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MONETARY POLICY WITH CONSTANT REAL STOCK OF BONDS

Abstract. *The equilibrium growth path stated in this paper is based on the IS-LM-SRAS model augmented with the dynamics of expected inflation and real wealth, developed by Sidrauski and Turnovski .*

In our previous papers, we computed the IS-LM equilibrium for the linear model and defined the dynamic system. In this work, we estimated the parameters using statistical data for Romania during 2001Q2-2014Q2 and computed the multipliers. These parameters and multipliers were further used to compute the coefficients of the two dimensional dynamic system.

We further augmented the dynamic system, with an objective function, reflecting the sum of the square differences of the real actual government expenditures, from the values established by fiscal policy. We applied the Pontryagin's Principle in order to compute the optimal conditions of the optimal control problem.

The overall dynamics of the economy is computed for short run, considering the monetary policy of constant stock of bound, using Runge-Kutta algorithm in MATLAB. The dynamics of the system was analyzed via phase diagram.

The results reflect the fact that the economy is in an unstable zone, with expected inflation decreasing and real wealth increasing, fact that determines us to add an objective function for minimizing the square differences of the government expenditures from the stationary ones during 2016Q1-2017Q4.

Keywords: *IS-LM-SRAS equilibrium, monetary policy, Phillips curve, optimal control, Pontryagin's Principle, phase diagram, Matlab, Runge-Kutta algorithm.*

JEL Classification: E52, E 59, C61, C62

1. Introduction

It is well known that a successful economy is one that has a high rate of economic growth. While the determination of the level and the rate of growth of the potential real output are the focus of growth theory, the determination of actual real output relative to potential real output is a concern of stability theory (Hossain & Chowdhury, 1996).

This paper is focused on the role of monetary policy with constant real stock of bonds policy, which is one of the macroeconomic instruments frequently employed for budget deficit financing. Our theoretical and empirical work is focused on an IS-LM- SARS model with adaptive expectations, based on the model developed by Sidrauski-Turnovski (Turnovski, 1996).

Theoretical and empirical concerns for the study of monetary policies and those using dynamic IS-LM models are numerous.

Unlike Keynesian models that work under the assumption of adaptive expectations of agents, the new Keynesian models contain a forward-looking approach meaning expectations of the agents.

Hicks and Hansen's IS-LM model was improved by Blanchard's contribution (Blanchard, 1981) who developed its dynamic version and introduced endogenous expectations of forward-looking agents.

In recent years dynamic IS-LM models have been used in the monetary policy analysis. Casares and McCallum (2000) extend the IS-LM framework by incorporating endogenous investments and including capital adjustment costs.

Li and Wu (2015) made a theoretical analysis of the limitations existing in the classic aggregate supply and demand in the IS-LM model, of which we mention: the lack of micro foundation, short-term static nature, inconsistent logic, functional defect and research divergence. In order to overcome these limitations, dynamic rational expectations and Phillips curve are introduced in the IS-LM model.

Hai and Minh Trang (2015) develop a VAR model to study the channels of monetary policy mechanism in Vietnam: exchange rate, interest rate and price shocks. Altăr (2008) analyzes the stability of a dynamic IS-LM model with adaptive expectations under the assumption that the aggregate supply is of Lucas type and the monetary policy is of Friedman type. Oprescu and Andrei (2009) find a quantification of the economic growth, based on the dynamic IS-LM-SRAS model.

Andrei et al. (2014) continue the previous research, by providing a short-term analytical solution of the static and dynamic IS-LM-SRAS equilibrium.

The model of Sidrauski and Turnovski is an example of classical monetary model that highlights the monetary transmission mechanism in a context of fully flexible prices of goods and labour. The solution of the dynamic model is obtained in terms of expected inflation rate and real wealth (particularly composed of bonds stock and monetary aggregate M1), which evolve according to the dynamic equations. Inflationary expectations are assumed adaptive and accumulation of financial assets occurs through the mechanism of financing budget deficits.

Our original contribution consists in expressing the implicit form of the Sidrausky and Turnovsky model in linear form, estimating the static IS-LM-SRAS model using statistical data series, computing the multipliers, and constructing, analising and solving the dynamic system. In order to improve the paths of the state variables (wealth and expected inflation) we attached an objective functional to the dynamic model and solved the optimal control problem resulted, using the Pontryagin Principle.

The paper is structured as follows. Section 2 refers to the short run dynamics of the economy with the constant real stock of bonds monetary policy for deficit financing. We studied the optimal dynamics of expected inflation and of the real wealth by minimizing the squares of the actual government expenditures gaps from their desired/political values.

Section 3 contains an empirical IS-LM-SRAS estimation for Romania using quarterly data during 2001q2-2014q2. The reesults of the IS-LM-SRAS equilibrium, were used for the computation of the dynamic model. We analyzed the dynamics of the model, computed and predicted the paths using Runge-Kutta algorithm and MATLAB software. Section 4 contains conclusions and further research.

2. The short run dynamics of the economy with the monetary policy of constant stock of bonds $b(t)=\overline{b}$

We started with the IS-LM-SARS equilibrium, analytically determined in Andrei et al, 2014:

$$\tilde{y}(t)=\hat{k}_g\cdot g(t)+\hat{k}_a\cdot a(t)-\hat{k}_g\cdot i_r^{'}\cdot\pi^e(t)+\hat{k}_g\cdot d_{ISLM}^{a}, \tag{1}$$

$$with: d_{ISLM}^{a}=\underline{c}+\underline{i}^{R}-\hat{k}_m\cdot(\underline{m}-\overline{m})$$

$$\tilde{i}(t)=-\frac{m_y^{'}}{m_i^{'}}\cdot\tilde{y}(t)-\frac{m_a^{'}}{m_i^{'}}\cdot a(t)+\frac{1}{m_i^{'}}(\overline{m}-\underline{m}) \tag{2}$$

$$\pi(t)=\pi^e(t)+\gamma(\tilde{y}(t)-\overline{y(t)}),\ \ \gamma>0 \tag{3}$$

where: $\tilde{y}(t)$ is the equilibrium real GDP, $g(t)$ is the real government expenditures, $\hat{k}_g$ is the corresponding multiplier computed under the constant real stock of bonds assumption, $b(t)=\bar{b}$, $a(t)$ is the real wealth, $\hat{k}_a$ is the corresponding multiplier, computed under the same assumption, $i_r^{'}$ is the sensitivity of the real investments to the real interest rate *r*, $\pi^e(t)$ is the expected inflation rate, d_{ISLM}^{a} -is the autonomous demand on the goods and money markets, $\tilde{i}(t)$ is the equilibrium nominal interest rate, $m_y^{'}$ is the sensitivity of the real money demand to the real GDP, $m_i^{'}$ is the sensitivity of the money demand to the nominal interest rate, $m_a^{'}$ is the sensitivity of the money demand to the real wealth, $\underline{m}$ is the autonomous money demand, $\overline{m}$ is the constant money supply as a monetary policy instrument, $\overline{y}(t)$ is the potential real GDP, $\pi(t)$ is the actual inflation rate.

Based on the IS-LM-SARS equilibrium computed analytically in the previous work mentioned above and on the Sidrausky-Turnovsky model, we built the dynamic model that considers the dynamics of the economy given by two state variables: expected inflation $\pi^e(t)$ and real wealth $a(t)$:

$$\begin{cases} \dot{\pi}^e(t)=\alpha(\pi(t)-\pi^e(t)), \quad \alpha>0 \\ \dot{a}(t)=g(t)-\tau\cdot y(t)+i(t)\cdot b(t)-\pi(t)\cdot a(t) \end{cases} \tag{4}$$

The first equation in (4) emphasizes the fact that the expected inflation dynamics is given by the gap between actual and expected inflation (adaptive inflationary expectations), with the adjustment speed given by the parameter α.

The wealth dynamics $\dot{a}(t)=\dot{m}(t)+\dot{b}(t)$ is given by the monetary policy for budget deficit $DB(t)=g(t)-\tau y(t)+i(t)b(t)$ financing, with $g(t)$ real government expenditures and τ tax rate.

The budget deficit $DB(t)$ is set up in the primary deficit $(g(t)-\tau y(t))$ and the debt service (interest payments) on government bonds $i(t)b(t)$.

In addition, the real current wealth is eroded by inflation, so, it is reduced by $(-\pi(t)\cdot a(t))$.

In conclusion, the economy is characterized by five endogenous variables $(y(t), i(t), \pi(t), \pi^e(t), a(t))$ from which the last two $(\pi^e(t), a(t))$ are state variables.

The instrumental variables are:

1. either - $m(t)$ the money issue, or - $b(t)$ bonds issue, depending on the monetary policy of the government (so that in $a(t)=m(t)+b(t)$ the other variable results to be endogenous);

2. $g(t)$ instrumental variable used by government.

We notice that the expected inflation dynamics $\dot{\pi}^e(t)$ is deduced in relation to an unobservable variable "Okun gap", using SRAS (see Oprescu, Andrei, 2009):

$$\dot{\pi}^e(t)=\alpha\gamma(y(t)-\overline{y(t)}) \ . \tag{4.1}$$

The SRAS equation was determined using expectations augmented Phillips curve, that was estimated using the gaps resulted from the difference between actual real GDP and potential GDP.

The specification of the dynamic model on the short run is obtained by replacing in the equations (4) the equilibrium paths $\tilde{y}(t)=\tilde{y}(a(t),\pi^e(t),g(t),\bar{b})$ and $\tilde{i}(t)=\tilde{i}(a(t),\pi^e(t),g(t),\bar{b})$, so that:

$$\begin{cases}\dot{a}(t)=g(t)-(\gamma\cdot a(t)+\tau)\tilde{y}(t)+(\gamma\cdot\bar{y}-\pi^e(t))a(t)+\tilde{i}(t)\cdot\bar{b}\\ \dot{\pi}^e(t)=\alpha\gamma(\tilde{y}(t)-\overline{y(t)})\end{cases} \tag{5'}$$

Taking into account the equilibrium paths (1), (2) and SRAS curve (3), we obtain the system of differential nonlinear equations that gives the dynamics of the economy in equilibrium:

$$\begin{cases}\dot{a}(t)=\alpha_1\pi^e(t))\cdot a(t)-\alpha_2\cdot a^2(t)+\alpha_3\pi^e(t)+\alpha_4\cdot g(t)+\alpha_5\\ \dot{\pi}^e(t)=\beta_1\cdot a(t)-\beta_2\pi^e(t)+\beta_3\cdot g(t)+\beta_4\end{cases} \tag{6}$$

The state variables are expected inflation $\pi^e(t)$ and real wealth $a(t)$ and the instrumental variable is the real government expenditures $g(t)$.

2.1 The optimal path of the system

We consider as government expenditures the value that corresponds to the null extended budget deficit:

$$DB(t)=g(t)-\tau y(t)+i(t)b(t)=0 \tag{7}$$

Consider further the equilibrium paths of the real GDP, the equilibrium path of the nominal interest rate and the constant stock of bonds assumption:

$$y(t)=\tilde{y}(t),\quad i(t)=\tilde{i}(t),\quad b(t)=\vec{b}$$

so that, the path of the real government expenditures becomes:

$$g^*(t) = \tau\,\tilde{y}(t) - \tilde{i}(t)\,\bar{b}$$

We can now formulate an optimal control problem that consists in the minimization of the squares of: government expenditures gaps from their policy values $(g(t) - g^*(t))^2$:

$$\begin{aligned}
&\min \int_0^T \left[(g(t) - g^*(t))^2\right] dt \\
&\dot{a}(t) = \alpha_1 \pi^e(t) \cdot a(t) - \alpha_2 \cdot a^2(t) + \alpha_3 \pi^e(t) + \alpha_4 \cdot g(t) + \alpha_5 \\
&\dot{\pi}^e(t) = \beta_1 \cdot a(t) - \beta_2 \pi^e(t) + \beta_3 \cdot g(t) + \beta_4 \\
&\textit{with } a(0),\ \pi^e(0) \ \textit{given}
\end{aligned} \tag{8}$$

The Hamiltonian function is:

$$\begin{aligned}
&H(a(t), \pi^e(t), g(t), \lambda_1(t), \lambda_2(t)) = (g(t) - g^*(t))^2 + \lambda_1(t)(\alpha_1 \pi^e(t) \cdot a(t) - \\
&- \alpha_2 \cdot a^2(t) + \alpha_3 \pi^e(t) + \alpha_4 \cdot g(t) + \alpha_5) + \lambda_2(t)(\beta_1 \cdot a(t) - \beta_2 \pi^e(t) + \beta_3 \cdot g(t) + \beta_4)
\end{aligned} \tag{9}$$

The optimal path of the real government expenditures:

$$\frac{\partial H(.)}{\partial g(t)} = 0 \Rightarrow g(t) = -\frac{1}{2}(\lambda_1(t)\alpha_4 + \lambda_2(t)\beta_3) + g^*(t) \tag{10}$$

is replaced in the Hamiltonian system, obtaining:

$$\begin{aligned}
&\dot{\lambda}_1(t) = -\lambda_1(t)\alpha_1\pi^e(t) + 2\lambda_1(t)\alpha_2 \cdot a(t) - \lambda_2(t)\beta_1 \\
&\dot{\lambda}_2(t) = -\lambda_1(t)\alpha_1 a(t) - \lambda_1(t)\alpha_3 + \lambda_2(t)\beta_2 \\
&\dot{a}(t) = \alpha_1\pi^e(t)) \cdot a(t) - \alpha_2 \cdot a^2(t) + \alpha_3\pi^e(t) - \frac{1}{2}\alpha_4^2 \cdot g(t)\lambda_1(t) - \frac{1}{2}\alpha_4\beta_3\lambda_2(t) + g^*(t) + \alpha_5 \\
&\dot{\pi}^e(t) = \beta_1 \cdot a(t) - \beta_2\pi^e(t) - \frac{1}{2}\beta_3\alpha_4\lambda_1(t) - \frac{1}{2}\beta_3^2\lambda_2(t) + g^*(t) + \beta_4 \\
&\textit{with } a(0),\ \pi^e(0),\ \lambda_1(T),\ \lambda_2(T) \ \textit{given}
\end{aligned} \tag{11}$$

This is a system of nonlinear differential equations, with mixed Cauchy conditions, that could be linearized around the stationary point $(\lambda_1^*, \lambda_2^*, a^*, \pi^{e*})$ and solved as a linear dynamic first order system, or could be solved numerically using MATLAB Runge-Kutta algorithm.

3. Empirical study for Romania

Our goal is to estimate the dynamic model given by (6), resulted from the assumption on the evolution of the expected inflation and real wealth, with the specification that the analysis is on short run. In the construction of the dynamic model we used the IS-LM-SRAS equilibrium values for real income and nominal interest rate.

The theoretical aspects of the IS-LM-SRAS static equilibrium for the specified linear functions, have been treated in Andrei et al. (2014).

3.1 IS-LM-SRAS model estimate

We initiated the empirical work with the IS model computation, for which we first estimated the investment and the consumption functions.

The regressions are performed using quarterly real data during 2001Q2-2014Q2 for actual GDP, GDP deflator, gross capital formation (National Institute of Statistics data), CPI (Consumption Price Index), actual inflation rate, expected inflation rate (under the assumption of adaptive inflationary mechanism), reference interest rate NBR data.

As it can be seen in Figure 1, the actual quarterly data of real investments exhibit repeating seasonal pattern.

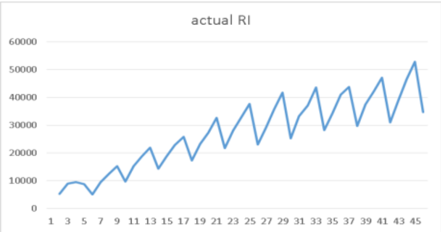


Figure 1: Real investments, quarterly data 2001Q2-2014Q2

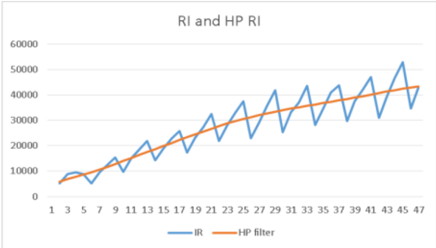


Figure 2: Actual and HP filtered real investments, quarterly data 2001Q2-2014Q2

In order to remove the seasonality we tried the simplest deterministic seasonal model, using some seasonal dummy models with intercept:

$$i^R(t) = \alpha + \delta_2 D_{2t} + \delta_3 D_{3t} + \delta_4 D_{4t}$$

$$i^R(t) = \alpha + \delta_1 D_{1t} + \delta_2 D_{2t} + \delta_3 D_{3t}$$

where the dummy variable is $D_{it} = 1$ for the season $i = 1,...,4$ and $D_{jt} = 0,\ j = 1,..,4,\ j \neq i$. All the estimations were rejected because of the unsatisfactory regression statistics.

We then tried to remove the seasonality applying the HP filter to the actual real investment data and obtained the trend free of seasonality influences, as could be seen in Figures 2 and 3.

$$i^R(t) = i'_y \cdot y(t) + i'_r \cdot (i(t) - \pi^e(t)) + \underline{i}^R = 0.293587y(t) - 1762.37(i(t) - \pi^e(t)) + c_t,$$
$$i'_y = 0.293587, \quad i'_r = -1762.37, \qquad \underline{i}^R = 0, \quad c_t = cyclical\ component$$
(12)

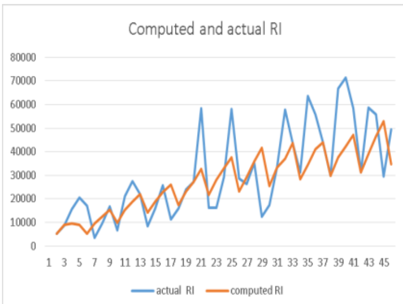


Figure 3: Computed and actual real investments quarterly data 2001Q2-2014Q2

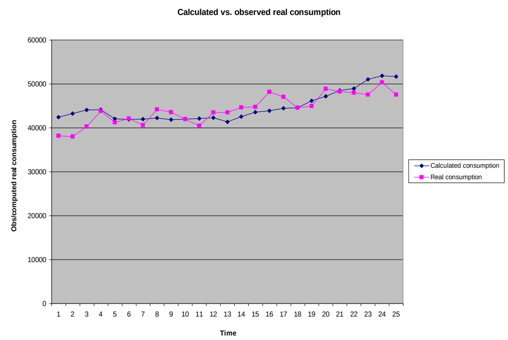


Figure 4: Computed vs. observed real consumption quarterly data 2001Q2-2014Q2

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.943072							
R Square	0.889385							
Adjusted F	0.863557							
Standard E	10146.45							
Observatio	45							
ANOVA								
	df	*SS*	*MS*	*F*	*ignificance F*			
Regressior	2	3.56E+10	1.78E+10	172.8687	5.36E-21			
Residual	43	4.43E+09	1.03E+08					
Total	45	4E+10						
	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	0	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
X Variable	0.293587	0.014522	14.84443	1.58E-18	0.186284	0.244857	0.186284	0.244857
X Variable	-1762.37	27121.89	1.612111	0.014254	-10973	98419.99	-10973	98419.99

Table 1: Regression statistics

From the regression statistics in the table above, we conclude that, after removing the seasonality, the model fits well the data and the parameters could be considered good estimators for the model.

So, $Multiple \quad R = 0.943072$ assures us that the dependent variable (real investments) is correlated with the independent variables (real GDP and real interest rate).

The coefficient of multiple determination, $R^2 = 0.889385$ shows that the regression model is a good fit of the data, meaning that it explains much of the variability of the data.
The Adjusted R shows that 86.3557% of variation is explained by the significant independent variables. The Significance F is very small, $5.36x10^{-21}$, hence the results are reliable.
All *p*-values are less than 0.05 so, we can reject the null hypothesis.
From the economic point of view, we obtained some results that deserve some attention. So, the coefficient $i_y^{'} = 0.293587$ symbolizes the marginal and average sensitivity of real investments $i^R(t)$ to real income $y(t)$ and its value reflects an important accumulation process in Romania.
The sensitivity of investments to real interest rate $i_r' = -1762.37$ has a highly negative value, such that each variation of the real interest rate is transferred with opposite effect, upon the real investments. The highly negative value of i_r' transfers and amplifies the volatility of the real interest rate, to volatility of the real investments, as could be seen in Figure 1.

We then estimated the Keynesian type consumption function, augmented with real wealth:

$$\begin{aligned} c(t) &= \underline{c} + c_y' y^d(t) + c_a' a(t) = \\ &= 9330.47 + 0.593354 y^d(t) + 0.130667 a(t), \quad c_y' = 0.593354, \quad \underline{c} = 9330.47, \quad c_a' = 0.130667 \end{aligned} \tag{13}$$

We recall that $c(t)$ is the real consumption, $\underline{c}$ is the autonomous consumption, $y^d(t)$ is the disposal income c_y', c_a' are the corresponding marginal propensity of consumption to income and wealth respectively.
The data used are: consumption expenditures of the population, disposal income (computed below), real wealth computed as a sum of real bonds (taken as portfolio investments balance of bonds nature from the payments balance of Romania) and real stock of money (considered as M1 monetary base composed by cash and demand deposits) NBR data, CPI (Consumption Price Index, NBR data).
The regression statistics assures us that the estimators could be considered satisfactory.
The disposal income was computed using NIS data for GDP, divided by deflator, real stock of bonds and real wealth (mentioned above), reference interest rate NBR data, expected inflation (adaptive mechanism) NBR data, taxes as percentage in GDP, World Bank data:

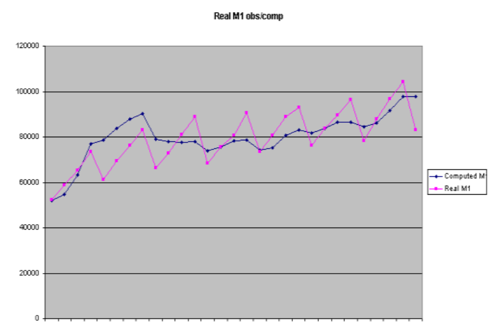
$$y^d(t) = (1-\tau)y(t) + i(t)\cdot b(t) - \pi^e(t)\cdot a(t) + (\underline{\theta} - \underline{t}) = (1-\tau)y(t) + i(t)\cdot b(t) - \pi^e(t)\cdot a(t) \tag{14}$$

The parameter $\tau = 0.179$ (computed as average, World Bank data) $\underline{\theta}$ fixed transfers, $\underline{t}$ fixed taxes are assumed to be equal.

The actual and computed real consumption have close paths, which again point to the fact that the consumption function (13) fits well the data, as it can be seen in Figure 4.

From Figure 4, we notice the decrease in both actual and the computed consumption during the crisis period (30Q-42Q). For given values of fiscal policy instruments: $\tau = 0.179$ tax rate, $g^* = 6870.524$ tens million lei, constant real government expenditures and for $\pi^e = 0.0199$ computed as the average value of the last 12 quarters, for the real stock of bonds $\bar{b} = 1590.90$ million lei (average value computed from NBR data), and for the autonomous demand computed as a sum of the autonomous investments, autonomous consumption and government expenditures ($g^* + d_{IS}^a$) the obtained IS function is:

$$IS: \qquad 0.205076y(t) + 766.3244i(t) = 0.404737a(t) + 13633.661 \qquad (15)$$



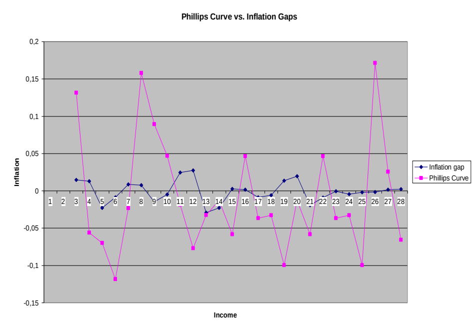


Figure 5: Actual/computed real inflation gaps quarterly data 2001Q2-2014Q2

Figure 6: The actual and computed money supply quarterly data 2001Q2-2014Q2

We proceed then to compute the LM curve, using as dependent variable the real monetary aggregate M1, and, as independent variables, the real income, the nominal interest rate and the real wealth. The estimated function LM is:

$$m(t) = \underline{m} + m'_a a(t) + m'_i i(t) + m'_y y(t) = 98878.38 + 0.77352a(t) - 1701.88i(t) + 0.3345y(t) \quad (16)$$

The regression statistics leads us to believe that the function adjusts well statistical data (see Figure 5).

The sensitivity of money demand to the nominal interest rate $m'_i = -1701.88$ has a highly negative value so that, a small variation in the nominal interest rate implies an important variation in the money stock in the opposite sense. The parameter $m'_y = 0.3345$, the sensitivity of the money stock to the real income, reflects the fact that an important share of the money stock is used for actual transactions. Finally, $m'_a = 0.77352$ the sensitivity of money stock to the real wealth, shows a tight interrelation between the financial and monetary markets.

Going forward with the application of the model, we estimated the SRAS curve as:

$$\pi(t) - \pi^e(t) = \gamma(y(t) - \overline{y(t)}) = 0.0000022(y(t) - \overline{y(t)}) \qquad (17)$$

We used as dependent variable the actual inflation gaps from the expected value, considering adaptive mechanism of expectations and, as the independent variable, the output gaps between actual real GDP and potential GDP, using the results obtained by Andrei, Paun (2014). The coefficient γ was estimated using regression without intercept. The actual and computed inflation gaps could be seen in figure 5.

The equilibrium path for the computed real income is:

$$\tilde{y}(t) = 1.56g(t) + 1.0331m(t) + 0.02779a(t) + 228824.064\pi^e(t) + 7312.5632 \qquad (18)$$

where the autonomous demand of goods and money $d^a_{ISLM} = 7312.5632$ *million lei* .

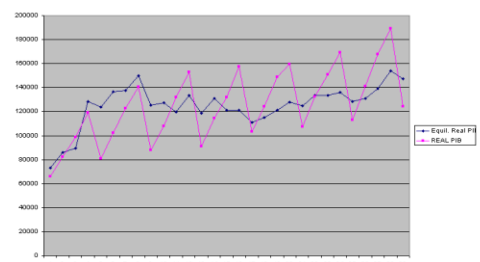


Figure 7: Equilibrium vs observed real GDP quarterly data 2001Q2-2014Q2

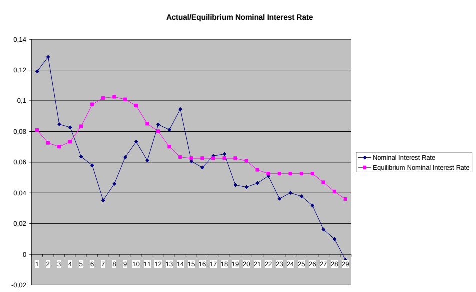


Figure 8: Equilibrium and actual nominal interest rates, quarterly data 2001Q2-2014Q2

We observe that during the period considered, 2001Q2-2014Q2, the equilibrium path maintains the tendency of the actual data, so, we concluded that the model is well specified.

The equilibrium nominal interest rate path, computed with the equilibrium real GDP, real wealth and real monetary aggregate is:

$$\tilde{i}(t) = 0.000019655\tilde{y}(t) + 0.00002099a(t) - 0.00005875m(t) + 5.8091 \qquad (19)$$

The computed equilibrium nominal interest rate trend meets the current values, showing the fact that the model matches the studied economic processes.

3.2 The dynamic model

Using the results of the static IS-LM equilibrium and SRAS curve under assumptions of constant real stock of bonds, and the dynamic mechanism of the adaptive inflationary expectations respectively, we obtained the dynamic system given analytically by equations (6).

The dynamic system, computed using (6), (18), (19) is:

$$\begin{cases} \dot{a}(t) = 0.021479462 \ a(t)\pi^e(t) - 3.94444x10^{-8} \ a^2(t) - 23305.8366\pi^e(t) - 1058.9089 \\ \dot{\pi}^e(t) = 2.53451x10^{-6} \ a(t) + 0.97446 \ \pi^e(t) - 0.2046366 \end{cases} \quad (20)$$

with the stationary paths:

$$\dot{a}(t) = 0 \Rightarrow \pi^e(t) = 8.913537x10^{-7} a(t) - 1.685749x10^{-12} a^2(t) - 0.0465943$$

$$\dot{\pi}^e(t) = 0 \Rightarrow \pi^e(t) = 2.593721x10^{-6} a(t) - 0.229547$$

(21)

and the computed (using MATLAB) stationary pair:

$$a^* = 87931.13956 \, million \ \ lei$$

$$\pi^{e*} = 0.022135 \quad (22)$$

The resulting phase diagram could be seen in Figure 9.

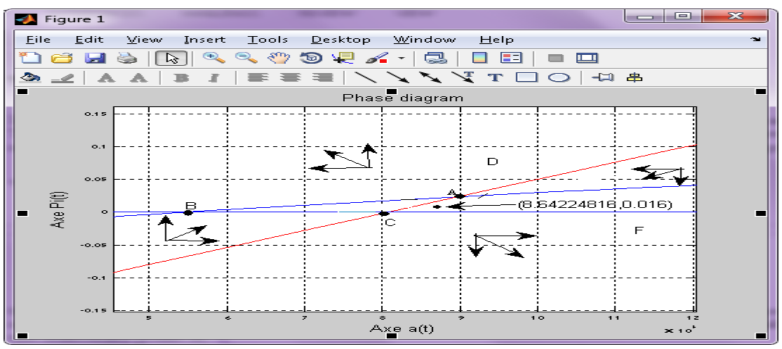


Figure 9: Phase diagram of the dynamic system (detail)

In the phase diagram we see that there are two stability areas and two instability areas delimited by the stationary paths given by (21). In the ABC stability area, the wealth and expected inflation increase, and the force vector shows that the path asymptotically converges to the stationary point A. In the opposite side, the FAD area, the real wealth and the expected inflation decrease, the force vector shows that the path asymptotically converges to the stationary point A. In the two stationary zones, the stationary paths run from opposite directions, forming a saddle stationary trajectory. The BAD and CAF areas are unstable that means that the paths diverge from the stationary point A.

Considering the initial values of the real wealth and expected inflation at the level of Q4 2007, $a_{2007Q4} = 86422.4816 \text{ million lei}$; $\pi^{e}_{2007Q4} = 0.0162$ respectively, we notice that the economic system is situated in the CAF zone of the phase space, so, in an unstable zone. In the CAF zone, the wealth increases and the inflationary expectations fall.

In the phase space, hence, the path starts at the point $a_{2007Q4} = 86422.4816; \pi^{e}_{2007Q4} = 0{,}0162$ and continues with increasing values of wealth and decreasing values of inflationary expectations.

We further calculated the trajectories of the two indicators, based on dynamic system (20), using MATLAB Runge-Kutta algorithm subroutine for nonlinear differential equations systems.

We also made a prediction of two state indicators for the period 2014Q2-2017Q4. The predicted values for the end of the period are: $a_{2017Q4} = 199630.497779 \text{ million lei } \pi^{e}_{2017Q4} = -0.00531$ that reflect also the tendencies deduced in the state phase analysis. The graphs of the two paths are reflected in Figures 10 (a), (b) below

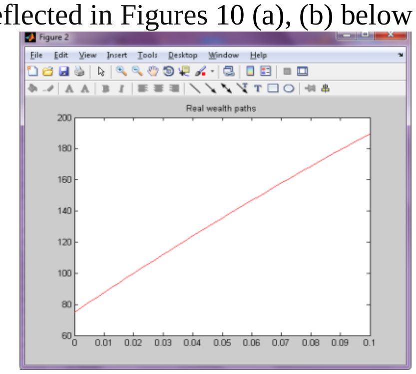


Figure 10: (a) Real wealth path 2007Q4-2017Q2

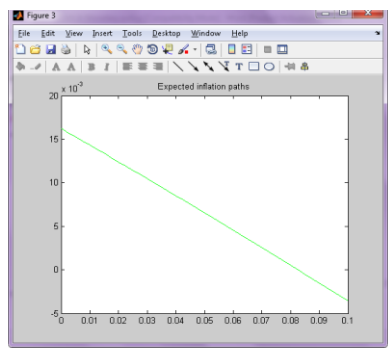


Figure 10: (b) Expected inflation path 2007Q4-2017Q2

The applied model on the data leads to results consistent with the trend of the actual values, proving that the model is well specified and that estimated functions capture the current data trends.

From the economic point of view, though the results reflect the actual tendencies of the Romanian corresponding indicators, they could not be accepted on long run.

Having in view that the model includes two macroeconomic policies, namely, constant issue of bonds for budget deficit financing and constant government expenditures, we tried to drive out the system's path from the unstable zone and from the deflationary tendency, attaching an objective quadratic function to (6) and solving the optimal problem (8) (Andrei et al. 2016).

The solution could be seen graphically in Figure 11 (a), (b) below.

The optimal solution drives to increasing trajectories for both state variables. This proves that the optimal control problem is well specified for the goal to out the economy from a deflationary tendency, maintaining a long time economic growth and moving the trajectory to a stable area.

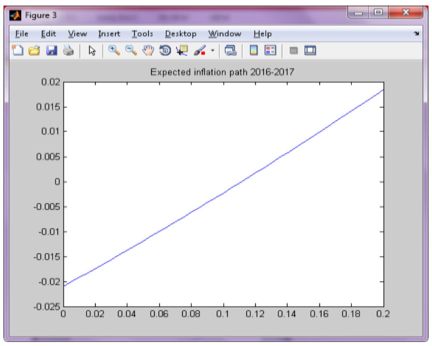


Figure 11.(a) Expected inflation path 2016Q1-2017Q4

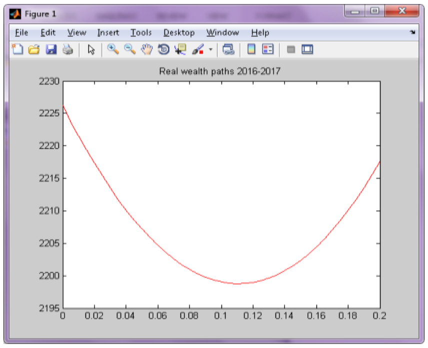


Figure 11 (b) Real wealth path 2016Q1-2017Q4

In Figure 11(a), it could be seen that the expected inflation has an increasing trend, starting from the initial value of $\pi^e_{2016Q1} = -2.1\%$, and continously increasing to $\pi^e_{2017Q4} = 2.0\%$ revealing the fact that the policy mix assumed has the desired effect.

The real wealth path has a parabolic trend, starting from $a_{2016Q1} = 2226.2392$ hundreds of millions lei and reaching the value $a_{2016Q4} = 2198.465$ hundreds of millions lei in Q4 of 2016, then rising until the end of 2017, reaching approximately the starting value.

The minimum value of the real wealth is reached in the same period when the expected inflation passes from negative to positive values.

The optimal actual government expenditures path has a decreasing trend from the initial value to the policy value $g^* = 6870.524$ tens million lei.

4 Conclusions and further research

Theoretically the authors use in this article a linear version of IS-LM-SARS with wealth model, developed in a previous paper (Andrei et al. 2014), build the dynamic model using equilibrium paths resulted from the static model and formulate an optimal control problem by attaching to the dynamic model an objective function.

Empirically, the authors estimate the static model, compute the multipliers and the parameters of the dynamic model, determined also the stationary vector, analyze the dynamics of the economy with the space phase diagram, and solved the dynamic system's trajectory, using the MATLAB facilities.

From the static equilibrium analysis, we conclude that the linear IS-LM-SARS wealth augmented model adjusts well the statistical data, being well specified and that it can be considered as the basis for dynamic analysis.

The analysis of the dynamic model reveals the fact that the paths are situated in an unstable zone with increasing deflation and increasing real wealth. In order to move the economy from the unstable area and to stop deflationary trend installed in the last quarter of last year, we introduced a fiscal policy (Andrei et al. 2016), by means of the objective function that consists in the minimization of the squared differences between the actual and stationary government expenditures during 2016Q1-2017Q4.

The results where satisfactory and economically feasible, obtaining a slightly increase of the expected inflation from negative values to positive ones that generated a decrease followed by a slight increase of the real wealth.

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