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A MODEL FOR OPTIMAL INVESTMENT PROJECT CHOICE USING FUZZY PROBABILITY

Abstract: *In this paper we present a model for classifying exclusive investments. The model uses Bellman and Zadeh's decision-making criterion, determining the degree of convergence when the objective is to maximize the net present value of the project under the constraint of minimizing risk.*
The original aspect of this work consists in incorporating uncertainty into the model by considering variables such as project life, net income and capitalization rate as uncertain in order to determine net present value and risk.
The concept of a fuzzy event is used to calculate the net present value and assess the risk of each investment project. This allows us to establish the degree to which a project is a good investment, understanding this as a fuzzy event and establishing the degree to which a project has a high net present value, understood as another fuzzy event.
Keywords*: fuzzy logic, fuzzy set theory, decision-making, fuzzy arithmetic, probability of a fuzzy event, investment.*
JEL Classification: C02, C51, C61

1. Introduction

Nowadays, numerous methods and techniques are available to help an investor obtain a prediction of possible return on investment. Remer and Nieto (1995a, 1995b) presented a comparison of a number of methods grouped into the following five types: net present value methods, rate of return methods, ratio methods, payback methods, and accounting methods.

However, such methods cannot be considered realistic investment selection models for use in scenarios where there is risk due to the fact that they are deterministic in nature, in the sense that the different magnitudes used in the calculation are considered well-known, an assumption that greatly simplifies actual economic reality.

There are several ways to include uncertainty in the magnitudes used in such methods. The most common is to assume that the probabilities of the possible values of the magnitudes are known. Suárez (1995) and Ross, Westerfield & Jaffe (1999) summarize various methods under this assumption.

With the birth of fuzzy thinking (Zadeh, 1965), a new way of incorporating uncertainty into the initial variables became possible (Georgescu, 2014; Gil-Lafuente et al., 2015; Dong et al., 2015; Ghorabaee et al., 2016). The theory of fuzzy subsets accepts that an economic variable does not have to be seen as an exact number but rather can take different values, assigning a level of possibility to each.

Zadeh's extension principle (Zadeh, 1975; Nguyen, 1978) extended the usual mathematical operations of certainty to the use of uncertain quantities represented by fuzzy numbers. Dubois and Prade (1978) proposed normal working methods with such operations, which constitute the arithmetical key to solving many economic problems in which the variables are highly uncertain in nature (Merigó, 2014; Scherger et al., 2015; Linares-Mustarós et al., 2015).

Fuzzy subset theory thus offers an alternative to classic probabilistic treatment, allowing the creation of new ordering rules based on non-probabilistic subjective assumptions.

This paper presents a numerical investment selection model supported by the ideas of fuzzy subset theory. The model allows the ordering of various exclusive investment projects, that is, ones that cannot be carried out simultaneously. Bellman and Zadeh's decision criterion (Bellman and Zadeh, 1970) is used to formulate the model by converging between the objective of determining the project that represents a good investment, in the sense of minimizing risk, and the constraint that the investment project has a high net present value. The central value for the net present value is calculated for each investment project by using Heilpern's formula (Heilpern, 1992) to determine the expected value. The concept of the probability of a fuzzy event (Zadeh, 1968) is used to assess risk, which allows us to establish the degree to which a project is a good investment, understanding it as a fuzzy event.

The paper is structured as follows. In Section 2, we present a procedure for calculating the probability of the fuzzy event "the project is a good investment" when estimated returns (also called cash flows or operating profit), capitalization rate and project life are considered to be uncertain. Given the uncertainty surrounding the variables, project risk is a fuzzy concept and it has therefore been modeled on the basis of the fuzzy probability methods created by Zadeh. In Section 3, we detail the process for calculating the possibility of the fuzzy event "the project has a high net present value" when, as in the previous section, estimated returns, capitalization rate and project life are considered to be uncertain. In Section 4, we formulate the model for choosing between several alternative investment projects. This section presents a numerical simulation with a fuzzy-type input data set. The final sections of the paper are comprised of the conclusions and references.

2. Calculating the probability of the fuzzy event "the project is a good investment"

In most different types of investment projects, project life is considered to be one of the variables subject to greatest uncertainty for various reasons. A methodology based on the use of fuzzy logic to apply the criterion of net present value enables uncertainty regarding project life to be incorporated for a particular investment project by associating a possibility distribution to it, usually denoted by "n". Thus, we can consider project life to be expressed through a fuzzy number of discrete nature $\tilde{n}$, that is, with support within the set of natural numbers, which allows us to interpret the fuzzy expression obtained from the net present value in several different ways.

Let us assume that $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n$ are fuzzy numbers representing the estimated net returns for a given project with a fixed life cycle of "n", and that $\tilde{i}$ is the triangular fuzzy number representing the uncertain capitalization rate, and $\tilde{P}$ is the uncertain investment payment generally also expressed through a triangular fuzzy number, even though this is often the variable with less degree of uncertainty. As established by Kaufmann and Gil Aluja (Kaufmann and Gil Aluja, 1986), we can obtain the uncertain expression of the net present value ($N\tilde{P}V$) by applying fuzzy arithmetic techniques to the following formula:

$$N\tilde{P}V = \frac{\tilde{A}_1}{1+\tilde{i}} + \frac{\tilde{A}_2}{(1+\tilde{i})^2} + \ldots + \frac{\tilde{A}_n}{(1+\tilde{i})^n} - \tilde{P} \tag{1}$$

In practice, determining the membership function is difficult, and it is much more appropriate to determine the fuzzy number through its α-cuts, which in our case, because the expression is growing with respect to $\tilde{A}_1$ and decreasing with respect to $\tilde{i}$ and $\tilde{P}$, will be determined by the following expressions:

$$\underline{NPV}(\alpha) = \frac{\underline{A_1}(\alpha)}{1+\bar{i}(\alpha)} + \frac{\underline{A_2}(\alpha)}{(1+\bar{i}(\alpha))^2} + \ldots + \frac{\underline{A_n}(\alpha)}{(1+\bar{i}(\alpha))^n} - \bar{P}(\alpha)$$
$$\overline{NPV}(\alpha) = \frac{\overline{A_1}(\alpha)}{1+\underline{i}(\alpha)} + \frac{\overline{A_2}(\alpha)}{(1+\underline{i}(\alpha))^2} + \ldots + \frac{\overline{A_n}(\alpha)}{(1+\underline{i}(\alpha))^n} - \underline{P}(\alpha) \tag{2}$$

In order to incorporate uncertainty into the project life, which we represent by "*n*" and assume to be expressed in number of years, we will consider said uncertain variable to be expressed through a discrete fuzzy number, as follows:

$$\tilde{n} = \{ (n_1, \mu_{\tilde{n}}(n_1)), (n_2, \mu_{\tilde{n}}(n_2)), \ldots, (n_m, \mu_{\tilde{n}}(n_m)) \} \quad (3)$$

with $n_1, n_2, \ldots, n_m$ being consecutive natural numbers.

From this perspective, we will consider that the net present value is a second order fuzzy subset (Kaufmann et al., 1994), which presents as support the various fuzzy numbers expressing the net present value for each possible value of the project life, and each with the degree of possibility that matches the corresponding degree of possibility of the value "n_k" (k=1, 2, ..., m), which we use to calculate the net present value.

We will symbolize this set through $N\tilde{\tilde{P}}V$, and express it through:

$$N\tilde{\tilde{P}}V = \{ (N\tilde{P}V_1, \mu_{\tilde{n}}(n_1)), (N\tilde{P}V_2, \mu_{\tilde{n}}(n_2)), \ldots, (N\tilde{P}V_m, \mu_{\tilde{n}}(n_m)) \} \quad (4)$$

With $N\tilde{P}V_k \;\; (k = 1, 2, \ldots, m)$ being the corresponding fuzzy number obtained from the formula (1), calculated for a project life value of $n = n_k$.

With this construction we will not have a single risk value for the investment project, rather we will have different risk values, each with a degree of possibility that will be determined on the basis of the possibility distribution associated with the life cycle of project $\tilde{n}$.

Thus, for each possible n_k value of the project life, we define the risk of the project as the share of negative values in the membership function of the net present value over the total. This value is determined by the following formula[1]:

$$R_k = \frac{\int_{\underline{NPV_k}(0)}^{0} \mu_{N\tilde{P}V_k}(x)dx}{\int_{\underline{NPV_k}(0)}^{\overline{NPV_k}(0)} \mu_{N\tilde{P}V_k}(x)dx} \qquad k = 1,2,3,\ldots,m \quad (5)$$

[1] A practical example of this type of construction for measuring risk and its theoretical calculation using triangular fuzzy numbers can be found in Linares et al.(2013).

Since for each possible *k* value of the project life we obtain a risk value of between 0 and 1 from formula (5), we can interpret the risk of the investment project as a fuzzy set $\tilde{R}$ whose support is given by:

$$S(\tilde{R}) = \{R_1, R_2, R_3, \ldots, R_m\} \tag{6}$$

and whose membership function is given by:

$$\mu_{\tilde{R}}(R_k) = \begin{cases} \mu_{\tilde{n}}(n_k) & \text{if } R_j \neq R_k \ \forall\ j \neq k \\ \mu_{\tilde{n}}(n_{j_1}) \vee \mu_{\tilde{n}}(n_{j_2}) \vee \ldots \vee \mu_{\tilde{n}}(n_k) & \text{if } \exists\ j_1, j_2, \ldots, j_m \neq k \text{ with } R_{j_i} = R_k \end{cases} \tag{7}$$

Let us remember that our aim in this section is to determine a probability function for the fuzzy event "the project is a good investment". We will now see how said function is constructed on the basis of the definition provided by Zadeh (1968), restated below.

If we consider a probability space (E, Σ, P) (where E is an ordinary set, Σ is the σ-algebra of all the parts of E, and P: $\Sigma \longrightarrow R$ is a probability measure), we can define a fuzzy event as a fuzzy subset $\tilde{A}$ of E, whose membership function $\mu_{\tilde{A}}$ is Σ-measurable for the measure P.

In the specific case where E is a finite set $E=\{x_1,x_2,\ldots,x_n\}$ and $\{p_1,p_2,\ldots,p_n\}$ is a probability distribution on the elements of E, then we can write the probability of a fuzzy event $\tilde{A}$ like this:

$$P(\tilde{A}) = \sum_{i=1}^{n} \mu_{\tilde{A}}(x_i) \cdot p_i \tag{8}$$

Note that in the case where $\tilde{A}$ is a crisp event, then $\mu_{\tilde{A}}(x) \in \{0,1\}$ and the above definition matches the ordinary concept of the probability of an event in a finite set.

In our work, the associated probability distribution we use will be determined from the possibility distribution of the investment risk, which we interpret as a fuzzy subset.

One way to address the problem is to consider that if we estimate the project life as n_k years with a degree of possibility $\mu_{\tilde{n}}(n_k)$, then we will consider the investment to be "good" (understanding this concept as a fuzzy event, since it is determined by a linguistic variable) with a degree $1-R_k$, R_k being the risk value calculated from formula (5). This risk and acceptance value will have a degree of possibility given by $\mu_{\tilde{n}}(n_k)$.

If we express the investment project's level of risk through the linguistic equivalence given in Table 1[2], we will be able to establish, the degree to which an "investment is good", understanding the degree of acceptance by the investor as that indicated in Table 2.

Table 1. Measure scale for risk

Risk value "R_k"	Linguistic equivalence
$R_k = 0$	Zero risk (Z)
$0 < R_k \le 0.2$	Very small risk (VS)
$0.2 < R_k \le 0.4$	Small risk (S)
$0.4 < R_k < 0.6$	Intermediate risk (I)
$0.6 \le R_k < 0.8$	High risk (H)
$0.8 \le R_k < 1$	Very high risk (VH)
$R_k = 1$	Total risk (T)

Table 2. Degree that the investment is good according to level of risk

Level of risk	Degree of acceptance for *"the investment is good"*
Z	1
VS	0.9
S	0.7
I	0.5
H	0.3
VH	0.1
T	0

With this interpretation of risk, let us now suppose that we have an investment project X with uncertain characteristics (return on investment, capitalization rate, project life and net returns), and that, based on risk calculations according to

[2] In this work, we use a scale with seven levels, in accordance with Miller's observation on the average capacity of human rote retention (Miller, 1956)

project life and its possibility distribution, we can associate the risk level of the investment project through a fuzzy set $\tilde{R}$ given through a set, as follows:

$$\tilde{R} = \{(Z,\mu_1),(VS,\mu_2),(S,\mu_3),(I,\mu_4),(H,\mu_5),(VH,\mu_6),(T,\mu_7)\} \quad (9)$$

Then we can define the fuzzy risk associated to a particular investment project X as a fuzzy set whose support is comprised of values that define the level of risk, and the respective membership values match the degree of acceptance of the project according to the risk level explained in Table 2. Thus, we will determine the fuzzy event $\tilde{G}$ comprised by *the good investments.* If we have $X_1, X_2, \ldots, X_m$ investments to analyze, then we need to assign a degree to determine the membership function that establishes the degree to which each X_i (i=1,2,...,m) belongs to the set $\tilde{G}$.

For this purpose, if we consider a particular investment project X from the ordinary set $\{X_1, X_2, \ldots, X_m\}$, then using the uncertain risk associated to X which will have been determined in (9), we construct a probability distribution that will later be used to calculate the probability that X is a good investment. We will see that this probability will give us a degree that the investment is good. To do this, let us first consider the cardinal of fuzzy subset $\tilde{R}$:

$$|\tilde{R}| = \sum_{k=1}^{7} \mu_k$$

and normalizing the possibility distribution given in (9), we construct the following probability distribution:

$$p_k = \frac{\mu_k}{|\tilde{R}|}. \quad (10)$$

Thus, $\{p_k : 1 \le k \le 7\}$ is a probability distribution defined on the elements of the support of the fuzzy set $\tilde{R}$ associated to X. We will use this distribution to calculate the probability of investment X being good. This probability is calculated in this way:

$$P(X \in \tilde{G}) = 1\frac{\mu_1}{|\tilde{R}|} + 0.9\frac{\mu_2}{|\tilde{R}|} + 0.7\frac{\mu_3}{|\tilde{R}|} + 0.5\frac{\mu_4}{|\tilde{R}|} + 0.3\frac{\mu_5}{|\tilde{R}|} + 0.1\frac{\mu_6}{|\tilde{R}|} + 0\frac{\mu_7}{|\tilde{R}|}$$

$$= \frac{1}{\left|\tilde{R}\right|}(1\mu_1 + 0.9\mu_2 + 0.7\mu_3 + 0.5\mu_4 + 0.3\mu_5 + 0.1\mu_6 + 0\mu_7) \qquad (11)$$

Note that evidently we have $0 \le P(X \in \tilde{G}) \le 1$. Thus, by performing these calculations we can compare various investment projects from the perspective of risk. The probability that "X is a good investment" will be higher, the lower the uncertain level of risk. Specifically, we note that if an investment project is such that the level of risk is zero, that is $\mu_1 = 1 \text{ y } \mu_k = 0 \;\; k \neq 1$, then the probability of "X is a good investment" is 1, $P(X \in \tilde{G}) = 1$, and in the case where the level of risk is total, that is $\mu_7 = 1 \text{ y } \mu_k = 0 \;\; k \neq 7$, then the probability that "X is a good investment" is 0, that is, $P(X \in \tilde{G}) = 0$.

3. Calculating the possibility of the fuzzy event "the project has a high net present value"

Our starting point in this section is calculating a central value for the net present value of each investment project under uncertainty.

We start with the mathematical expectation from the bundle of fuzzy numbers comprising those numbers that constitute the support of the second order fuzzy subset, obtained in formula (4). To calculate this, we use the weighting given by the probability distribution that we have established in (10). The expectation of the project's net present value, which in our case is a fuzzy number denoted by $\tilde{E}$, is determined by its α-cuts (Kaufmann & Gupta, 1991), as follows:

$$E_\alpha = \left[\sum_{k=1}^{m} q_k \cdot \underline{NPV_k}(\alpha)\,,\, \sum_{k=1}^{m} q_k \cdot \overline{NPV_k}(\alpha)\right] \qquad (12)$$

where the weighting coefficients in this case are given by :

$$q_k = \frac{\mu_{\tilde{n}}(n_k)}{\sum_{k=1}^{m} \mu_{\tilde{n}}(n_k)}$$

On the basis of this fuzzy number, we calculate its expected value using Heilpern's formula[3] as follows:

[3] Let us remember that given a fuzzy number $\tilde{A}$, Heilpern's formula proposes the following expected value:

$$VE(\tilde{E}) = \frac{1}{2} \cdot \int_0^1 \underline{E}(\alpha) \cdot d\alpha + \frac{1}{2} \cdot \int_0^1 \overline{E}(\alpha) \cdot d\alpha \tag{13}$$

To assign the degree to which a particular investment project has a high net present value, we will define the fuzzy subset $\tilde{C}$ of the reference value for the set of real numbers, given the vague statement: "*high net present value*", on the basis of a membership function of the type indicated in Figure 1, where M represents a threshold value by virtue of which the decision-maker believes that if the net present value of the investment exceeds said M value, then the investment has maximum interest from the point of view of current net value.

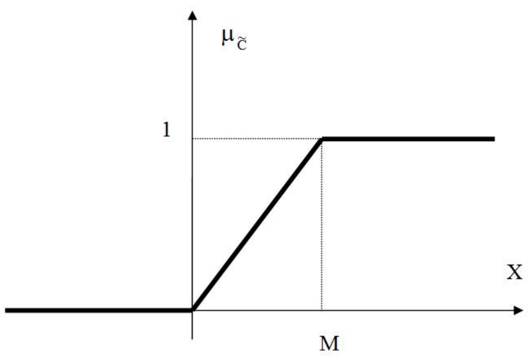


Figure 1. Membership function for the vague statement: "the investment has a high net present value"

Therefore, the possibility that a project has a high net present value would be given by the following expression:

$$Poss\left(X \in \tilde{C}\right) = \begin{cases} 0 & \text{si } VE(\tilde{E}) < 0 \\ \dfrac{VE(\tilde{E})}{M} & \text{si } 0 \leq VE(\tilde{E}) \leq M \\ 1 & \text{si } VE(\tilde{E}) > M \end{cases} \tag{14}$$

$VE(\tilde{A}) = \beta \cdot \int_0^1 \underline{A}(\alpha) \cdot d\alpha + (1-\beta) \cdot \int_0^1 \overline{A}(\alpha) \cdot d\alpha$ where β is a value between 0 and 1 and depends on the investor's outlook, that is, whether it is more optimistic or more cautious. We have proposed $\beta = 0.5$ so as to situate ourselves in an intermediate position.

4. Confluence between maximizing the value of the project and minimizing risk

Bellmann and Zadeh proposed a decision-making model in a fuzzy environment based on maximizing a fuzzy goal subject to a fuzzy constraint. In our case, the fuzzy goal will be to choose the best investment (in the expressed sense of minimizing risk) subject to the fuzzy constraint of the high current net value presented by the investment. To this end, these authors proposed assigning a degree of membership for each alternative given by the "*minimum*" operator which models the logical conjunction "and" represented in Figure 2.

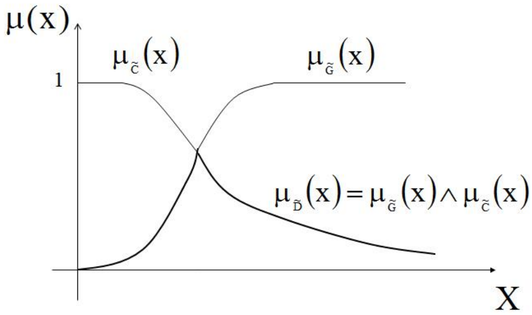


Figure 2. Confluence between the goal and the constraint according to Bellmann and Zadeh's criterion

To employ Bellman and Zadeh's criterion, for each investment project we need to determine one degree for the fuzzy goal "*project* X *is a good investment*", identified by means of a confidence measure $\mu_{\tilde{G}}(X)$, and another for the fuzzy constraint "*project* X *has a high net present value*", identified by means of $\mu_{\tilde{C}}(X)$. The degree of acceptance for the decision will then be determined by $\mu_{\tilde{D}}(X) = \mu_{\tilde{G}}(X) \wedge \mu_{\tilde{C}}(X)$, where $\wedge$ is the minimum operator.

Given that the measure to determine the degree of each confidence measure must verify the following axioms (Dubois and Prade, 1988)

1. $\mu(\Phi) = 0$ if Φ is an impossible event
2. $\mu(\Omega) = 1$ if Ω is a sure event
3. $\mu(A) \leq \mu(B)$ if $A \subseteq B$

We propose:

1) that the definition for the degree of fuzzy goal X *is a good investment* is $\mu_{\tilde{G}}(X) = P(X \in \tilde{G})$ expressed in (11). Kaufmann (1977) shows that 1, 2 and 3 are met if we have a probability measure.
2) and that the degree of fuzzy constraint *X has a high current net value* is $\mu_{\tilde{C}}(X) = \text{Poss } (X \in \tilde{C})$. Dubois and Prade (1988) show that possibility measures also meet 1, 2 and 3.

To illustrate the theory, we will now present a practical numerical example that models some of the initial conditions of economic reality using triangular fuzzy numbers.

We should mention here that if the variables that appeared in the right hand of formula (1) are estimated as triangular fuzzy numbers, the fuzzy number $N\tilde{P}V$ does not have a triangular structure due to the appearance of the product and the division (Kaufmann et al., 1991). For practical purposes, we would determine whether an approximation via the triangular fuzzy number

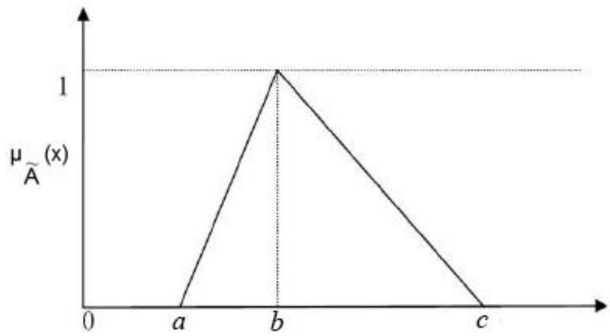
$$(\underline{NPV}(0), \underline{NPV}(1) = \overline{NPV}(1), \overline{NPV}(0))$$

would be admissible. The error between the fuzzy number and its approximation may be a good indicator[4]of the validity of the approximation.

Let us remember that this type of fuzzy number can be represented by four lines, two of which are horizontal in 0. Figure 3depicts an example of how it is represented graphically. Its detailed use in this paper to capture uncertain magnitudes is justified by both its ease of use, providing a very quick idea of the range of values that are possible, and its similarity with the idea of the pessimistic, optimistic and maximum level of confidence thinking, with which humans tend to deal with specific problems in the economic sphere.

Given that the values "*a*", "*b*" and "*c*", with $a \le b \le c$, characterize the triangular fuzzy number, it will be common to identify a triangular fuzzy number $\tilde{A}$ with the expression (*a, b, c*), as we have already done.

[4] Jiménez and Rivas' (1996) proposal is interesting in this respect: it suggests that with a fixed scale of p levels, for the approximation of a fuzzy number $\tilde{X}$ to be admissible for its triangular approximation $\tilde{X}t$, there must be no semantic difference between the two representations; that is, the approximation is admissible if $\forall x \in \text{Support}(\tilde{X})$ verifies the inequality: $\left|\mu_{\tilde{X}}(x) - \mu_{\tilde{X}_t}(x)\right| \le \frac{1}{p-1}$



$$\mu_{\tilde{A}}(x) = \begin{cases} 0 \text{ si } x < a \\ \dfrac{x-a}{b-a} \text{ si } a \le x < b \\ \dfrac{c-x}{c-b} \text{ si } b \le x \le c \\ 0 \text{ si } x > c \end{cases}$$

Figure 3. Representation of a triangular fuzzy number

It is possible to immediately test whether the fuzzy numbers that constitute the support for $N\tilde{\tilde{P}}V$ are triangular numbers in the form $N\tilde{P}V_k \approx (a_k, b_k, c_k)$; the fuzzy number $\tilde{E}$, being a linear combination of triangular numbers (Kaufmann et al., 1991), is also a triangular fuzzy number that can be expressed by:

$$\tilde{E} = \left(\sum_{k=1}^{n} q_k a_k, \sum_{k=1}^{n} q_k b_k, \sum_{k=1}^{n} q_k c_k \right) \tag{15}$$

where q_k are the weighting coefficients considered in (12).
If $\tilde{E}$ is a triangular fuzzy number, identified by the expression (*a*, *b*, *c*), then this results in:

$$VE(\tilde{E}) = \frac{a + 2b + c}{4} \tag{16}$$

Since this process is completely programmable, we will now present a numerical example completed in Adobe Flash Professional CS6. The program[5, 6] calculates all degrees of different investment projects by displaying all relevant data of the process on-screen.
Running the program displays a table of values (see Figures 4, 5 and 6), where the data for the following three investments have been entered:

INVESTMENT	DATA

[5] The program can be viewed at the following address: http://web2.udg.edu/grmfcee/fuzzyinvestment.exe

[6] The programming codes can be viewed by opening the following file in the program Adobe Flash Professional CS6: http://web2.udg.edu/grmfcee/fuzzyinvestment.fla

X_1	$\tilde{n} = \{(1,0),(2,0.2),(3,0.6),(4,1),(5,0.8),(6,0.4)\}$ $\tilde{i} = (0.06,0.08,0.09)$ M=1000 P = 6000 m.u. $\tilde{A}_i = (1500,2000,2500)$ $i = 1,2,3,4,5,6$
X_2	$\tilde{n} = \{(1,0.2),(2,0.3),(3,0.7),(4,1),(5,1),(6,0.2)\}$ $\tilde{i} = (0.06,0.08,0.09)$ M=1000 P = 5000 m.u. $\tilde{A}_i = (1200,1900,2300)$ $i = 1,2,3,4,5,6$
X_3	$\tilde{n} = \{(1,0.4),(2,0.5),(3,0.8),(4,1),(5,0.3),(6,0.1)\}$ $\tilde{i} = (0.06,0.08,0.09)$ M=1000 P = 5000 m.u. $\tilde{A}_i = (1400,2010,2700)$ $i = 1,2,3,4$ $\tilde{A}_i = (500,800,900)$ $i = 5,6$

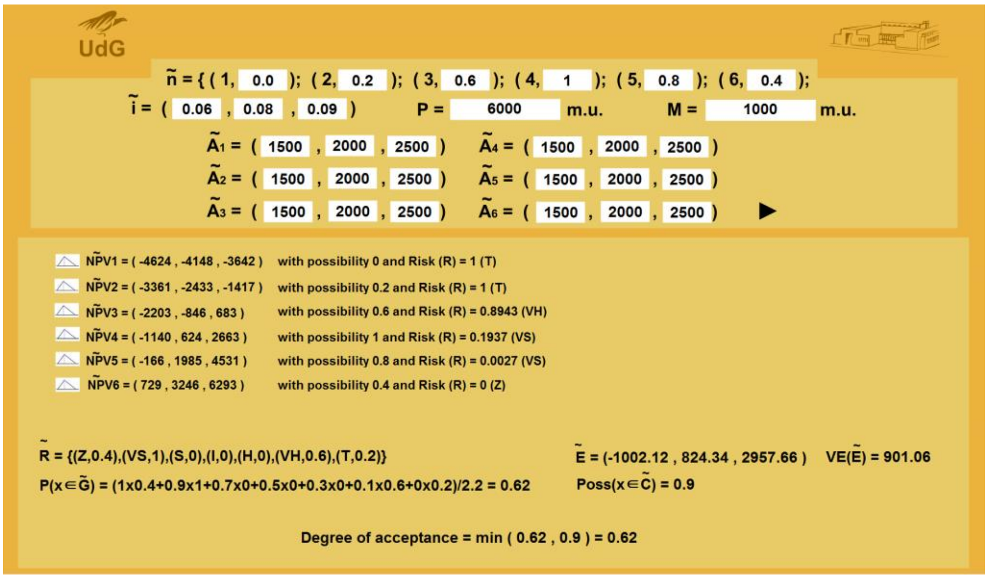


Figure 4. Screenshot of the program for the investment X_1

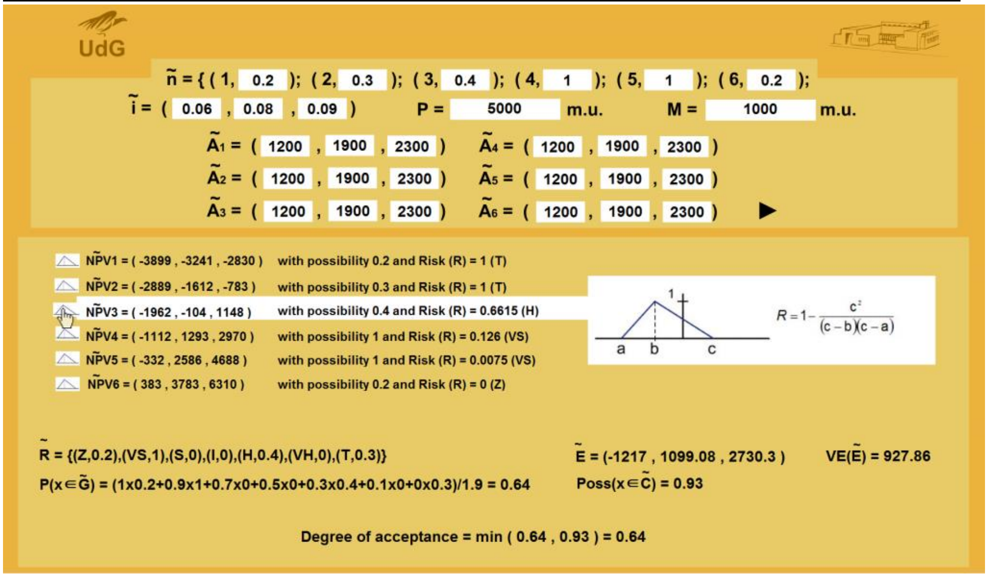


Figure 5. Screenshot of the program for the investment X_2

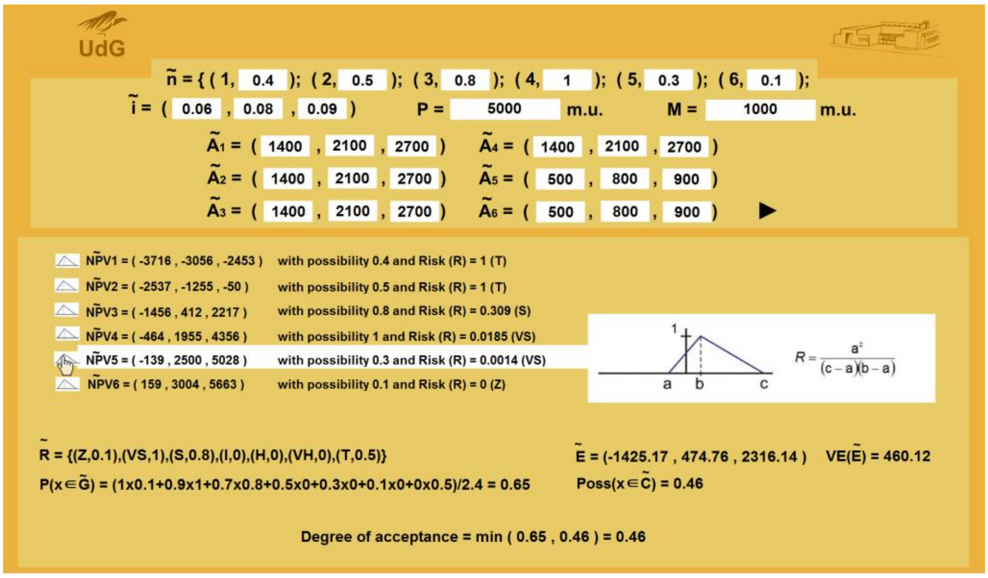


Figure 6. Screenshot of the program for the investment X_3

The program[7] gives us a degree of acceptance for each investment project. We can see that the degree for the first investment project is 0.62, for the second 0.64, and for the third 0.46. Therefore, the second project is best for our purposes.

5. Conclusions

In this paper, we have employed the concept of the probability of a fuzzy event to establish the degree to which a particular investment project can be accepted by the decision-maker from the point of view of minimizing risk. The proposed measure meets the necessary objectives for the criterion to be consistent, in the sense that it decreases respect the level of risk and its range of variation is within the interval [0,1]. When the risk is zero, the investment project is accepted with degree 1, and when the risk is total the degree of acceptance of the project is 0. From this measure, we have established a method to classify various alternative investment projects.

For situations in which the project life is uncertain, we have proposed interpreting the net present value as a fuzzy subset of a second order type 2; that is, one whose support elements are fuzzy numbers and which allows a possibility distribution to be established for the various uncertain values that can be assumed by the net present value of a particular investment project. Based on the values of the membership function of these second order fuzzy subsets, we can propose a system for choosing between the various alternative investment projects.

Finally, in order to study the feasibility of an investment project under the general hypotheses of uncertainty established here, we must be able to, with the same situation regarding level of risk (for example, in the absence of risk), choose the project with the highest net present value. In the most general case analyzed in this paper, we identify the best project using Bellmann and Zadeh's decision-making criterion in the context of uncertainty by determining the fuzzy intersection that corresponds to the confluence of the goal of having the most viable project (in the sense of minimizing the risk) and having a high net present value. It is also important to note that the criterion could be established in very different ways depending on the operator chosen to model the intersection between fuzzy sets. Using this methodology, we have been able to generalize the classic criterion based on the evaluation of current net value and establish a relationship between different alternative investment projects in a coherent way, taking into consideration maximization of current net value and minimization of risk.

[7] The data can be modified to perform other numerical tests. Modifying the values in the input tables and pressing the calculation button will automatically display the relevant new calculations and provide new arrangements for selection. Hovering over the image that represents the fuzzy number "net present value" reveals the formula used to calculate the respective value for the risk level.

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