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# THE MULTI-CRITERIA FRACTIONAL TRANSPORTATION PROBLEM WITH FUZZY "BOTTLENECK" CONDITION

***Abstract.*** *The paper proposes a new approach to the multi-criteria fractional transportation problem with the same bottleneck denominators, additionally criterion for is also time constraint, i.e. the type bottleneck. We propose to study the case when the bottleneck criterion is not deterministic, but of fuzzy type. In this paper I propose an iterative algorithm for solving the model. It generates the crowds efficient model solutions for different types of approaches to the time required for transport from optimistic to pessimistic, using for this purpose the possible ranges of variation thereof. The algorithm was tested on several examples and was found to be quite effective.*

***Keywords:*** *Fuzzy programming, fractional multi-criteria transportation model, "bottleneck" criterion, efficient solution, coefficient of optimism.*

**JEL Classification: 90C29, 90C70.**

## 1. Introduction

It's well known, the increasing of criteria number leads only to increasing of solution accuracy for multi-criteria optimization problems. This is why the interest of the multi- criteria optimization problem is on the rise, including the multi-criteria transportation model, which has numerous practical applications. The efficient solutions of the multi-criteria transportation problem of liner type can be achieved using various algorithms developed in [8], [11], [13], [21] and many others. From practical applicability point of view, imposing of minimal time to realise the solution of model appears as a logical condition which would surely improve its quality. In the speciality literature the criterion of minimizing the maximum time is called a "bottleneck" criterion. A large variety of algorithms have been proposed for different kinds of multi-criteria transportation problems of "bottleneck" type. Thus, for solving the three-criteria transportation problem, including the "bottleneck" Aneja and Nair in [1] developed an efficient algorithm, but Wild and Karwan in [20] proposed an efficient algorithm for solving the generalized *r*-criteria transportation problem of the same type. It's important to mention, that many of economical decision problems lead to the fractional optimization models, because that a lot of important characteristics of these may be evaluated really using only some ratio relations. The time-constraining criterion is, obviously, one of conditions so much important for major optimization problems. A particular case, but quite often meted is of identical denominators like the “bottleneck” time function. Moreover, we studied various cases when "bottleneck" denominator function is included as a separate criterion in the optimization model. The efficient algorithms for solving these types of models are proposed by Sharma and Swarup in [14] for one-criterion fractional transportation model of “bottleneck” type and by Tkacenko in [17] for multi-criteria fractional transportation model of the same type. Because in real life, often, some parameters and coefficients of the optimization models are of indeterminate in [18] A.Tkacenko develop a case study when the cost coefficients of multi-criteria fractional transportation “bottleneck” model are of fuzzy type. In this paper is studied the case when the time characteristics of the of multi-criteria fractional transportation “bottleneck” model are of fuzzy type.

## 2. Problem formulation

Since for any type of mathematical optimization model, the time coefficients have greatest influence on both the optimal solution and the value of objective function, we propose to investigate the multi-criteria transportation model, in which the coefficients are of fuzzy type. We propose to include in the model the “bottleneck” criterion separate, which is quite important for any decisional situation especially from practical point of view. The mathematical model of multi-criteria fractional transportation problem of “bottleneck” type with fuzzy time coefficients is the following:

$$\min\ F_1 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 x_{ij}}{\max\limits_{i,j}\left\{\tilde{t}_{ij} \mid x_{ij} > 0\right\}} \tag{1}$$

$$\dots\dots\dots\dots$$

$$\min\ F_r = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij}}{\max\limits_{i,j}\left\{\tilde{t}_{ij} \mid x_{ij} > 0\right\}} \tag{2}$$

$$\min\ F_{r+1} = \max_{i,j}\left\{\tilde{t}_{ij} \mid x_{ij} > 0\right\} \tag{3}$$

*in conditions* :

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m} \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n} \tag{5}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{6}$$

$$x_{ij} \geq 0,\ i = \overline{1,m},\ j = \overline{1,n} \tag{7}$$

*where :* $c_{ij}^{k}$ *, k=1,2…r, i=1,2,…m, j=1,2,…n are costs or other amounts corresponding to concrete interpretations of those criteria being of deterministic type,* $\tilde{t}_{ij}$ *- necessary unit transportation time from source i to destination j, which is of fuzzy type,* $a_i$ *- disposal at source i,* $b_j$ *- requirement of destination j,* $x_{ij}$ *- amount transported from source i to destination j, that is only positive.*

We can observe, that in model (1)-(7) the first "*r*" criteria are of linear-fractional type, moreover, with identical denominators. The denominator function appears again as a separate the *(r+1)* criterion, being a time-constraining criterion, met in special literature as a bottleneck-type criterion. Because this criterion is of a non-linear type, obviously, the model (1)-(7) becomes more complex from the solving point of view, the more it is undetermined type. At first it looks like a contradiction between the first r criteria and the *(r+1)* criterion, but this confusion disappears while analysing its practical significance. We can mention that among the first *r* criteria there may be the one of maximum type with the following interpretation: the maximization of the total shipping quantity in time unit that is in coherence, without any doubt with maximization of entire shipping quantity criterion. In fact, this doesn't not make the optimization model more complicated as using some elementary transformations the maximum types of criteria can be modified into minimal types as they appear in the model (1)-(7). Particular cases of the model (1)-(7) were analysed in the paper [7], [8].

## 3. Theoretical analysis of fuzzy time multi-criteria model

As we can see, the time parameter has a direct influence on the structure of efficient solutions of the model (1) - (7), and therefore on the objective functions of the model. Unfortunately, this characteristic, unlike the characteristics of: price, benefit is most at risk. That's why it welcomed the decision maker to be able to assess the transport costs on a whole variation interval of time, characteristic of each route in order to ensure efficient financial management. Assuming time as a continuous variable, each decision maker can assign to each routes a priori a corresponding needed interval of time. Using a coefficient of subjective nature, which in fact characterizes the individual risk aversion, we can obtain different values of deterministic time for each route. We propose to calculate the route time characteristic $t_{ij}$ for the model (1) – (7) using the optimistic coefficient $p$, by applying the formula:

$$t_{ij} = b_{ij} - \rho(b_{ij} - a_{ij}) \tag{8}$$

*where:* $a_{ij}, b_{ij}$ *- are the limit values of variation interval for each time coefficient* $t_{ij}$ *where:* $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$.

We can observe that $p \in [0;1]$ *for* $\forall(i, j, k)$, *where* $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$. Moreover, any values of time coefficients according of optimistic coefficient $p$ belongs to its corresponding interval $t_{ij} \in [a_{ij}, b_{ij}]$.

Agreeing to the formula (8), the parameters $p$ can be considered as the probabilistic parameters of belonging for every value of time coefficients $\{t_{ij}\}$ from their corresponding variation intervals [7].

*Supposing that the variables* $\{t_{ij}\}$ *for* $\forall(i, j, k)$ *where* $i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,r}$ *are continuous on theirs corresponding intervals, the parameters* $p$ *appear as the distribution functions of these variable. Therefore the functions* $p$ *enjoys all properties of distribution functions including the monotony and continuity property.*

***Thereby, with increasing value of the coefficient of optimism of decision maker who can be considered a probabilistic size, the value of time required to achieve the route decreases, also belonging to its characteristic time interval.***

***Analogical, with the decreasing value of the coefficient of optimism decision maker who can be considered a probabilistic size, the amount of time necessary to achieve the route will increase, also belonging to its characteristic time interval.***

**Thus are true the next relations:**

$$\begin{aligned}
&\textit{for } \forall(p_1 \,\&\, p_2),\ \textit{from } [0;1],\ \textit{where } p_1 \neq p_2\\
&1)\quad \textit{if } p_1 \le p_2 \Rightarrow t_{ij}^1 \ge t_{ij}^2 \textit{ for}\\
&\qquad i \in \{1,2,...,m\},\ j \in \{1,2,...,n\},\ t_{ij}^l \in [a_{ij}; b_{ij}],\ l = 1,2.\\
&2)\quad \textit{if } p_1 \ge p_2 \Rightarrow t_{ij}^1 \le t_{ij}^2 \textit{ for}\\
&\qquad i \in \{1,2,...,m\},\ j \in \{1,2,...,n\},\ t_{ij}^l \in [a_{ij}; b_{ij}],\ l = 1,2.
\end{aligned} \tag{9}$$

By fuzzy linear programming we mean the appliance of the fuzzy set theory to linear multi-criteria decision making problems. In multi-criteria decision making problems, the objective functions are represented by fuzzy sets, but the decision set is defined as the intersection of all fuzzy sets and constraints. The decision rule is to select the solution having the highest membership of the decision set. Zadeh [2] introduced the basic concepts of fuzzy set theory. Zimmermann in [23] made an innovation in the field of multi-criteria decision making. He first applied fuzzy set theory concept with suitable choices of membership functions and derived a fuzzy linear programming. He shows that obtained solution using the fuzzy linear programming is always efficient one, further it can find a optimal compromise solution.

***Definition 1*** *An element $x$ has a degree of membership in a set A, denoted by a membership function $\mu_A(x)$. The rang of the membership function is [0,1].*

According with the definition (1), the coefficient of optimism of the decision maker for the model (1) - (7) can be interpreted as a function of belonging of the required time to achieve the route to its corresponding interval.

## 4. The Initialization Procedure

The basic idea of the algorithm, that we propose is to transform fuzzy type model (1) - (7) into one of deterministic type, based on determining of the whole lot of time characteristic for all model routes for any value of the coefficient of optimism of the decision maker. By imposing of the “bottleneck” criterion separate, we will obtain the set of all efficient solutions of model (1) - (7), for time value restriction.

**1.** We will suppose, that the set of all routes time variation intervals is given by their variation limit values, they are the following: $\{[a_{ij};b_{ij}]\}_{i=\overline{1,m},\, j=\overline{1,n}}$ ;

**2.** We assume that the decision maker has estimated a certain amount of coefficient of optimism, let it is $p_1$, then we may calculate by applying the formula (8) the values of all time parameters that is the following: $t_{ij} = b_{ij} - \rho_1(b_{ij} - a_{ij})$, for $i = \overline{1,m},\ j = \overline{1,n}.$

**3.** By applying the step 1 and 2 of the procedure we reduce the model (1) - (7) to a deterministic models such as:

$$\min F_1 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{\ 1} x_{ij}}{\max\limits_{i,j}\{t_{ij} \mid x_{ij} > 0\}} \tag{10}$$

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$$\min\ F_r = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}{}^{r} x_{ij}}{\max_{i,j}\{t_{ij} \mid x_{ij} > 0\}} \tag{11}$$

$$\min\ F_{r+1} = \max_{i,j}\{t_{i,j} \mid x_{i,j} > 0\} \tag{12}$$

*in conditions* :

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m} \tag{13}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n} \tag{14}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{15}$$

$$x_{ij} \geq 0,\ i = \overline{1,m},\ j = \overline{1,n} \tag{16}$$

*with the same means of parameters like the (1) - (7) model just that of deterministic type.*

**5. Some Reasons and Statements**

Since the model (10) - (16) is of multi-criteria type, as we know, these rarely admit optimal solutions. For solving these usually it builds a set of efficient solutions, known also as Pareto-optimal or non-dominated solutions, solutions of the "optimal compromise". In order to investigate the model of multiple criteria, we should propose firstly the definition of efficient solution for the deterministic type of model. We will consider the next multiple-criteria transportation model of "bottleneck" type with deterministic data, without affecting the generality we assume that all $r$ criteria are of minimum type:

$$\min\ Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{1} x_{ij},\ \min\ Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{2} x_{ij}$$

$$\cdots\cdots\cdots\cdots$$

$$\min\ Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}\ , \min\ Z_{r+1} = \max_{i,j}\{t_{i,j} \mid x_{i,j} > 0\} \tag{17}$$

*in conditions:*

$$\sum_{j=1}^{n} x_{ij} = a_i\ , \ \forall i = \overline{1,m}\ , \ \sum_{i=1}^{m} x_{ij} = b_j\ , \ \forall j = \overline{1,n}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ , } x_{ij} \ge 0 \text{ , } i = \overline{1,m}\text{ , } \; j = \overline{1,n}$$

*with the same significance of the model parameters as in model (10) - (16).*

Let suppose that: $(\overline{X},\overline{T})$ is one basic solution for the model (17), where:

$$\overline{T} = \max_{i,j} \{\bar{t}_{ij} / \bar{x}_{ij} > 0\}.$$

***Definition 2*** *The basic solution $(\overline{X},\overline{T})$ of the model $(17)$ is a basic efficient one if and only if for any other basic solution $(X,T) \ne (\overline{X},\overline{T})$ for which exists at least one index $j_1 \in (1,...r)$ for which the relation $Z_{j_1}(X) \le Z_{j_1}(\overline{X})$ is true, there immediately exists another, at least, one index $j_2 \in (1,....r)$, where $j_2 \ne j_1$, for which at least, one of the both relations $Z_{j_2}(\overline{X}) < Z_{j_2}(X)$ or $\overline{T} < T$ is true. If all of these three inequalities are verified simultaneously with the equal sign, it means that the solution is not unique.*

***Definition 3*** *The basic solution $(\overline{X},\overline{T})$ of the model $(17)$ is one of the best compromise solution for a certain time $\overline{T}$, if the solution $\overline{X}$ is located closest to the optimal solutions of each criterion.*

Analysing the model (10) - (16) and the model (17), it is evident, that both models have the same set of basic solutions, because of the same availability domains. The next theorem demonstrates that the model (10) - (16) and (17) have also the same set of basic efficient solutions.

***Theorem 1.*** The set of the basic efficient solutions of the model (10) - (16) and of the model (17) coincide.

*Proof.* We will prove the any basic efficient solution $(\overline{X},\overline{T})$ for the model (17) is also basically efficient for the model (10) - (16).

Let suppose, that $(\overline{X},\overline{T})$ is an basic efficient solution for model (17). According the above definition the result is the following:

for any other basic solution $(X,T) \ne (\overline{X},\overline{T})$ for which the relation $F_{j_1}(X) \le \langle F_{j_1}(\overline{X})$ is true, there immediately exists is at least one index $\exists j_2 \in (1,...r)$, for which the relation $F_{j_2}(\overline{X}) \langle F_{j_2}(X)$ or $\overline{T} < T$ is true. (it is not essential, but for simplicity there will be excluded the case with of multiple solutions).

*case a*) $T \le \overline{T}$ ;

Let suppose that $(\overline{X}, \overline{T})$ is not the basic efficient solution for model (10) - (16). It results, that the following relation is true:

$$\frac{F_j(X)}{T} \le \frac{F_j(\overline{X})}{\overline{T}}, \qquad \text{for any indexes} \quad j \in (1,...r), \tag{18}$$

and, at least for one index, let it be $j_2$, the inequality (18) is quite strict.

Multiplying inequalities (18) by $T$ and supposing $k = \frac{T}{\overline{T}}$, we obtain the following true relations:

$$F_j\ (X) \le k * F_j\ (\overline{X}), \qquad \text{for any } j \in (1,...r) \tag{19}$$

and at least for index $j_2$, where $j_2 \in (1,...r)$ the corresponding relation from (19) is quite strict.

Because, it is obvious that $k \le 1$, from the relation (19) immediately results the following true relations:

$$F_j(X) \le F_j(\overline{X}) \tag{20}$$

for any indexes $j \in (1,...r)$, and for index $j_2$, the relation is surely strict.

The obtained relations (20) contradict the supposition, that the solution $(\overline{X}, \overline{T})$ is basically efficient for the model (17), resulting that the basic efficient solution $(\overline{X}, \overline{T})$ for the model (17) is also basically efficient for the model (10)-(16), the fact needed to prove.

*case b*) $\overline{T} < T$;

In this case, because there still exists one value registration at least for one criterion (time criterion), which is better for the solution $(\overline{X}, \overline{T})$ than for the solution $(X, T)$ for the model (17) and especially for the model (10) - (16), resulting that the solution $(\overline{X}, \overline{T})$ is basically efficient for the both models, that we intended to demonstrate.

It can be proved analogously that each efficient solution of the model (10) - (16) is also an efficient solution for the model (17).

This theorem is proved.

I'd like to mention that the multi-criteria models with all linear criteria were comparatively much more investigated in the specialized literature as in [3],[4],[10],[11],[12] .

***So for each time level allowing placement of the basic solution for the model (17), we can determine its corresponding optimal compromise solution.***

## 6. Fuzzy techniques

In order to solve the model (17) by applying the fuzzy technique we propose the next

***Ordering Algorithm***:

***Step 1*** Ordering the time matrix – *T( p )* according to cell values in ascending order and assigning for each cell a serial number, thus we will get all $(m \times n)$ ordered cells.

***Step 2*** Selecting the firsts at least *(m + n-1)* cells according to the arrangement order, until we can place the initial basic solution for the model (17), supposing that the other cells are blocked.

***Step 3*** By applying the ***algorithm of fuzzy technique*** for the problem with unlocked cells, we get the optimal compromise solution for the model (17) using only the unlocked cells, which corresponds to the following time: $t^* = \min \max_{i,j} \{ t_{i,j} | x_{i,j} > 0 \}$.

***Step 4*** Unlocking iteratively in increasing order of time the next matrix cell (or cells with the same time and cost values), we will return to the step 3 of the algorithm and we will find the next optimal compromise solution of model with time of its realization, obviously, higher than the previous $t^*$ time.

The step 4 is repeated until all of cells in the matrix of time will be unblocked.

Thus, the proposed algorithm will highlight a finite set of optimal compromise solutions for the model (17), each of them corresponding to the smallest time possible of its realisation.

Because the problem has finite dimensions, the algorithm is realized in a finite number of steps.

We will apply the fuzzy linear programming technique [3] for solving the model (17). By applying of fuzzy linear programming technique to the multi-objective linear transportation model (17), we will find its optimal compromise solution for one certain time level.

At the first we assign for each objective function two values $U_k$ and $L_k$ as upper and lower bounds for the objective function $Z_k$ :

$L_k$ - aspired level of achievement for objective *k;*

$U_k$ - highest acceptable level of achievement for objective *k;*

$d_k = U_k - L_k$ is obviously a degradation allowance for objective *k.*

We build the fuzzy model, because of aspiration and degradation levels for each objective have been specified. On the next step we will transform the fuzzy model into one of deterministic type model of linear programming.

**The solving fuzzy technique is the following:**

***Step 1*** Solving of $r$ one-criterion transportation problems.
***Step 2*** Building the table of values, in which are registered values of the all objective functions in the optimal solutions of every objective function.
***Step 3*** According to the table of values we may choose the best - $L_k$ and the worst $U_k$ values from the set of solutions.

The initial fuzzy model is built keeping the aspirations of each criterion, as the follows:

$$Z_k \underset{\sim}{<} L_k,\ k=\overline{1,r},$$
$$\sum_{j=1}^{n} x_{ij} = a_i,\quad \forall i=\overline{1,m}\ ,\quad \sum_{i=1}^{m} x_{ij} = b_j,\quad \forall j=\overline{1,n}\,, \tag{21}$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad x_{ij} \ge 0\ \ for\ all\ \ i\ and\ \ j$$

The membership function $\mu_k(X)$ is defined as the next:

$$\mu_k(x) = \begin{cases} 1, & if \quad Z^k(x) \le L_k \\ \dfrac{U_k - Z_k}{U_k - L_k}, & if \quad L_k < Z^k(x) < U_k \\ 0, & if \quad Z^k(x) \ge U_k \end{cases} \tag{22}$$

Taking into account the relations (21) and the above definition of the membership function $\mu_k(X)$, the equivalent linear programming problem for the multi-objective transportation problem (17) for one available time level is the next:

Max $\lambda$

*in conditions:*

$$\lambda \le \frac{U_k - Z_k}{U_k - L_k},\ \ k=\overline{1,r},$$
$$\sum_{j=1}^{n} x_{ij} = a_i,\quad \forall i=\overline{1,m}\ ,\quad \sum_{i=1}^{m} x_{ij} = b_j,\quad \forall j=\overline{1,n}\,, \tag{23}$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad x_{ij} \ge 0\ \ for\ all\ \ i\ and\ \ j,\ \ \lambda \ge 0.$$

By simplifying the model (23), we will obtain the next linear programming optimization model:

Max $\lambda$

*in conditions:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda \cdot \left(U_k - L_k\right) \le U_k, \ k = \overline{1, r}, \tag{24}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1, m}, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1, n},$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad x_{ij} \ge 0 \ \textit{for all } i \textit{ and } j, \ \lambda \ge 0.$$

Thus, we can say, that using the fuzzy technique for solving the model (10)-(16), we easily find a compromise solution for the multi-objective transportation model (17) for one available time level. By modifying the time level, we can obtain the set of all compromise solutions corresponding to all time levels.

***Remark 1*** The above described algorithm is applicable to all types of multi-objective transportation problems as well to the vector minimum as to the vector maximum problems.

***Remark 2*** The optimal compromise solution of the model (17) for one certain available time level doesn't necessarily to be of integer type.

***Remark 3*** Iteratively modifying the decision maker's coefficient of optimism, we get the multitudes of optimal compromise solutions by using the ***Ordering Algorithm***, then the ***Technics Fuzzy.***

Thus, the proposed algorithm will highlight a finite set of optimal compromise solutions for the model (17), each of them corresponding to the smallest time possible of its realisation.

Particular cases of the model (1), without of the "bottleneck" criterion were analysed in [4] by Chanas and Kuchta. The authors proposed a method of interval for solving one criterion transportation model with fuzzy cost coefficients. The idea be applied to multi-criteria problem [5], but it leads to considerable increasing of the number of objective functions, which really complicates the solving process of the problem. In the papers [6],[9],[10],[12],[15],[16],[18],[19], [22] are proposed certain analyses of various points of view about the multiple criteria transportation model with fuzzy parameters and are developed different algorithms in order to its solving. It should be noted the practical impossibility of solving these types of models using some parametric methods.

We can solve the model (10)-(16) also by finding of a set of efficient solutions for each value of decision maker's coefficient of optimism. This algorithm is more difficult, but for each value of this parameter, it offers for the decider one large set of efficient solutions of integer type, which are very important in elaboration of correct managerial strategies.

## 7. The Combinatorial Algorithm

Primarily we will perform the first three steps of the **Initialization Procedure** for a certain value of decision maker's coefficient of optimism $p$, then the following procedure will be applied iteratively:

***The first step.*** We arrange the values $\{t_{ij}\}$ from the matrix T in increasing order using for this an ordering index, let it be *h*. We outline that in the model (17) there are $n$ supplies and $m$ demands and $(r+1)$ objective-functions (including time criterion).

***The second step.*** We try to find an initial basic solution using one of the criteria, namely the matrix of this criterion, from that we will use only the cells according to ordering increasing *s*-index. Obviously, the initial basic solution will be placed in at least ($m+n-1$) cells. Thus, in the matrix in which we placed the initial basic solution will be unblocked $s_0$ cells, where $s_0 \geq m+n$-1. The obtained solution at this iteration will mark the 0-level of the logical tree of efficient solutions. We will consider that the following cells with ordering indexes greater than $s_0$ are blocked. For the 0-level we will calculate the corresponding $T_0$ according to the following formula:

$$T_0 = \max_{i,j}\{t_{i,j}(\rho) | x_{i,j} > 0\} \tag{25}$$

***The third step*** *(exploration of the deep branch).* We shall try to improve the solution from the actual level, using for this only the unblocked cells. For this purpose we shall calculate the values: $\Delta_{ij} = (u_i + v_j) - c_{ij}$. All configurations of basic solutions can be recorded at the next level *l=1*. Thus the logical tree will contain on level 1 no more than $s_1$ branches, where $s_1 \geq s_0 - (m+n-1)$. The procedure from the $3^{rd}$ step is iterative one and explores the possibility to increase the number of logical branches on the next level using every the branches from the previous level. If all possibilities of placement have been explored as to improve at least one of criteria using for this purpose just the $s_0$ cells ( according the described ordering) , then one can go to the $4^{th}$ step.

***The fourth step.*** We will unblock the next *(*$s_0$ *+1)* cell and will obtain a new achievement time for a new efficient solution, which will be obviously greater or equal than the previous time. I'd like to outline that, after each unblocking iteratively procedure there are again ($m \times n - s_0$) blocked cells, because after every unlocking we consider: $s_0 = s_0 + 1$. If the relation $\Delta_{i,j} \geq 0$ is true at least for one criterion, for this cell, we will repeat the procedure of the $3^{rd}$ step, otherwise we shall continue to unblock the next cell, according to the ordering index $h$ until we will get $s = m \times n$. After finishing up the $4^{th}$ step, the set of all basic solutions of model (1) - (7) will have been recorded , out of which we can easily select the ones that are basic and efficient.

One can see, that the logical solving tree iteratively increase its branches by exploration of a new configurations of the basic solutions on every level. The increasing of both the numerous of branches of each level as well as the number of levels is constrained by the fact that the problem is of finite dimensions on the one hand and on the other hand by the request that the new solution configuration should not be repeated. The correctness of the above algorithm is based from the following theorem.

***Theorem 2.*** The set of all efficient basic solutions for the multiple criteria fractional transportation problem with fuzzy “bottleneck” criterion (1) - (7) for any value of decision maker’s coefficient of optimism is found by applying the ***Combinatorial Algorithm.***

*Proof.* We will assume that, by applying t**he Initialization Procedure** to the model (1) - (7) for a certain value of decision maker’s coefficient of optimism $p$ , we reduce it to a deterministic model after (10) - (16), after that the model been reduced to model (17).

Let $L_T$ be a list of basic efficient solutions of model (1) - (7) being found by applying the above Combinatorial Algorithm for a certain value of decision maker’s coefficient of optimism $p$ . We suppose, that exists one basic efficient solution $S_{j1}$ for the model (1) – (7), that was found using another algorithm different of the above one, so it results that $S_{j1} \notin L_T$ . Let $S_{j1}$ corresponds to $T_{j1}$. We will fix it on the branch that corresponds to the $T_{j1}$ beginning with the level 0. Wide exploration of the fixed branch leads to the registration of all basic solutions of the branch $T_{j1}$. So, all the basic solutions that correspond to time $T_{j1}$ belong to this set. We will separate in the set $L_{T_{jl}}$ the efficient basic solutions, that correspond to time $T_{j1}$. It is obvious that $L_{j1} \subset L_T$ . As a result, if $S_{j1} \in L_{T_{j1}}$ , then $S_{j1}$ is a basic efficient solution found by applying the above algorithm or if $S_{j1} \notin L_{T_{j1}}$ , then $S_{j1}$ is not a basic solution and moreover, it is not one basic efficient. So, is true the following: either $S_{j1}$ is a basic efficient solution and it belongs in the list $L_T$ or it is not a basic efficient solution. We proved that for a certain value of decision maker’s coefficient of optimism $p$ we obtained the set of all corresponding efficient solutions for the model (1) - (7). Thereby, modifying the coefficient of optimism $p$ by applying the formula (8), we get a lot of new efficient solutions for model (1) - (7), which are restricted to a new time By building the set of efficient solutions for model (1) - (7) for any value of the coefficient of optimism $p$ in the interval [0,1], in fact, we fully solve the proposed model. The theorem is proved.

## 8. Conclusions

In this paper is developed an integrate multistage procedure to solve the multi-objective fractional transportation problem with fuzzy of “bottleneck” restriction. By applying the hypothesis about the interconnection between the time required for transport and coefficient of optimism of decision maker, which of course is of subjective type, we reduce the model to one of deterministic type. After, for each of possible time level we construct its corresponding set of efficient solutions. I would like to emphasize, that at this stage we may apply the fuzzy technique for finding the optimal compromise solution, corresponding to the early established time level. However, as it’s known, the set of efficient solutions offers several options for developing optimal management strategies. By modifying of the time level, depending on the decision maker's coefficient of optimism, we can obtain all sets of efficient solutions by applying combinatorial algorithm, each of them corresponding to its time of realization. In dependence of the economic stability, the parameter may following different laws of distribution. Finally, we conclude, that these kind of models are very actually and utile especially from the decisional and managerial point of view.

**Example:**

*Let be the 2-criteria fractional transportation problem with fuzzy "bottleneck" condition with 3 supplies and 4 demands. Supposing that we know the data of unit costs for the first two criteria of minimum type, which are the next:*

***Cost 1, 2 =***

| | | | |
|---|---|---|---|
| 1<br>4 | 2<br>4 | 7<br>3 | 7<br>4 |
| 1<br>5 | 9<br>8 | 3<br>9 | 4<br>10 |
| 8<br>6 | 9<br>2 | 4<br>5 | 6<br>1 |

The third criterion of the problem is of type "bottleneck" by time, similar to the model (1) – (7). The supply and demand are the next:
$A = \{8,19,17\}$; $B = \{11,3,14,16\}$.

The variation intervals of fuzzy time on each route are as follows:

$[a_{11}; b_{11}] = [5; 15]$; $[a_{21}; b_{21}] = [63; 73]$; $[a_{31}; b_{31}] = [32; 42]$;

$[a_{12}; b_{12}] = [90; 100]$; $[a_{22}; b_{22}] = [61; 71]$; $[a_{32}; b_{32}] = [58; 68]$;

$[a_{13}; b_{13}] = [68; 78]$; $[a_{23}; b_{23}] = [25; 35]$; $[a_{33}; b_{33}] = [18; 28]$;

$[a_{14}; b_{14}] = [47; 57]$; $[a_{24}; b_{24}] = [16; 26]$; $[a_{34}; b_{34}] = [12; 22]$;

We want to build the set of all efficient solutions for the value of decision maker's coefficient of optimism $p = 0.5$.

***Solution procedure:***

Knowing the value $p = 0.5$, we will apply the formula (8): $t_{ij} = b_{ij} - \rho(b_{ij} - a_{ij})$ in order to determine the set of deterministic data $\{t_{ij}\}_{i=\overline{1,m},\, j=\overline{1,n}}$. We obtain the next data for the time parameters:

***Time=***

| | | | | |
|---|---|---|---|---|
| 10 | 95 | 73 | 52 | 8 |
| 68 | 66 | 30 | 21 | 19 |
| 37 | 63 | 19 | 23 | 17 |
| 11 | 3 | 14 | 16 | $b_j \backslash a_i$ |

By using the above proposed **Combinatorial Algorithm** we have found the following 11 efficient basic solutions:

$$X^1 = (x_{11} = 8, x_{21} = 3, x_{22} = 2, x_{23} = 14, x_{33} = 1, x_{34} = 16),\ S^1 = \left(\frac{176}{68}, \frac{207}{68}, 68\right);$$

$$X^2 = (x_{11} = 8, x_{21} = 3, x_{22} = 3, x_{24} = 13, x_{33} = 14, x_{34} = 3),\ S^2 = \left(\frac{164}{68}, \frac{276}{68}, 68\right);$$

$$X^3 = (x_{11} = 6, x_{14} = 2, x_{21} = 5, x_{23} = 14, x_{32} = 3, x_{34} = 14),\ S^3 = \left(\frac{178}{68}, \frac{203}{68}, 68\right);$$

$$X^4 = (x_{11} = 8, x_{21} = 3, x_{23} = 14, x_{24} = 2, x_{32} = 3, x_{34} = 14),\ S^4 = \left(\frac{172}{68}, \frac{213}{68}, 68\right);$$

$$X^5 = (x_{11} = 8, x_{21} = 3, x_{24} = 16, x_{32} = 3, x_{33} = 14),\ S^5 = \left(\frac{158}{68}, \frac{283}{68}, 68\right);$$

$$X^6 = (x_{13} = 8, x_{21} = 11, x_{22} = 2, x_{23} = 6, x_{32} = 1, x_{34} = 16),\ S^6 = \left(\frac{208}{73}, \frac{167}{73}, 73\right);$$

$$X^7 = (x_{13} = 6, x_{14} = 2, x_{21} = 11, x_{23} = 8, x_{32} = 3, x_{34} = 14),\ S^7 = \left(\frac{202}{73}, \frac{173}{73}, 73\right);$$

$$X^8 = (x_{12} = 2, x_{13} = 6, x_{21} = 11, x_{23} = 8, x_{32} = 1, x_{34} = 16),\ S^8 = \left(\frac{186}{95}, \frac{171}{95}, 95\right);$$

$$X^9 = (x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16),\ S^9 = \left(\frac{176}{95}, \frac{175}{95}, 95\right);$$

$$X^{10} = (x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{24} = 13, x_{33} = 14, x_{34} = 3),\ S^{10} = \left(\frac{143}{95}, \frac{265}{95}, 95\right);$$

$$X^{11} = (x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{23} = 13, x_{33} = 1, x_{34} = 16),\ S^{11} = \left(\frac{156}{95}, \frac{200}{95}, 95\right).$$

We observe, that the data of the model (1) – (7) for the decision maker's coefficient of optimism $p = 0.5$ coincide with the data from the example of Aneja and Nair from the article [1]. We can mention that, using the proposed **Combinatorial Algorithm** we obtained with 2 efficient basic solutions more compared as the authors' results from this article. By modifying the decision marker's coefficient of optimism, and therefore for the new time level of achievement for transport routes, we get a lot of new efficient solutions for the model (1) – (7).

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