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AN EXACT SOLUTION FOR JOINT OPTIMIZATION OF INVENTORY AND ROUTING DECISIONS IN BLOOD SUPPLY CHAINS: A CASE STUDY

***Abstract.** In this paper, blood inventory-routing problem (Blood-IRP) is for the first time introduced and its unique characteristics that distinguish the problem from traditional IRPs are discussed. The perishability of blood products, the need to use older-first policy, the necessity of establishing order-up-to level policy for safety issues, the need for minimizing blood wastages, and the existence of new concepts such as assigned and unassigned inventories, crossmatch release period and crossmatch to transfusion ratio are among the especial aspects of Blood-IRP. Afterward, a strong mixed integer linear programming (MILP) formulation for solving the Blood-IRP is proposed. Moreover, a novel iterative Branch-and-Cut (B&C) is developed to solve a real case study with 8 customers, 3 vehicles, 3 periods and a product shelf life of 2 periods to optimality. Furthermore, the shortcomings of traditional IRP models with regard to blood logistics are discussed. The blood product considered in this paper is platelet; the highest perishable blood product. The obtained results emphasize the need for conducting this study. Finally, a conclusion is provided and future research directions are discussed.*

***Keywords:** Blood inventory-routing, Multi-vehicle IRP, perishable products, platelet, branch-and-cut algorithm.*

JEL Classification: C61; I11

1. Introduction

In Iran, community blood centers (CBC) are responsible for collecting whole blood units, decomposing them into their components and delivering the components to the demand points including hospital blood banks (HBB) and other medical centers and laboratories based on a prescheduled demand pattern. When these products are received at HBBs, a number of units are crossmatched with the patients' blood and kept reserved somewhere called assigned inventory. The crossmatched units remain in the assigned inventory for a maximum period of time called crossmatch release period (CRP) until they are either transfused to patients or turned back to the unassigned inventory for future utilizations. The ratio of number of crossmatched units to transfused units is called crossmatch to transfusion ratio. In this process, the units that become spoiled are considered as wastage and are discarded.

Because of the criticality and perishability of the product and the unique features of blood supply chain mentioned above, CBCs should deliver the products such that no shortage occurs at hospitals, the amount of spoiled blood units be kept at a minimum, and the units turn back to the unassigned inventory be taken into account while inventory and routing costs are minimized. To do so, in this paper, an inventory-routing problem (IRP) is developed which simultaneously determines when, how, and how much products should be delivered to the HBBs by integrating inventory and routing decisions and considering the distinctive characteristics of blood supply chains. Throughout the paper, the proposed model will be referred to as Blood-IRP that is a special case of perishable inventory-routing problems (PIRP).

This paper makes several scientific contributions. First of all, Blood-IRP is for the first time introduced and its unique characteristics that distinguish the problem from traditional IRPs are discussed. Second of all, a strong mixed integer linear programming (MILP) formulation for solving the Blood-IRP is proposed. All the distinctive characteristics of the Blood-IRP, including perishability, older-first policy, order-up-to level policy, assigned and unassigned inventory, crossmatch release period and crossmatch to transfusion ratio are incorporated into the MILP formulation. Moreover, a novel iterative Branch-and-Cut (B&C) is developed to solve the problem to optimality. Also, a case study is considered and solved to optimality by the proposed MILP formulation. Finally, the shortcomings of traditional IRP models with regard to blood logistics are discussed. To our best knowledge, this is the first time a Blood-IRP is defined and modeled in the context of IRPs.

The remainder of paper is organized as follows. In section 2, we review previous studies on the IRP and blood supply chain in the existing literature. In section 3 the problem under study is formally described and delineated in detail. In

Section 4 the MILP formulation is provided. In Section 5 the proposed branch-and-cut approach is elaborated. Section 6 defines the case study in detail. Section 7 provides the numerical results and discusses the shortcomings of traditional IRP models with regard to blood logistics. Finally, conclusion remarks and future research directions are drawn in section 8.

2. Background and related work

The work of Bell et al. (1983) is the first research conducted on IRP, 31 years ago. Since then the concept has evolved and many of its variants are introduced to handle the inventory-routing needs of the real world.

To solve IRP models, some authors developed exact approaches. Archetti et al. (2007) proposed a mixed-integer linear programming formulation for a single vehicle OU-policy IRP and then made the linear relaxation of the problem stronger by introducing new additional valid inequalities. Furthermore, a Branch-and-cut (B&C) algorithm was developed to solve the model to optimality. Adulyasak, Cordeau, and Jans (2012) proposed a benders decomposition approach for a production routing problem (PRP). Coelho and Laporte (2013b) proposed a B&C algorithm for a MMIRP problem. The first article that is considered MIRP for a heterogeneous fleet of vehicles is the work of Coelho and Laporte (2013a). In their work, a number of different variants of MIRP problems are considered and a branch-and-cut approach is provided to solve them to optimality. In their work, MIRP with and without homogeneous vehicles, MIRP with transshipment and MIRP with added consistency features (see Coelho, Cordeau, and Laporte (2012b)) are solved for a variety of benchmark instances. Their proposed branch-and-cut outperforms the only heuristic introduced by Coelho, Cordeau, and Laporte (2012a) for the single vehicle case of the problem. In the most recent study, Adulyasak, Cordeau, and Jans (2014) provide several formulations with and without a vehicle index for both maximum level (ML) and order-up-to (OU) level policies. Also, they provide branch-and-cut (B&C) algorithms to solve instances with up to 30 customers, three periods and three vehicles for OU policy within two hours. An adaptive large neighborhood algorithm was used to provide the proposed B&C with initial solutions.

Scrutinizing the literature of IRP since its very infancy, the authors could not find any work with a special focus on blood-IRP. It is despite the fact that due to perishability of the blood products and its criticality for the customers, the importance of integrating inventory and routing decisions in blood supply chains (BSC) is magnified. Although there are several recent studies that have taken perishability into account (e.g., (Coelho and Laporte 2013c); (Le et al. 2013); (Al Shamsi, Al Raisi, and Aftab 2014); (Jia et al. 2014) and (Coelho and Laporte 2014)), none of which could be directly applied to a Blood-IRP.

However, some good studies that have taken into account the distinctive features of blood logistics are briefly reviewed as follows.

Pierskalla (2004) developed an integrated regional and central blood bank location-allocation model and used a sweep algorithm for its routing part. They showed some useful ways of calculating a target inventory level (TIL) for hospital blood banks (HBB). Also, they analyzed different delivery strategies that a community blood center (CBC) should consider. Şahin, Süral, and Meral (2007) proposed a location-allocation model for the regionalization of blood services on the part of the Turkish Red Crescent Society.

Hemmelmayr et al. (2009) examined the cost-effectiveness of switching from a vendee-managed inventory setting to a VMI system for Austrian Red Cross blood bank network in Eastern Austria using an integer programming formulation. Later, to tackle the stochastic demand of products, Hemmelmayr et al. (2010) extended their previous work. Gunpinar (2013) developed three different mathematical formulations for modeling blood supply chains.

Based on the literature review conducted on the IRP context and studying the most recent review papers (Andersson et al. (2010) and Coelho, Cordeau, and Laporte (2013)) we find out that Blood-IRP is already an untouched area and there is a need for a pioneer study to narrow the existing gap in the literature. We believe that our study contributes to bridging the existing gap to a little extent.

3. Problem description

In this section the problem is formally described.

Consider a graph $G = (V, E)$,

Where $V = \{0, \dots, n\}$ is the vertex set and $E = \{(i, j) : i, j \in V, i < j\}$ is the arc set. Vertex 0 stands for the CBC and the other vertices $V' = V \setminus \{0\}$ are symbols of HBBs. Also, consider the period set $T = \{1, \dots, lp\}$ where lp shows the length of planning horizon. At each period $t \in T$ a set of $K = \{1, \dots, nv\}$ of vehicles are available at the CBC. Also, an age group g is defined for each unit of blood product where $g \in SL = \{0, \dots, sl\}$ and the product shelf life is sl , i.e. the unit becomes outdated if its age is older than sl . When one unit of blood becomes spoiled a wastage cost of oc is incurred. Each HBB has a predefined target inventory level TIL_i that should be met whenever it is visited. At the beginning of planning horizon the CBC is aware of the initial inventories of bloods at each HBB and knows the deterministic demand de_i^t of each hospital at each period. Also, r^t quantity of fresh blood units becomes available at CBC at the start of each period. When a vehicle k departs from the CBC to meet the demands of a set $S \subseteq V'$ of

HBBs its capacity Qu must be respected. A routing cost rc_{ij} is associated with each arc $(i, j) \in E$. When HBB i is visited by vehicle k in period t a quantity of $\sum_{g \in SL} q_i^{gkt}$ blood units is delivered such that the OU policy is respected. After realization of demand de_i^t , from age group g in inventory, d_i^{gt} quantity of units is used such that the equality $de_i^t = \sum_{g \in SL} d_i^{gt}$ is respected. From this quantity, a ratio of $(1-p)$ returns back to the unassigned inventory after R periods where p is transfusion to crossmatch ratio and R is crossmatch release period. For each unit of age g that remains at inventory at the end of period hc_i^g Unit holding cost is incurred. The problem is to determine, when to visit each HBB, how much to deliver to each of them and the route of each hospital such that inventory costs, transportations costs and wastage costs are minimized.

The proposed MILP formulation provided in the next section uses the following notation:

Indices:

- | | |
|--------------------------------|---|
| $i, j \in V = \{0, \dots, n\}$ | Indices of locations where 0 represents CBC and $V' = V \setminus \{0\}$ is set of HBBs |
| $g \in SL = \{0, \dots, sl\}$ | Index of age groups that belongs to the discrete set SL (shelf life) where 0 represents fresh blood units |
| $t \in T = \{1, \dots, lp\}$ | Index of time period where lp is the length of planning horizon |
| $k \in K = \{1, \dots, nv\}$ | Index of vehicles. A set K of vehicles are available. |

Parameters:

- | | |
|-----------|---|
| n | Total number of HBBs |
| sl | Shelf life of blood units after which the blood units are outdated and become unsuitable for transfusion. |
| lp | The length of planning horizon |
| nv | Total number of available vehicles |
| hc_i^g | Unit holding cost of blood of age g for HBB i |
| oc | Unit wastage cost of blood |
| rc_{ij} | Routing cost associated with edge $(i, j) \in E$ where $E = \{(i, j) : i, j \in V, i < j\}$ |
| TIL_i | Target inventory level determined for each HBB |

r^t	Number of fresh blood units CBC receives at each time period
de_i^t	Demand of each HBB i for each time period t
Qu	Vehicle capacity (a homogeneous fleet of vehicles are considered)
P	Transfusion to crossmatch ratio
R	Crossmatch release period
<i>Variables:</i>	
Inv_i^{gt}	Inventory level of blood units of age g at the end of time t at each HBB
u_i^t	Number of outdated blood units at each HBB at the end of period t
x_{ij}^{kt}	Number of times edge (i, j) is traversed by vehicle k in period t
q_i^{gkt}	The quantity of blood units of age g delivered by vehicle k in period t to HBB i
y_i^{kt}	Binary variable equal to one if and only if HBB i is visited by vehicle k in period t
z_i^t	Binary variable equal to one if and only if HBB i is visited in period t
β_i^{gt}	Number of blood units of age g returned from assigned inventory of HBB i to its unassigned inventory at the beginning of time t
d_i^{gt}	Number of blood units of age g used to satisfy the demand of HBB i in period t
L_i^{gt}	Binary variable equal to one if and only if blood units of age g are used to satisfy demand in time period t
α_i^{gt}	An auxiliary variable associated with age group g in time t that captures the number of blood units in an age group left to be utilized for the subsequent period if all available blood in this age group is not completely used to meet the demand of present period

4. Mathematical formulation

In this section the proposed model, which is based on the following assumptions, is presented:

- The products considered in the model are perishable. It means that a product shelf life is defined after which the products become spoiled.
- This is a deterministic model. Since the major part of blood demand at HBBs are related to the elective surgeries that are previously scheduled, this assumption can be justifiable.
- Older-first policy is considered. It means that while a product of older age is available, the use of the fresher units is not permitted. It is worth noting that for some special procedures such as cardiopulmonary bypass surgeries, oncology,

and hematology patients, fresh blood is only accepted; a fact that should be well handled in practice.

d) The inventory management policy at HBBs is set to order-up-to level policy. Hence, a target inventory level is defined. Each time the HBB is visited by a vehicle, its inventory level should reach to the target level.

e) The concepts of assigned and unassigned inventory, crossmatch release period and crossmatch to transfusion ratio are incorporated into the model.

f) The CBC has enough inventory to meet the demands of all HBBs.

Figure 1 shows the inventory flow of product of age g at HBB i in period t .

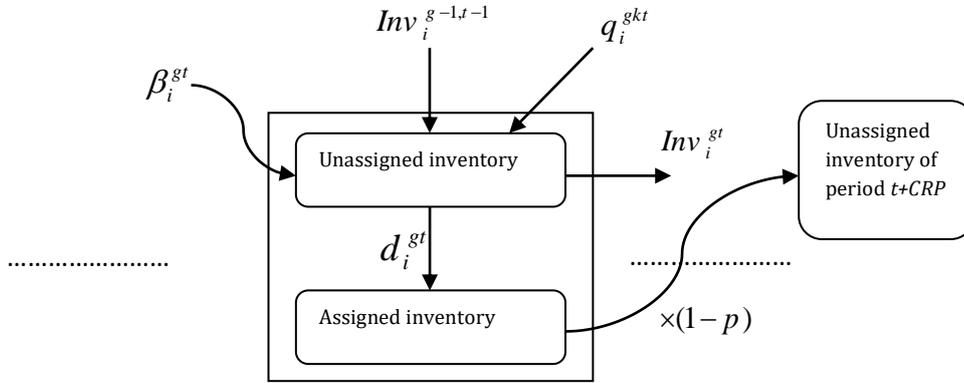


Figure 1. Inventory flow of product of age g at HBB i in period t (p is transfusion to crossmatch ratio)

4.1 MILP formulation

$$\min \sum_{i \in V} \sum_{g \in SL} \sum_{t \in T} hc_i^g \cdot Inv_i^{gt} + \sum_{i \in V'} \sum_{t \in T} oc \cdot u_i^t + \sum_{i \in V} \sum_{\substack{j \in V \\ i < j}} \sum_{k \in K} \sum_{t \in T} rc_{ij} \cdot x_{ij}^{kt} \quad (1)$$

subject to:

$$q_i^{gkt} \leq TIL_t \cdot y_i^{kt} \quad \begin{matrix} i \in V', g \in SL \\ k \in K, t \in T \end{matrix} \quad (2)$$

$$\sum_{g \in SL} \sum_{k \in K} q_i^{gkt} + \sum_{g \in SL} Inv_i^{g-1,t-1} + \sum_{g \in SL} \beta_i^{gt} \leq TIL_i \quad i \in V', t \in T \quad (3)$$

$$\sum_{g \in SL} \sum_{k \in K} q_i^{gkt} \geq TIL_t \cdot z_i^t - \sum_{g \in SL} Inv_i^{g-1,t-1} - \sum_{g \in SL} \beta_i^{gt} \quad i \in V', t \in T \quad (4)$$

$$Inv_0^{gt} = Inv_0^{g-1,t-1} - \sum_{i \in V'} \sum_{k \in K} q_i^{gkt} \quad g \in SL \setminus \{0\}, t \in T \quad (5)$$

$$Inv_0^{0t} = r^t \quad t \in T \quad (6)$$

$$Inv_i^{gt} = Inv_i^{g-1,t-1} + \sum_{k \in K} q_i^{gkt} + \beta_i^{gt} - d_i^{gt} \quad \begin{matrix} i \in V', g \in SL \setminus \{0\} \\ , t \in T \end{matrix} \quad (7)$$

$$Inv_i^{0t} = \sum_{k \in K} q_i^{0kt} - d_i^{0t} \quad i \in V', t \in T \quad (8)$$

$$d_i^{gt} \leq TIL_i \cdot L_i^{gt} \quad \begin{matrix} i \in V', g \in SL, \\ t \in T \end{matrix} \quad (9)$$

$$L_i^{g-1,t} \leq L_i^{gt} \quad \begin{matrix} i \in V', g \in SL \setminus \{0\}, \\ t \in T \end{matrix} \quad (10)$$

$$TIL_i \cdot (1 - L_i^{g-1,t}) \geq \sum_{j=g}^{sl} Inv_i^{j-1,t-1} + \sum_{j=g}^{sl} \sum_{k \in K} q_i^{jkt} + \sum_{j=g}^{sl} \beta_i^{jt} - de_i^t + 1 \quad \begin{matrix} i \in V', g \in SL \setminus \{0\}, \\ t \in T \end{matrix} \quad (11)$$

$$\sum_{g \in SL} Inv_i^{gt} \leq TIL_i \quad i \in V', t \in T \quad (12)$$

$$de_i^t = \sum_{g \in SL} d_i^{gt} \quad i \in V', t \in T \quad (13)$$

$$\sum_{i \in V'} \sum_{g \in SL} q_i^{gkt} \leq Qu \cdot y_0^{kt} \quad k \in K, t \in T \quad (14)$$

$$\alpha_i^{gt} \leq \left(Inv_i^{g-1,t-1} + \sum_{k \in K} q_i^{gkt} + \beta_i^{gt} \right) \cdot (L_i^{gt} - L_i^{g-1,t}) \quad \begin{matrix} i \in V', g \in SL \setminus \{0\}, \\ t \in T \end{matrix} \quad (15)$$

$$Inv_i^{gt} = (1 - L_i^{gt}) \cdot \left(Inv_i^{g-1,t-1} + \sum_{k \in K} q_i^{gkt} + \beta_i^{gt} \right) + (L_i^{gt} - L_i^{g-1,t}) \cdot \alpha_i^{gt} \quad \begin{matrix} i \in V', g \in SL \setminus \{0\}, \\ t \in T \end{matrix} \quad (16)$$

$$\beta_i^{gt} = \left[\left[\left(Inv_i^{g-R-1,t-R-1} + \sum_{k \in K} q_i^{g-R,k,t-R} + \beta_i^{g-R,t-R} \right) L_i^{g-R,t-R} - \alpha_i^{g-R,t-R} \right] \cdot (1-p) \right] \quad (17)$$

$$\begin{matrix} g = 1 + R, \dots, sl + R, \\ t = R + 1, \dots, T, i \in V' \end{matrix}$$

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$$\sum_{j \in V, i < j} x_{ij}^{kt} + \sum_{j \in V, j < i} x_{ji}^{kt} = 2 \cdot y_i^{kt} \quad i \in V, k \in K, t \in T \quad (18)$$

$$\sum_{i \in S} \sum_{j \in S, i < j} x_{ij}^{kt} \leq \sum_{i \in S} y_i^{kt} - y_m^{kt} \quad S \subseteq V', k \in K, \quad (19)$$

$$\sum_{k \in K} y_i^{kt} \leq 1 \quad t \in T, m \in S \quad (20)$$

$$Inv_i^{gt} = \beta_i^{gt} \quad i \in V', t \in T \quad (21)$$

$$\beta_i^{gt} = 0 \quad g = sl + 1, \dots, sl + R, \quad (22)$$

$$\beta_i^{gt} = 0 \quad g = 1 + R, \dots, sl + R, \quad (23)$$

$$u_i^t = \sum_{j=0}^R Inv_i^{sl+j,t} \quad i \in V', t \in T \quad (24)$$

$$z_i^t \leq \sum_{k \in K} y_i^{kt} \quad i \in V', t \in T \quad (25)$$

$$z_i^t \geq \frac{1}{nv} \cdot \sum_{k \in K} y_i^{kt} \quad i \in V', t \in T \quad (26)$$

$$Inv_i^{gt}, d_i^{gt}, q_i^{gkt}, \alpha_i^{gt}, \beta_i^{gt}, u_i^t \in \square^+ \quad i \in V', g \in SL, \quad (27)$$

$$x_{0i}^{kt} \in \{0, 1, 2\} \quad t \in T, k \in K, \quad (28)$$

$$x_{ij}^{kt} \in \{0, 1\} \quad i, j \in V', k \in K, \quad (29)$$

$$L_i^{gt} \in \{0, 1\} \quad t \in T \quad (30)$$

$$y_i^{kt} \in \{0, 1\} \quad i \in V, k \in K, t \in T \quad (31)$$

The objective function (1) minimizes total costs including inventory, outdate and routing costs. Constraints (2) – (4) are the corresponding order-up-to level constraints that guarantee the total quantity of blood units delivered to each

hospital blood bank (HBB) at each time period is either a positive value that makes its inventory level equal to the predetermined target inventory level (TIL) if the HBB is served at that period, and zero otherwise. Constraints (5) show the inventory balance for the community blood center (CBC) for each period and each age group and constraints (6) indicates that at each period t , CBC receives an amount of r^t fresh blood units. Constraints (7) and (8) show the inventory balance at each HBB. The latter constraints are specifically for fresh units while the former ones are for non-fresh items. The mentioned old first (OF) policy is imposed by constraints (9) – (11). These constraints ensure that while an older item is not already transfused making use of the fresher units is not permissible. Constraints (12) ensure that the capacity constraints at each HBB are respected. Constraints (13) are known by definition. Capacity constraints on vehicle are enforced by Constraints (14). Constraints (15) and (16) compute the quantities of variables α_i^{st} for each HBB and each age group at each period. In order to calculate the amount of blood units returned back from assigned inventory to unassigned inventory at the beginning of each period, constraint (17) together with constraints (22) and (23) are included. Constraints (18) and (19) are degree constraints and subtour elimination constraints respectively. Constraints (20) limit the number of vehicles visit each customer in each period to at most one vehicle thereby prevent from split deliveries. Constraints (21) imply that HBBs would not use or keep the expired blood units. Constraints (24) compute the quantities of outdated units in HBBs. Inequalities (25) and (26) establish the relation between z and y variables. This variable is employed in the objective function for calculating wastage costs. Constraints (27) – (31) put into effect integrality and non-negativity conditions on variables.

4.2 Linearization

The mixed integer programming formulation developed in subsection 4.1 contains nonlinear terms in inequalities (15) – (17). This is due the presence of products of binary and integer variables and floor function. These nonlinearities, however, can be easily avoided and converted to their equivalent linear counterpart using well-known approaches. The obtained model is called MILP formulation.

4.3 Valid inequalities

In order to solve the proposed model to optimality, using a branch-and-cut algorithm, it is necessary to strengthen the model through adding several valid inequalities:

$$x_{0i}^{kt} \leq 2 \cdot y_i^{kt} \quad i \in V, k \in K, t \in T \quad (32)$$

$$x_{ij}^{kt} \leq y_i^{kt} \quad i, j \in V', k \in K, t \in T \quad (33)$$

$$y_i^{kt} \leq y_0^{kt} \quad i \in V', k \in K, t \in T \quad (34)$$

$$y_0^{kt} \leq y_0^{k-1,t} \quad k \in K \setminus \{1\}, t \in T \quad (35)$$

$$y_i^{kt} \leq \sum_{j < i} y_j^{k-1,t} \quad i \in V, k \in K \setminus \{1\}, t \in T \quad (36)$$

Since the fleet of vehicles is homogeneous, vehicle symmetry breaking constraints (35) along with constraints (36) for HBB vertices are valid.

5. Branch-and-cut algorithm

There are some similarities and some differences between our B&C and B&Cs of the literature. Both approaches drop SECs from the model and try to add them to the model whenever they are violated. However, the algorithms in the literature add them for any integer solution that violates them while our approach adds them to the model only when the optimum solution is attained. This new approach can reduce the solution time of the problem significantly. In the following, the proposed B&C algorithm with its main three steps is elaborated:

Step 1. Drop subtour elimination constraints (19) from the proposed mathematical formulation and solve the rest of the formulation using CPLEX software.

Step 2. Suppose z^* is the optimum solution found by the previous step and $f(z^*)$ is its objective function value.

Check if z^* has any violated subtour elimination constraints (VSEC):

If the answer is no:

z^* is also the optimum solution of the original problem;
Terminate the algorithm;

Else

Use the *subtour identification procedure* (explained below) to find all of the connected graphs of each vehicle in each period.

Add the corresponding SECs into the model.

Go to Step 3;

Step 3. Set $f(z^*)$ as the lower bound of this new model (with added SECs).

Set z^* as the initial infeasible solution of the new model.

Solve the new model using CPLEX software with z^* as its initial infeasible solution and $f(z^*)$ as its lower bound.

Go to Step 2;

In order to make the best use of CPLEX capabilities in Steps 1 and 3 some remarks should be made. Firstly, it should be kept in mind that valid inequalities (32 – 36) should also be added to the model. These inequalities are very useful in helping CPLEX generate new cuts. Secondly, it is recommended that branching priorities be defined for the problem. It is clear by definition that a higher priority should be assigned to y variables compared to x variables.

In Step 3 when z^* is entered as an initial solution to the CPLEX software, CPLEX starts several heuristics to repair the solution and find a feasible solution. Since z^* is the optimal solution of the problem in the absence of SECs, the repaired solution is expected to be close enough to the lower bound of the new model (which has set to be $f(z^*)$). This mechanism increases the effectiveness of the proposed B&C notably.

5.1 Subtour identification procedure

The core task of Subtour identification procedure is to identify the subtours of a given solution and produces the VSECs.

In this procedure, for each vehicle k in each time period t a tour is opened. This tour starts from the supplier and tries to meet all of the nodes i for which $y_i^{kt} = 1$ only once and in a sequential manner and return back to the supplier. If such a tour was available there would be no VSEC for vehicle k in period t . Otherwise, the vehicle will reach to its starting point before meeting all the nodes. In such condition it is clear that at least two SECs are violated. Then again a new tour is opened from one of the nodes that are not met in the previous tours and traverses through its connected nodes until reaching to its starting point. This procedure continues until all of the nodes are already met. At this stage equal to the number of tours of VSECs for vehicle k in period t are found. For all vehicles this procedure runs and a list of VSECs is produced.

6. Case study

To show the applicability of the proposed model a set of data from a real case of Sari blood center was collected. Sari is a big city in province of Mazandaran in northern Iran wherein a community blood center is located. This center provides the needed blood products for a number of hospital blood banks located in an approximated radius of 150 kilometers. These HBBs are located at 8 cities including Sari itself. The cities are Galugah, Behshahr, Neka, Juybar, Babol, Qaemshahr and Zirab.

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At the end of each week, HBBs predict their needs of blood products for the following 7 days based on their elective procedures and their sense of emergency demands. Then they inform the CBC of their predicted demands for each day of the coming week separately. For 3 days, the demand for a total number of 21 HBBs was recorded. Among the 21 HBBs, 9 are located in Sari, 4 are in Behshahr, two are in Neka and two are in Qaemshahr and 1 in each of the other cities, Galugah, Juybar, Babol and Zirab. Table 1 shows the distances between each pair of cities considered in the study.

Three vehicles are available at the CBC for meeting the demands of HBBs. When demand predicts are received by the CBC it plans an experienced-based schedule for the upcoming week by which the time of visit of each city, the quantity to deliver and the assignment of HBBs to vehicle routes are determined. For this aim, they always put the HBBs located in the same city in the same route and because of practical purposes, a plan that does not respect this constraint is not considered as applicable. Thus, in order to apply our model, we considered aggregated data for each city and reduced the number of demand location to 8 aggregated demand points. These aggregated data are illustrated in Table 2.

Table 1. Distances between each pair of cities (kilometers)

City	Behshahr	Sari	Galugah	Neka	Juybar	Qaemshahr	Zirab	Babol
Behshahr	0	51	25	23	72	75	118	94
Sari	51	0	75	26	21	22	65	42
Galugah	25	75	0	52	97	99	143	143
Neka	23	26	52	0	48	50	94	70
Juybar	72	21	97	48	0	24	84	31
Qaemshahr	75	22	99	50	24	0	38	21
Zirab	118	65	143	94	84	38	0	59
Babol	94	42	143	70	31	21	59	0

Table 2. Aggregated demand data for different periods

City	Periods		
	1	2	3
Behshahr	3	2	3
Sari	79	97	89
Galugah	0	1	1
Neka	6	7	7
Juybar	4	7	6

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Qaemshahr	32	37	36
Zirab	3	3	3
Babol	1	0	1

The demand data shown in Table 2 are for a special blood product, i.e. platelet. To solve the case study, the following parameters are also considered:

$$R = 1 \text{ day}, p = 0.5, r^1 = 293, r^2 = 350, r^3 = 334; Qu = 140; oc = 10000$$

$$TIL_1 = 5, TIL_2 = 100, TIL_3 = 5, TIL_4 = 10, TIL_5 = 10, TIL_6 = 45, TIL_7 = 5, TIL_8 = 3$$

Unit holding costs are considered as is provided in Table 3.

Table 3. Unit holding costs

City	age group 1	age group 2	age group 3
Behshahr	8.19	2.79	4.86
Sari	5.22	3.96	7.02
Galugah	4.59	1.44	1.89
Neka	2.16	3.33	10.62
Juybar	4.41	9.27	4.32
Qaemshahr	3.51	3.87	5.13
Zirab	8.64	1.26	6.66
Babol	0.09	3.6	9.27
Behshahr	4.05	4.95	4.59

Parameter $rc_{ij} = 80 * dc_{ij}$ is considered as a linear function of dc_{ij} where dc_{ij} is the distance between node i and node j given in Table 1. The case problem can be coded as follows: BLOOD-IRP-n-sl-nv-lp (BLOOD-IRP-8-2-3-3)

Using the B&C technique developed in Section 5 the case has been solved to optimality within 4 seconds. It is worth noting that all computations were done on a personal computer with two cores of 2.2 GHz and a 2 GB RAM and no parallel computing technology was accessible for the authors. The optimum solution along a number of numerical results is provided in section 7.

7. Numerical results and discussions

Based on the numerical data provided in the previous section, the optimum objective function value found by B&C is 75872.52 unit costs of which about 73% are for routing costs, 14% are for inventory costs and the remaining are for outdate costs. The 10000 unit cost for outdate costs show that in the optimal solution only 1 platelet unit is wasted. Now, let see which role β_i^{st} plays in decreasing the total costs and making the BLOOD-IRP a more preferred model. To do so, we made a

comparison between our proposed model and the one proposed by Coelho and Laporte (2014). To make the evaluation fair, we dropped outdated costs from our model's objective function, and sales revenue from that of Coelho and Laporte (2014) and changed their model to minimization of sum of routing and inventory costs. Also, since their model is based on a maximum level (ML) inventory policy, we dropped inequalities (4) from our model. After making these changes we had the two models solved to optimality by our proposed B&C. The obtained results were interesting. The best cost of the BLOOD-IRP was 64726.97 while the best cost of Coelho and Laporte (2014) model was about 12% higher because of a significant 15% increase in routing costs in exchange for an only 4% reduction in inventory costs.

The significant cost increase will get more tangible if one checks the values of parameter β_i^{st} in the optimum solution of BLOOD-IRP. For example, $\beta_2^{33} = 48$ in the optimum BLOOD-IRP solution. It means that at the start of period 3, 48 units return back to unassigned inventory and become available for use during this period. However, such capability does not exist in the work of Coelho and Laporte (2014); the model which we believe has been the most inclusive perishable IRP formulation that has been developed by now. Thus, their model is forced to deliver the more units through transportation resulting in a remarkable increment in routing costs.

Now let see how the proposed iterative B&C works. To do so, the objective function values were recorded in each iteration and the percent of increase in the objective function value whenever new cuts were added was observed. Since the above-mentioned case study was solved in 4 iterations, we made use of a more complicated problem with $n=10$, $sl=3$, $nv=1$ and $lp=6$ that takes 8 iterations to solve. The objective function values and the percent of increase in the values are demonstrated using Figures 2 and 3 respectively.

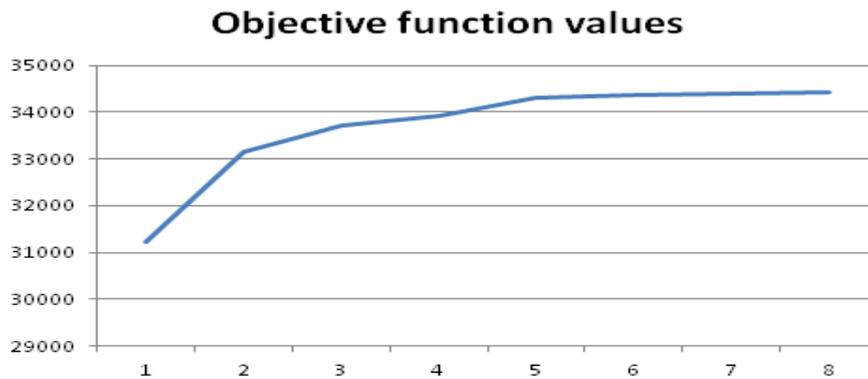


Figure 2. Objective function values per iteration of the proposed B&C method

Figure 2 and Figure 3 illustrate that in the initial iteration adding cuts makes a sharp increase in the hitherto found objective function value. However, after a number of iterations the percent of changes in OFVs becomes nearly zero. It is an indication of the fact that the optimum objective function of the real problem will not be far from the objective function of the on hand problem. It is worth mentioning that the amount of objective function in each iteration is always a lower bound for the optimum solution. Thus, by this algorithm we can claim that after a few iterations we can obtain strong lower bounds. Such high-quality lower bounds can be utilized as the measurements against which the performance of many heuristic and metaheuristic algorithms can be evaluated.

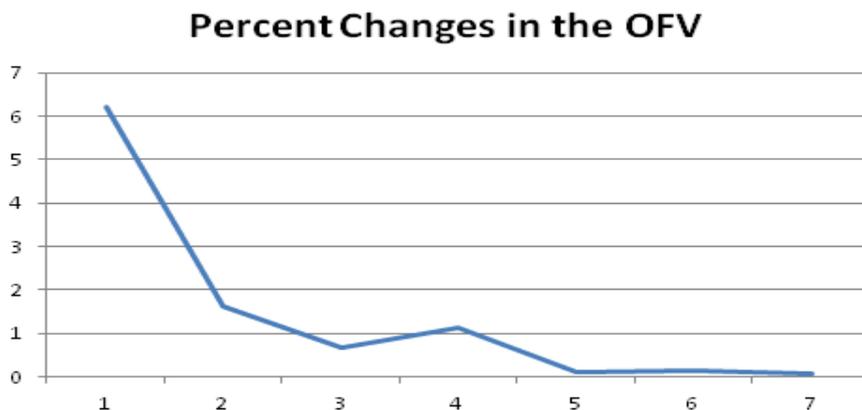


Figure 3. Percent of changes in the OFV per iteration of the proposed B&C method

8. Conclusions and directions for future research

In this paper, for the first of time, blood inventory-routing problem has been officially defined and formulated. Since in the inventory management of blood products the concepts of assigned inventory, crossmatch issuing policies, crossmatch to transfusion ratio and crossmatch release period are frequently used, the current literature on IRP does not provide a good model for blood related activities. Moreover, an iterative B&C method was developed that could solve the BLOOD-IRPs of not excessive size to optimality within reasonable times. The most important feature of this method is to provide strong lower bounds for large-scale problems in a relatively small number of iterations. These lower bounds could be further used for performance evaluation of new heuristic and metaheuristic approaches. Moreover, by studying a case study with real data and presenting the numerical results we showed how the BLOOD-IRP can reduce the costs of a blood supply chain in comparison with other non-blood IRPs. This fact stressed the need for developing IRP formulations that are more responsive to blood logistics. Although we do not believe the proposed model is inclusive enough to address all the challenges that could emerge in inventory-routing of blood products, we believe that it has at least narrowed the existing gap in the literature to a small extent and has made the way more straightforward for the future interested researchers. Furthermore, the area for improving the current study is widely open. For example, considering uncertainties of demands are of high interest. For this aim, development of new robust techniques or making use of robust possibilistic programming theory can be taken into consideration.

REFERENCES

- [1] **Bell, Walter J., Louis M Dalberto, Marshall L. Fisher, Arnold J. Greenfield, Ramchandran Jaikumar, Pradeep Kedia, Robert G Mack and Paul J. Prutzman (1983),***Improving the Distribution of Industrial Gases with an On-Line Computerized Routing and Scheduling Optimizer*; *Interfaces*13 (6):4 – 23;
- [2] **Archetti, Claudia, Luca Bertazzi, Gilbert Laporte and Maria Grazia Speranza (2007),** *A Branch-and-cut Algorithm for a Vendor-managed Inventory-routing Problem*; *Transportation Science* 41 (3):382 – 391;
- [3] **Adulyasak, Yossiri, Jean-François Cordeau and Raf Jans (2012),***Benders Decomposition for Production Routing under Demand Uncertainty*; *In.: GERAD Tech. Rep. G-2012-57, HEC Montréal, Canada*;

- [4] Coelho, Leandro C. and Gilbert Laporte (2013b), *A Branch-and-cut Algorithm for the Multi-product Multi-vehicle Inventory-routing Problem*; *International Journal of Production Research*, **51** (23-24):7156 – 7169; doi: 10.1080/00207543.2012.757668;
- [5] Coelho, Leandro C. and Gilbert Laporte (2013a), *The Exact Solution of Several Classes of Inventory-routing Problems*; *Computers & Operations Research* **40** (2):558 – 565;
- [6] Coelho, Leandro C., Jean-François Cordeau and Gilbert Laporte (2012b), *Consistency in Multi-vehicle Inventory-routing*; *Transportation Research Part C: Emerging Technologies* **24** (0):270-87. doi: <http://dx.doi.org/10.1016/j.trc.2012.03.007>;
- [7] Coelho, Leandro C., Jean-François Cordeau, and Gilbert Laporte (2012a), *The Inventory-routing Problem with Transshipment*. *Computers & Operations Research*, **39** (11):2537-48;
- [8] Adulyasak, Yossiri, Jean-François Cordeau, and Raf Jans (2014), *Formulations and Branch-and-cut Algorithms for Multivehicle Production and Inventory Routing Problems*; *INFORMS Journal on Computing*, **26** (1):103- 120;
- [9] Coelho, Leandro Callegari and Gilbert Laporte (2013c), *Optimal Joint Replenishment and Delivery of Perishable Products*; Technical Report;
- [10] Le, Tung, Ali Diabat, Jean-Philippe Richard and Yuehwen Yih (2013), *A Column Generation-based Heuristic Algorithm for an Inventory Routing Problem with Perishable Goods*. *Optimization Letters*, **7** (7):1481 – 1502;
- [11] Al Shamsi, Ahmed, Ammar Al Raisi and Muhammad Aftab (2014), *Pollution-Inventory Routing Problem with Perishable Goods*. In *Logistics Operations, Supply Chain Management and Sustainability*, 585-596. Springer;
- [12] Jia, Tao, Xiaofan Li, Nengmin Wang and Ran Li (2014), *Integrated Inventory Routing Problem with Quality Time Windows and Loading Cost for Deteriorating Items under Discrete Time*; *Mathematical Problems in Engineering*, vol. 2014, Article ID 537409, 14 pages, Doi:10.1155/2014/537409;
- [13] Coelho, Leandro C. and Gilbert Laporte (2014), *Optimal Joint Replenishment, Delivery and Inventory Management Policies for Perishable Products*; *Computers & Operations Research*, 47:42 – 52;
- [14] Pierskalla, WilliamP. (2004), *Supply Chain Management of Blood Banks*; In *Operations Research and Health Care*, edited by MargaretL Brandeau, François Sainfort and WilliamP Pierskalla, 103-45. Springer US;
- [15] Şahin, Güvenç, Haldun Süral and Sedef Meral (2007), *Locational Analysis for Regionalization of Turkish Red Crescent Blood Services*; *Computers & Operations Research*, **34** (3):692 – 704;

- [16] **Hemmelmayr, Vera, Karl F Doerner, Richard F Hartl and Martin WP Savelsbergh (2009)**, *Delivery Strategies for Blood Products Supplies*; *OR spectrum*, **31** (4):707 – 725;
- [17] **Hemmelmayr, Vera, Karl F. Doerner, Richard F. Hartl, and Martin W.P. Savelsbergh (2010)**, *Vendor Managed Inventory for Environments with Stochastic Product Usage*; *European Journal of Operational Research*, **202** (3):686 – 695;
- [18] **Gunpinar, Serkan (2013)**, *Supply Chain Optimization of Blood Products*. *University of South Florida*;
- [19] **Andersson, Henrik, Arild Hoff, Marielle Christiansen, Geir Hasle and Arne Løkketangen (2010)**, *Industrial Aspects and Literature Survey: Combined Inventory Management and Routing*; *Computers & Operations Research*, **37** (9): 1515- 1536;
- [20] **Leandro, C. Coelho, Jean-François, Cordeau, Gilbert Laporte (2014)**, *Thirty Years of Inventory Routing*; *Transportation Science*, **48** (1): 1 – 19.