Ciprian Ioan RUJESCU[1], Andrea FEHER[2]

[1]"King Michael I of Romania" University of Agricultural Sciences and Veterinary Medicine from Timişoara, Faculty of Agricultural Management, Timişoara, Romania,
[2]Romanian Academy, Timişoara Branch, Research Centre for Sustainable Rural Development of Romania,
rujescu@usab-tm.ro
feherandrea.usab@gmail.com

THE OPTIMAL DISTRIBUTION OF CERTAIN CROPS STARTING FROM A FIXED CAPITAL

ABSTRACT

The study presents an optimization model on the distribution of "n" crops and the total area to be cultivated, starting from a fixed capital that will be fully and optimally used. First a model for two agricultural crops is presented, then the model is generalized to "n" crops. The model will be also described by a concrete example, the related algorithm taking the fixed capital as input data, and will generate the optimal distribution of wheat and corn crops (where n = 2) and the total necessary area to be cultivated; the economic calculations are based on available data from Romania, in the year 2014.

Key words: optimum area, agriculture, mathematical model.

JEL Classification: C61.

1. INTRODUCTION

The purpose of the paper is to determine the optimum production area starting from a fixed capital, optimal to be fully used. This aspect represents a subject of usual interest for the beneficiaries of certain projects in the agricultural sector. The utilized economic values are from the year 2014, being relative and only orientative values. The related expenses depend to a certain extent on local particularities, hence their re-evaluation is necessary.

2. STATE OF KNOWLEDGE

The studies on the optimization of certain economic processes already represent a classical subject for the specialty literature (Floudas 2009). However, the permanent variability of certain defining elements of the economic phenomenon most often induces the need to re-evaluate the size of production factors. Unlike the case of monoculture farming, previously studied by the author (Rujescu et al., 2005–2015), if the farmer decides to establish two or more crops, besides the total optimum area, there is another problem that has to be taken into consideration, i.e. the distribution by optimum individual areas; moreover, in this case, we do not have a clear determination of the total initial area to be cultivated and there will be no issue concerning the saving up of the allocated amounts; the entire capital has to be distributed on land plots with initially unknown sizes – values to be returned by means of the presented algorithm. The model presupposes that the production functions for each crop are known. From a mathematical point of view, it is based on the classical theory of determining the extremes of real functions with a real variable (Steven N. 2008, Intriligator MD, 2002).

3. MATHEMATICAL METHOD

Mathematical model for two agricultural crops (case n = 2). We may consider two agricultural crops, C_1 and C_2, known from a production response point of view, based on the allocated nitrogen quantity (or the nitrogen quantity existing in soil at a certain moment). N_1 and N_2 shall be considered as variable quantities of nitrogen to be allocated. Hence the production functions are known and expressed by the following formula:

(1) $Q_1 = a_1 N_1^2 + b_1 N_1 + c_1,\ a_1 < 0$

(2) $Q_2 = a_2 N_2^2 + b_2 N_2 + c_2,\ a_2 < 0$.

We shall consider p_N the nitrogen unit price, p_1 and p_2 capitalization prices for agricultural productions, s_{f1}, s_{f2} fixed amounts per area unit necessary for crop establishment and maintenance. Consequently, the profit functions for each crop become:

$$Pr_1 = a_1 N_1^2 p_1 + b_1 N_1 p_1 + c_1 p_1 - p_N N_1 - s_{f1},\ a_1 < 0$$

$$Pr_2 = a_2 N_2^2 p_2 + b_2 N_2 p_2 + c_2 p_2 - p_N N_2 - s_{f2},\ a_2 < 0$$

We may now proceed with establishing the optimal fertilization level for the unit of area, for each agricultural crop. This is based on canceling the 1st degree derivative, leading to:

(3) $N_{1\ optim} = \dfrac{p_N - p_1 b_1}{2 p_1 a_1},\quad N_{2\ optim} = \dfrac{p_N - p_2 b_2}{2 p_2 a_2}$

The producer has a fixed initial amount “S” available (fixed capital) to be used and strictly allocated for the two crops. Then the capital restriction is:

$$N_{1\ optim} \cdot p_N \cdot s_{1\ optim} + s_{f_1} \cdot s_{1\ optim} + N_{2\ optim} \cdot p_N \cdot s_{2\ optim} + s_{f2} \cdot s_{2\ optim} = S$$

which leads to:

$$\frac{p_N - b_1 p_1}{2a_1 p_1} \cdot p_N \cdot s_{1\ optim} + s_{f_1} \cdot s_{1\ optim} + \frac{p_N - b_2 p_2}{2a_2 p_2} \cdot p_N \cdot s_{2\ optim} + s_{f2} \cdot s_{2\ optim} = S,$$

$$\left(\frac{p_N - p_1 b_1}{2p_1 a_1} \cdot p_N + s_{f_1}\right) \cdot s_{1\ optim} + \left(\frac{p_N - p_2 b_2}{2p_2 a_2} \cdot p_N + s_{f2}\right) \cdot s_{2\ optim} = S$$

$$\frac{p_N{}^2 - p_1 b_1 p_N + 2p_1 a_1 s_{f_1}}{2p_1 a_1} \cdot s_{1\ optim} + \frac{p_N{}^2 - p_2 b_2 p_N + 2p_2 a_2 s_{f_1}}{2p_2 a_2} \cdot s_{2\ optim} = S$$

We may note that

$$c_1 = \frac{p_N{}^2 - p_1 b_1 p_N + 2p_1 a_1 s_{f_1}}{2p_1 a_1},$$

$$c_2 = \frac{p_N{}^2 - p_2 b_2 p_N + 2p_2 a_2 s_{f_1}}{2p_2 a_2}$$

and the above formula becomes:

(4) $c_1 s_1 + c_2 s_2 = S$.

Denoting by T the total area that the farmer can make available, imposing both the capital and maximum land area restrictions, the problem lies in determining the individual areas, s_1 and s_2, so that:

$$\begin{cases} c_1 s_1 + c_2 s_2 = S \\ s_1 + s_2 < T \end{cases}$$

The sequence from the logical scheme that describes the solutions to the above-mentioned equation is shown in Figure 1.

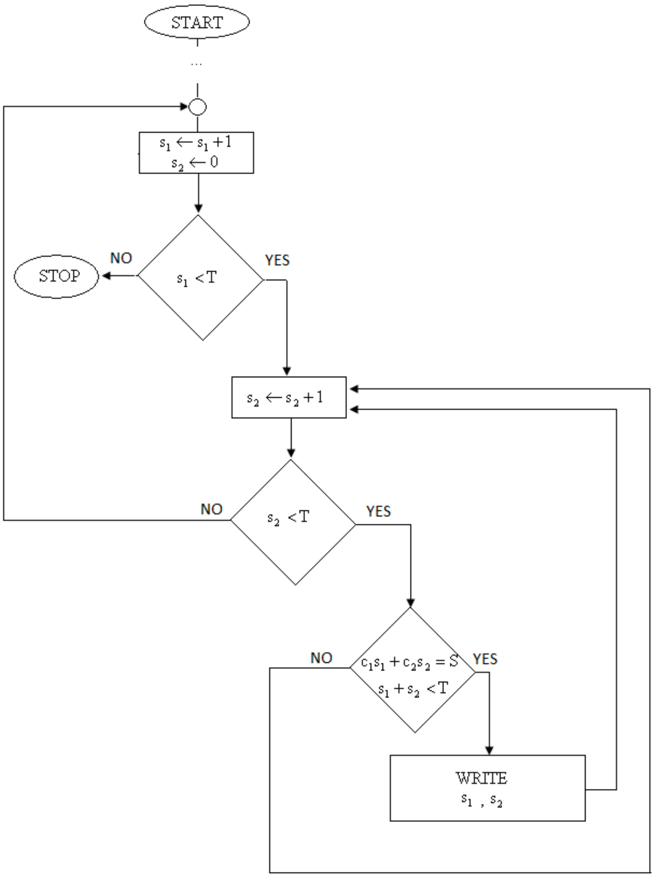

Figure 1. Logical scheme sequence for determining
the optimum distribution of areas for two crops

Generalization for n agricultural crops. We shall now consider “n” crops denoted by C_1, C_2, …,C_n or briefly denoted by C_i, the selling prices for each agricultural product related to them p_i, N_i is the level of nitrogen fertilization for the “i” crop, $s_{f\,i}$ fixed amounts per unit of area, i = 1,…,n.

Let us presume that the production functions are known and expressed as:

$$Q_i = a_i N_i^2 + b_i N_i + c_i ,\ a_i < 0$$

for the crop C_i, i = 1,…,n, and profit functions,

$$Pr_i = a_i N_i^2 p_i + b_i N_i p_i + c_i p_i - p_N N_i - s_{fi} ,\ a_i < 0 .$$

By derivation, we shall obtain:

$$(2a_i N_i + b_i) p_i - p_N = 0 \Rightarrow N_{i\ optim} = \frac{p_N - b_i p_i}{2 a_i p_i} ,\ i = 1,2,...,n$$

The capital restriction becomes:

$$\left(\frac{p_N - b_1 p_1}{2a_1 p_1}\cdot p_N + s_{f_1}\right)\cdot s_1 + \left(\frac{p_N - b_2 p_2}{2a_2 p_2}\cdot p_N + s_{f2}\right)\cdot s_2 + \ldots$$

$$+\left(\frac{p_N - b_n p_n}{2a_n p_n}\cdot p_N + s_{fn}\right)\cdot s_n = S$$

or limited:

$$\sum_{i=1}^{n}\frac{p_N{}^2 - b_i p_i p_N + 2a_i p_i s_{fi}}{2a_i p_i}\cdot s_i = S$$

We shall note afterwards

$$c_i = \frac{p_N{}^2 - b_i p_i p_N + 2a_i p_i s_{fi}}{2a_i p_i}\ ,\ i = 1,2,\ldots,n$$

and we obtain: $\sum_{i=1}^{n} c_i \cdot s_i = S$.

The following production functions are given:

$Q_1 = -0.052\ N^2 + 11.32\ N + 4183$,

for wheat and,

$Q_2 = -\ 0.0975\ N^2 + 14.2\ N + 6412$

for corn.

The data for the year 2014 indicate the price of the 33.4% ammonium nitrate fertilizer at about €340/ton, while the fixed costs per one hectare of wheat are €280. We assume a set price for wheat, p_1 = €0.17/kg. For corn, the fixed costs per hectare are €320, with the selling price p_2 = €0.18/kg.

4. RESULTS AND DISCUSSIONS

Replacing the data in the model for two crops, we shall obtain:

a) For wheat

$$Q(N) = -0.00884\,N^2 + 1.5844\,N + 431.11$$

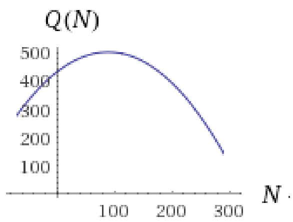

Fig. 2

$$\frac{d\,\mathrm{Pr}_1}{dN} = -2\cdot 0.00884N + 1.5844 = 0$$

$$\Leftrightarrow -0.01768N = -1.5844$$

$$N_{1\ optim} = \frac{1.5844}{0.01768} = 89.6\ kg/ha$$

$$\Pr_1 = \left(-0.052N^2 + 11.32N + 4183\right)\cdot 0.17 - 0.34\cdot N - 280$$

$$\Pr_1 = -0.00884N^2 + 1.5844N + 431.11$$

$$c_1 = 89.6\cdot 0.34 + 280 \cong 310\,E/ha$$

b) For corn

$$Q(N) = -0.01755\,N^2 + 2.216\,N + 834.16$$

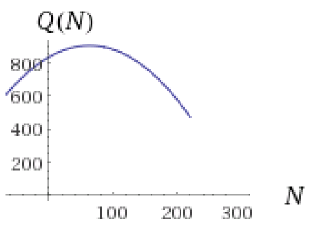

Fig. 3

$$\frac{d\Pr_1}{dN} = -2\cdot 0.01755N + 2.216 = 0$$

$$\Leftrightarrow -0.0351N = -2.216$$

$$N_{1\ optim} = \frac{2.216}{0.0351} = 63.13\ kg/ha$$

$$\Pr_2 = \left(-0.0975N^2 + 14.2N + 6412\right)\cdot 0.18 - 0.34\cdot N - 320$$

$$\Pr_1 = -0.01755N^2 + 2.216N + 834.16$$

$$c_2 = 63.16\cdot 0.34 + 320 \cong 341E/ha$$

Equation (4) becomes: $310s_1 + 341s_2 = S$.

For a few values of the available capital S, i.e. iterating by values of the area allocated for the first crop (30 iterations), the data below is obtained.

Table 1

Optimum distribution of two agricultural crops vs. fixed capital,
s_1 – area cultivated with wheat, s_2 – area cultivated with corn

	S= 10000 E	**S= 15000 E**	**S= 20000 E**	**S= 25000 E**	**S= 30000 E**
s_1 (ha)	**s_2 (ha)**	**s_2 (ha)**	**s_2 (ha)**	**s_2 (ha)**	**s_2 (ha)**
1	28	43	58	72	87
2	28	42	57	71	86
3	27	41	56	71	85
4	26	40	55	70	84
5	25	39	54	69	83
6	24	39	53	68	83
7	23	38	52	67	82
8	22	37	51	66	81
9	21	36	50	65	80
10	20	35	50	64	79
11	19	34	49	63	78
12	18	33	48	62	77

Table 1 (continued)

13	18	32	47	61	76
14	17	31	46	61	75
15	16	30	45	60	74
16	15	29	44	59	73
17	14	29	43	58	73
18	13	28	42	57	72
19	12	27	41	56	71
20	11	26	40	55	70
21	10	25	40	54	69
22	9	24	39	53	68
23	8	23	38	52	67
24	8	22	37	51	66
25	7	21	36	51	65
26	6	20	35	50	64
27	5	19	34	49	63
28	4	19	33	48	63
29	3	18	32	47	62
30	2	17	31	46	61
…	…	…	…	…	…

Source: own calculations

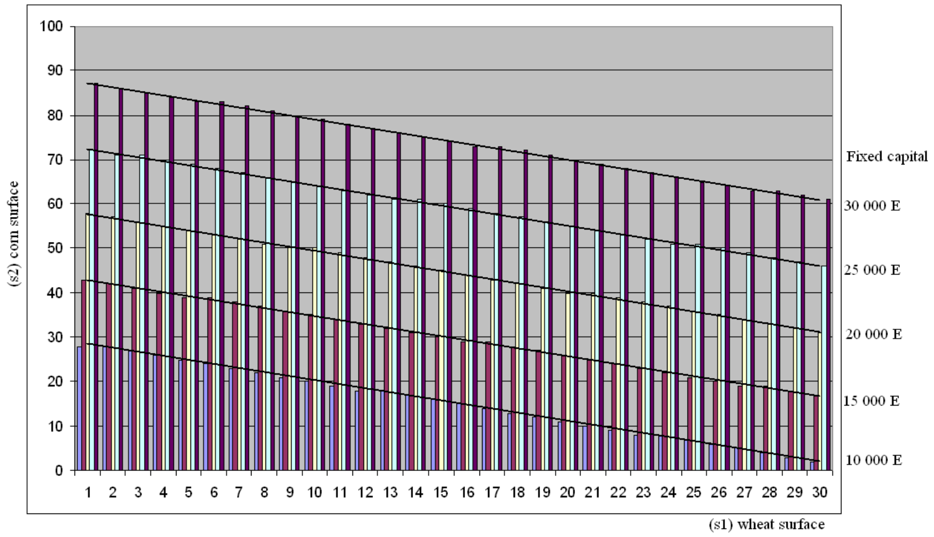

Figure 2. Nomogram establishing the optimum areas under wheat and corn for different values of fixed capital

5. CONCLUSIONS

The values obtained by simulation represent different optimum distributions of the two crops, wheat and corn, for different values of the allocated fixed capital to be used.

The multitude of solutions is not complete in the above-mentioned example, the iteration being performed for values allocated for the wheat crop from 1 to 30 hectares, corresponding to a capital ranging from €10000 to €30000.

It can be noticed that there are various distribution modalities and we can definitely assert that these cannot be randomly chosen or only based on using formal calculations.

REFERENCES

1. Borlan, Z.; Hera, Cr. (1982), *Tabele şi nomograme agrochimice (Agrochemical Tables and Charts)*, Ceres Printing House, Bucharest.
2. Boldea, M., Sala, F., Radulov, I., Crista, F., & Berbecea, A., (2010), *On the Functional Dependence between the Agricultural Yield and the Fertilization with w Single Type of Fertilizer*. Research Journal of Agricultural, 42(3), pp. 38–43.
3. Floudas, Christodoulos A.; Pardalos, Panos (EDS.) (2009), *Encyclopedia of Optimization,* 2nd ed., Springer.
4. Găvruţă Adrian, Rujescu Ciprian (2008), *Algorithm for Optimizing the Size of the Cultivated Areas for an Agricultural Exploitation*, Archives of the University of Craiova, Agriculture, montanology, land registry, vol. XXXVII/B.
5. Goian M., Otiman P.I., Borza I.,Gergen I., Goian Maria, Sâmbotin L., Rădulescu Hortensia, Sala F. (1993), *200 probleme de fertilizare rezolvate (200 Fertilisation Issues Resolved)*, Mera and Prouniversitaria Printing Houses.
6. Hera CR., Eliade Gh., Ghina L., Popescu Ana, 1984, *Asigurarea azotului necesar culturilor agricole (Ensuring Necessary Nitrogen for Agricultural Crops)*, Ceres Printing House, Bucharest.
7. Intriligator Michael David (2002), *Mathematical optimization and economic theory*, Philadelphia: SIAM, Society for industrial and applied mathematics, cop. 2002.
8. Otiman P.I., Creţ F. (2002), *Elemente de matematici aplicate în economia agroalimentară (Applied Mathematics Elements in the Agri-food Economy)*, Agroprint Printing House, Timişoara.
9. Rujescu Ciprian Ioan (2005*), Optimization model for the dimensions of the agricultural exploitation in the case the production selling price varies*, Proceedings of International workshop on Applied Mathematics – on the occasion of opening the National Seminar of Applied Mathematics, Timisoara August 26–27, 2004, p.152–155, Mirton Printing House.
10. Rujescu Ciprian Ioan, Ciolac Ramona, Martin Simona Cristina, Butur Marius (2014), *The use of a two-factor model for the fertilization of agricultural crops in order to determine the optimal production area*, LSMA vol. XVI (1).
11. Rujescu Ciprian Ioan, Ciolac Mariana Ramona, Martin Simona Cristina, Butur Marius (2014), *Minimization plan of nitrogen fertilizers allocation, dictated by the agricultural market*, SGEM 2014 Conference Proceedings, Vol.3, pp. 277–284.
12. Rujescu Ciprian Ioan, Mateia Adrian Nicolae, Martin Simona, Ciolac Ramona (2015), *Determination of the optimum productions surface (monofactorial model) having an initial fixed capital*, Quaestus Multidisciplinary Research Journal, 149.
13. Steven N. Durlauf and Lawrence E. Blume., EDS. (2008), *The New Palgrave Dictionary of Economics.* Second Edition. Palgrave Macmillan.
14. www.apia.org.ro/.
15. http://statistici.insse.ro.