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## **PRICING OF MEDIA PLATFORMS WITH VERTICAL DIFFERENTIATION**

***Abstract.** Quality difference prevails among media firms featuring the two-sided property. This paper builds a two-sided duopoly model with vertical differentiation where consumers are heterogeneous in their preferences to high quality. We find that the media platform with high quality does not necessarily charge a higher subscription fee or advertising fee than the media platform with low quality in the presence of cross externalities between consumers and advertisers. In the consumer market, if the cross externality is sufficiently strong relative to the vertical differentiation in the consumer market and the horizontal differentiation on the advertising market, then the high-quality media firm charges a lower subscription fee than the low-quality media firm. In the advertising market, it is also possible for the high-quality firm to charge a lower subscription fee if the externality of consumers is weaker than the externality of advertisers.*

***Keywords:** Media market; Vertical differentiation; Pricing; Two-sided platforms.*

**JEL Classification: D43, L11, L13, L82**

### **1. Introduction**

Media platforms create value by connecting media consumers and advertisers. Examples include newspapers and magazines. Unlike traditional firms who make profit by selling products to consumers only, media firms have two revenue sources. First, media firms sell consumers media products and charge them a subscription fee. In addition, they also generate revenues from advertisers by charging an advertising fee. Consumers care about the advertising intensity, and advertisers are concerned about the number of consumers who are potential consumers of their products. Consequently, media firms behave as two-sides platforms, which determine the pricing to both consumers and advertisers taking into account the cross externalities between consumers and advertisers.

In addition, quality difference prevails among media platforms. Different platforms vary in their product quality with respect to content, printing, and design, etc. In a one-sided market, the price of a high-quality product is generally higher than that

of a low-quality product because higher quality raises the value of the product to consumers. In a two-sided media market, however, it remains to be a question whether high quality guarantees a higher price to consumers. Besides quality, the cross externalities between consumers and advertisers also play a crucial role in determining the pricing of media platforms. In addition, it is also worth investigating the prices to advertisers, which depend on the number of consumers. Does the media platform with high quality always charge a higher price to consumers and advertisers than the media platform with low quality?

The above-mentioned question motivates us to study how the two-sidedness affects the price competition among vertically differentiated platforms. To address this issue, this paper establishes a two-sided duopoly model where two platforms are vertically differentiated, namely, with high quality and low quality in the consumer market, and horizontally differentially in the advertising market.

Consumers are heterogeneous in their preferences to high quality. Moreover, both consumers and advertisers single-home, that is, they only choose one platform. We investigate how the pricing rules of media platforms to consumers and advertisers are complicated by the comparison between vertical differentiation and indirect externalities. We find that the media platform with high quality does not necessarily charge a higher subscription fee or advertising fee than the media platform with low quality owing to the presence of cross externalities between consumers and advertisers. In the consumer market, if the cross externality is sufficiently strong relative to the vertical differentiation in the consumer market and the horizontal differentiation on the advertising market, then the high-quality media firm charges a *lower* subscription fee than the low-quality media firm. In the advertising market, it is also possible for the high-quality firm to charge a *lower* subscription fee if the externality of consumers is weaker than the externality of advertisers.

The rest of this paper proceeds as follows. In Section 2, we establish a two-sided duopoly model with vertical differentiation and two single-homing groups. In Section 3, we derive the equilibrium prices and profits of the two vertically differentiated platforms. Section 4 concludes.

## **2. Background and related work**

The present paper relates to two strands of literature. First, it builds upon the works on the pricing behavior of firms in two-sided markets. Caillaud and Jullien (2003) analyze the case of non-differentiated platforms that provide pure intermediation service to homogenous users and charge both a lump-sum fee and a proportional fee, arguing that proportional fees act as a form of risk sharing between the platform and the agents. However, they do not consider the case when consumers are heterogeneous in their preferences, which is prevalent in the media market. Rochet and Tirole (2003) and Armstrong (2006) investigate the pricing behavior of two-sided

platforms with heterogeneous consumers. The former focuses on the case of pure usage (proportional) fees and provide the classical argument on two-sided markets that one group's agents may subsidize the other group, while the latter places an emphasis on lump-sum fees. In general, the above literature provides pioneering analyses on the properties of two-sided markets. However, these works assume symmetry among two-sided platforms. The present paper contributes to the first strand of literature by introducing vertical differentiation across platforms.

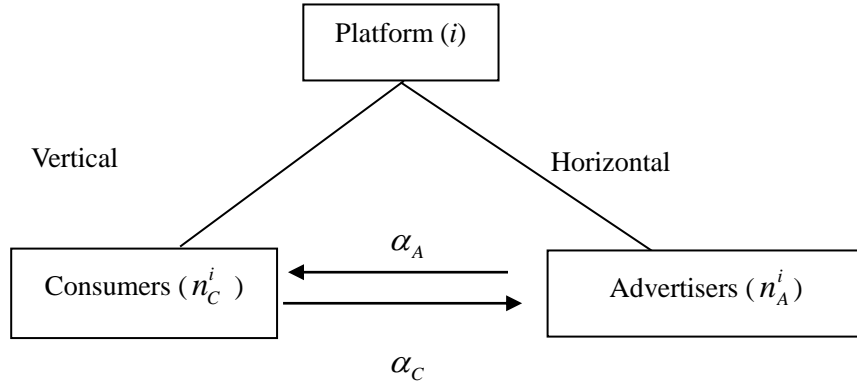
Second, this paper is also related to the literature on the pricing in the media market. Most of the existing literature on the media market concentrates on the broadcasting industry. Gabszewicz et al. (2001, 2002) assume that viewers are indifferent to advertising intensity, so no externality from advertisers to consumers is considered in these works. Gabszewicz et al. (2004) analyzes the free-to-air TV market where advertisers are homogeneous. The seminal work by Coate and Anderson (2005) considers the competition between two broadcasting platforms that are horizontally differentiated to viewers/consumers. Peitz and Velletri (2008) further contributes by introducing vertical differentiation between a Pay TV and a free-to-air TV in the consumer market and heterogeneous advertisers. Reisinger (2011) assumes that platforms are differentiated from the consumers' perspective but are homogenous for advertisers. This paper complements this strand of literature as follows. First, unlike the above literature, which generally consider advertisements as nuisance, this paper relaxes their assumption so that the externality from advertisers to consumers may be positive or negative. The positive externality is especially reasonable for paper media because consumers value the information provided in advertisements when they read magazines, such as automobile magazines, sports magazines and fashion magazines. Second, our model introduces heterogeneity in consumers' preferences on quality, which is also a realistic setting that captures the difference among consumers with respect to their valuation of quality.

## 2. The Model

Suppose there are two groups of agents, consumers and advertisers, who are connected by two competing media platforms, which are platform *L* and platform *H*, respectively. Group *C* and group *A* represent consumers and advertisers, respectively. Each consumer and advertiser choose to register on a single platform.<sup>1</sup> Figure 1 provides a sketch of the model.

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<sup>1</sup>The reasons for both groups to single-home include strong differentiation of the platform and limited resources of the advertisers. The empirical works on the media market, represented by Kaiser and Wright (2006) and Argentesi and Filistrucchi (2007), show that few firms advertise on more than one newspaper or magazine. This assumption is also widely employed in theoretical literature on two-sided markets, such as Belleflamme and Toulemonde (2009) and Kotsogiannis and Serfes (2010).



**Figure 1. A Sketch of the Model**

The two platforms are vertically differentiated in the consumer market. Platform  $L$  provides the product of low quality  $s_L$ , and platform  $H$  provides the product of high quality  $s_H$ . We assume that  $s_H$  and  $s_L$  are exogenously given. All consumers agree that high quality is preferable to low quality but they are heterogeneous in the degree of valuing quality. To capture this heterogeneity, a preference parameter for quality is assigned as  $\theta \in [\underline{\theta}, \bar{\theta}] \subset \mathbf{R}_+$ , in which consumers are uniformly distributed. The consumer with a higher  $\theta$  has a stronger preference to high quality. For simplicity without generality, we assume  $\underline{\theta} = 0$  and  $\bar{\theta} = 1$ , so that consumers are of mass 1.

The utility of consumer  $\theta$  on platform  $i$  can be described by

$$u_C^i = r + \theta s_i + \alpha_A n_A^i - p_C^i, i=H, L, \quad (1)$$

where  $u_C^i$  is the utility of the consumer if she chooses platform  $i$  ( $i=H, L$ ). The consumer obtains a reservation utility from either platform, which is represented by  $r$ . In addition, the consumer receives a premium value that depends on the quality of platform  $i$ , which is measured by  $\theta s_i$ . Advertisers on platform  $i$  bring consumers an externality of  $\alpha_A n_A^i$ , where  $\alpha_A$  represents the externality that an advertiser brings to the consumer on the same platform, which can be positive or negative, and  $n_A^i$  is the number of advertisers on platform  $i$ . Platform  $i$  charges consumers a subscription fee

$p_C^i$ . To ensure full market coverage, we assume  $r$  is sufficiently large so that all consumers participate.

According to equation (1), the consumer that is indifferent between the two platforms satisfies  $r + \hat{\theta}s_L + \alpha_A n_A^L - p_C^L = r + \hat{\theta}s_H + \alpha_A n_A^H - p_C^H$ . Therefore, the number of consumers on platform  $L$  and platform  $H$  are

$$n_C^L = \hat{\theta} = \frac{(p_C^H - p_C^L) + \alpha_A(n_A^H - n_A^L)}{S}, \quad (2)$$

$$n_C^H = 1 - \hat{\theta} = 1 + \frac{(p_C^L - p_C^H) + \alpha_A(n_A^L - n_A^H)}{S}. \quad (3)$$

where  $S \equiv s_H - s_L > 0$  denotes the quality difference between platform  $H$  and platform  $L$ .

On the advertiser's side, the media platforms are horizontally differentiated because advertisers are mainly concerned about the number of consumers that they can reach. Advertisers obtain a utility of  $u_A^i$  if they join platform  $i$  ( $i=L, H$ ). The utility of an advertiser consists of the intrinsic value of the platform and the external benefits brought by consumers on the same platform. An advertiser obtains an intrinsic value of  $v$  from either platform, and enjoys a positive externality from consumers on the same platform, which is measured by  $\alpha_C n_C^i$ . Here  $\alpha_C > 0$  is the externality brought by a consumer, and  $n_C^i$  is the number of consumers on platform  $i$ . Platform  $i$  charges each advertiser an advertising fee of  $p_A^i$ . The effect of advertising may vary across different types of advertisers. In a sports magazine, for instance, an advertisement of sports facilities may receive more attention than an advertisement of electronics. To capture the heterogeneity of advertising effects, we employ the Hotelling specification. Advertisers are of mass one, and are uniformly distributed along a unit interval of  $[0,1]$ . The two media platforms are located at the two endpoints of the interval, with platform  $L$  located at 0 and platform  $H$  located at 1. The advertiser of type  $x$  incurs a disutility  $t|x-l^i|$  if he or she chooses platform  $i$  ( $i=L, H$ ), where  $t$  is the differentiation parameter,  $x$  represents the type of the advertiser, and  $l^i$  is the location of the platform. Therefore, the utility of an advertiser when s(he) joins platform  $i$  ( $i=L, H$ ) is

$$u_A^i = v + \alpha_C n_C^i - p_A^i - t|x-l^i|, \quad i=L, H. \quad (4)$$

To ensure full market coverage, we assume  $v$  is sufficiently large so that all advertisers participate.

With the Hotelling specification, the numbers of advertisers on platform  $i$  are

$$n_A^i = \frac{1}{2} + \frac{u_A^i - u_A^j}{2}. \quad (5)$$

Substituting expressions (5) into expressions (4), and using  $n_A^L + n_A^H = 1$ , yields the market shares of platform  $L$  and platform  $H$  in the advertising market, respectively:

$$n_A^L = \frac{1}{2} + \frac{\alpha_C(n_C^L - n_C^H) + (p_A^H - p_A^L)}{2t}, \quad (6)$$

$$n_A^H = \frac{1}{2} + \frac{\alpha_C(n_C^H - n_C^L) + (p_A^L - p_A^H)}{2t}. \quad (7)$$

Rearranging equations (3), (4), (6) and (7), we can express the numbers of consumers and advertisers on platform  $i$  ( $i=L, H$ ) in terms of prices and qualities:

$$n_C^L = \frac{t(p_C^H - p_C^L) + \alpha_A(p_A^H - p_A^L) - \alpha_C\alpha_A}{tS - 2\alpha_C\alpha_A}, \quad (8)$$

$$n_C^H = \frac{tS + t(p_C^L - p_C^H) + \alpha_A(p_A^L - p_A^H) - \alpha_C\alpha_A}{tS - 2\alpha_C\alpha_A}, \quad (9)$$

$$n_A^L = \frac{1}{2} + \frac{2\alpha_C(p_C^H - p_C^L) + (p_A^H - p_A^L - \alpha_C)S}{2(tS - 2\alpha_C\alpha_A)}, \quad (10)$$

$$n_A^H = \frac{1}{2} + \frac{2\alpha_C(p_C^L - p_C^H) + (p_A^L - p_A^H + \alpha_C)S}{2(tS - 2\alpha_C\alpha_A)}. \quad (11)$$

Both platforms incur zero marginal cost to both groups. Platform  $H$  incurs a fixed cost  $F$  to produce the product of high quality, while the fixed cost of platform  $L$  is zero. We can express the profits of platforms  $L$  and  $H$  as

$$\pi^L = p_C^L n_C^L + p_A^L n_A^L, \quad (12)$$

$$\pi^H = p_C^H n_C^H + p_A^H n_A^H - F. \quad (13)$$

The two platforms compete simultaneously in price.

### 3. The Equilibrium

This section discusses the optimal prices of the media platforms.

Substituting equations (8), (9), (10) and (11) into equations (12) and (13), we can express the profits of platform  $L$  and platform  $H$  in terms of the prices:

$$\begin{aligned} \pi^L = & p_C^L \frac{t(p_C^H - p_C^L) + \alpha_A(p_A^H - p_A^L) - \alpha_C \alpha_A}{tS - 2\alpha_C \alpha_A} \\ & + p_A^L \left[ \frac{1}{2} + \frac{2\alpha_C(p_C^H - p_C^L) + (p_A^H - p_A^L - \alpha_C)S}{2(tS - 2\alpha_C \alpha_A)} \right], \end{aligned} \quad (14)$$

$$\begin{aligned} \pi^H = & p_C^H \frac{tS + t(p_C^L - p_C^H) + \alpha_A(p_A^L - p_A^H) - \alpha_C \alpha_A}{tS - 2\alpha_C \alpha_A} \\ & + p_A^H \left[ \frac{1}{2} + \frac{2\alpha_C(p_C^L - p_C^H) + (p_A^L - p_A^H - \alpha_C)S}{2(tS - 2\alpha_C \alpha_A)} \right] - F. \end{aligned} \quad (15)$$

We make the following assumption to provide a meaningful solution:

**Assumption:**  $9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) > 0$ .

This assumption provides a sufficient condition for the existence of a market-sharing equilibrium. (See Appendix A) Note that this condition always holds if

$$-2\alpha_C < \alpha_A < -\frac{1}{2}\alpha_C. \quad ^2$$

#### 3.1 Pricing Behavior

Platform  $i$  ( $i=L, H$ ) maximizes its profit with respect to its price to consumers and advertisers, given its rival's pricing behavior. The equilibrium prices of platform  $L$  and platform  $H$  are expressed in the following lemma.

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<sup>2</sup>Recall that we assume  $\alpha_C > 0$  to capture the positive externality from consumers to advertisers.

**Lemma 1** In equilibrium, platform  $L$ 's prices to consumers and to advertisers are

$$p_C^L = \frac{3tS - (\alpha_C + 2\alpha_A)(\alpha_C + \alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)} S - \alpha_C$$

$$p_A^L = t - \alpha_A + \frac{(\alpha_A - \alpha_C)tS}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

Platform  $H$ 's prices to consumers and to advertisers are

$$p_C^H = \frac{6tS - (3\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)} S - \alpha_C$$

$$p_A^H = t - \alpha_A + \frac{(\alpha_C - \alpha_A)tS}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

**Proof:** See Appendix A. *Q.E.D*

Comparing the prices to consumers and advertisers between platform  $H$  and platform  $L$  in Lemma 1, we find that platform  $H$  does not necessarily charge a higher price than platform  $L$ . Proposition 1 summarizes our findings.

**Proposition 1:** (i) To consumers, platform  $H$  charges a higher price than platform  $L$  if the vertical differentiation in the consumer market and the horizontal differentiation on the advertising market is sufficiently strong relative to the cross externality, and platform  $L$  charges a higher price than platform  $H$  otherwise. That is,

$$\text{if } 3tS > 2\alpha_C(\alpha_C + 2\alpha_A), \text{ then } p_C^H > p_C^L,$$

$$\text{if } 3tS < 2\alpha_C(\alpha_C + 2\alpha_A), \text{ then } p_C^H < p_C^L,$$

$$\text{if } 3tS = 2\alpha_C(\alpha_C + 2\alpha_A), \text{ then } p_C^H = p_C^L;$$

(ii) To advertisers, platform  $H$  charges a higher price than platform  $L$  if the externality of consumers is stronger than the externality of advertisers, and platform  $L$  charges a higher price than platform  $H$  otherwise. That is

$$\text{if } \alpha_C > \alpha_A, \text{ then } p_A^H > p_A^L,$$

$$\text{if } \alpha_C < \alpha_A, \text{ then } p_A^H < p_A^L,$$

$$\text{if } \alpha_C = \alpha_A, \text{ then } p_A^H = p_A^L.$$

From Proposition 1, we can see that platform  $H$  does not necessarily charge a



higher price than platform  $L$  to either consumers or advertisers although it provides the product with a higher quality. The reason is the presence of the externalities across consumers and advertisers. If there is no cross externality, that is,  $\alpha_C = \alpha_A = 0$ , then it is straightforward that platform  $H$  charges a higher price to consumers than platform  $L$ . However, the cross-group externalities intensify the competition between the two platforms and decrease the advantage of platform  $H$ . When the cross externalities are sufficiently high, the opportunity cost of losing consumers or advertisers is very large. Therefore, platform  $H$  has strong incentives to reduce its price to gain more market share. Note that this may happen even when the externality from advertisers ( $\alpha_A$ ) is negative as long as the positive externality from consumers ( $\alpha_C$ ) is sufficiently large.

The intuition for the pricing to advertisers is straightforward. If the externality from consumers ( $\alpha_C$ ) is higher than externality from advertisers ( $\alpha_A$ ), inviting more consumers has a higher payoff than placing more advertisements, thus platform  $H$  would charge advertisers a higher price. The same logic applies if the opposite condition holds.

### 3.2 Market Share

Now we compare the market shares of the two platforms in both the consumer market and the advertising market. Based on Lemma 1, we obtain the market shares of the platforms, as summarized in the following lemma.

**Lemma 2** In equilibrium, the market shares of Platform  $L$  in the consumer and advertising market are

$$n_C^L = \frac{3tS - (2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

$$n_A^L = \frac{1}{2} - \frac{S}{2} \frac{\alpha_C + 2\alpha_A}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

And the market shares of Platform  $H$  in the consumer and advertising market are

$$n_C^H = \frac{6tS - (2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

$$n_A^H = \frac{1}{2} + \frac{S}{2} \frac{\alpha_C + 2\alpha_A}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}$$

Comparing the market shares in the above lemma, we obtain the following proposition.

**Proposition 2** In equilibrium, the market share of Platform  $H$  is larger than that of Platform  $L$  in both the consumer market and the advertising market. That is,  $n_C^H > n_C^L$  and  $n_A^H > n_A^L$ .

Despite the ambiguous pricing behavior, proposition 2 shows that Platform  $H$  always occupies a larger market share than Platform  $L$  in both markets. However, the cross externality plays a role in changing the market shares of both platforms in the consumer market compared to the benchmark case of no externality. If there is no cross externality, that is,  $\alpha_C = \alpha_A = 0$ , then  $n_C^L = 1/3$  and  $n_C^H = 2/3$ . If

$-2\alpha_C < \alpha_A < -\frac{1}{2}\alpha_C$ , then  $n_C^L > 1/3$  and  $n_C^H < 2/3$ , which implies that the market share of Platform  $H$  decreases when the externality from advertisers to consumers is moderately negative. If  $\alpha_A < -2\alpha_C$  or  $\alpha_A > -\frac{1}{2}\alpha_C$ , then  $n_C^L < 1/3$  and  $n_C^H > 2/3$ , which implies that the market share of Platform  $H$  rises when the externality from advertisers to consumers is strongly negative, weakly negative or positive.

### 3.3 Profits

Based on Lemma 1 and Lemma 2, we solve the maximized profits of the media platforms, as summarized in the following lemma.

**Lemma 3** The maximized profit of Platform  $L$  is

$$\pi^L = \frac{1}{2} + \frac{2tS^2 + 2(\alpha_C + \alpha_A)(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) - S\alpha_C(\alpha_C + 2\alpha_A) - 2tS(4\alpha_C + 5\alpha_A)}{2[9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)]}$$

and the maximized profit of Platform  $H$  is

$$\pi^H = \frac{1}{2} + \frac{8tS^2 + 2(\alpha_C + \alpha_A)(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) - S(3\alpha_C + 2\alpha_A)(\alpha_C + 2\alpha_A) - 2tS(5\alpha_C + 4\alpha_A)}{2[9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)]}$$

Comparing the profits in the above lemma, we obtain the results in the following position.

**Proposition 3.** Platform  $H$  earns a higher profit than Platform  $L$  if

$$3tS - (\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) - t(\alpha_C - \alpha_A) > 0,$$

and earns a lower profit otherwise.

**Proof:** By Lemma 3, the difference between platform  $H$ 's profit and platform  $L$ 's profit is  $\pi^H - \pi^L = \frac{3tS - (\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) - t(\alpha_C - \alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)} S$ . Because  $9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) > 0$  and  $S > 0$  by the Assumption,  $\pi^H - \pi^L \geq (<)0$  if  $3tS - (\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) - t(\alpha_C - \alpha_A) \geq (<)0$ . **Q.E.D.**

Proposition 3 shows that Platform  $H$  may earn a lower profit than Platform  $L$  in a few cases. If the horizontal differentiation in the advertising market ( $t$ ) is sufficiently high and the externality from consumers ( $\alpha_C$ ) is much stronger than the externality from advertisers ( $\alpha_A$ ), while the quality of Platform  $H$  is not sufficient high compared to Platform  $L$ , then Platform  $H$  may price so aggressively that its profit is lower than that of Platform  $H$ .

#### 4. Conclusion

Media firms are vertically differentiated two-sided platforms. This paper compares the optimal prices and profits of two-sided platforms which are vertically differentiated in the consumer market and horizontally differentiated in the advertising market. In the presence of cross externality between consumers and advertisers, the high-quality platform may not charge a higher price than the low-quality platform in both consumer and advertising markets. When the cross externalities are sufficiently high, the opportunity cost of losing consumers or advertisers is very large. Therefore, Platform  $H$  has strong incentives to reduce its price to gain more market share. In addition, Platform  $H$  always occupies a higher market share than Platform  $L$  in both consumer and advertising markets. Finally, Platform  $H$  does not necessarily earn a higher profit than Platform  $L$ . Our results may provide a new perspective of understanding the pricing behaviors of vertically differentiated media platforms, especially of newspapers and magazines.

The present paper is restricted to the case when both groups single-home. In reality, however, some groups may multi-home. For instance, consumers may read news from more than one platform. Therefore, a possible extension is to investigate the endogenous choice of price instruments when at least one group multi-homes. All in all, the present paper is just attempt to cast light on the pricing instruments choices among media firm.

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**Appendix A:** Proof of Lemma 1 and Assumption

Platform  $i(i=L,H)$  decides on the prices to the two groups in order to maximize its profits:

$$\text{Max}_{p_C^i, p_A^i} \pi^i = p_C^i n_C^i + p_A^i n_A^i, \quad i=L,H$$

F.O.C of the two profits yield:

$$\begin{pmatrix} -2t & t & -(\alpha_1 + \alpha_2) & \alpha_2 \\ -2(\alpha_1 + \alpha_2) & 2\alpha_1 & -2S & S \\ t & -2t & \alpha_2 & -(\alpha_1 + \alpha_2) \\ 2\alpha_1 & -2(\alpha_1 + \alpha_2) & S & -2S \end{pmatrix} \begin{pmatrix} p_1^1 \\ p_1^2 \\ p_2^1 \\ p_2^2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \alpha_2 \\ 2\alpha_1 \alpha_2 - (t - \alpha_1) S \\ \alpha_1 \alpha_2 - t S \\ 2\alpha_1 \alpha_2 - (t + \alpha_1) S \end{pmatrix} \quad (\text{A-1})$$

The calculation of the above matrix yields the determinant:

$$\Delta = -(tA - 2\alpha_1 \alpha_2)[9tA - 2(2\alpha_1 + \alpha_2)(\alpha_1 + 2\alpha_2)]$$

The sufficient and necessary condition for a market-sharing equilibrium to exist is the second order condition of the platforms' profits should be negative, that is,

$$J = \begin{pmatrix} \pi_{C_L C_L}^L & \pi_{C_L A_L}^L & \pi_{C_L C_H}^L & \pi_{C_L A_H}^L \\ \pi_{A_L C_L}^L & \pi_{A_L A_L}^L & \pi_{A_L C_H}^L & \pi_{A_L A_H}^L \\ \pi_{C_H C_L}^H & \pi_{C_H A_L}^H & \pi_{C_H C_H}^H & \pi_{C_H A_H}^H \\ \pi_{A_H C_L}^H & \pi_{A_H A_L}^H & \pi_{A_H C_H}^H & \pi_{A_H A_H}^H \end{pmatrix}$$

should be negative definite. Here  $(\pi_{M_i N_j}^i = \frac{\partial(\partial \pi^i / \partial p_M^i)}{\partial p_N^j})$ ,  $i = L, H$ ,  $j = L, H$ ,  $i \neq j$ ;  $M = C, A$ ,  $N = C, A$ , and  $M \neq N$ ).

By equations (14) and (15), we can derive the Jacobian matrix:

$$J = (tS - 2\alpha_C\alpha_A) \begin{pmatrix} -2t & -(\alpha_C + \alpha_A) & t & \alpha_A \\ -2(\alpha_C + \alpha_A) & -2S & 2\alpha_C & S \\ t & \alpha_C & -2t & -(\alpha_C + \alpha_A) \\ 2\alpha_C & S & -2(\alpha_C + \alpha_A) & -2S \end{pmatrix}.$$

$J$  is negative definite if and only if:

- a.  $|J_1| = -2t < 0$ ,
- b.  $|J_2| = 9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) > 0$ .

To ensure that a market-sharing equilibrium exists, the sufficient and necessary condition is

$$9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A) > 0.$$

This proves the Assumption, by which it is direct that the determinant  $\Delta < 0$ .

Using matrix (A-1), we can compute the equilibrium prices of both platforms to consumer and advertisers:

$$\begin{aligned} p_C^L &= \frac{3tS - (\alpha_C + 2\alpha_A)(\alpha_C + \alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)} S - \alpha_C, \\ p_A^L &= t - \alpha_A + \frac{(\alpha_A - \alpha_C)tS}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}, \\ p_C^H &= \frac{6tS - (3\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)} S - \alpha_C, \\ p_A^H &= t - \alpha_A + \frac{(\alpha_C - \alpha_A)tS}{9tS - 2(2\alpha_C + \alpha_A)(\alpha_C + 2\alpha_A)}. \end{aligned}$$