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MULTICRITERIA DECISION-MAKING METHOD BASED ON COSINE SIMILARITY MEASURES BETWEEN INTERVAL-VALUED FUZZY SETS WITH RISK PREFERENCE

***Abstract**. This paper presents the cosine similarity measure between IVFSs with risk preference and gives its decision making method using the cosine similarity measure depending on decision makers' optimistic, neutral, and pessimistic natures for the subjective judgments that accompany the decision making process. Through the weighted cosine similarity measure between an alternative and the ideal alternative corresponding to one of optimistic, neutral, and pessimistic choices desired by decision makers, we can determine the ranking order of alternatives and the best one. This choosing feature corresponding to decision makers' preference makes the proposed method not only more flexible, but also more suitable for many practical applications. Finally, an illustrative example is presented to demonstrate the feasibility and applicability of the proposed method.*

***Keywords**: interval-valued fuzzy set, cosine similarity measure, multicriteria decision making.*

JEL Classification: C44, D8, D81

1. Introduction

In many real-world situations, the decision maker cannot provide deterministic alternative values because the decision information given by decision makers is often imprecise or uncertain due to a lack of data, time pressure, or the decision makers' limited attention and information processing capabilities. This kind of uncertainty in multicriteria decision making can be handled using fuzzy set theory. Then, fuzzy sets are ideally suited for solving decision making problems with uncertain information. Bellman and Zadeh (1970) first proposed the fuzzy decision-making model. Since then, great numbers of studies on fuzzy multicriteria decision problems have most often been performed in a fuzzy environment (Chenand Hwang, 1992; Chen, 2000; Wang and Parkan, 2005; Xu, 2007; Fu, 2008). In addition, because it may be difficult for decision makers to exactly quantify their opinions as a number in the interval [0, 1], it is more suitable to represent this degree of certainty by an interval. Therefore, Zadeh (1975) first proposed the concept of an interval-valued fuzzy set (IVFS). IVFSs are suitable for capturing imprecise or uncertain decision information. After that, IVFSs have been applied to multicriteria decision-making problems (Xu, 2006; Ashtiani et al., 2009). On the other hand, optimism and pessimism, the concepts developed by Scheier and Carver (1985), are fundamental constructs that reflect how people respond to their perceived environment and how they form subjective judgments. Although theories differ in their specifics, a common idea is that optimists and pessimists diverge in their explanations and predictions of future events. Recently, Chen (2011) presented a new method to reduce cognitive dissonance and to relate optimism and pessimism in multicriteria decision analysis in an interval-valued fuzzy decision environment.

The similarity measure is one of important tools for the degree of similarity between objects. Functions expressing the degree of similarity of items or sets are used in physical anthropology, numerical taxonomy, ecology, information retrieval, psychology, citation analysis, and automatic classification. In fact, the degree of similarity or dissimilarity between the objects under study plays an important role. In the query expansion, various term-term similarity measures based on the collocation have been suggested to select the additional search terms. In vector space, the cosine similarity measure (Salton and McGill, 1987) is often used for this purpose. However, these similarity measures cannot deal with the similarity measures for intuitionistic fuzzy information. For this purpose, Ye (2011) proposed the cosine similarity measure of intuitionistic fuzzy sets (IFSs) and applied it to pattern recognition and medical diagnosis in intuitionistic fuzzy environment. Then, Ye (2012) proposed the Jaccard, Dice, and cosine similarity measures between trapezoidal intuitionistic fuzzy numbers (TIFNs) that are treated as continuous and applied them to multicriteria group decision-making problems. To overcome some disadvantages of the cosine similarity measure of IFSs, Shi and Ye (2013) further presented an improved cosine similarity measure of vague sets by considering degree of hesitation and applied it to the fault diagnosis of turbine. Furthermore, Ye (2013) put forward interval-valued intuitionistic fuzzy cosine similarity measure and its application to multiple attribute decision-making problems with interval-valued intuitionistic fuzzy information.

Until now, to the best of the author's knowledge, one does not pay attention to the cosine similarity measure of IVFSs with risk preference and its multicriteria decision-making method. Therefore, the main purpose of this paper extends the cosine similarity measure (Salton and McGill, 1987) in fuzzy vector space to handling imprecise data or uncertain information represented as IVFSs and proposes the cosine similarity measure between IVFSs with risk preference, and then establishes a decision-making method based on the weighted cosine similarity measure with risk preference, which utilizes the optimistic, neutral, and pessimistic subjective judgments corresponding to decision-makers' preference to deal with difficult decision-making problems in some cases. However, we usually determine the ranking order of alternatives through the weighted cosine similarity measure between an alternative and the ideal alternative corresponding to one of optimistic, neutral, and pessimistic choices desired by decision makers. This choosing feature corresponding to decision-makers' preference makes the proposed method not only more flexible, but also more suitable for many practical applications.

The remainder of this paper is organized as follows. Section 2 briefly describes some concepts of fuzzy set, IVFSs, and the cosine similarity measure of fuzzy sets. Section 3 presents cosine similarity measures of IVFSs with risk preference. In Section 4, a decision-making method is established based on the weighted cosine similarity measure between an alternative and the ideal alternative with risk preference, which utilizes the optimistic, neutral, and pessimistic subjective judgments corresponding to decision-makers' preference to deal with interval-valued fuzzy decision-making problems. In Section 5, an illustrative example is presented to demonstrate the feasibility and applicability of the developed method. Section 6 gives conclusions and future research direction.

2. Preliminaries

In this section, we introduce some basic concepts and definitions related to fuzzy sets, IVFSs, and a cosine similarity measure for fuzzy sets, which will be needed in the following analysis.

2.1 Fuzzy sets and IVFSs

Definition 1. Zadeh(1965)defined a fuzzy set A in the universe of discourse X as follows:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}, \tag{1}$$

which is characterized by a membership function $\mu_A(x)$: $X \rightarrow [0, 1]$, where $\mu_A(x)$ indicates the membership degree of the element x to the set A.

In fuzzy set theory, it is often difficult for an expert to exactly quantify his or her opinion as a number in interval [0, 1]. Therefore, it is more suitable to represent this degree of certainty by an interval. From such point of view, Zadeh (1975) further proposed the concept of an IVFS.

Definition 2. An IVFS A in the universe of discourse X was given by Zadeh (1975):

$$A = \left\{\left\langle x, \left[\mu_A^-(x), \mu_A^+(x)\right]\right\rangle \mid x \in X\right\}, \tag{2}$$

where $\mu_A^-(x)$: $X\to[0,1]$ and $\mu_A^+(x)$: $X\to[0,1]$ are called a lower limit of membership degree and a upper limit of membership degree of the element x to the set A, respectively, with the condition $0 \le \mu_A^-(x) \le \mu_A^+(x) \le 1$. For convenience, a basic element in an IVFS is denoted by $a = \left[\mu_A^-(x), \mu_A^+(x)\right]$, which is called an interval-valued fuzzy element (IVFE).

2.2 Cosine similarity measure for fuzzy sets

A cosine similarity measure for fuzzy sets (Salton and McGill, 1987) is defined as the inner product of two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vector representations of the two fuzzy sets.

Assume that $A = (\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n))$ and $B = (\mu_B(x_1), \mu_B(x_2), \ldots, \mu_B(x_n))$are two fuzzy sets in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. A cosine similarity measure (angular coefficient) betweenA and B can be defined as follows (Salton and McGill, 1987):

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A^2(x_i)}\sqrt{\sum_{i=1}^{n} \mu_B^2(x_i)}}. \tag{3}$$

The cosine similarity measure takes value in the interval [0,l]. It is undefined if $\mu_A(x_i) = 0$ and/or$\mu_B(x_i) = 0$ ($i = 1, 2, \ldots, n$). Thus, let the cosine measure value be zero when $\mu_A(x_i) = 0$ and/or$\mu_B(x_i) = 0$ ($i = 1, 2, \ldots, n$).

3. Cosine similarity measures between IVFSs

Assume that there are two IVFSs $A = \left\{\left\langle x_i, \left[\mu_A^-(x_i), \mu_A^+(x_i)\right]\right\rangle \mid x_i \in X\right\}$ and $B = \left\{\left\langle x_i, \left[\mu_B^-(x_i), \mu_B^+(x_i)\right]\right\rangle \mid x_i \in X\right\}$in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. The elements in A and Bcan be considered as two pairs of vector representations with the length of n elements:

$$L_A = \left(\mu_A^-(x_1), \mu_A^-(x_2), \ldots, \mu_A^-(x_n)\right) \text{ and } U_A = \left(\mu_A^+(x_1), \mu_A^+(x_2), \ldots, \mu_A^+(x_n)\right), \tag{4}$$

$$L_B = \left(\mu_B^-(x_1), \mu_B^-(x_2), \ldots, \mu_B^-(x_n)\right) \text{ and } U_B = \left(\mu_B^+(x_1), \mu_B^+(x_2), \ldots, \mu_B^+(x_n)\right). \tag{5}$$

Based on the extension of the cosine similarity measure for fuzzy sets (Salton and McGill, 1987), a cosine similarity measure between L_A and L_B is defined in the vector space as follows:

$$C^{-}(L_A, L_B) = \frac{\sum_{i=1}^{n} \mu_A^{-}(x_i)\mu_B^{-}(x_i)}{\sqrt{\sum_{i=1}^{n} (\mu_A^{-}(x_i))^2}\sqrt{\sum_{i=1}^{n} (\mu_B^{-}(x_i))^2}} . \tag{6}$$

Another cosine similarity measure between U_A and U_B is defined in the vector space as follows:

$$C^{+}(U_A, U_B) = \frac{\sum_{i=1}^{n} \mu_A^{+}(x_i)\mu_B^{+}(x_i)}{\sqrt{\sum_{i=1}^{n} (\mu_A^{+}(x_i))^2}\sqrt{\sum_{i=1}^{n} (\mu_B^{+}(x_i))^2}} . \tag{7}$$

Thus, the cosine similarity measure between A and B is proposed in the vector space as follows:

$$C_{IVFS}(A, B) = \alpha C^{-}(L_A, L_B) + (1-\alpha) C^{+}(U_A, U_B) , \tag{8}$$

where $\alpha \in [0, 1]$ expresses the risk preference desired by decision makers in decision making. By adjusting the risk preference value of αwe can obtain the similarity measure corresponding to the decision makers' risk preference value. Especially if $\alpha = 1$, $\alpha = 0$, and $\alpha = 0.5$, respectively, we have the pessimistic, optimistic and neutral similarity measures selected by the decision makers. When $\alpha = 1$ for the pessimistic choice, Eq. (8) degenerates to Eq. (6); then when $\alpha = 0$ for the optimistic choice, Eq. (8) degenerates to Eq. (7); while $\alpha = 0.5$ for neutral choice, Eq. (8) is an average similarity measure of Eq. (6) and Eq. (7).

The cosine similarity measure between IVFSs A and B satisfies the following properties:

(P1) $0 \leq C_{IVFS}(A, B) \leq 1$;

(P2)$C_{IVFS}(A, B) = C_{IVFS}(B, A)$;

(P3) $C_{IVFS}(A, B) = 1$ if $A = B$, i.e., $\mu_A^{-}(x_i) = \mu_B^{-}(x_i)$ and $\mu_A^{+}(x_i) = \mu_B^{+}(x_i)$ for $i = 1, 2,..., n$.

Proof.

(P1)It is obvious that the property is true according to cosine values for Eqs. (6) and (7).

(P2) It is obvious that the property is true.

(P3) When $A = B$, there are $\mu_A^{-}(x_i) = \mu_B^{-}(x_i)$ and $\mu_A^{+}(x_i) = \mu_B^{+}(x_i)$ for $i = 1, 2, ..., n$. So there is $C_{IVFS}(A, B) = 1$.

Therefore, we complete the proofs.

If we consider the weight of x_i (i =1, 2, …, n) with $w_i \in$ [0, 1] and $\sum_{i=1}^{n} w_i = 1$, the weighted cosine similarity measure between IVFSs A and B is proposed as follows:

$$W_{IVFS}(A,B) = \alpha W^{-}(L_A, L_B) + (1-\alpha) W^{+}(U_A, U_B), \tag{9}$$

where

$$W^{-}(L_A, L_B) = \frac{\sum_{i=1}^{n} w_i^2 \mu_A^{-}(x_i)\mu_B^{-}(x_i)}{\sqrt{\sum_{i=1}^{n}(w_i\mu_A^{-}(x_i))^2}\sqrt{\sum_{i=1}^{n}(w_i\mu_B^{-}(x_i))^2}}, \tag{10}$$

$$W^{+}(U_A, U_B) = \frac{\sum_{i=1}^{n} w_i^2 \mu_A^{+}(x_i)\mu_B^{+}(x_i)}{\sqrt{\sum_{i=1}^{n}(w_i\mu_A^{+}(x_i))^2}\sqrt{\sum_{i=1}^{n}(w_i\mu_B^{+}(x_i))^2}}. \tag{11}$$

Similarly, the weighted cosine similarity measure between IVFSs A and B also satisfies the following properties:

(P1) $0 \le W_{IVFS}(A, B) \le 1$;

(P2) $W_{IVFS}(A, B) = W_{IVFS}(B, A)$;

(P3) $W_{IVFS}(A, B) = 1$ if $A = B$, i.e., $\mu_A^{-}(x_i) = \mu_B^{-}(x_i)$ and $\mu_A^{+}(x_i) = \mu_B^{+}(x_i)$ for i = 1, 2, ..., n.

By similar proof method, we can prove that the properties (P1)-(P3).

If we take $w_i = 1/n$ for i =1, 2, …, n, there are $W^{+}(A, B) = C^{+}(A, B)$, $W^{-}(A, B) = C^{-}(A, B)$, and $W_{IVFS}(A, B) = C_{IVFS}(A, B)$.

4. Decision-making method based on the cosine similarity measure

For an interval-valued fuzzy multicriteria decision-making problem, the evaluations of each alternative with respect to each criterion for the fuzzy concept ‘‘excellence” can be given by the form of IVFEs. Suppose that there exists a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$. Each alternative is assessed on $n$criteria, which are denoted by $C = \{C_1, C_2, \ldots, C_n\}$. The evaluation value of a criterion C_j (j = 1, 2, …, n) on an alternative A_i (i = 1, 2, …, m) is represented by an IVFE $d_{ij} = [\mu_{ij}^{-}(C_j), \mu_{ij}^{+}(C_j)]$ (i = 1, 2, …, m; j = 1, 2, …, n) given by the decision maker or expert according to some evaluated criteria. Thus we can obtain an interval-valued fuzzy decision matrix $D = (d_{ij})_{m\times n}$, whichis defined as the following form:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} [\mu_{11}^-, \mu_{11}^+] & [\mu_{12}^-, \mu_{12}^+] & \cdots & [\mu_{1n}^-, \mu_{1n}^+] \\ [\mu_{21}^-, \mu_{21}^+] & [\mu_{22}^-, \mu_{22}^+] & \cdots & [\mu_{2n}^-, \mu_{2n}^+] \\ \vdots & \vdots & \vdots & \vdots \\ [\mu_{m1}^-, \mu_{m1}^+] & [\mu_{m2}^-, \mu_{m2}^+] & \cdots & [\mu_{mn}^-, \mu_{mn}^+] \end{bmatrix} \end{array}. \tag{12}$$

In multicriteria decision-making environments, the concept of ideal point has been used to help the identification of the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct to evaluate alternatives. Therefore, we define an ideal IVFE for each criterion in the ideal alternative A^* as $d_j^* = [1, 1]$, $j = 1, 2, \ldots, n$.

The weight vector of criteria for the different importance of each criterion is given as the weight vector $w = (w_1, w_2, \ldots, w_n)$, where any weight $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Thus the weighted cosine similarity measure between an alternative A_i and the ideal alternative A^* represented by the IVFSs is given as the follows:

$$W_i(A^*, A_i) = \alpha W_i^-(L_{A^*}, L_{A_i}) + (1-\alpha) W_i^+(U_{A^*}, U_{A_i}) \text{ for } \alpha \in [0, 1], \tag{13}$$

where

$$W_i^-(L_{A^*}, L_{A_i}) = \frac{\sum_{j=1}^{n} w_j^2 \mu_{A_i}^-(x_j)}{\sqrt{\sum_{j=1}^{n} (w_j)^2} \sqrt{\sum_{j=1}^{n} (w_j \mu_{A_i}^-(x_j))^2}}, \tag{14}$$

$$W_i^+(U_{A^*}, U_{A_i}) = \frac{\sum_{j=1}^{n} w_j^2 \mu_{A_i}^+(x_j)}{\sqrt{\sum_{j=1}^{n} (w_j)^2} \sqrt{\sum_{i=1}^{n} (w_j \mu_{A_i}^+(x_j))^2}}. \tag{15}$$

The weighted cosine similarity measure value of $W_i(A^*, A_i)$ is within the values between 0 and 1. By adjusting the risk preference value of α, we can obtain the similarity measure corresponding to the decision makers' risk preference. Especially when α = 0, α = 0.5, and α = 1, respectively, we have the three decision choices: the optimistic weighted cosine similarity measure for Eq. (15), and the neutral weighted cosine similarity measure (the average similarity measure of Eq. (14) and Eq. (15)), and the pessimistic weighted cosine similarity measure for Eq. (14). Then, in the decision-making process, the decision choice of the similarity measures, depends on the optimistic or neutral or pessimistic nature for decision makers.

The weighted cosine similarity measure provides the global evaluation for each alternative regarding all the criteria from Eq. (13). The larger the value of the weighted cosine similarity measure, the better the alternative. Through the weighted cosine similarity measure with one of three decision choices desired by the decision makers, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well. Hence the proposed method indicates its flexibility. Then, its advantage is to overcome the difficulty of the ranking order and decision-making when there may exist the same measure values of some alternatives in the decision-making process and to provide the decision makers with more flexible choices in real applications.

5. Illustrative example

The following practical example involves a supplier selection problem in a supply chain discussed in Chen (2011). The authorized decision maker in a small enterprise attempts to reduce the supply chain risk and uncertainty to improve customer service, inventory levels, and cycle times, which results in increased competitiveness and profitability. The decision maker considers various criteria involving (i) C_1: performance (e.g., delivery, quality, and price); (ii) C_2: technology (e.g., manufacturing capability, design capability, and ability to cope with technology changes); and (iii) C_3: organizational culture and strategy (e.g., feeling of trust, internal and external integration of suppliers, compatibility across levels, and functions of the buyer and supplier). Using the supplier rating system, the decision maker evaluates five suppliers $A = \{A_1, A_2, \ldots, A_5\}$, based on three criteria $C = \{C_1, C_2, C_3\}$. The lower extreme $\mu_{ij}^{-}(C_j)$ and upper extreme $\mu_{ij}^{+}(C_j)$ of the membership degrees for the supplier $A_i \in A$ with respect to the criterion $C_j \in C$ are given and formed as the following decision matrix (Chen, 2011):

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad \end{array} \\ \begin{bmatrix} [0.21, 0.36] & [0.43, 0.60] & [0.51, 0.76] \\ [0.03, 0.84] & [0.33, 0.78] & [0.48, 0.71] \\ [0.81, 0.92] & [0.01, 0.76] & [0.33, 0.58] \\ [0.32, 0.68] & [0.15, 0.34] & [0.36, 0.97] \\ [0.59, 0.87] & [0.31, 0.64] & [0.14, 0.63] \end{bmatrix} \end{array}.$$

The weight vector of the three criteria is given as $w = (0.30, 0.23, 0.47)$ in Chen (2011). The decision-making process of this problem depending on the optimistic or neutral or pessimistic nature of the decision maker is described as the follows.

If the decision maker deals with this problem with the optimistic decision choice, by applying Eq. (15) we can obtain the values of the optimistic weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9669$, $W_2(A^*, A_2) = \mathbf{0.9973}$, $W_3(A^*, A_3) = 0.9785$, $W_4(A^*, A_4) = 0.9624$, and $W_5(A^*, A_5) = 0.9891$.

From the optimistic point of view, therefore, the alternatives can be ranked as $A_2>A_5>A_3>A_1>A_4$, which implies that the optimal alternative is A_2.

If the decision maker solves this problem with the neutral decision choice, by Eqs. (13)-(15) we can obtain the values of the neutral weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9628$, $W_2(A^*, A_2) = 0.9378$, $W_3(A^*, A_3) = 0.9100$, $W_4(A^*, A_4) = \mathbf{0.9690}$, and $W_5(A^*, A_5) = 0.9070$.

Therefore, from the neutral point of view the alternatives can be ranked as $A_4>A_1>A_2>A_3>A_5$, which implies that the optimal alternative is A_4.

If the decision maker handles this problem with the pessimistic decision choice, by using Eq. (14) we can obtain the values of the pessimistic weighted cosine similarity measure:

$W_1(A^*, A_1) = 0.9587$, $W_2(A^*, A_2) = 0.8783$, $W_3(A^*, A_3) = 0.8415$, $W_4(A^*, A_4) = \mathbf{0.9756}$, and $W_5(A^*, A_5) = 0.8249$.

Therefore, from the pessimistic point of view the alternatives can be ranked as $A_4>A_1>A_2>A_3>A_5$, which implies that the optimal alternative is also A_4.

As a choosing approach, when one uses different decision choices in the above decision-making problem, in general the choice of the optimal alternative will change accordingly. Through the weighted cosine similarity measure with one of three decision choices, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well. This choosing feature makes the proposed method not only efficient, but more suitable for many practical applications.

However, many decision-making problems are essentially humanistic and subjective in nature (Chen, 2011); hence there actually does not exist a unique or uniform criterion for decision making in an imprecise environment. However, the proposed method provides the decision makers more choosing schemes.

6. Conclusion

In this study, we proposed the cosine similarity measure of IVFSs with risk preference and gave a decision-making method using the cosine similarity measure depending on optimistic, neutral, and pessimistic natures for the decision makers under an interval-valued fuzzy decision environment. Through the weighted cosine similarity measure between an alternative and the ideal alternative corresponding to one of three decision choices, the ranking order of all the alternatives can be determined and the best alternative can be easily identified as well. The feasibility and effectiveness of the proposed multicriteria decision-making method were illustrated by an illustrative example. Its advantage is to overcome the difficulty of the ranking order and decision-making when there may exist the same measure values of some alternatives in some cases. Furthermore, this choosing feature makes the proposed method not only flexible, but more suitable for many practical applications of decision making in an imprecise environment.

To extend this work, one can apply the cosine similarity measures between IVFSs to other practical applications such as fault diagnosis and medical diagnosis, or discuss how to cope with group decision-making problems based on the cosine similarity measures under incomplete information.

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