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FUZZY LOGIC IN THE DESIGN OF PUBLIC POLICIES: APPLICATION OF LAW

***Abstract.** This paper presents a methodological proposal in the field of uncertainty management, FuzzyLogic, to support efficient and effective decision-making in the field of Legal Sciences. We aim to contribute to provide more elements to a better guidance in law application. We apply the methodology to assign a sanction in years of imprisonment to a given felony; in this case to parricide. This work aims to foster the design of new public policies to qualify the sentences of felonies in all areas of justice in order to establish databases of felonies characterizations to provide the judge a greater number of elements to make more deficient and effective decisions.*

***Keywords:** fuzzy logic, right, justice, uncertainty, public policy.*

JEL Classification: C02, C36, C63

1. Decision-Making in Law Sciences

Decision-making in law sciences also applies to jurisprudence systems. Just like in other application areas, the stages to consider for effective and efficient administration are: forecast, planning, integration, organization, and control. With these elements it is possible to establish the present state and the directions to follow to make the right decisions. This paper presents basic ideas on uncertainty useful in the field of law science. These ideas direct the public policy efforts to review the sentence levels for several cases of felonies against society, which by those means will be qualified more precisely, still maintaining the role of the responsible judge. Thus, the problem to be addressed is the study of fair criteria that allow decision making in the assignment of sentences according to the closest scenario to that of where the felony was committed, from all possible variants that exist.

Currently, fuzzy logic has not been extensively applied to law in terms of legal reasoning (Mazarese, 2012). Generally, legal decisions are a matter of fact,

and as principle questions, they must be of logical-deductive nature. That is, legal decisions must be made following the rules of inference of two-valued logics. The application of fuzzy logic, on the other hand, takes place on an uncertain environment, that is, by means of a multivalent logic. Legal decisions are characterized as judicial syllogism chains. Nowadays, the judicial syllogism theory is widely accepted; it is conceived as the main warranty of the rationality of judicial decisions, and as a necessary mean to assure that the value of judicial certainty can be achieved.

Any human being who dares to face the logical-deductive problems in legal decisions is considered as a defender of the subjectivism, which gives raise to many situations in legal power. Nevertheless this is considered a wrong idea, given that the analysis is performed using a two-valued logic. On the other hand, the scenarios and facts in most of the cases are given in uncertain environments. These environments are suitable for fuzzy logic and qualitative reasoning, drawn upon multi-valued logics.

Classically, a logical reconstruction of facts in a law case does not necessarily guarantees rationality. This reconstruction can be achieved by an explicative model that includes all scenarios to be evaluated by confidence intervals, established subjectively by a panel of law experts. After this, we need to use semantic labels, associated to a numeric scale that map from ambiguous to precise terms, allowing an effective and efficient label assignment, removing ambiguity from human decisions.

2. The Decision-Making Process

Decision-making must follow that the search for certainty is a search for peace, hunted by risk and fear. In life is not uncertainty that bothers humans, but the danger it entails.

Decisions can be made in strategic and operative contexts. Decision-making theory assumes responsible decision makers are willing to maximize their satisfaction level and decrease the risk. The decider basic analysis process is based on the administrative process, which contains the following stages:

- Problem statement
- Statement of possible alternatives
- Selection of the best solution
- Solution implantation
- Follow-up and evaluation
- Feedback

Decision thoughts are associated to:

1. Analyzing the causes and defining the facts
2. Detect problems
3. Visualize consequences and apply solutions

In the study of judicial decisions (Mazarese, 2012), the application of fuzzy logic will not go against the legal process. People may get confused in the use of a judicial language, considered as the set of necessary languages in judicial decisions, to formulate:

- The right issue – major premise of the judicial syllogism
- The fact issue – minor or factual premise of the judicial syllogism
- The case norm – conclusion of the judicial syllogism
- The justification of the judicial decision itself, when required in a judicial decision

These stages form a process that can be formulated by fuzzy logic. This process requires the study of historical files to determine the formation of a felony and its evolution to these days, when a law will be issued or modified to assign a sentence. These tools allow us to effectively and efficiently define the felony and the scenarios where it occurred; this way we can define semantic labels and establish a numeric relation with the sentence in the interval corresponding to the assessment of the felony. This mechanism allows us to assign a more rational sentence than a simple subjective personal valuation of the judge.

Judge decisions are based on a process that applies scientific research. For (Kaufman, 1987), the efforts are oriented towards: objective knowledge (facts) and ideal entities (whose existence resides in the human mind only).

These orientations classify the scientific knowledge in formal and empirical sciences. Thus, we can establish that reality is the foundation of empirical sciences; in this approach there is a group that acts on a social reality, called social sciences (law falls into this class) (González, 2000). Within social sciences there are measuring techniques that follow the concepts of Galileo Galilei, measuring what is measurable and trying to measure what is not measurable yet.

In justice administration not every concept of interest can be quantified. Nonetheless, it is possible to make a good approximation performing a fuzzy situational analysis.

When addressing a problem, the questions refer to the object of study. Knowing what or investigating how are questions whose answers involve the objects of study. It is important to note that justice management there is a possibility the directly or in the future, certain phenomena be quantified. This brings about the concept of equilibrium, whose determination conveys the computation of matrices like minimum, fair mean, and maximum.

Descriptive statistics is used to determine the different variables used in decision-making (minimum, mean, maximum). In law, these are terms used in setting sentences for a felony. In the case of writing the Penal Laws for the state of Michoacan, let us say, article W, to determine the sentence time for a given felony, we need to study the different scenarios in a given time interval. These scenarios provide a minimum and a maximum sentence time, from which we can compute mean time = (minimum time + maximum time)/2. Nevertheless, there is a big void

in the determination of the sentence time in the intervals between the extremes and the mean. This void calls for new analysis criteria and methodologies that allow us to determine that information and make more certain decisions for the case at hand.

Humans deal with imprecise estimations, such as terrible, bad, good, very good, and excellent, among others. Different approaches to deal with those estimations and their precision have been used in the past. Those estimations were somehow quantifies through classical mathematics, written in a two-valued logic. But, in order to be able to include those terms in reasoning mechanisms, we need to deal with uncertainty.

A logical reconstruction can be made using an explicative model that includes all scenarios, evaluating them by confidence intervals, using estimations from judicial experts. These intervals need to be associated to semantic tags, and to a numeric scale that allow us to transform imprecision into precise terms. Using this mechanism we can assign numerical values to the intervals between the extremes and the mean, in order to quantify the time sentence for a felony. All this can be accomplished using fuzzy logic.

1.1. Fuzzy Logic

One of the most important problems addressed by this kind of work is the sentence assignment (jail time). The classical procedures of sentence assignment use direct criteria and the judge experience, based on the sentence time interval for the felony under judgment. Additionally, the decision process is supported by descriptive statistics to determine the mean value between the limits that law establishes for the kind of felony in question.

As a consequence, sentence assignment yields very different results, depending on the judge. This fact makes justice look unfair for an outsider. The criteria used in the process are based on subjective facts to assign a sentence.

Within the sentence interval established by law, the central value lies in between the extreme points of the sentence assigned to a particular felony. A triangular fuzzy number, as shown in Figure 1, can represent this uncertain scenario. Sentence times are represented in the horizontal axis, and certainty levels from the judge (α) are represented in the ordinates, ranging in the $[0,1]$ (Kaufman, 1987).

Figure 1 shows that for each value $[0 \leq \alpha_k \leq 1]$ corresponds a confidence interval, given by $[r_k^\alpha, s_k^\alpha]$, which can be expressed as a function of (α_k), as shown in Equation (1)(González, 2000).

$$[r_k^\alpha, s_k^\alpha] = [r + (m - r)\alpha_k, s - (s - m)\alpha_k], \quad \alpha \in [0,1] \quad (1)$$

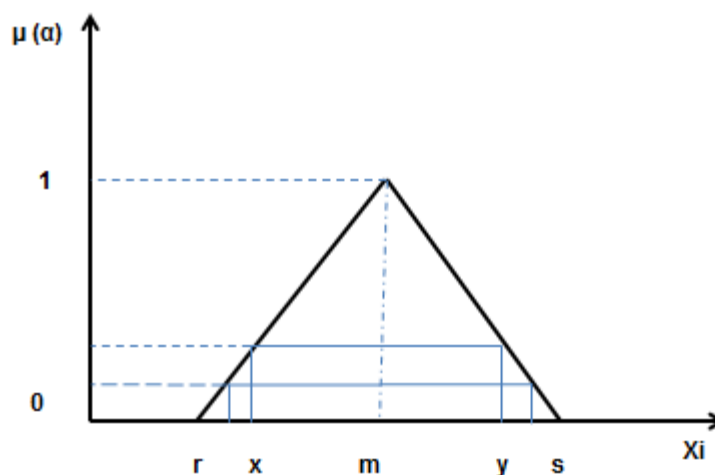


Figure 1. Triangular Fuzzy Number

This yields an interval where we consider the felony can take place; this relevance level, α_k , will be provided by a set of experts.

To avoid discretionary sentence assignments to a given felony, this work proposes a homogeneous interval distribution in 5 partitions in the given sentence interval. We assume all partitions to contain the same conditions and similar characteristics of committing a felony in the state, country, etc. We require to know when the felony was committed for the first time, and under what conditions, in order to group the occurrences by characteristics and sentence time. This allows the judge to assign a sentence time, according to the scenario, within the time interval assigned to that felony.

This process will provide the judge with a greater amount of information, in order to make better, fairer, more effective and efficient decisions. The assignment must correspond to one of the groups or classes established in the partition. In this class, the judge may assign a point in the partition interval, or just take its middle point. This procedure is certain to produce better decisions than just using common sense and discretion.

The decision making process must be supported by an information system containing an exhaustive study of all possible case. From the uncertainty point of view, a committee of experts in legal matters assigns values to determine ideal felony characterizations. This study leads to the analysis of all possible scenarios for that felony. These scenarios are obtained from historic research of the legal records for this felony, from the first time it is recorded and its evolution to the present time.

Therefore, the values assigned to each scenario correspond to the minimum, maximum, and mean sentence time. To determine the sentence levels

of previous cases, we use a reference scale to determine the felony level for the case in question.

As an example, an 11-value scale contains semantic labels assigned to each felony state, following the judge reference. This scale is shown in Table 1.

Table 1. 11-Value Reference Scale

Indicator	Label
1	Excellent
0.9	Extremely Good
0.8	VeryGood
0.7	Good
0.6	AlmostGood
0.5	Average
0.4	AlmostBad
0.3	Bad
0.2	Very Bad
0.1	Extremely Bad
0	Worst

The felony is assessed using the reference scale, where the maximum sentence is associated with the label Excellent. This label Excellent implies that the felony took place under all aggravating conditions, while the label Worst indicates that it could not be proved that the felony took place.

Felony characterization is important; for that matter we need to design a profile that approximates the ideal profile, which includes all possible behavior scenarios. Therefore, his characterization is different for each state, even though the felony has the same name. This fact implies that sentence times are different from state to state. The longer the history files we base this characterization, the more cases it considers, and the finer the fuzzy profile will be.

A felony profile contains the components $C = \{C_i, i = 1, 2, 3, \dots, n$, where C_i is the i -th behavior scenario (partition, P^i), $\mu^i =$ membership function (importance assigned by the judge), for $i = 1, 2, 3, \dots, n$. Graphically, we can see this as:

P^1	P^2	P^3	P^4					P^n
μ_1	μ_2	μ_3	μ_4	μ_n

The membership level is derived from the reference scale, which contains all possible characterizations, corresponding to different sentence times. The use triangular fuzzy numbers to represent each behavior scenario or partition (P_i) is shown in Fig. 1, where $\mu(\alpha)$ is the membership function of the partition or

scenario, corresponding to the level of justice application. X_i is the number of years in sentence time.

This enables the determination of each confidence interval using Equation (1), where the sentence time can be assigned in the analysis interval. E.g., the interval is $[r,s]$, where r is the minimum and s the maximum sentence times; m is the mean point, computed by Equation (2).

$$m = \frac{L_r + L_s}{2} = \frac{r+s}{2} \quad (2)$$

Using these tools and supported by an information system we can characterize almost perfectly the case to be judged. The information system must contain all information regarding the felony from the first time it has been recorded, considering sentence times and places. For instance, for Parricide, in the state of Michoacan, Mexico, the felony or crime is sentenced from 20 to 40 years in prison. The judge may assign different sentence times, according to the characterization of the felony. For that purpose, we can use the 11-value reference scale shown in Table 1. This procedure provides a more precise analysis and diminishes the level of variation in the assigned sentence times from judge to judge, which takes us to seek the establishment of processes under complete certainty, using probability models, and uncertain models, based on multivalued logic. Fuzzy logic is one of those multivalued logics, and allows us to effectively and efficiently characterize all possible scenarios in the felony under analysis.

3. Study Case

As a study case we will use the following case. This case presents the following information:

- Case number 1 000 000/2009.
- Felony: Parricide
- Accused: "W"
- Against: Mrs. "X"
- Sentence time: 25 years
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According to the Penal Law of the State of Michoacan (2009), Art. 283: Anyone that deprives of life to any consanguineous ancestor, legitimate or natural, knowing the delinquent that kinship, the felon will be sentenced to twenty to forty years in prison.

The judge's decision is highly influenced by his or her experience and subjective estimations. In this case, the felon is sentenced to 25 years in prison.

The expected sentence time $E(X) = 30$ is the mean sentence time. Those 25 years correspond to a subjective evaluation of the judge. Statistically, the corresponding sentence is the mean value, between the extremes.

For this analysis we assume that sentence time ranges from 20 to 40 years, as established by the law of the state of Michoacán México. For a non-expert person, the absolute reality would assign a scale of $[0, 40]$ years. 0 corresponds to the case where the felony was not proved, and 40 to having proved the existing of all aggravating conditions.

To avoid discretion to determine the sentence time to the accused for the committed felony, we propose to make a homogeneous distribution by intervals. We consider a partition with 5 intervals within the sentence time period the law assigns to this felony. Table 2 shows this distribution.

Table 2. Felony Characterization

Partition	Sentence Time Interval (Years)	Dangerousness of the Felon
1	$[0, 3.5, 7]$	Slightly dangerous
2	$[8, 11.5, 15]$	Dangerous
3	$[16, 19.5, 23]$	Compulsive dangerous
4	$[24, 27.5, 31]$	Very dangerous
5	$[32, 36, 40]$	Extremely dangerous

To determine the confidence intervals of each partition we use Equation (1), yielding Table 3. Table 4 provides the numeric values for the confidence intervals for different values of parameter α .

Table 3. Confidence Intervals

Partition	Confidence Interval (Years)
1	$[0+3.5\alpha, 7-3.5\alpha]$
2	$[8+3.5\alpha, 15-3.5\alpha]$
3	$[16+3.5\alpha, 23-3.5\alpha]$
4	$[24+3.5\alpha, 31-3.5\alpha]$
5	$[32+4\alpha, 40-4\alpha]$

Table 4. Confidence Intervals for different values of α

α	$0+3.5\alpha$	$7-3.5\alpha$	$8+3.5\alpha$	$15-3.5\alpha$	$16+3.5\alpha$	$23-3.5\alpha$	$24+3.5\alpha$	$31-3.5\alpha$	$32+4\alpha$	$40-4\alpha$
0.0	0.00	7.00	8.00	15.00	16.00	23.00	24.00	31.00	32.00	40.00
0.1	0.35	6.65	8.35	14.65	16.35	22.65	24.35	30.65	32.40	39.60
0.2	0.70	6.30	8.70	14.30	16.70	22.30	24.70	30.30	32.80	39.20
0.3	1.05	5.95	9.05	13.95	17.05	21.95	25.05	29.95	33.20	38.80
0.4	1.40	5.60	9.40	13.60	17.40	21.60	25.40	29.60	33.60	38.40
0.5	1.75	5.25	9.75	13.25	17.75	21.25	25.75	29.25	34.00	38.00
0.6	2.10	4.90	10.10	12.90	18.10	20.90	26.10	28.90	34.40	37.60
0.7	2.45	4.55	10.45	12.55	18.45	20.55	26.45	28.55	34.80	37.20

0.8	2.80	4.20	10.80	12.20	18.80	20.20	26.80	28.20	35.50	36.80
0.9	3.15	3.85	11.15	11.85	19.15	19.85	27.15	27.85	35.60	36.40
1.0	3.50	3.50	11.50	11.50	19.50	19.50	27.50	27.50	36.00	36.00

The determination of the confidence intervals is the basis for the assignment of the sentence time to a felon. Using the reasoning based on the analysis of the Parricide crime, we take as sentence time the mean value in each of the partitions, using the values provided by the confidence intervals and the discretion levels of the judge. These parameters are established for the case of an 11-value reference scale. The fuzzy mean value is determined using Equation (3).

$$\tilde{X} = \frac{\tilde{x}_i}{n} \tag{3}$$

where \tilde{X} is the mean value, for each partition, $\tilde{x}_i = lowerlimit + upperlimit$, and $n=2$.

Similarly, the discretion level of the judge for each partition is:

$$\alpha = [0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0]$$

The representation and interpretation of the sentence time in this case are associated to partition 2, corresponding to the scenario assigned to the felony. The graphic interpretation of the felony and its assigned sentence time are represented in Figure 2.

4. Results

The sentence time for the study case, using the classical procedure, was 25 years of imprisonment. The result using Fuzzy Logic with 5 intervals in the time scale, considering the mean times is shown in Table 5.

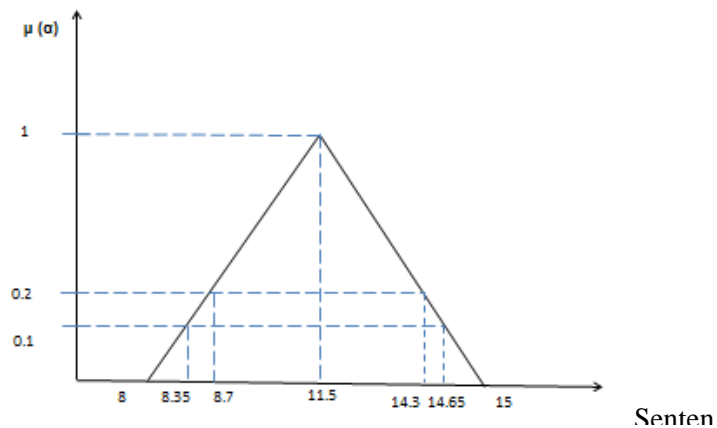


Figure 2. Partition number 2

Table 5. Felony Characterization

Partition	Sentence Time (Years)	Dangerousness of the Felon
1	3.5	Slightly dangerous
2	11.5	Dangerous
3	19.5	Compulsive dangerous
4	27.5	Very dangerous
5	36	Extremely dangerous

This characterization allows the decision maker in the legal area, to aim the effort to be more effective and efficient in the assignment of sentence times.

5. Conclusions

Following the form of the law to qualify sentences related to felonies, and that the decision is tied to the personal point of view of the judge (the justice decision maker). If the judge has a high level of experience in the qualification of some kind of felons, the decision approximates the mean of the interval corresponding the real scenario. Since this is regularly not happening, we recommend incorporating techniques based on uncertainty theory, more specifically, on fuzzy logic, to evaluate cases in the justice domain. We also recommend making public policies that can be translated to laws that allow the law professional to make decisions in a more effective and efficient way.

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