Calcedonia ENACHE

Academy of Economics Studies, Bucharest, Romania
calcedoniaenache@yahoo.com

A CROSS-SPECTRAL ANALYSIS OF ROMANIA'S FOREIGN TRADE IN AGRICULTURAL PRODUCTS

ABSTRACT

An interesting variant, from the point of view of the factor analysis in the economy, is represented by the Crossed Spectral Analysis. The simultaneous analysis of the oscillation of different frequencies for two or several data series can be regarded as an analytical approach, by constituent elements, of the correlation between two or several phenomena evolving in parallel in time. The objective of the present study consists in the analysis of the interdependency relations between Romania's agri-food exports and imports focusing on the relation intensity in the synchronous but mainly in asynchronous optics (phase lag) for the different frequency components.

Key words: spectral analysis, trade balance, agricultural products.

JEL Classification: C4, F19, F44.

1. INTRODUCTION

The problem of spectral analysis of time series is obviously of large interest to many applied scientists who utilize spectral analysis in their scientific research. The initial appearance of spectral analysis in the study of macroeconomics time series dates from the middle 1960s, encouraged by the necessity of a more insightful knowledge of the series structure and sustained by the contemporaneous progress in spectral estimation and computation. The first works centered on the problem of seasonal adjustment procedures and on the general spectral structure of economic data. Cross spectral methods were pointed out from the outset as being significant in determining and interpreting the relationship between economic variables.

After the early years, the range of application of such analysis was enlarged to the study of other econometric subjects, among which the controversial trend-cycle separation, the related problem of business cycles extraction and the analysis of co-movements among series, helpful in the study of international business cycles. By the mid 1970s time-domain time-series methods came into vogue, due to the appearance of the important book by Box Jenkins (1970). By using the time domain methodology and adjusting it to the computer, these authors and others have evolved a useful and flexible methodology for carrying out such important functions as prediction (or forecasting). Nold (1972) produced a bibliography of applications of spectral methods in economics covering much of the most active period, recording 101 papers by 68 diverse authors, although some of the references given are only marginally relevant.

Lately, spectral techniques have largely been out of favor by applied econometricians even if they are still used as one of the bundle of empirical techniques available for analysis of time series data. The theoretical aspects of the frequency-domain representations remain important when properties of these diverse methods are considered.

2. THE TIME SERIES MODEL: A BRIEF REVIEW

The model we cover here describes phenomena for which the generating mechanism can be considered, at least for relatively long stretches of time, to be unchanging with the passage of time. The model itself has this unchanging or stationarity property beginning with the infinite past and extending into the infinite future. If X(t) represents a numerical characteristic measured at time t, in the model X(t) is viewed as a random variable foe all times $-\infty < t < \infty$, along with their joint probability distributions, is a stochastic process. By imposing the physical conception of unchanging or stationarity on the probability distributions of the stochastic process, this model becomes stationary stochastic process. The spectral theory involves only the first two moments, the mean and the covariance, of the model. Consequently, it is really only necessary to impose the stationarity conditions on these moments. When this is done, the stochastic process is called weakly (or second-order) stationary.

The stationarity assumption implies that the process means EX(t), (where E denotes the expectation operator), are unchanging in time; $EX(t) \equiv_t m$. Only the behavior of the residuals of the process from this constant mean value is theoretical interest. We can shift to the residuals without changing notation simply by assuming m=0. This will be taken as the value of the mean hereafter. The only process parameter of interest is then the covariance function $R(t_1,t_2) = EX(t_1)X(t_2)$ which describes the stochastic relationship between measured values of the physical phenomena at pairs of time points t_1 and t_2.

The condition of stationarity implies that the physical phenomenon has no relevant origin. That is, for all t, $t_1$and t_2,

$$R(t_1,t_2) = R(t+t_1,t+t_2)$$

This being true, by taking $t=-t_1$, we see that $R(t_1,t_2)=R(0, t_2-t_1)$. That is, the covariance depends on t_1 and t_2 only through the time difference t_2-t_1. The covariance is then completely characterized by the function

$$C(\tau) = R(0,\tau), \ -\infty < \tau < \infty,$$

called the autocovariance function of the process.

The implication of accepting this model for the physical process under study is that all of the interesting and relevant information about the process is then contained in the values of C(τ). One such value is $C(0)=EX^2(t)$, the process variance. The variance represents the average "energy" or power of the process. It has the physical interpretation of a time average of energy because of the property

$$l\, im_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X^2(t)dt = C(0).$$

Without the factor of 1/2T in the last displayed expression, power resembles a sum of squares similar to the usual measure of variability seen in the 'analysis of variance'. The representation of the response vector makes it possible to decompose the total sum of squares into a sum of component sums of squares, each of them representing the contribution of a different factor in the model.

Spectral analysis performs precisely this same operation on time series. In the time series context, the orthogonal vectors of the decomposition are the cosine functions

$$A(\lambda)\cos(\lambda t + \theta(\lambda)),\ -\infty < t < \infty,$$

where, for given frequency λ (in radians per unit time), A(λ) represents the amplitude and θ(λ) the phase of the cosine function. The functions are viewed as being indexed by λ and functions with different values of this index are orthogonal. The fact that these same functions crop up in so many different mathematical contexts is what makes Fourier analysis such a rich field of study. Their appearance in the context of weakly stationary stochastic processes provides the mathematical foundation for the spectral analysis of time series.

3. SPECTRAL PARAMETERS FOR BIVARIATE TIME SERIES

Two weakly stationary process X(t) and Y(t) are said to be stationarily correlated if the covariance $R_{XY}(t_1,t_2) =EX(t_1)Y(t_2)$ depends only on t_1-t_2. The cross covariance function $C_{XY}(\tau)$ is then defined to be :

$$C_{XY}(\tau) = EX(t+\tau)Y(t).$$

A pair of stationarily correlated weakly stationary process constitutes a bivariate weakly stationary process. The cross-covariance function is the new time-domain parameter which, along with the autocovariance functions $C_X(\tau)$, completely describes the relevant properties of the bivariate process.

The corresponding spectral parameter $F_{XY}(d\tau)$, called the cross-spectral distribution or, more simply, the cross spectrum, satisfies the relation:

$$C_{XY}(\tau) = \int e^{i\lambda\tau} F_{XY}(d\lambda).$$

The cross spectrum has discrete and continuous components $p_{XY}(\lambda)$ and $f_{XY}(\lambda)$, called the cross-spectral function and cross-spectral density, for which:

$$F_{XY}(d\lambda)= p_{XY}(\lambda)+ f_{XY}(\lambda)d\lambda.$$

These functions will be non zero only where the corresponding spectral functions or spectral densities are nonzero for both component processes.

The input and output of a linear filter will always be stationarily correlated. Consequently, we can compute the cross spectrum of such series. It is convenient to use the fact that $F_{XY}(d\lambda)$ is the (complex) covariance of $Z_X (d\lambda)$ and $Z_Y(d\lambda)$:

$$F_{XY}(d\lambda) = EZ_X(d\lambda)\overline{Z_Y(d\lambda)}.$$

If Y(t)=L(X(t)) and L has transfer function $D(\lambda)$, then:

$$F_{XY}(d\lambda) = EZ_X(d\lambda)\overline{Z_Y(d\lambda)} = \overline{D(\lambda)}EZ_X(d\lambda)\overline{Z_X(d\lambda)} = \overline{D(\lambda)}F_X(d\lambda)$$

Thus the transfer function, complete with both gain and phase information, can be computed as:

$$D(\lambda) = \frac{\overline{F_{XY}(d\lambda)}}{F_X(d\lambda)}$$

This is only one possible use of the cross spectrum. In general, the cross spectrum contains information about the interrelationship between the components of a bivariate time series in much the same way that a covariance measures the linear relationship between two random variables. In fact, this analogy is much closer than one might imagine. In each frequency dimension λ, the cross spectrum *is* essentially the covariance of the two 'random variables' $Z_X (d\lambda)$ and $Z_Y(d\lambda)$. The chief difference is that these variables are complex valued, which makes the covariance complex-valued as well.

Two different real-valued representations of the cross spectrum are in common use, each depending on a particular expression for complex numbers. The cross spectrum is determined by the cross-spectral density $f_{XY}(\lambda)$. Representing $f_{XY}(\lambda)$ in Cartesian form (with a negative sign) leads to the equation:

$$f_{XY}(\lambda) = c(\lambda) - iq(\lambda),$$

where $c(\lambda)$ and $q(\lambda)$ are the cospectral density and quadrature spectral density, respectively. Thus one complete list of real-valued spectral parameters for the bivariate process would be $c(\lambda)$, $q(\lambda)$, $f_x(\lambda)$ and $f_y(\lambda)$.

A second set of parameters is obtained from applying the polar representation $z = re^{i\theta}$ to $f_{XY}(\lambda)$, where r = IzI and θ = arg z. Here, we let

$$\rho(\lambda) = \frac{|f_{XY}(\lambda)|}{\sqrt{f_X(\lambda) f_Y(\lambda)}} \quad and \quad \psi(\lambda) = \arg f_{XY}(\lambda)$$

These parameters are called the coherence and phase, respectively. Along with $f_X(\lambda)$ anf $f_Y(\lambda)$ they represent an alternate real-valued parameterization of the bivariate process.

Writing $Z_X(d\lambda)$ and $Z_Y(d\lambda)$ in polar form, we have

$$f_{XY}(\lambda) = EZ_X(d\lambda)\overline{Z_Y(d\lambda)} = E|Z_X(d\lambda)||Z_Y(d\lambda|e^{i(\theta_X(\lambda)-\theta_Y(\lambda))}$$

If the phases $\theta_X(\lambda)$ and $\theta_Y(\lambda)$ were constant, the exponential would factor out of the expectation giving

$$\psi(\lambda) = \theta_X(\lambda) - \theta_Y(\lambda)$$

In this case, ψ(λ) would represent the phase lead of the X(t) time series over that of the Y(t) series at frequency.

The coherence behaves almost exactly like the absolute value of a correlation coefficient. For example, 0≤p(λ)≤1, with values near 0 indicating a weak linear relationship at frequency λ and values near 1 a strong relationship. In the time series context, 'linear' refers to linear filters. That is, p(λ) measures the degree to which Y(t) can be represented as the output of a linear filter with input X(t). In fact, $p^{2}(\lambda)$ has precisely the interpretation of the coefficient of determination. It is the proportion of the variation (power) of Y(t) at λ that is attributable to its linear relationship with X(t) in the following sense: If $\hat{L}$ is the linear filter that minimizes the power, $E(Y(t)-L(X(t)))^2$ in the 'residual process' among all filters L, then $p^{2}(\lambda)$ is the ratio of the spectral density

$$f_{\hat{Y}}(\lambda) \text{ to } f_Y(\lambda), \text{ where } \hat{Y}(t) = \hat{L}(X(t)).$$

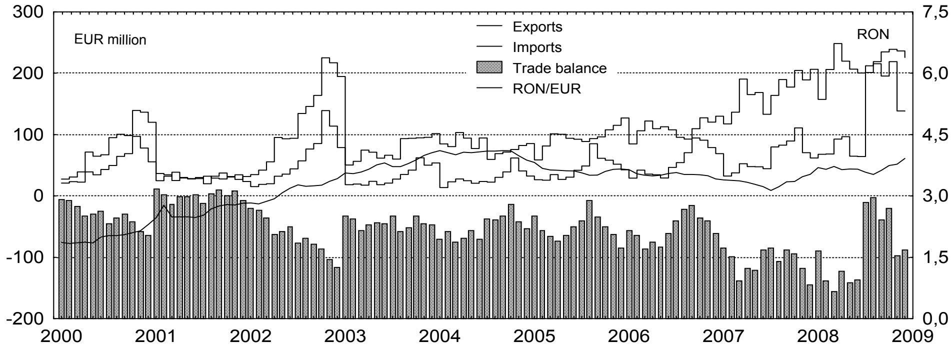
The process $\hat{Y}(t)$ represents the best approximation to Y(t) at frequency λ attributable to $\hat{Y}(t)$. Thus, for example, if Y(t) is exactly a linear function of X(t), Y(t)=L(X(t)), and if D(λ) is the transfer function of L, then we see from earlier calculations that

$$\rho^2(\lambda) = \frac{|f_{XY}(\lambda)|^2}{f_X(\lambda)f_Y(\lambda)} = \frac{|\overline{D(\lambda)}f_X(\lambda)|^2}{f_X(\lambda)(|D(\lambda)|^2 f_X(\lambda))} = 1.$$

Another important property of the absolute value of a correlation coefficient is its invariance under linear transformation. This property also holds for coherence. Thus, if X(t) and Y(t) have coherence function p(λ) and if $U(t)=L_1(X(t))$ and $V(t)=L_2(Y(t))$, where L_1 and L_2 are arbitrary linear filters, then p(λ) will also be the coherence of U(t) and V(t) at all frequencies for which the spectral densities $f_U(\lambda)$ and $f_V(\lambda)$ are both positive. These properties make it possible to translate one's intuition about correlation and simple linear regression directly to coherence for a frequency by frequency assessment of the association between two time series.

4. APPLICATION OF THE CROSS SPECTRUM

For the analysis of the interdependence relations between Romania's agri-food imports and exports I used the monthly data from the period 2000-2008.



Source: The Statistics Bulletins of Foreign Trade,NIS (2000-2009)

Figure 2. The evolution of the Romanian monthly foreign trade in agricultural products and the exchange rate RON/EUR, 2000–2008.

Here are my findings after studying the interdependence relations between Romania's exports and imports of agricultural products and after highlighting especially the connection in the synchronous and asynchronous approaches (phase lag) for the various frequency components:

– The power spectrum of the two foreign trade flows had the following maximum recorded values for the frequencies f = 12 ($f_{E\ (12)}$ = 36072.13) in the case of exports and f = 12 ($f_{I(12)}$ = 43543.08) in the case of imports;

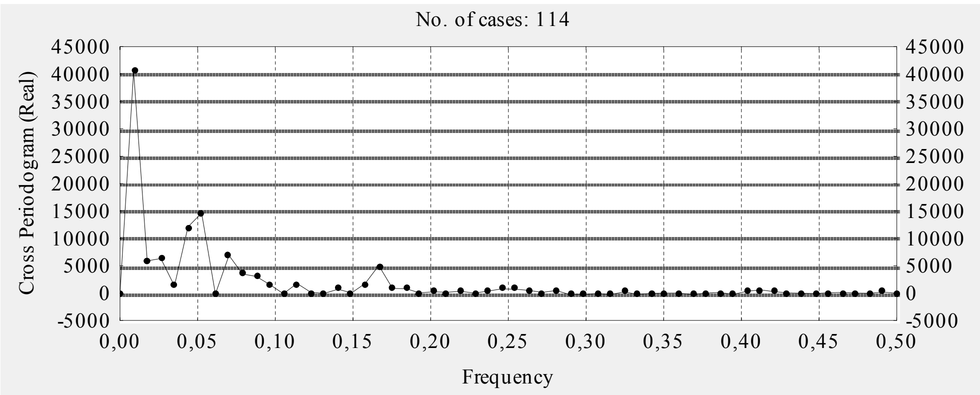


Figure 2. Cross Periodogram (Real).

– The cospectral density (c_j) and quadrature spectral density (q_j) had the following maximum levels: f_{12} = 39421.98, respectively q_{12} = 1957.61 which indicates the possible phase correlations related to those frequencies; namely $q_j \neq 0$ for j=1,2,… 11 shows that asynchronous correlation for such frequencies are possible;

– The phase spectrum values were very close for all frequencies, which confirm the persistence of "delayed" connection, among which there is the one that occurred after 6 months.

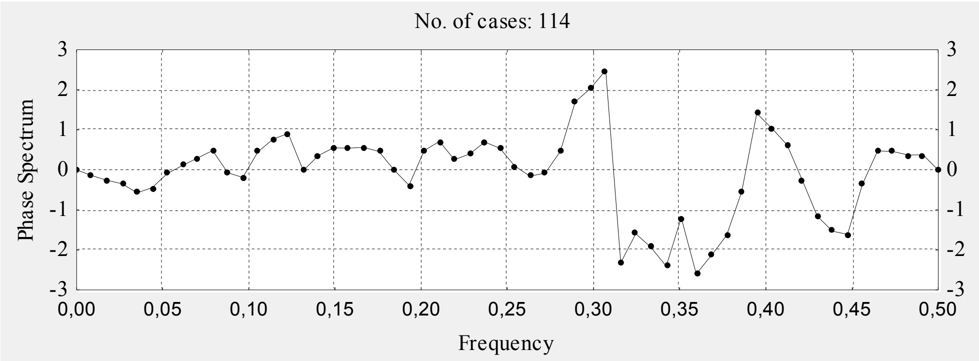


Figure 3. Phase Spectrum.

– The coherence value was significantly higher for j = 12 and thus illustrates the fact that those components describing one complete oscillations across the interval are the most intensively correlated. Therefore intense correlations for frequencies 6 and 4 resulted;

– When export increased by an oscillation, the growth average of the variation of import frequency 12 was 1.13.

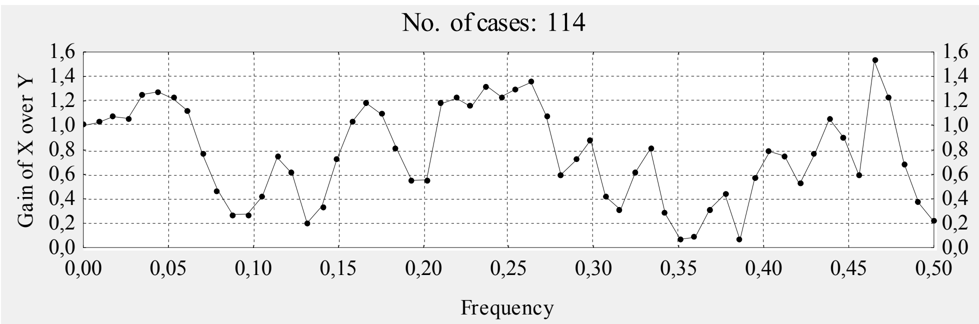


Figure 4. Gain of X over Y.

5. CONCLUSIONS

An important foreign trade characteristic is represented by the fluctuating nature of this activity. Corresponding to the spectral analysis optics, an evolution process in time can be regarded as an aggregate of systematic oscillations whose frequency and amplitude differ. The oscillations of different frequencies are overlapped, forming a complex fluctuating process, which can be investigated in the conditions of the existence of a large enough number of numerical values regarding the respective phenomenon evolution. By the analysis of interdependency relations between the two components of the agricultural products trade balance in which we focused upon the relation intensity in the synchronous and mainly in asynchronous optics (phase lag) for the different frequency components, the present paper highlights the main spectral analysis characteristics and their practical application.

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