot (1 + tm) - tp^2] - 4 \cdot tm > 0 \quad (6.33)$$

The equation (6.32) can also write as:

$$- tn \cdot tp^2 + tn \cdot (1 + tm) \cdot tp + tn \cdot (1 - tn) - tm = 0 \quad (6.34)$$

with the condition:

$$Dp = tn \cdot [- 4 \cdot tn^2 + (5 + 2 \cdot tm + tm^2) \cdot tn - 4 \cdot tm] > 0 \quad (6.35)$$

where:

$$0 < tn < tm < tp < 1 \quad (6.36)$$

The model permits to compute different alternatives of fiscal policy, when the accent moves from the instrument of normal tax policy to the instrument of penalty tax and their limits. Some simulation results present in Annex 6.2.

Other model to optimising the productivity of fiscal penalty activity, which correlated with the first model, should permit to choose the best policy from a set of possible fiscal policy alternatives. It can conceive, by supposing the registered GDP by penalty procedures (Yp), as being a concave function of the expenditure for fiscal penalty procedures. Therefore, we choose the following logistical type function:

$$Yp = Yh \cdot [(1 - e^{- a \cdot j}) / (1 + e^{- a \cdot j})] \quad (6.37)$$

where e is the natural logarithmic base; j - the necessary expenditure to apply penalty procedures; a - coefficient (which may be estimated econometrically).

In this case, the expression of total amount of taxes from penalty procedures can write as:

$$Tp = tn \cdot Y \cdot tp \cdot [(1 - e^{- a \cdot j}) / (1 + e^{- a \cdot j})] \quad (6.38)$$

A promising forthcoming path is to develop the model on an econometric base (13). In this paper we present in Annex 6.3 only a simulation result.

6.4 A Global Model Based on Labour Supply Method

Firstly, we consider a national economy having only two sectors: visible (or official) sector and invisible (or underground) sector. In case of visible sector the registered GDP supposes as having the following components:

$$Yv = A + Sv + Bv \quad (6.39)$$

where Yv is the GDP produced in the visible sector; A - consumption of fixed capital (only in visible sector); Sv - wages of employees in visible sector; Bv - profit of entrepreneurs (capitalists) in the visible sector. In the invisible sector the produced GDP will be:

$$Ya = Sa + Ba \quad (6.40)$$

where Ya is the GDP produced in the underground sector; Sa - wages of employees in the invisible sector; Ba - profit of entrepreneurs (capitalists) in the invisible sector. In case of invisible sector it supposes that there is only circulating capital.

Also, we consider that both the available time fund and total number of labour force distribute between two components, as following:

$$F = Fv + Fa \quad (6.41)$$

$$L = Lv + La \quad (6.42)$$

where F is the total available time fund for work within a calendar year; L - number of total potential working persons; Fv - time used for work in the visible sector by year; Fa - potential available time used by the persons having status of employees in visible sector for work as a double job in underground sector; Lv - number of employees working in visible sector; La - potential number of employees working in underground sector.

We mention that the available working time evaluates as average number of hours' values by a calendar year. For instance, they may consider as average number of hours worked by a person within a calendar year.

Now, we express the GDP created in the two sectors of a national economy as functions of productivity, which consider here as linear functions:

$$Yv = Lv \cdot Fv \cdot wv \quad (6.43)$$

$$Ya = (La \cdot F + Lv \cdot Fa) \cdot wa \quad (6.44)$$

where wv and wa are the average productivity by person by hour in visible sector and respectively in invisible sector.

To obtain the total number of hours worked by year in underground sector, we considered two categories. One of this encloses persons that work full-time in the underground sector (persons having status of non-employees in the visible sector) (La · F). The other come from the persons having the status of employees in visible sector, but working also in underground sector during the supplementary work time as a second job (Lv · Fa).

What is interesting for the agents or people are the level of disposable income or available GDP. In the two sectors of the economy the available income or GDP will give by the following relations:

$$Ydv = Yv - T = Yv \cdot (1 - t) \quad (6.45)$$

and respectively

$$Yda = Ya \quad (6.46)$$

which can be also written under the following forms:

$$Ydv = Lv \cdot Fv \cdot wv \cdot (1 - t) \quad (6.47)$$

$$Yda = (Lv \cdot Fa + La \cdot F) \cdot wa \quad (6.48)$$

where Ydv, Yda represent the available income in visible sector and respectively in invisible sector; T is the total amount of taxes paid (or of obligatory levying); t - average tax rate relatively to Yv.

Taking into account the structural relations (6.41 and 6.42) we can write now the expression of the total available income as following:

$$Yd = Lv \cdot Fv \cdot wv \cdot (1 - t) + [Lv \cdot (F - Fv) + (L - Lv) \cdot F] \cdot wa \quad (6.49)$$

or

$$Yd = (L - La) \cdot (F - Fa) \cdot wv \cdot (1 - t) + [(L - La) \cdot Fa + La \cdot F] \cdot wa \quad (6.50)$$

The first relation permits to analyse the impact of number of persons working in visible sector (Lv) and of their corresponding number of hours worked in this sector (Fv) on the total available income at national level. The second makes the same but concerning the number of persons working in invisible sector (La) and respectively the number of hours worked in invisible sector by persons having the status of employees in visible sector (Fa). We remember that in case of persons actually having the status of non-employees in visible sector (but having a potential to work by age and disposable free time criteria) it supposed that they allocate their entire available working time to work in the invisible sector (F). In the same time, the persons actually having the status of employees in visible sector are forced to divide the same entire disposable working time (F) between work in the visible sector (Fv) and work in invisible sector (Fa).

Maybe this total available income, greater than available income in visible sector, is responsible of some unexplainable macroeconomic non-correlation registered between some "official" indicators.

Now, we define the yearly national potential by the following relation:

$$P = F \cdot L \cdot wv \quad (6.51)$$

where P is the maximum level of GDP by year.

Due to the productivity in the invisible sector is on rule supposed weakly than productivity in the visible sector it results the following non-equality:

$$Yv + Ya < P \tag{6.52}$$

Now, we can express the actual total available income by year as:

$$Yd = P \cdot [\, m + lv \cdot fv \cdot (1 - t - m) \,] \tag{6.53}$$

where m is the ratio between the productivity in invisible sector and productivity in visible sector (wa/wv); lv - the share of employees in visible sector in the total number of potential working persons (Lv/L); fv - the share of time used to work in the visible sector in total available working time within a calendar year (Fv/F). On the other hand, if the entire activity would allocate in the visible sector the maximum level of available GDP would be:

$$Yd^* = P \cdot (1 - t) \tag{6.54}$$

Now, we suppose that the people chose the actual situation that is the actual distribution of the total capacity to work between sectors. This produces an available income greater than or at least equal to that would be produced by the above hypothetical case. Therefore, there will be the following restriction:

$$Yd > Yd^* \tag{6.55}$$

After some technical algebraic operations, it obtains the following equivalent restrictions:

$$m > 1 - t \tag{6.56}$$

$$wa > wv \cdot (1 - t) \tag{6.57}$$

We must now mention that our present model conceives as a simulation model in the goal to determine the variance interval thresholds of underground sector coming from the existing statistical data at macroeconomic level. In this case we considered the absolute values both for the total potential number of working persons (L) and for total number of potential working hours during a calendar year (F). This could seem a few graters exaggerated comparing with the real values registered in case of visible sector (Lv and Fv).

So, in case of invisible sector, we consider that the resulting levels of some indicators - productivity, profit rate - are smaller than in the real situation. We consider that this diminution must consider in the following way. It is because aside of the comprehension to obtain available income for instance in our model there is an implicit manner included the comprehension to leisure of the people having an available labour force. For instance, the actual available income computed by our simulation model is greater than the level that would obtain in the case of a full-time work in the visible sector ($Yd > Yd^*$). The difference must consider as implicitly including the satisfaction of leisure comprehension problem.

Now, we consider importantly evaluating the variation interval of the underground sector dimension. In this way, we write the share of invisible sector in the national economy as:

$$ya (wa) = Ya (wa) / Y (wa) \tag{6.58}$$

where Ya (wa) is given in relation (6.44) and Y is the total yearly GDP:

$$Y (wa) = Yv + Ya (wa) \tag{6.59}$$

For the productivity in the invisible sector we chose the following extreme values:

$$wa_{min} = (1 - t) \cdot wv \tag{6.60}$$

$$wa_{max} = wv \quad (6.61)$$

to which correspond the following extreme values of the share of invisible sector in national economy:

$$ya_{min} = 1 - \{ (lv \cdot fv) / [1 - t \cdot (1 - lv \cdot fv)] \} \quad (6.62)$$

$$ya_{max} = 1 - lv \cdot fv \quad (6.63)$$

Within this interval we shall analyse the diverse alternatives regarding the average wages and profits in the underground sector relating to the situation registered in the visible sector.

In case of visible sector, to evaluate the average wage and the average rate of profit, we considered the following relations:

$$sv = [Sv / (Lv \cdot Fv)] = [Yv - (T + Bv + A)] / (Lv \cdot Fv) \quad (6.64)$$

$$bv = [Bv / (T + A + Sv)] = [Yv - (T + Sv + A)] / (T + A + Sv) \quad (6.65)$$

where sv is the average wage by person by hour of work in visible sector; bv - the average profit rate in visible sector; Sv - total amount of the yearly salaries in visible sector; Bv - total amount of the yearly profits in visible sector.

On the other hand, in invisible sector the corresponding relations are:

$$sa = [Sa / (La \cdot F + Lv \cdot Fa)] = (Ya - Ba) / (La \cdot F + Lv \cdot Fa) \quad (6.66)$$

$$ba = [Ba / Sa] = (Ya - Sa) / Sa = (wa - sa) / sa \quad (6.67)$$

where sa is the average wage by person by hour of work in invisible sector; ba - the average profit rate in invisible sector; Sa - total amount of the yearly salaries in invisible sector; Ba - total amount of the yearly profits in invisible sector.

In case of the following applications we replaced the consumption of fixed capital (A) by investment. Moreover the average wage in the invisible sector considered comprised between the values 0.5 and 2 relating to the average wage in the visible sector of the national economy. The detailed tables and calculus find in Annexes 6.4. Here we present only some conclusions resulting from the model's application in case of Romania's economy.

For the each year of the period 1989-1993 the number of employees in the visible sector (Yv) is that from the available official statistical publications. To evaluate the yearly time of work in visible sector for each year we firstly diminished the number of days of year by weekend days and vacancies. Then we multiply the result with legal time working per day (8 hours) and then by an average coefficient that designs the average using degree of the total legal working time.

For both total potential numbers of employees (L) and total number of hours by year by person (F) we considered three versions that designed by maximum (I), intermediary (II), and minimum (III). Also, in case of the maximal version, L includes the employees in the visible sector (Lv), official unemployed, school population of age over 15 years, and retired population of age under 70 years. In case of intermediary version, it added to Lv the same number as in precedent case, but divided by two. In case of minimal version, it added to Lv only a half of unemployment number.

In case of F, the maximal version obtained by multiplying the calendar year number of days with eight (representing hours of work by day). To determine the intermediate value of F, we added 52 (representing the number of Saturdays by year) to the average number of days worked per year by a person in the visible sector (Fv). Moreover, we added a half of number of days used for vacancies in visible sector. Then we multiply the result with eight (number of hours worked by day). In case of minimal version we chose the same number of days worked in visible sector but multiplied only with eight hours per day.

In case of each variant (I, II, and III), we calculated the two threshold values - minim and maxim - of underground economy by using formulas (6.62 and 6.63). Also, within this interval, we separated a sub-interval. This covers, on the one hand, the intersection between the curve of salary in underground sector and the value of salary in official sector (inferior limit). On the other hand, it covers the intersection between the curve of profit rate in underground sector and the value of profit rate in official sector (superior limit). This sub-interval represents, when it adopted one of the three variants for analysing, the set enclosing the most probably values of underground economy share. A synthetic presentation of simulation results there is in Table 6.1. Also, in Figure 6.1, we show the dynamics of underground economy in Romania in the period 1989-1993, according to the three variants.

Variant I, derived from the theoretical limits of the productive national potential, can suggest the maxim values to which the underground sector should extend. It is useful for long run forecasts. The versions II and III seem to produce estimates that are nearly of the figures commonly used in literature for to appoint the size of underground economy. One of the most difficult problems remains the separation between the preference of people to use their available time for work in underground economy and that to use the available time for leisure. In any case, the variant I can produce a satisfactory estimation for the total available time (including both components).

Table 6.1

Variant I:

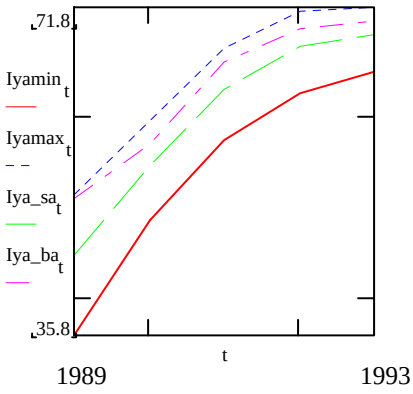
$Year_t :=$	$Iyamin_t :=$	$Iyamax_t :=$	$Iya\_sa_t :=$	$Iya\_ba_t :=$
1989	35.8	51.3	44.6	50.8
1990	48.4	59.4	54.5	56.8
1991	57.2	67.3	62.8	65.8
1992	62.3	71.3	67.5	69.4
1993	64.8	71.8	68.7	70.4

Variant II:

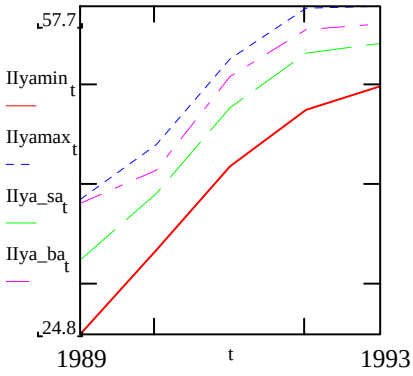
$Year_t :=$	$IIyamin_t :=$	$IIyamax_t :=$	$IIya\_sa_t :=$	$IIya\_ba_t :=$
1989	24.8	38.4	32.3	37.9
1990	33.3	43.7	38.9	41.2
1991	41.7	52.4	47.6	50.6
1992	47.4	57.6	53.0	55.4
1993	49.6	57.7	54.0	56.0

Variant III:

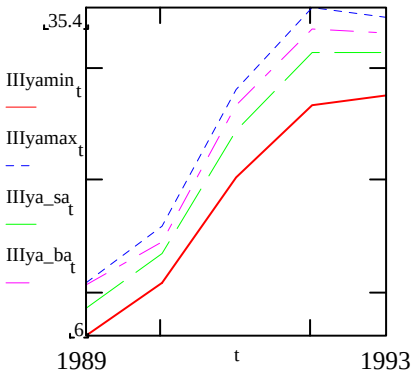
$Year_t :=$	$IIIyamin_t :=$	$IIIyamax_t :=$	$IIIya\_sa_t :=$	$IIIya\_ba_t :=$
1989	6.0	10.8	8.5	10.6
1990	10.7	15.8	13.3	14.4
1991	20.1	28.0	24.3	26.6
1992	26.7	35.4	31.3	33.4
1993	27.6	34.6	31.3	33.1



Variant I:



Variant II:



Variant III:

Figure 6.1

6.5 A Generalized Model for the Allocation of Time

Now, we recall the two hypotheses of the model for the time allocation, relations (6.1) and (6.2). Also, we present in Figure 6.2 a graphical representation of this model.

The main implication of this model, also largely used in literature, is that in point M of graphic the average salary in underground sector equals to that in visible sector. This, translated in terms of our model, can be expressed by following relations:

$$Sa_M = A_1 \cdot hv^{\theta} = Sv_M = sv \cdot hv \quad (6.68)$$

On the graphic, also one may observe that the difference between Sa and Sv is maximal only in one point of abscise noted ha*, which represents the optimal number of hours worked in underground sector.

To evaluate this optimal level of underground work, we write the function, which must be maximized as:

$$G(ha) = A_1 \cdot ha^{\theta} - sv \cdot ha \quad (6.69)$$

and the first-order derivative as:

$$G'(ha) = \theta \cdot A_1 \cdot ha^{\theta - 1} - sv \quad (6.70)$$

However, at the national level, the function that concerns is:

$$H(ha) = S - S^* = (A_1 \cdot ha^{\theta} + sv \cdot ha) - sv \cdot (hv + ha) \quad (6.71)$$

Here, we can observe the equality between H and G. So, the two functions have the same solution for maxim.

After some algebraic operations, we obtained the following remarkable value for ha:

$$h^* = hv \cdot \theta^{[1/(1-\theta)]} \quad (6.72)$$

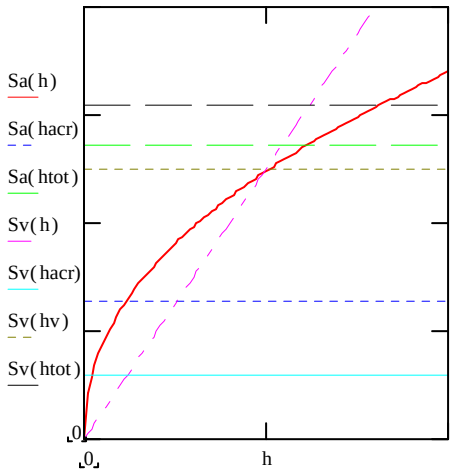
that then we permitted to express the optimal proportion of underground economy within the national economy by the following relation:

$$(h^* / h) = \{\theta^{[1/(1-\theta)]}\} / \{1 + \theta^{[1/(1-\theta)]}\} \quad (6.73)$$

where, h is total number of hours worked in a country in a year.

On base of this relation we calculated the share of underground economy for various values of parameter θ (see Annex 6.5). An interesting result is that, at limit, the maximal share of underground economy (in shown optimal conditions) is only about 26.9% (i.e. 1/(1+e)).

Developing this model, we can obtain the map of the entire process of allocation of time, as it is shown in Figure 6.2. Also, here is represented the migration of some cohorts from the so-called "army of reserve" (leisure time) to the underground sector. For instance, this may be provoked by an increase in number of hours worked in visible sector, which will move at a higher level the curve of wage in underground sector. This will make more attractive work in underground activity and it may lead to a re-allocation of available time of households. In these terms it may be considered the potential labour force and the maximal values of underground sector in case of variant I from precedent section.



$h := 0..(hmax)$

$La := L - Lv$
$Fa := F - Fv$
$hv := Lv \cdot Fv$

$hacr := hv \cdot \theta^{\frac{1}{1-\theta}}$

(hacr=h*)

$htot := hv + hacr$
$hrez := hmax - htot$

$Sv(h) := sv \cdot h$
$Sa(h) := A \cdot h^{\theta}$
$Sacr := A \cdot hacr^{\theta}$
$SvM := sv \cdot hv$
$Stot(h) := Sv(h) + Sacr$

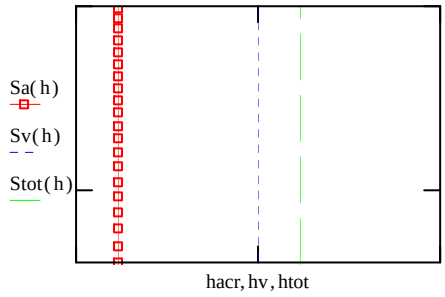


Figure 6.2

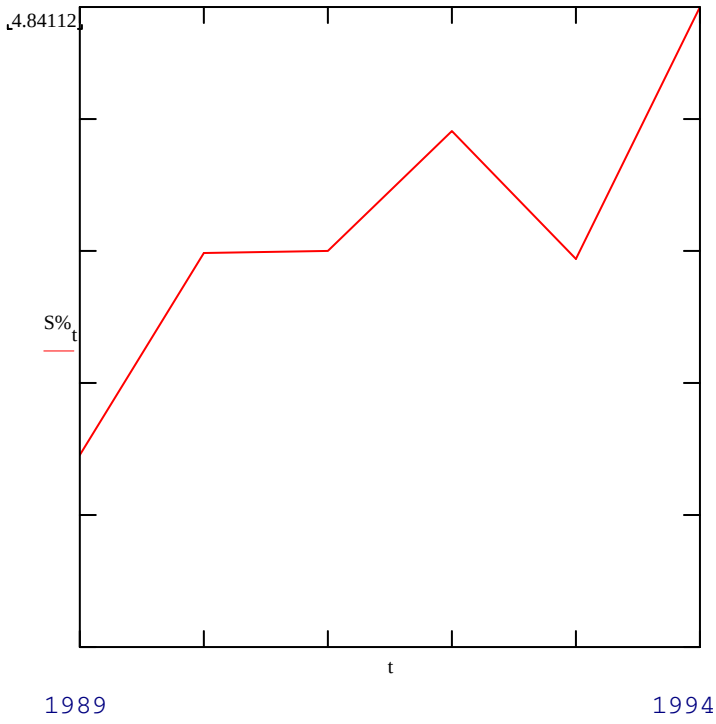
NOTES

(1) See Fortin and Frechette (1987) for a full description of the data set (Fortin and Frechette, 1986). Related surveys that did in Europe reported in Pestieau (1985) and Ginsburg et al. (1987) for Norway; and Van Eck and Kazemir (1988) for Netherlands.
(2) Most empirical studies in North America based on data from the Compliance Measurement Program (Witte and Woodbury, 1985; Feinstein, 1991; Smith, 1985; Portes et al., 1989).
(3) The two measurement approaches, micro surveys and tax audits, should thus be viewed as complementing each other.
(4) A similar approach it used by Gronau (1977) to explain the allocation time among home production, market work, and leisure. He postulates that the value of home production is a concave function of hours worked at home, which explains why hours worked at home are negatively related to the market wage, just as hours in the underground sector are negatively related to the regular-sector wage in the Lemieux's article. One explanation for the concavity of the earnings' function in the underground sector is based on the principle that the informal nature of economic activities in that sector imposes a limit on the scope of these activities. Because of these market limitations, the underground-sector workers face a downward-sloping demand for his output. As hours of work and output expand, the output price goes down, which tends to reduce the value of the marginal product of labour when there are constant (or decreasing) returns to scale in production. Labor earnings in the underground sector are thus typically a concave function of hours worked in that sector (Lemieux et al., 1994).

(5) An interesting review of the debates surrounding the Laffer curve there is in Fullerton (1982).
(6) In specialized literature, it is typically assumed that farmers face the choice between cultivating their land according to a concave production function or working in the market at a given wage rate that does not depend on hours of work (Rosenzweig, 1980; Benjamin, 1992).
(7) These results, obtained by Lemieux et al., based on 93 valid observations in regular and underground sector. The net wage is the regular wage net of an imputed marginal tax rate that takes account of income taxes, payroll taxes, and tax-back rates (embodied in the social transfer system).
(8) This formulation was first proposed by Yitzhaki (1974).
(9) However, Fluet (1987) has shown that, under the assumption that $W_0 < W_1$, there necessarily exists a positive relationship between the tax rate and labour supply in the underground sector. Fluet assumes that the degree of absolute risk aversion is decreasing in consumption and that there is an interior solution for h_0 and h_1.
(10) One justification for the use of the tax variable only is that this variable has by far the strongest impact on the size of the underground economy in all studies known by us. An exception is the study by Frey and Weck-Hannemann (1984), where the variable "tax immorality" has a quantitatively larger and statistically higher influence in the model than the direct tax share. In a study on the U.S. underground economy, Pommerehne and Schneider (1985) showed that the tax variable has a dominating influence and contributes roughly 70-78% to the size of the underground economy.
(11) In studies for European countries, Kirchgassner (1983) and Schneider (1986) reach the conclusion that the estimation results for Germany, Denmark, Norway and Sweden are quite robust when using this approach.
(12) Moreover, in first years of transition, at least in Romania's economy there was a "dollarization" phenomenon that evades from official statistics. Some estimations show that the stock of foreign currency held by population is over one thousand million dollars, many transactions being made directly in dollars or other foreign currency (Dobrescu, 1994).
(13) An interesting forthcoming way may be to investigate the conditions when the fiscal policies enter chaotic regimes. It seems that we already found a possible similarity between the so-called Henon's model and the penalty model. So, a preliminary canonical form of this correspondence should write as following: $t_i/(A \cdot tn_i) = 1 - n^2 \cdot (t_{i-1})^2 + (n - 1) \cdot t_{i-2}$, where A = Y/Yv ; n = tp/tn ; i = years; comparing with the Henon model: $X_i = 1 - a \cdot (X_{i-1})^2 + b \cdot X_{i-2}$.

$Year_t :=$

	$S\%_t$
1989	1.459
1990	2.978
1991	2.992
1992	3.908
1993	2.934
1994	4.841



$tn := 0.2$ $n := 10$ $i := tn \cdot 10 .. n$ $tp_i := \frac{i}{n}$ $Y := 100$

$tm(tp) := \left[tn \cdot \frac{(1 - tn) + tp \cdot (1 - tp)}{(1 - tn) + tn \cdot (1 - tp)} \right]$

$Yn := Y \cdot (1 - tn)$

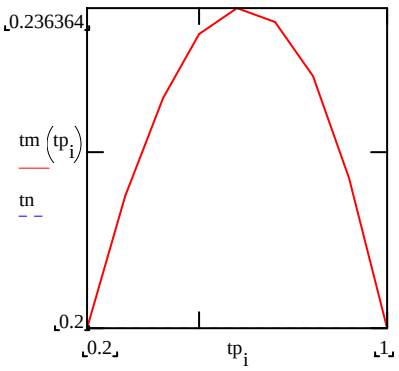
$Yh := Y \cdot tn$

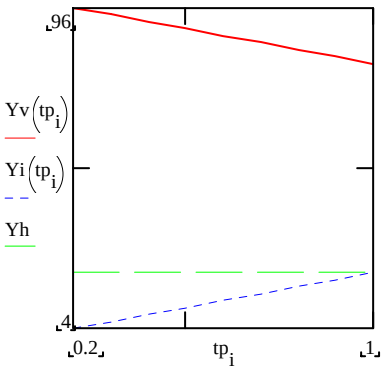
$Yp(tp) := Yh \cdot (1 - tp)$ $Yv(tp) := Yn + Yp(tp)$ $Tn := tn \cdot Yn$

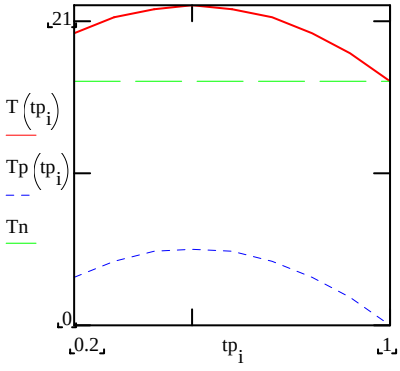
$Yi(tp) := tp \cdot Yh$ $T(tp) := tm(tp) \cdot Yv(tp)$ $Tp(tp) := tp \cdot Yp(tp)$

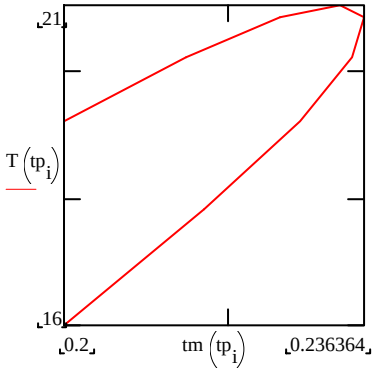
$Yn = 80$
$Yh = 20$
$Tn = 16$

tp_i	$tm(tp_i)$	$Yp(tp_i)$	$Yi(tp_i)$	$T(tp_i)$	$Tp(tp_i)$
0.2	0.2	16	4	19.2	3.2
0.3	0.215	14	6	20.2	4.2
0.4	0.226	12	8	20.8	4.8
0.5	0.233	10	10	21	5
0.6	0.236	8	12	20.8	4.8
0.7	0.235	6	14	20.2	4.2
0.8	0.229	4	16	19.2	3.2
0.9	0.217	2	18	17.8	1.8
1	0.2	0	20	16	0





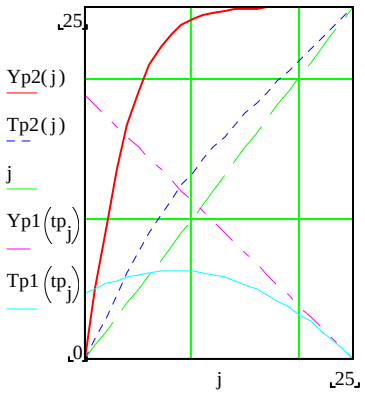


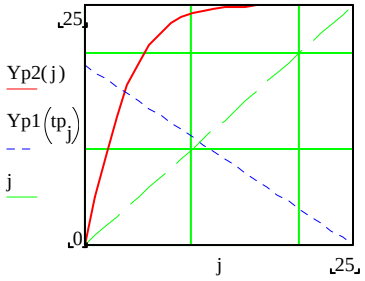


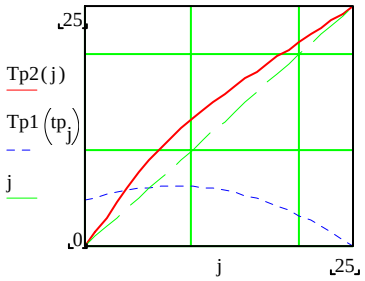
$tn := 0.25$ $Y := 100$ $a := 0.4$ $Ya := Y \cdot tn$ $Ya = 25$

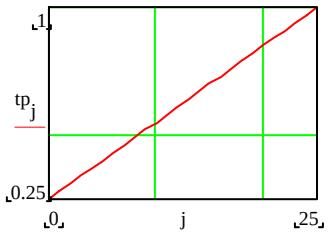
$Yp1(tp) := Ya \cdot (1 - tp)$ $Tp1(tp) := Ya \cdot (1 - tp) \cdot tp$ $j := 0..Ya$

$tp_j := tn + \frac{1 - tn}{Ya} \cdot j$ $Yp2(j) := Ya \cdot \left(\frac{1 - e^{-a \cdot j}}{1 + e^{-a \cdot j}} \right)$ $Tp2(j) := Ya \cdot \left(\frac{1 - e^{-a \cdot j}}{1 + e^{-a \cdot j}} \right) \cdot tp_j$









Vmax-1989

$Yv := 800 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.339 \cdot 10^4$

$Fvc := 290$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 253.084$

$FvCO := 22$

$Fvl := Fvc - FvCO$ $Fvl = 268$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 273.86$

$kF := 0.9$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.93 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$ $wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 38.036$

$Svndir := 1920 \cdot 12 \cdot 3 \cdot 10^6 + 3063 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 3.595 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.298 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 17.092$ $T := 376.7 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 3.595 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 17.092$ $AsiBv := Ydv - Sv$

$AsiBv = 6.381 \cdot 10^{10}$ $INVv := 236.4 \cdot 10^9 - 223.9 \cdot 10^9$

$Bv := AsiBv - A$ $Bv = 5.131 \cdot 10^{10}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 365 \cdot 8$ $L := 14.8 \cdot 10^6$

$Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

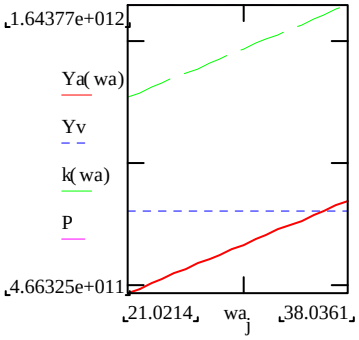
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

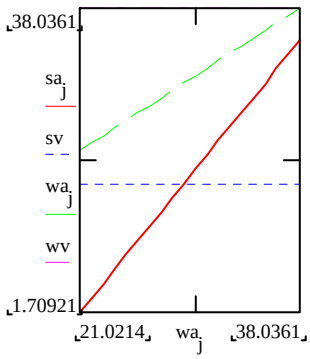
$Yv = 8 \cdot 10^{11}$ $T = 3.767 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$ $A = 1.25 \cdot 10^{10}$

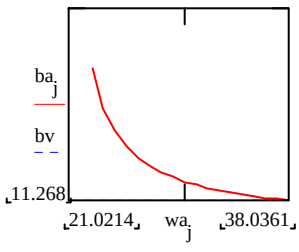
$Vv = 4.108 \cdot 10^{11}$ $Sv = 3.595 \cdot 10^{11}$ $Bv = 5.131 \cdot 10^{10}$ $F = 2.92 \cdot 10^{3}$

$wv = 38.036$ $t = 0.471$ $sv = 17.1$ $bv = 13.8$

$ya(wa)$	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.368	21	0.13	0.05	2	1129.9
0.378	22	0.13	0.05	3	541.1
0.387	23	0.13	0.05	5	344.9
0.397	24	0.13	0.05	7	246.8
0.406	25	0.13	0.05	9	187.9
0.414	25	0.14	0.06	10	148.6
0.423	26	0.14	0.06	12	120.6
0.431	27	0.14	0.06	14	99.6
0.439	28	0.14	0.06	15	83.2
0.446	29	0.14	0.06	17	70.1
0.454	30	0.15	0.07	19	59.4
0.461	31	0.15	0.07	21	50.5
0.468	32	0.15	0.07	22	43
0.475	33	0.15	0.07	24	36.5
0.482	34	0.15	0.07	26	30.9
0.489	34	0.16	0.08	27	26
0.495	35	0.16	0.08	29	21.7
0.501	36	0.16	0.08	31	17.8
0.507	37	0.16	0.08	32	14.4
0.513	38	0.16	0.08	34	11.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.736$ $fv = 0.661$ $yamax = 0.513$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax} \right) \right]$ $yamin = 0.358$

Vmax-1990

$Yv := 857.9 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.871 \cdot 10^4$

$Fvc := 265$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 297.005$

$FvCO := 25$

$Fvl := Fvc - FvCO$ $Fvl = 240$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 327.943$

$kF := 0.85$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.632 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 48.227$

$Svndir := 2740 \cdot 12 \cdot 3 \cdot 10^6 + 3381 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 4.192 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.845 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 23.563$ $T := 307.4 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 4.192 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 23.563$ $AsiBv := Ydv - Sv$

$AsiBv = 1.313 \cdot 10^{11}$ $INVv := 168.4 \cdot 10^9 - 155.7 \cdot 10^9$

$Bv := AsiBv - A$ $Bv = 1.186 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 365 \cdot 8$

$L := 15 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$ $wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

$Yv = 8.579 \cdot 10^{11}$ $T = 3.074 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$ $A = 1.27 \cdot 10^{10}$

$Vv = 5.378 \cdot 10^{11}$ $Sv = 4.192 \cdot 10^{11}$ $Bv = 1.186 \cdot 10^{11}$ $F = 2.92 \cdot 10^{3}$

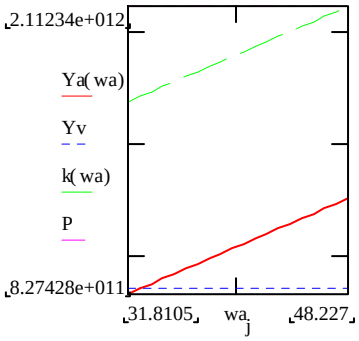
$wv = 48.227$ $t = 0.358$ $sv = 23.6$ $bv = 27.5$

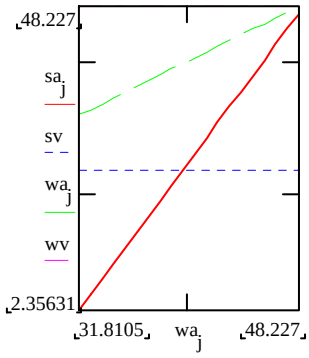
ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.491	32	0.17	0.08	2	1250
0.498	33	0.17	0.08	5	593.3
0.504	34	0.17	0.09	7	374.5
0.511	34	0.18	0.09	9	265
0.517	35	0.18	0.09	12	199.3
0.523	36	0.18	0.09	14	155.6
0.529	37	0.18	0.1	16	124.3
0.534	38	0.18	0.1	19	100.8
0.54	39	0.19	0.1	21	82.6
0.546	40	0.19	0.1	24	68
0.551	40	0.19	0.11	26	56.1
0.556	41	0.19	0.11	28	46.1
0.561	42	0.2	0.11	31	37.7
0.566	43	0.2	0.11	33	30.5
0.571	44	0.2	0.11	35	24.2
0.576	45	0.2	0.12	38	18.8
0.58	46	0.2	0.12	40	13.9
0.585	46	0.21	0.12	42	9.6
0.589	47	0.21	0.12	45	5.8
0.594	48	0.21	0.13	47	2.3

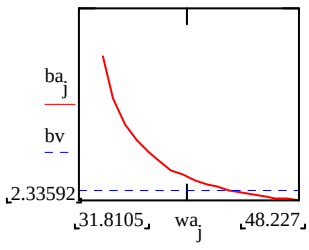
$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.727$ $fv = 0.559$ $yamax = 0.594$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.484$







Vmax-1991

$Yv := 2203.9 \cdot 10^{9}$ $Lv := 10.8 \cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 2.041 \cdot 10^{5}$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 797.128$

$FvCO := 26$

$Fvl := Fvc - FvCO$ $Fvl = 230$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 887.238$

$kF := 0.74$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.362 \cdot 10^{3}$ $Lv = 1.08 \cdot 10^{7}$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 149.871$

$Svndir := 5496 \cdot 12 \cdot 3.1 \cdot 10^{6} + 7460 \cdot 12 \cdot 7.7 \cdot 10^{6}$ $Svndir = 8.938 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 8.276 \cdot 10^{4}$

$sv := \frac{svan}{Fv}$ $sv = 60.778$ $T := 773.9 \cdot 10^{9}$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 8.938 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 60.778$ $AsiBv := Ydv - Sv$

$AsiBv = 5.362 \cdot 10^{11}$ $INVv := 314 \cdot 10^{9} - 208.9 \cdot 10^{9}$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 4.311 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 365 \cdot 8$

$L := 15.4 \cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

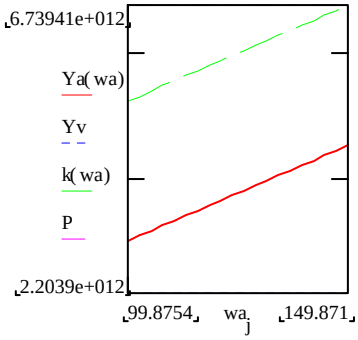
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

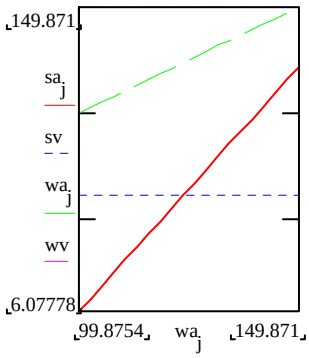
$Yv = 2.204 \cdot 10^{12}$ $T = 7.739 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$ $A = 1.051 \cdot 10^{11}$

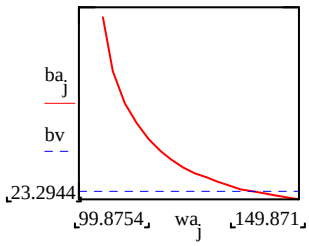
$Vv = 1.325 \cdot 10^{12}$ $Sv = 8.938 \cdot 10^{11}$ $Bv = 4.311 \cdot 10^{11}$ $F = 2.92 \cdot 10^{3}$

$wv = 149.871$ $t = 0.351$ $sv = 60.8$ $bv = 43.2$

ya(wa)	wa_j	$k(wa)/10^{13}$	$Ya(wa)/10^{13}$	sa_j	ba_j
0.578	100	0.52	0.3	6	1543.3
0.585	103	0.53	0.31	12	743.3
0.591	105	0.54	0.32	18	476.6
0.597	108	0.55	0.33	24	343.3
0.603	110	0.55	0.33	30	263.3
0.608	113	0.56	0.34	36	210
0.614	116	0.57	0.35	43	171.9
0.619	118	0.58	0.36	49	143.3
0.624	121	0.59	0.37	55	121.1
0.629	124	0.59	0.37	61	103.3
0.634	126	0.6	0.38	67	88.7
0.639	129	0.61	0.39	73	76.6
0.643	131	0.62	0.4	79	66.4
0.648	134	0.63	0.41	85	57.6
0.652	137	0.63	0.41	91	50
0.657	139	0.64	0.42	97	43.3
0.661	142	0.65	0.43	103	37.4
0.665	145	0.66	0.44	109	32.2
0.669	147	0.67	0.45	115	27.5
0.673	150	0.67	0.45	122	23.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.701$ $fv = 0.466$ $yamax = 0.673$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax} \right) \right]$ $yamin = 0.572$

Vmax-1992

$Yv := 6083.5\,10^{9}$ $Lv := 10.6\cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 5.739\cdot 10^{5}$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 2.242\cdot 10^{3}$

$FvCO := 27$

$Fvl := Fvc - FvCO$ $Fvl = 229$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 2.506\cdot 10^{3}$

$kF := 0.67$ $kzef := 8$

$Fv := Fvl\cdot kF\cdot kzef$

$Fv = 1.227\cdot 10^{3}$ $Lv = 1.06\cdot 10^{7}$

$wv := \frac{Yv}{Lv\cdot Fv}$ $wv = 467.571$

$Svndir := 12653\,12\cdot 3.3\cdot 10^{6} + 20140\,12\cdot 7.3\cdot 10^{6}$ $Svndir = 2.265\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 2.137\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 174.11$ $T := 2051.2\cdot 10^{9}$ $Ydv := Yv - T$

$Sv := sv\cdot Fv\cdot Lv$ $Sv = 2.265\cdot 10^{12}$ $Ydv = 4.032\cdot 10^{12}$

$sv := \frac{Sv}{Lv\cdot Fv}$ $sv = 174.11$ $AsiBv := Ydv - Sv$

$AsiBv = 1.767\cdot 10^{12}$ $INVv := 888.6\,10^{9} - 706.7\cdot 10^{9}$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 1.585\cdot 10^{12}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv\cdot 100}{A + Sv}$

$F := 366\,8$

$L := 15.5\cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\cdot F\cdot wv$

$wa_j := (1 - t)\cdot wv + j\cdot\left(\frac{wv\cdot t}{r}\right)$

$Ya(wa) := (Lv\cdot Fa + La\cdot F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

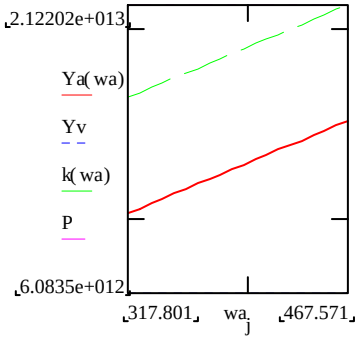
$ba_j := \left(\frac{wv\cdot mw_j}{ms_j\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

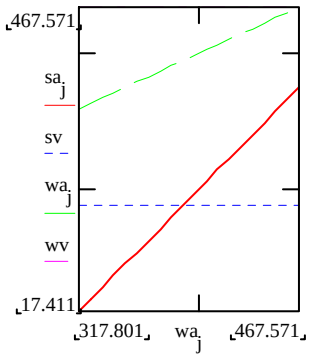
$Yv = 6.084 \cdot 10^{12}$ $T = 2.051 \cdot 10^{12}$ $Ydv = 4.032 \cdot 10^{12}$ $A = 1.819 \cdot 10^{11}$

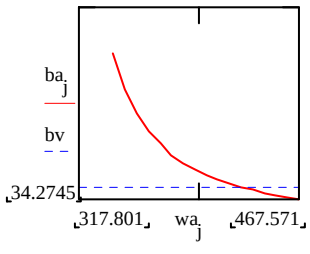
$Vv = 3.85 \cdot 10^{12}$ $Sv = 2.265 \cdot 10^{12}$ $Bv = 1.585 \cdot 10^{12}$ $F = 2.928 \cdot 10^{3}$

$wv = 467.571$ $t = 0.337$ $sv = 174.1$ $bv = 64.8$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.628	318	1.64	1.03	17	1725.3
0.634	326	1.66	1.05	35	835.3
0.64	334	1.69	1.08	52	538.6
0.645	341	1.71	1.11	70	390.3
0.65	349	1.74	1.13	87	301.3
0.655	357	1.76	1.16	104	241.9
0.66	365	1.79	1.18	122	199.6
0.665	373	1.82	1.21	139	167.8
0.67	381	1.84	1.23	157	143.1
0.674	389	1.87	1.26	174	123.3
0.679	397	1.89	1.28	192	107.1
0.683	405	1.92	1.31	209	93.6
0.687	412	1.94	1.34	226	82.2
0.691	420	1.97	1.36	244	72.4
0.695	428	1.99	1.39	261	63.9
0.699	436	2.02	1.41	279	56.5
0.703	444	2.05	1.44	296	50
0.706	452	2.07	1.46	313	44.2
0.71	460	2.1	1.49	331	39
0.713	468	2.12	1.51	348	34.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.684$ $fv = 0.419$ $yamax = 0.713$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.623$

Vmax-1993

$Yv := 19737.5\,10^{9}$ $Lv := 10.2\,10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 1.935\cdot 10^{6}$

$Fvc := 255$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 7.588\cdot 10^{3}$

$FvCO := 28$

$Fvl := Fvc - FvCO$ $Fvl = 227$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 8.524\cdot 10^{3}$

$kF := 0.68$ $kzef := 8$

$Fv := Fvl\,kF\,kzef$

$Fv = 1.235\cdot 10^{3}$ $Lv = 1.02\cdot 10^{7}$

$wv := \frac{Yv}{Lv\,Fv}$ $wv = 1.567\cdot 10^{3}$

$Svndir := 34920\,12\,3.4\,10^{6} + 59717\,12\,6.8\,10^{6}$ $Svndir = 6.298\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 6.174\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 499.981$ $T := 5491.8\,10^{9}$ $Ydv := Yv - T$

$Sv := sv\,Fv\,Lv$ $Sv = 6.298\cdot 10^{12}$ $Ydv = 1.425\cdot 10^{13}$

$sv := \frac{Sv}{Lv\,Fv}$ $sv = 499.981$ $AsiBv := Ydv - Sv$

$AsiBv = 7.948\cdot 10^{12}$ $INVv := 2821.8\,10^{9} - 1666.8\,10^{9}$

$Bv := AsiBv - A$ $Bv = 6.793\cdot 10^{12}$ $t := \frac{T}{Yv}$ $A := INVv$

$Bv := Vv - Sv$ $Vv := Ydv - A$

$F := 365\,8$ $bv := \frac{Bv\,100}{A + Sv}$

$L := 15.3\,10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\,F\,wv$

$wa_j := (1 - t)\,wv + j\cdot\left(\frac{wv\,t}{r}\right)$

$Ya(wa) := (Lv\,Fa + La\,F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\,sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

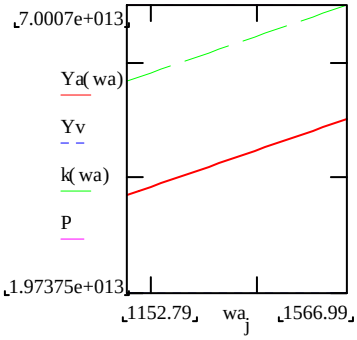
$ba_j := \left(\frac{wv\cdot mw_j}{ms\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

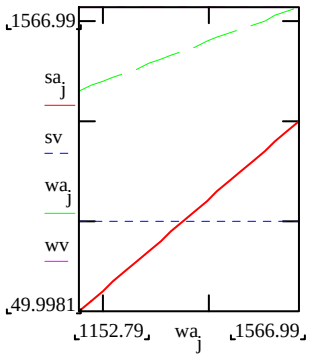
$Yv = 1.974 \cdot 10^{13}$ $T = 5.492 \cdot 10^{12}$ $Ydv = 1.425 \cdot 10^{13}$ $A = 1.155 \cdot 10^{12}$

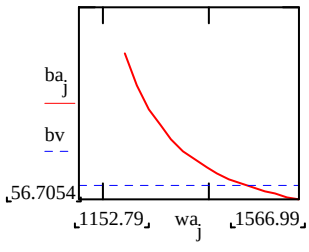
$Vv = 1.309 \cdot 10^{13}$ $Sv = 6.298 \cdot 10^{12}$ $Bv = 6.793 \cdot 10^{12}$

$wv = 1.567 \cdot 10^{3}$ $t = 0.278$ $sv = 500$ $bv = 91.1$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.652	1153	5.67	3.7	50	2205.7
0.656	1175	5.74	3.77	100	1074.6
0.66	1196	5.81	3.84	150	697.6
0.664	1218	5.88	3.91	200	509.1
0.668	1240	5.95	3.98	250	396
0.672	1262	6.02	4.05	300	320.6
0.676	1284	6.09	4.12	350	266.8
0.68	1305	6.16	4.19	400	226.4
0.683	1327	6.23	4.26	450	194.9
0.687	1349	6.3	4.33	500	169.8
0.69	1371	6.37	4.4	550	149.2
0.694	1393	6.44	4.47	600	132.1
0.697	1414	6.51	4.54	650	117.6
0.7	1436	6.58	4.61	700	105.2
0.703	1458	6.65	4.68	750	94.4
0.706	1480	6.72	4.75	800	85
0.709	1502	6.79	4.82	850	76.7
0.712	1523	6.86	4.89	900	69.3
0.715	1545	6.93	4.96	950	62.7
0.718	1567	7	5.03	1000	56.7







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.667$ $fv = 0.423$ $yamax = 0.718$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.648$

Vint-1989

$Yv := 800 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.339 \cdot 10^4$

$Fvc := 290$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 253.084$

$FvCO := 22$

$Fvl := Fvc - FvCO$ $Fvl = 268$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 273.86$

$kF := 0.9$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.93 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 38.036$

$Svndir := 1920 \cdot 12 \cdot 3 \cdot 10^6 + 3063 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 3.595 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.298 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 17.092$ $T := 376.7 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 3.595 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 17.092$ $AsiBv := Ydv - Sv$

$AsiBv = 6.381 \cdot 10^{10}$ $INVv := 236.4 \cdot 10^9 - 223.9 \cdot 10^9$

$Bv := AsiBv - A$ $Bv = 5.131 \cdot 10^{10}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := (268 + 52 + 11) \cdot 8$

$L := 12.9 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r}\right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

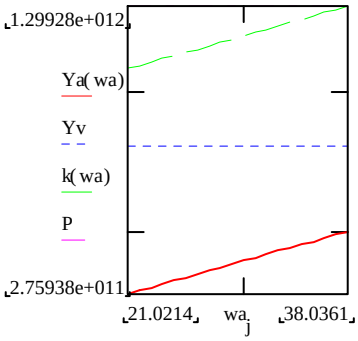
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

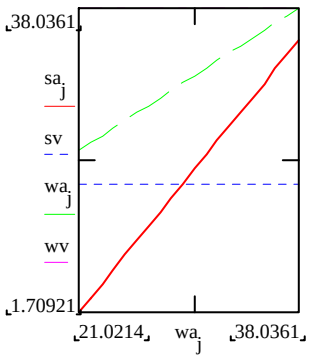
$Yv = 8 \cdot 10^{11}$ $T = 3.767 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$ $A = 1.25 \cdot 10^{10}$

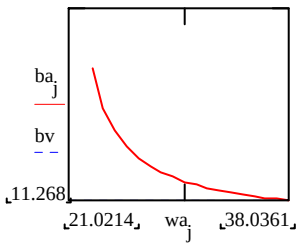
$Vv = 4.108 \cdot 10^{11}$ $Sv = 3.595 \cdot 10^{11}$ $Bv = 5.131 \cdot 10^{10}$ $F = 2.648 \cdot 10^{3}$

$wv = 38.036$ $t = 0.471$ $sv = 17.1$ $bv = 13.8$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.256	21	0.11	0.03	2	1129.9
0.264	22	0.11	0.03	3	541.1
0.272	23	0.11	0.03	5	344.9
0.28	24	0.11	0.03	7	246.8
0.288	25	0.11	0.03	9	187.9
0.295	25	0.11	0.03	10	148.6
0.302	26	0.11	0.03	12	120.6
0.309	27	0.12	0.04	14	99.6
0.316	28	0.12	0.04	15	83.2
0.323	29	0.12	0.04	17	70.1
0.33	30	0.12	0.04	19	59.4
0.336	31	0.12	0.04	21	50.5
0.343	32	0.12	0.04	22	43
0.349	33	0.12	0.04	24	36.5
0.355	34	0.12	0.04	26	30.9
0.361	34	0.13	0.05	27	26
0.367	35	0.13	0.05	29	21.7
0.373	36	0.13	0.05	31	17.8
0.379	37	0.13	0.05	32	14.4
0.384	38	0.13	0.05	34	11.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.845$ $fv = 0.729$ $yamax = 0.384$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.248$

Vint-1990

$Yv := 857.9 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.871 \cdot 10^4$

$Fvc := 265$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 297.005$

$FvCO := 25$

$Fvl := Fvc - FvCO$ $Fvl = 240$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 327.943$

$kF := 0.85$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.632 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 48.227$

$Svndir := 2740 \cdot 12 \cdot 3 \cdot 10^6 + 3381 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 4.192 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.845 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 23.563$ $T := 307.4 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 4.192 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 23.563$ $AsiBv := Ydv - Sv$

$AsiBv = 1.313 \cdot 10^{11}$ $INVv := 168.4 \cdot 10^9 - 155.7 \cdot 10^9$

$Bv := AsiBv - A$ $Bv = 1.186 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := (240 + 52 + 12) \cdot 8$

$L := 13 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

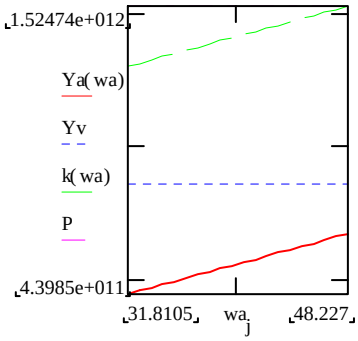
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

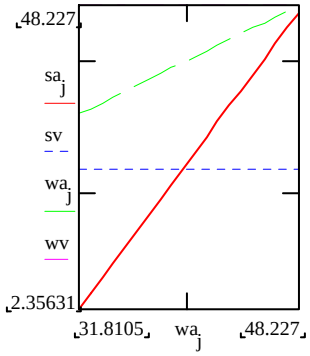
$Yv = 8.579 \cdot 10^{11}$ $T = 3.074 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$ $A = 1.27 \cdot 10^{10}$

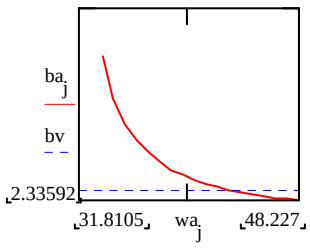
$Vv = 5.378 \cdot 10^{11}$ $Sv = 4.192 \cdot 10^{11}$ $Bv = 1.186 \cdot 10^{11}$ $F = 2.432 \cdot 10^{3}$

$wv = 48.227$ $t = 0.358$ $sv = 23.6$ $bv = 27.5$

ya(wa)	wa_j	$k(wa)/10^{13}$	$Ya(wa)/10^{13}$	sa_j	ba_j
0.339	32	0.13	0.04	2	1250
0.345	33	0.13	0.05	5	593.3
0.351	34	0.13	0.05	7	374.5
0.357	34	0.13	0.05	9	265
0.362	35	0.13	0.05	12	199.3
0.368	36	0.14	0.05	14	155.6
0.374	37	0.14	0.05	16	124.3
0.379	38	0.14	0.05	19	100.8
0.384	39	0.14	0.05	21	82.6
0.39	40	0.14	0.05	24	68
0.395	40	0.14	0.06	26	56.1
0.4	41	0.14	0.06	28	46.1
0.405	42	0.14	0.06	31	37.7
0.41	43	0.15	0.06	33	30.5
0.414	44	0.15	0.06	35	24.2
0.419	45	0.15	0.06	38	18.8
0.424	46	0.15	0.06	40	13.9
0.428	46	0.15	0.06	42	9.6
0.433	47	0.15	0.07	45	5.8
0.437	48	0.15	0.07	47	2.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.838$ $fv = 0.671$ $yamax = 0.437$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax} \right) \right]$ $yamin = 0.333$

Vint-1991

$Yv := 2203.9 \cdot 10^9$ $Lv := 10.8 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 2.041 \cdot 10^5$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 797.128$

$FvCO := 26$

$Fvl := Fvc - FvCO$ $Fvl = 230$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 887.238$

$kF := 0.74$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.362 \cdot 10^3$ $Lv = 1.08 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 149.871$

$Svndir := 5496 \cdot 12 \cdot 3.1 \cdot 10^6 + 7460 \cdot 12 \cdot 7.7 \cdot 10^6$ $Svndir = 8.938 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 8.276 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 60.778$ $T := 773.9 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 8.938 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 60.778$ $AsiBv := Ydv - Sv$

$AsiBv = 5.362 \cdot 10^{11}$ $INVv := 314 \cdot 10^9 - 208.9 \cdot 10^9$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 4.311 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := (230 + 52 + 13) \cdot 8$

$L := 13.1 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

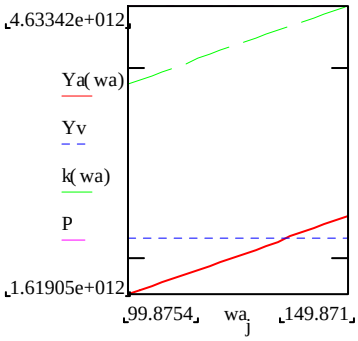
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

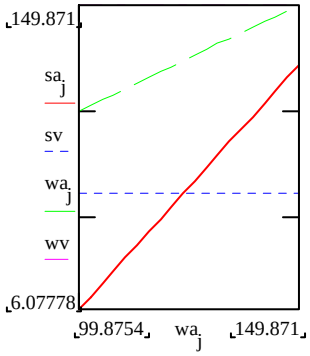
$Yv = 2.204 \cdot 10^{12}$ $T = 7.739 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$ $A = 1.051 \cdot 10^{11}$

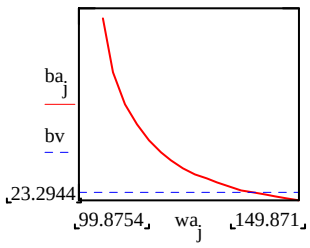
$Vv = 1.325 \cdot 10^{12}$ $Sv = 8.938 \cdot 10^{11}$ $Bv = 4.311 \cdot 10^{11}$ $F = 2.36 \cdot 10^{3}$

$wv = 149.871$ $t = 0.351$ $sv = 60.8$ $bv = 43.2$

ya(wa)	wa_j	$k(wa)\ 10^{13}$	$Ya(wa)\ 10^{13}$	sa_j	ba_j
0.424	100	0.38	0.16	6	1543.3
0.43	103	0.39	0.17	12	743.3
0.436	105	0.39	0.17	18	476.6
0.442	108	0.4	0.17	24	343.3
0.448	110	0.4	0.18	30	263.3
0.454	113	0.4	0.18	36	210
0.46	116	0.41	0.19	43	171.9
0.465	118	0.41	0.19	49	143.3
0.471	121	0.42	0.2	55	121.1
0.476	124	0.42	0.2	61	103.3
0.481	126	0.42	0.2	67	88.7
0.487	129	0.43	0.21	73	76.6
0.492	131	0.43	0.21	79	66.4
0.497	134	0.44	0.22	85	57.6
0.501	137	0.44	0.22	91	50
0.506	139	0.45	0.23	97	43.3
0.511	142	0.45	0.23	103	37.4
0.515	145	0.45	0.23	109	32.2
0.52	147	0.46	0.24	115	27.5
0.524	150	0.46	0.24	122	23.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.824$ $fv = 0.577$ $yamax = 0.524$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.417$

Vint-1992

$Yv := 6083.5\,10^{9}$ $Lv := 10.6\cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 5.739\cdot 10^{5}$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 2.242\cdot 10^{3}$

$FvCO := 27$

$Fvl := Fvc - FvCO$ $Fvl = 229$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 2.506\cdot 10^{3}$

$kF := 0.67$ $kzef := 8$

$Fv := Fvl\cdot kF\cdot kzef$

$Fv = 1.227\cdot 10^{3}$ $Lv = 1.06\cdot 10^{7}$

$wv := \frac{Yv}{Lv\cdot Fv}$ $wv = 467.571$

$Svndir := 12653\,12\cdot 3.3\cdot 10^{6} + 20140\,12\cdot 7.3\cdot 10^{6}$ $Svndir = 2.265\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 2.137\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 174.11$ $T := 2051.2\,10^{9}$ $Ydv := Yv - T$

$Sv := sv\cdot Fv\cdot Lv$ $Sv = 2.265\cdot 10^{12}$ $Ydv = 4.032\cdot 10^{12}$

$sv := \frac{Sv}{Lv\cdot Fv}$ $sv = 174.11$ $AsiBv := Ydv - Sv$

$AsiBv = 1.767\cdot 10^{12}$ $INVv := 888.6\,10^{9} - 706.7\cdot 10^{9}$

$Bv := AsiBv - A$ $Bv = 1.585\cdot 10^{12}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv\cdot 100}{A + Sv}$

$F := (229 + 52 + 14)\cdot 8$

$L := 13\cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\cdot F\cdot wv$

$wa_j := (1 - t)\cdot wv + j\cdot\left(\frac{wv\cdot t}{r}\right)$

$Ya(wa) := (Lv\cdot Fa + La\cdot F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

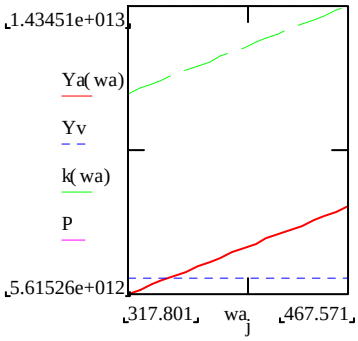
$ba_j := \left(\frac{wv\cdot mw_j}{ms_j\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

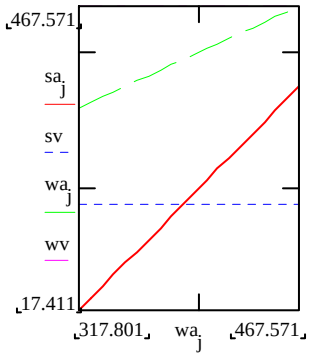
$Yv = 6.084 \cdot 10^{12}$ $T = 2.051 \cdot 10^{12}$ $Ydv = 4.032 \cdot 10^{12}$ $A = 1.819 \cdot 10^{11}$

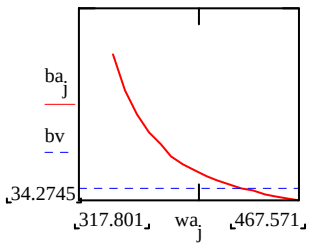
$Vv = 3.85 \cdot 10^{12}$ $Sv = 2.265 \cdot 10^{12}$ $Bv = 1.585 \cdot 10^{12}$ $F = 2.36 \cdot 10^{3}$

$wv = 467.571$ $t = 0.337$ $sv = 174.1$ $bv = 64.8$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.48	318	1.17	0.56	17	1725.3
0.486	326	1.18	0.58	35	835.3
0.492	334	1.2	0.59	52	538.6
0.498	341	1.21	0.6	70	390.3
0.504	349	1.23	0.62	87	301.3
0.509	357	1.24	0.63	104	241.9
0.515	365	1.25	0.65	122	199.6
0.52	373	1.27	0.66	139	167.8
0.525	381	1.28	0.67	157	143.1
0.53	389	1.3	0.69	174	123.3
0.535	397	1.31	0.7	192	107.1
0.54	405	1.32	0.71	209	93.6
0.545	412	1.34	0.73	226	82.2
0.55	420	1.35	0.74	244	72.4
0.554	428	1.36	0.76	261	63.9
0.559	436	1.38	0.77	279	56.5
0.563	444	1.39	0.78	296	50
0.568	452	1.41	0.8	313	44.2
0.572	460	1.42	0.81	331	39
0.576	468	1.43	0.83	348	34.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.815$ $fv = 0.52$ $yamax = 0.576$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.474$

Vint-1993

$Yv := 19737.5\,10^{9}$ $Lv := 10.2\cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 1.935\cdot 10^{6}$

$Fvc := 255$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 7.588\cdot 10^{3}$

$FvCO := 28$

$Fvl := Fvc - FvCO$ $Fvl = 227$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 8.524\cdot 10^{3}$

$kF := 0.68$ $kzef := 8$

$Fv := Fvl\cdot kF\cdot kzef$

$Fv = 1.235\cdot 10^{3}$ $Lv = 1.02\cdot 10^{7}$

$wv := \frac{Yv}{Lv\cdot Fv}$ $wv = 1.567\cdot 10^{3}$

$Svndir := 34920\,12\cdot 3.4\cdot 10^{6} + 59717\cdot 12\cdot 6.8\cdot 10^{6}$ $Svndir = 6.298\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 6.174\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 499.981$ $T := 5491.8\,10^{9}$ $Ydv := Yv - T$

$Sv := sv\cdot Fv\cdot Lv$ $Sv = 6.298\cdot 10^{12}$ $Ydv = 1.425\cdot 10^{13}$

$sv := \frac{Sv}{Lv\cdot Fv}$ $sv = 499.981$ $AsiBv := Ydv - Sv$

$AsiBv = 7.948\cdot 10^{12}$ $INVv := 2821.8\,10^{9} - 1666.8\,10^{9}$

$Bv := AsiBv - A$ $Bv = 6.793\cdot 10^{12}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv\cdot 100}{A + Sv}$

$F := (227 + 52 + 14)\cdot 8$

$L := 12.7\cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\cdot F\cdot wv$

$wa_j := (1 - t)\cdot wv + j\cdot\left(\frac{wv\cdot t}{r}\right)$

$Ya(wa) := (Lv\cdot Fa + La\cdot F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

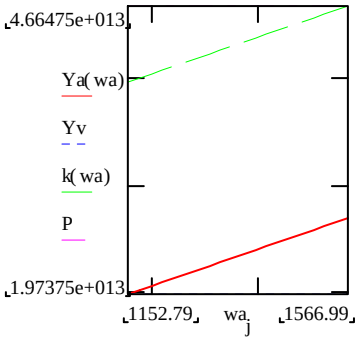
$ba_j := \left(\frac{wv\cdot mw_j}{ms_j\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

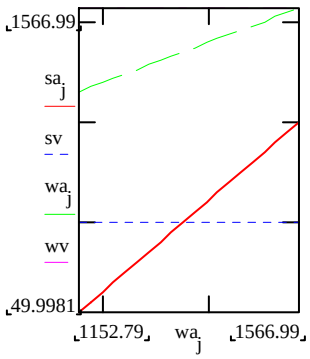
$Yv = 1.974 \cdot 10^{13}$ $T = 5.492 \cdot 10^{12}$ $Ydv = 1.425 \cdot 10^{13}$ $A = 1.155 \cdot 10^{12}$

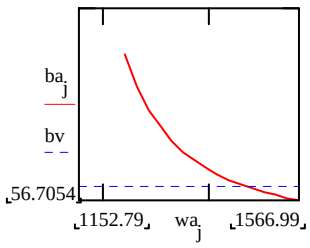
$Vv = 1.309 \cdot 10^{13}$ $Sv = 6.298 \cdot 10^{12}$ $Bv = 6.793 \cdot 10^{12}$

$wv = 1.567 \cdot 10^{3}$ $t = 0.278$ $sv = 500$ $bv = 91.1$

ya(wa)	wa_j	k(wa) 10^{13}	Ya(wa) 10^{13}	sa_j	ba_j
0.501	1153	3.95	1.98	50	2205.7
0.505	1175	3.99	2.02	100	1074.6
0.51	1196	4.03	2.05	150	697.6
0.515	1218	4.07	2.09	200	509.1
0.519	1240	4.1	2.13	250	396
0.523	1262	4.14	2.17	300	320.6
0.528	1284	4.18	2.2	350	266.8
0.532	1305	4.22	2.24	400	226.4
0.536	1327	4.25	2.28	450	194.9
0.54	1349	4.29	2.32	500	169.8
0.544	1371	4.33	2.35	550	149.2
0.548	1393	4.37	2.39	600	132.1
0.552	1414	4.4	2.43	650	117.6
0.555	1436	4.44	2.47	700	105.2
0.559	1458	4.48	2.5	750	94.4
0.563	1480	4.52	2.54	800	85
0.566	1502	4.55	2.58	850	76.7
0.57	1523	4.59	2.62	900	69.3
0.573	1545	4.63	2.65	950	62.7
0.577	1567	4.66	2.69	1000	56.7







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.803$ $fv = 0.527$ $yamax = 0.577$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax} \right) \right]$

$yamin = 0.496$

Vmin-1989

$Yv := 800 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.339 \cdot 10^4$

$Fvc := 290$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 253.084$

$FvCO := 22$

$Fvl := Fvc - FvCO$ $Fvl = 268$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 273.86$

$kF := 0.9$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.93 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 38.036$

$Svndir := 1920 \cdot 12 \cdot 3 \cdot 10^6 + 3063 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 3.595 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.298 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 17.092$ $T := 376.7 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 3.595 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 17.092$ $AsiBv := Ydv - Sv$

$AsiBv = 6.381 \cdot 10^{10}$ $INVv := 236.4 \cdot 10^9 - 223.9 \cdot 10^9$

$Bv := AsiBv - A$ $Bv = 5.131 \cdot 10^{10}$ $t := \frac{T}{Yv}$ $A := INVv$

$Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 268.8$

$L := 11 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r}\right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

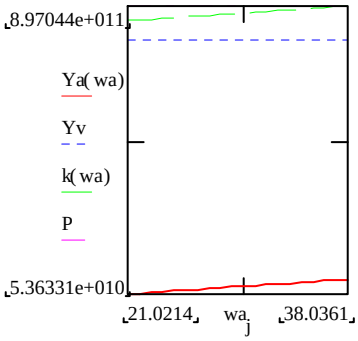
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

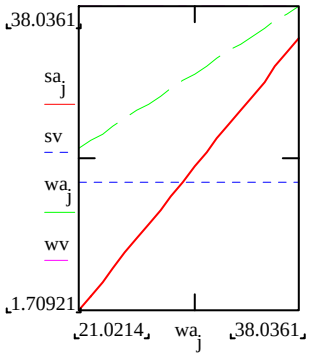
$Yv = 8 \cdot 10^{11}$ $T = 3.767 \cdot 10^{11}$ $Ydv = 4.233 \cdot 10^{11}$ $A = 1.25 \cdot 10^{10}$

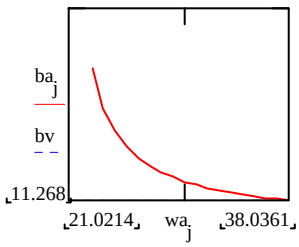
$Vv = 4.108 \cdot 10^{11}$ $Sv = 3.595 \cdot 10^{11}$ $Bv = 5.131 \cdot 10^{10}$ $F = 2.144 \cdot 10^{3}$

$wv = 38.036$ $t = 0.471$ $sv = 17.1$ $bv = 13.8$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.063	21	0.09	0.01	2	1129.9
0.065	22	0.09	0.01	3	541.1
0.068	23	0.09	0.01	5	344.9
0.07	24	0.09	0.01	7	246.8
0.073	25	0.09	0.01	9	187.9
0.075	25	0.09	0.01	10	148.6
0.078	26	0.09	0.01	12	120.6
0.08	27	0.09	0.01	14	99.6
0.082	28	0.09	0.01	15	83.2
0.085	29	0.09	0.01	17	70.1
0.087	30	0.09	0.01	19	59.4
0.09	31	0.09	0.01	21	50.5
0.092	32	0.09	0.01	22	43
0.094	33	0.09	0.01	24	36.5
0.097	34	0.09	0.01	26	30.9
0.099	34	0.09	0.01	27	26
0.101	35	0.09	0.01	29	21.7
0.104	36	0.09	0.01	31	17.8
0.106	37	0.09	0.01	32	14.4
0.108	38	0.09	0.01	34	11.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.991$ $fv = 0.9$ $yamax = 0.108$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.06$

Vmin-1990

$Yv := 857.9 \cdot 10^9$ $Lv := 10.9 \cdot 10^6$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 7.871 \cdot 10^4$

$Fvc := 265$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 297.005$

$FvCO := 25$

$Fvl := Fvc - FvCO$ $Fvl = 240$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 327.943$

$kF := 0.85$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.632 \cdot 10^3$ $Lv = 1.09 \cdot 10^7$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 48.227$

$Svndir := 2740 \cdot 12 \cdot 3 \cdot 10^6 + 3381 \cdot 12 \cdot 7.9 \cdot 10^6$ $Svndir = 4.192 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 3.845 \cdot 10^4$

$sv := \frac{svan}{Fv}$ $sv = 23.563$ $T := 307.4 \cdot 10^9$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 4.192 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 23.563$ $AsiBv := Ydv - Sv$

$AsiBv = 1.313 \cdot 10^{11}$ $INVv := 168.4 \cdot 10^9 - 155.7 \cdot 10^9$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 1.186 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 240 \cdot 8$

$L := 11 \cdot 10^6$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

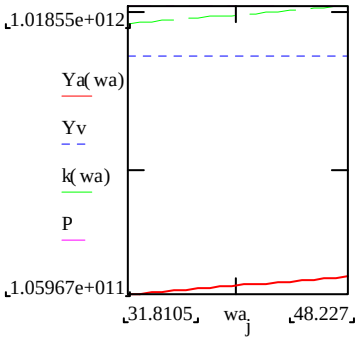
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

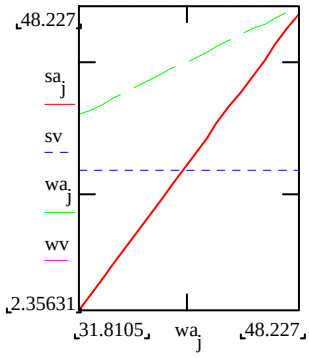
$Yv = 8.579 \cdot 10^{11}$ $T = 3.074 \cdot 10^{11}$ $Ydv = 5.505 \cdot 10^{11}$ $A = 1.27 \cdot 10^{10}$

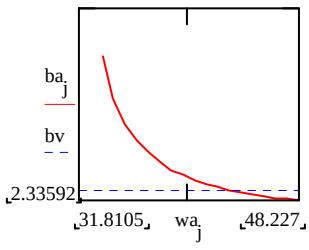
$Vv = 5.378 \cdot 10^{11}$ $Sv = 4.192 \cdot 10^{11}$ $Bv = 1.186 \cdot 10^{11}$ $F = 1.92 \cdot 10^{3}$

$wv = 48.227$ $t = 0.358$ $sv = 23.6$ $bv = 27.5$

$ya(wa)$	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.11	32	0.1	0.01	2	1250
0.113	33	0.1	0.01	5	593.3
0.115	34	0.1	0.01	7	374.5
0.118	34	0.1	0.01	9	265
0.12	35	0.1	0.01	12	199.3
0.123	36	0.1	0.01	14	155.6
0.126	37	0.1	0.01	16	124.3
0.128	38	0.1	0.01	19	100.8
0.131	39	0.1	0.01	21	82.6
0.133	40	0.1	0.01	24	68
0.136	40	0.1	0.01	26	56.1
0.138	41	0.1	0.01	28	46.1
0.141	42	0.1	0.01	31	37.7
0.143	43	0.1	0.01	33	30.5
0.146	44	0.1	0.01	35	24.2
0.148	45	0.1	0.01	38	18.8
0.151	46	0.1	0.02	40	13.9
0.153	46	0.1	0.02	42	9.6
0.155	47	0.1	0.02	45	5.8
0.158	48	0.1	0.02	47	2.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.991$ $fv = 0.85$ $yamax = 0.158$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.107$

Vmin-1991

$Yv := 2203.9 \cdot 10^{9}$ $Lv := 10.8 \cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 2.041 \cdot 10^{5}$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 797.128$

$FvCO := 26$

$Fvl := Fvc - FvCO$ $Fvl = 230$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 887.238$

$kF := 0.74$ $kzef := 8$

$Fv := Fvl \cdot kF \cdot kzef$

$Fv = 1.362 \cdot 10^{3}$ $Lv = 1.08 \cdot 10^{7}$

$wv := \frac{Yv}{Lv \cdot Fv}$ $wv = 149.871$

$Svndir := 5496 \cdot 12 \cdot 3.1 \cdot 10^{6} + 7460 \cdot 12 \cdot 7.7 \cdot 10^{6}$ $Svndir = 8.938 \cdot 10^{11}$

$svan := \frac{Svndir}{Lv}$ $svan = 8.276 \cdot 10^{4}$

$sv := \frac{svan}{Fv}$ $sv = 60.778$ $T := 773.9 \cdot 10^{9}$ $Ydv := Yv - T$

$Sv := sv \cdot Fv \cdot Lv$ $Sv = 8.938 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$

$sv := \frac{Sv}{Lv \cdot Fv}$ $sv = 60.778$ $AsiBv := Ydv - Sv$

$AsiBv = 5.362 \cdot 10^{11}$ $INVv := 314 \cdot 10^{9} - 208.9 \cdot 10^{9}$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 4.311 \cdot 10^{11}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv \cdot 100}{A + Sv}$

$F := 230 \cdot 8$ $Fa := F - Fv$

$L := 11.1 \cdot 10^{6}$ $La := L - Lv$ $r := 20$ $j := 1 .. r$

$P := L \cdot F \cdot wv$

$wa_j := (1 - t) \cdot wv + j \cdot \left(\frac{wv \cdot t}{r} \right)$

$Ya(wa) := (Lv \cdot Fa + La \cdot F) \cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j \cdot \frac{2 \cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

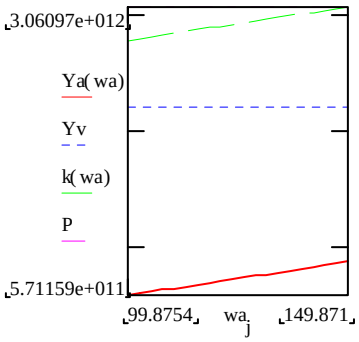
$ba_j := \left(\frac{wv \cdot mw_j}{ms_j \cdot sv} \cdot 100 \right) - 100$ $mb := \frac{ba}{bv}$

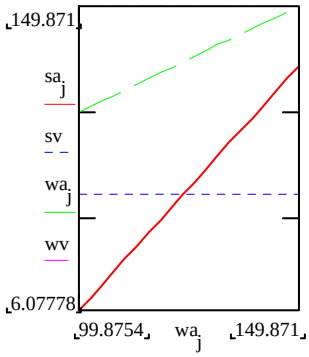
$Yv = 2.204 \cdot 10^{12}$ $T = 7.739 \cdot 10^{11}$ $Ydv = 1.43 \cdot 10^{12}$ $A = 1.051 \cdot 10^{11}$

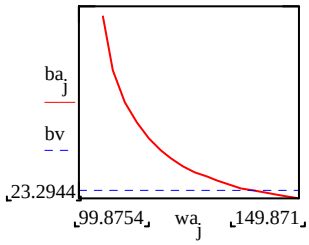
$Vv = 1.325 \cdot 10^{12}$ $Sv = 8.938 \cdot 10^{11}$ $Bv = 4.311 \cdot 10^{11}$ $F = 1.84 \cdot 10^{3}$

$wv = 149.871$ $t = 0.351$ $sv = 60.8$ $bv = 43.2$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.206	100	0.28	0.06	6	1543.3
0.21	103	0.28	0.06	12	743.3
0.214	105	0.28	0.06	18	476.6
0.219	108	0.28	0.06	24	343.3
0.223	110	0.28	0.06	30	263.3
0.227	113	0.29	0.06	36	210
0.231	116	0.29	0.07	43	171.9
0.235	118	0.29	0.07	49	143.3
0.239	121	0.29	0.07	55	121.1
0.243	124	0.29	0.07	61	103.3
0.247	126	0.29	0.07	67	88.7
0.251	129	0.29	0.07	73	76.6
0.254	131	0.3	0.08	79	66.4
0.258	134	0.3	0.08	85	57.6
0.262	137	0.3	0.08	91	50
0.266	139	0.3	0.08	97	43.3
0.269	142	0.3	0.08	103	37.4
0.273	145	0.3	0.08	109	32.2
0.276	147	0.3	0.08	115	27.5
0.28	150	0.31	0.09	122	23.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.973$ $fv = 0.74$ $yamax = 0.28$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.201$

Vmin-1992

$Yv := 6083.5\,10^{9}$ $Lv := 10.6\cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 5.739\cdot 10^{5}$

$Fvc := 256$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 2.242\cdot 10^{3}$

$FvCO := 27$

$Fvl := Fvc - FvCO$ $Fvl = 229$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 2.506\cdot 10^{3}$

$kF := 0.67$ $kzef := 8$

$Fv := Fvl\cdot kF\cdot kzef$

$Fv = 1.227\cdot 10^{3}$ $Lv = 1.06\cdot 10^{7}$

$wv := \frac{Yv}{Lv\cdot Fv}$ $wv = 467.571$

$Svndir := 12653\,12\cdot 3.3\cdot 10^{6} + 20140\,12\cdot 7.3\cdot 10^{6}$ $Svndir = 2.265\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 2.137\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 174.11$ $T := 2051.2\,10^{9}$ $Ydv := Yv - T$

$Sv := sv\cdot Fv\cdot Lv$ $Sv = 2.265\cdot 10^{12}$ $Ydv = 4.032\cdot 10^{12}$

$sv := \frac{Sv}{Lv\cdot Fv}$ $sv = 174.11$ $AsiBv := Ydv - Sv$

$AsiBv = 1.767\cdot 10^{12}$ $INVv := 888.6\,10^{9} - 706.7\cdot 10^{9}$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 1.585\cdot 10^{12}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv\cdot 100}{A + Sv}$

$F := 229\,8$

$L := 11\cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\cdot F\cdot wv$

$wa_j := (1 - t)\cdot wv + j\cdot\left(\frac{wv\cdot t}{r}\right)$

$Ya(wa) := (Lv\cdot Fa + La\cdot F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

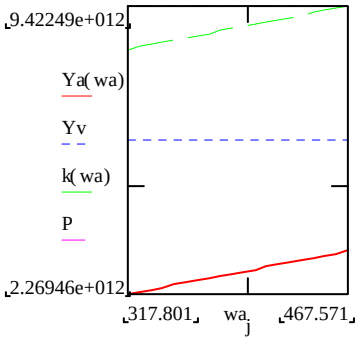
$ba_j := \left(\frac{wv\cdot mw_j}{ms_j\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

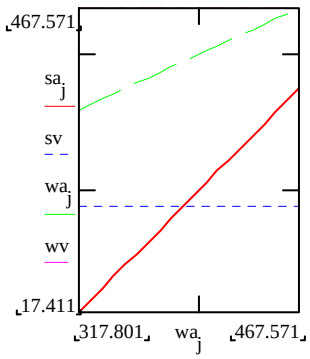
$Yv = 6.084 \cdot 10^{12}$ $T = 2.051 \cdot 10^{12}$ $Ydv = 4.032 \cdot 10^{12}$ $A = 1.819 \cdot 10^{11}$

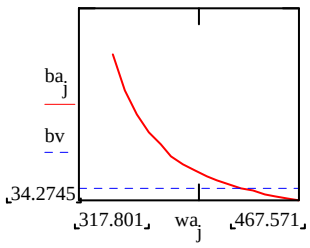
$Vv = 3.85 \cdot 10^{12}$ $Sv = 2.265 \cdot 10^{12}$ $Bv = 1.585 \cdot 10^{12}$ $F = 1.832 \cdot 10^{3}$

$wv = 467.571$ $t = 0.337$ $sv = 174.1$ $bv = 64.8$

ya(wa)	wa_j	k(wa) 10^{13}	Ya(wa) 10^{13}	sa_j	ba_j
0.272	318	0.84	0.23	17	1725.3
0.277	326	0.84	0.23	35	835.3
0.281	334	0.85	0.24	52	538.6
0.286	341	0.85	0.24	70	390.3
0.291	349	0.86	0.25	87	301.3
0.295	357	0.86	0.26	104	241.9
0.3	365	0.87	0.26	122	199.6
0.305	373	0.87	0.27	139	167.8
0.309	381	0.88	0.27	157	143.1
0.313	389	0.89	0.28	174	123.3
0.318	397	0.89	0.28	192	107.1
0.322	405	0.9	0.29	209	93.6
0.326	412	0.9	0.29	226	82.2
0.33	420	0.91	0.3	244	72.4
0.334	428	0.91	0.31	261	63.9
0.339	436	0.92	0.31	279	56.5
0.343	444	0.93	0.32	296	50
0.347	452	0.93	0.32	313	44.2
0.35	460	0.94	0.33	331	39
0.354	468	0.94	0.33	348	34.3







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.964$ $fv = 0.67$ $yamax = 0.354$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.267$

Vmin-1993

$Yv := 19737.5\,10^{9}$ $Lv := 10.2\cdot 10^{6}$ $wvLv := \frac{Yv}{Lv}$ $wvLv = 1.935\cdot 10^{6}$

$Fvc := 255$ $wvc := \frac{wvLv}{Fvc}$ $wvc = 7.588\cdot 10^{3}$

$FvCO := 28$

$Fvl := Fvc - FvCO$ $Fvl = 227$ $wvz := \frac{wvLv}{Fvl}$ $wvz = 8.524\cdot 10^{3}$

$kF := 0.68$ $kzef := 8$

$Fv := Fvl\cdot kF\cdot kzef$

$Fv = 1.235\cdot 10^{3}$ $Lv = 1.02\cdot 10^{7}$

$wv := \frac{Yv}{Lv\cdot Fv}$ $wv = 1.567\cdot 10^{3}$

$Svndir := 34920\,12\cdot 3.4\cdot 10^{6} + 59717\cdot 12\cdot 6.8\cdot 10^{6}$ $Svndir = 6.298\cdot 10^{12}$

$svan := \frac{Svndir}{Lv}$ $svan = 6.174\cdot 10^{5}$

$sv := \frac{svan}{Fv}$ $sv = 499.981$ $T := 5491.8\,10^{9}$ $Ydv := Yv - T$

$Sv := sv\cdot Fv\cdot Lv$ $Sv = 6.298\cdot 10^{12}$ $Ydv = 1.425\cdot 10^{13}$

$sv := \frac{Sv}{Lv\cdot Fv}$ $sv = 499.981$ $AsiBv := Ydv - Sv$

$AsiBv = 7.948\cdot 10^{12}$ $INVv := 2821.8\,10^{9} - 1666.8\,10^{9}$

$A := INVv$

$Bv := AsiBv - A$ $Bv = 6.793\cdot 10^{12}$ $t := \frac{T}{Yv}$ $Vv := Ydv - A$

$Bv := Vv - Sv$

$bv := \frac{Bv\cdot 100}{A + Sv}$

$F := 227\cdot 8$

$L := 10.6\cdot 10^{6}$ $Fa := F - Fv$ $La := L - Lv$ $r := 20$ $j := 1..r$

$P := L\cdot F\cdot wv$

$wa_j := (1 - t)\cdot wv + j\cdot\left(\frac{wv\cdot t}{r}\right)$

$Ya(wa) := (Lv\cdot Fa + La\cdot F)\cdot wa_j$ $Yda(wa) := Ya(wa)$ $k(wa) := Yv + Ya(wa)$

$ya(wa) := \frac{Ya(wa)}{k(wa)}$ $sa_j := j\cdot\frac{2\cdot sv}{r}$ $mw := \frac{wa}{wv}$ $ms := \frac{sa}{sv}$

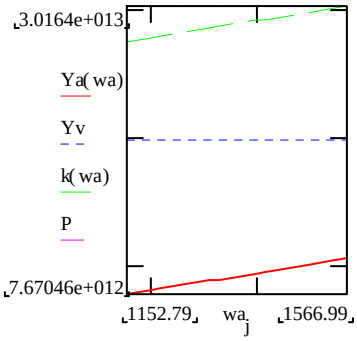
$ba_j := \left(\frac{wv\cdot mw_j}{ms_j\cdot sv}\cdot 100\right) - 100$ $mb := \frac{ba}{bv}$

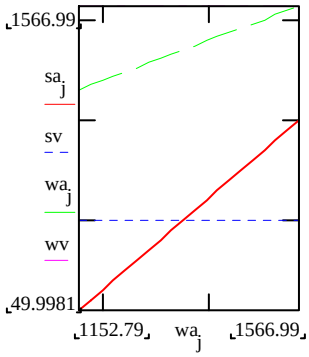
$Yv = 1.974 \cdot 10^{13}$ $T = 5.492 \cdot 10^{12}$ $Ydv = 1.425 \cdot 10^{13}$ $A = 1.155 \cdot 10^{12}$

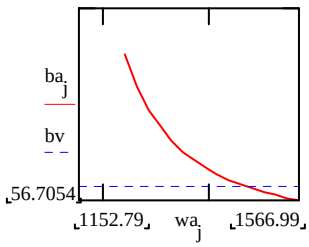
$Vv = 1.309 \cdot 10^{13}$ $Sv = 6.298 \cdot 10^{12}$ $Bv = 6.793 \cdot 10^{12}$

$wv = 1.567 \cdot 10^{3}$ $t = 0.278$ $sv = 500$ $bv = 91.1$

ya(wa)	wa_j	$\frac{k(wa)}{10^{13}}$	$\frac{Ya(wa)}{10^{13}}$	sa_j	ba_j
0.28	1153	2.74	0.77	50	2205.7
0.284	1175	2.76	0.78	100	1074.6
0.287	1196	2.77	0.8	150	697.6
0.291	1218	2.78	0.81	200	509.1
0.295	1240	2.8	0.83	250	396
0.298	1262	2.81	0.84	300	320.6
0.302	1284	2.83	0.85	350	266.8
0.306	1305	2.84	0.87	400	226.4
0.309	1327	2.86	0.88	450	194.9
0.313	1349	2.87	0.9	500	169.8
0.316	1371	2.89	0.91	550	149.2
0.319	1393	2.9	0.93	600	132.1
0.323	1414	2.91	0.94	650	117.6
0.326	1436	2.93	0.96	700	105.2
0.33	1458	2.94	0.97	750	94.4
0.333	1480	2.96	0.98	800	85
0.336	1502	2.97	1	850	76.7
0.339	1523	2.99	1.01	900	69.3
0.342	1545	3	1.03	950	62.7
0.346	1567	3.02	1.04	1000	56.7







$lv := \frac{Lv}{L}$ $fv := \frac{Fv}{F}$ $yamax := 1 - (lv \cdot fv)$

$lv = 0.962$ $fv = 0.68$ $yamax = 0.346$

$yamin := \left[1 - \left(\frac{lv \cdot fv}{1 - t \cdot yamax}\right)\right]$ $yamin = 0.276$

$hmax := 100$ $h := 100$ $n := 40$ $i := 1..(n-1)$ $\theta_i := \frac{i}{n}$

$hacr(\theta) := h \cdot \theta^{\frac{1}{1-\theta}}$ $hacr\%(\theta) := \frac{hacr(\theta)}{h + hacr(\theta)}$

$hacr\%(\theta_i)$

0.0222
0.041
0.0573
0.0719
0.085
0.0969
0.1079
0.118
0.1273
0.1361
0.1442
0.1519
0.1591
0.1659
0.1723
0.1784
0.1842
0.1897
0.195
0.2
0.2048
0.2094
0.2138
0.2181
0.2221
0.226
0.2298
0.2335
0.237
0.2404
0.2436
0.2468
0.2499
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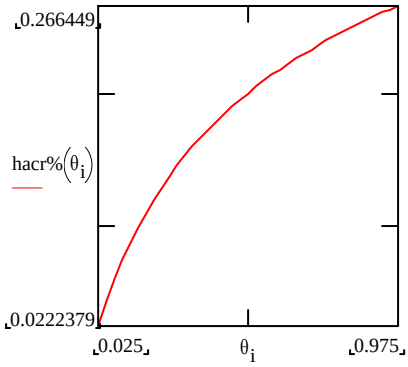
0.2585
0.2612
0.2639
0.2664



7. FINAL CONCLUSIONS

The transition towards a free market economic system in Central and East-European countries is doubled (followed) by a profound economic crisis. Generally, in these countries the production in the official sector has been decreased rapidly after 1989 and official unemployment grew up to levels even greater than those currently registered in Western countries. Also, the income from the work in official sector decreased dramatically during a historically short period. In these conditions, an increasing share of labour force supply moves to the black labour market. Contrary to the hypothesis of greater wages in underground sector than those in official sector, many suppliers of labour force on black market accept wages smaller than in the official sector case ($sa < sv$).

On the other hand, the profit rate in the underground sector is always greater than the net profit rate in the visible sector ($ba > bv$). This is at least due to the two reasons:

- in the underground sector there is no payment as taxes or other social duties;
- the expenditures for investments in fixed capital in underground sector are minimally (nearly zero) (they used exclusively only circulating capital and they operate under the rule of the profit maximization in the short run, which, within a situation as how is at present in the transition period, of penury of capital funds, doubled by an increasing inflation and a depreciation of national currency, is accentuated.

In the transition period, there is a relative penury of consumption goods and services yet and the production of heavy industries and generally of capital goods remains again monopoly of the state enterprises. Consequently, in the underground sector, comparing with visible sector, the smaller amounts of disposable capital from small entrepreneurs go mainly to the production of consumption goods and services, where there is an uncovered demand for employment, the initial necessary amount of capital is smaller being exclusively destined to assure circulating capital funds, the velocity of expended monetary funds and rate of return to capital are greater.

Moreover, in the first years of transition when the inflation rate was greater than interest rate (namely a negative interest rate in real terms) and the former fiscal legislation collapsed there was a massive migration from visible sector to underground activities.

Also, in conditions of an average small income per household, a supplementary supply of labour comes from some categories of population traditionally non-included in the labour force (such as: pupils, students, retired persons, and domestic women) and from official unemployed.

In transition period has been developed some new differences among groups of population: one is that between employed people and unemployed people, other is that between people employed only in official sector and those employed in underground sector or in both sectors. In conditions of a deepen crisis of production in the visible sector, the winners will be those employed in underground sector (including persons employed in both sectors). In a first stage, the disposable income providing from such differences will be accumulated and then oriented (by reasons of efficiency) to underground activities. But in a second stage, when the amount of capital accumulated became greater than the power of absorption of unofficial sector, there will be an inverse tendency: the new riches people coming from underground activities will desire to make official their new status and consequently they will try to "whiten" their money and then to become "respectable" people, to enter into the legal activities. This last phase we may consider it as the end of a cyclical sequence and the entire movement as a wave. After the stage of "primitive accumulation of capital" the Eastern economies will register a similar evolution as Western economies. On the general background of the restraint of share of the underground economy, it will be registered some fundamental restructurings within the invisible sector. The share of more sophisticate underground activities will increase comparing with the share of rudimentary and artisan underground activities.

Since it will be possibly to enter a new-spiralled cycle, it remains a question for the future time. Therefore, we consider that the description of the general process of economic development in long run based on a model of endogenous cycle, which explain the general

economic fluctuations by a set of successive waves or overlapping cyclical phases relating to the report between underground economy and visible economy, may be as a plausible alternative to the standard models based only the dynamics of the official sector of economy in short run. In this case, the historical economic evolution can be seen as permanent but fluctuating overlapping transfers of cohorts of economic factors (labour, capital) from official sector to underground sector and reverse. During the instability periods, as crises, wars or transitions there is an intensification of transfers in double sense between the two sectors of economy, that corresponds to certain great structural re-adjustments of the economic system as well as the entire society.

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