MODELING ASYMMETRIC VOLATILITY IN THE CHICAGO BOARD OPTIONS EXCHANGE VOLATILITY INDEX

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Abstract

Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry. This paper considers the ability of the asymmetric GARCH-type models (TGARCH, EGARCH, APGARCH) to capture the stylized features of volatility in the Chicago Board Options Exchange Volatility Index (VIX). We analyzed daily VIX returns for the period September 26th, 2012 - September 27th, 2017. The results of this paper suggest that in the presence of asymmetric responses to innovations in the market, the EGARCH (1,1) Student-t model which accommodates the kurtosis of VIX return series is preferred.

Keywords: asymmetry, volatility, response to market innovation

JEL Classification: C22, C58, G15

1. Introduction

VIX is the ticker symbol for the Chicago Board Options Exchange[1] (CBOE) Volatility Index, which represents market expectations of volatility over the next 30 days (CBOE, 2017a). VIX indices are computed for various instruments. The most important VIX index is the S&P 500 VIX index, which is computed using data

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[1] The first exchange to list standardized, exchange-traded stock options began its first day of trading on April 26, 1973, in a celebration of the 125th birthday of the Chicago Board of Trade (CBOT).

from S&P 500 options contracts. Option contract prices depend on many factors, the most important of which are the strike price, the price of the underlying instrument, the time to maturity and the expected future price volatility of the underlying instrument. When expected volatility is high, option prices are high. Carefully chosen averages of option prices thus can estimate volatility. VIX options give traders a way to trade volatility without having to factor in the price changes of the underlying instrument, dividends, interest rates or time to expiration - factors that affect volatility trades using regular equity or index options. VIX options allow traders to focus almost exclusively on trading volatility (Ahoniemi, 2006: 2-3).

The VIX uses prices of various S&P 500 options with expirations between 23 and 37 days to measure traders' expectations of volatility. The VIX helps us measure sentiment by telling us how much traders are willing to pay for these options. Typically, the VIX rises when traders are worried about downside risk. Because when traders are worried about downside risk, they'll pay higher prices for downside protection through options. This illustrates how the VIX rises when traders are scared and markets are coming under pressure. Again, because traders were willing to pay up big for downside protection through S&P 500 options (CBOE, 2017b).

The VIX was the first successful attempt at implementing a volatility index. When the index was first conceived in 1993, the methodology was based on a Black-Scholes pricing model given a known market option price (a weighted measure of the implied volatility of eight S&P 100 at-the-money put and call options). The method of calculation for the VIX has varied through time. In 2004, the VIX expanded to use options based on a broader index, the S&P 500, which allows for a more accurate view of investors' expectations on future market volatility (Hancock, 2012: 284-285).

Whaley (2000) points out on the CBOE's 'investor fear gauge' index; it is the forward-looking measure of future stock market volatility, and this index is constructed by market participants through observed option prices. The highest level of VIX implies greater investor's fear. Whaley argues that VIX is more a barometer of invertors' fear (investor sentiment) of the downside risk. Higher VIX levels indicate that the market's expectation of 30-day forward volatility is increased.

One advantage of the VIX is its negative correlation with the movements in the market (S&P 500). According to the CBOE's own website, since 1990 the VIX has moved opposite the S&P 500 Index (SPX) 88% of the time. The inverse relationship between market volatility and stock market returns suggest a diversification benefit which can significantly reduce portfolio risk (Dennis et al., 2006: 382-383; Brandt and Kang, 2004). The national and international economic, political and/or social problems (shocks) affect especially the financial markets with high liquidity and increase the volatility of these markets. Movements of the VIX are largely dependent on market reactions. This means global investors saw uncertainty in the market and decided to take profits/gains or realize/stop losses.

The purpose of this study is to examine the comparative performance of asymmetric volatility models (TGARCH, EGARCH and APGARCH) under Student-*t* and GED distributions by using daily returns of CBOE Volatility Index (VIX). The remainder of this paper proceeds as follows. The section 2 details the Asymmetric GARCH-type models (TGARCH, EGARCH, APGARCH) methodology. The section 3 describes the VIX-CBOE Volatility Index returns data to be used in this study and presents the empirical results. The robustness of these findings is assessed using the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Hannan-Quinn Criterion (HQC), log-likelihood (LL) values. The section 4 contains some concluding remarks.

2. Methodology

Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry in volatility. Asset returns are approximately uncorrelated but not independent through time as large (small) price changes tend to follow large (small) price changes. This temporal concentration of volatility is commonly referred to as volatility clustering and it was not fully exploited for modeling purposes until the introduction of the ARCH model by Engle (1982) and Generalized ARCH (GARCH) model by Bollerslev (1986).

Both the ARCH and GARCH models allow taking the first two characteristics into account, but their distributions are symmetric and therefore fail to model the third stylized fact, namely the “leverage effect” (see Black 1976, Christie 1982 and Nelson 1991). Almost all financial returns data commonly exhibits an asymmetry in that positive and negative shocks to the market do not bring forth equal responses. The underlying concept is that negative shocks increase conditional volatility more than positive shocks, hence there is asymmetry on the impact of good and bad news on the riskiness of the stock market.

Due to an increasing number of empirical evidences saying that negative (positive) returns are generally associated with upward (downward) revisions of conditional volatility, this phenomenon is often referred to as asymmetric volatility in the literature (Goudarzi, 2011). To solve this problem, many nonlinear extensions of the GARCH model have been proposed. Among the most widely spread asymmetric volatility models are the GJRGARCH (Glosten, Jagannathan and Runkle GARCH)) or TGARCH (Threshold-GARCH), EGARCH (Exponential GARCH) and APGARCH (Asymmetric Power GARCH) models. Here, the basic definitions and theoretic properties of the models are discussed.

***TGARCH Model*:** In order to verify the existence of asymmetric volatility in VIX returns, one of the model were introduced independently by Zakoian (1994) and Glosten, Jaganathan, and Runkle (1993). By assigning a dummy variable to negative returns, they were able to allow asymmetric effects of good and bad news on conditional volatility. It is also known as Threshold GARCH (TGARCH) model since we consider $\varepsilon_{t-1} = 0$ as a point of separation of the impacts of negative and positive shocks (Enders, 2004). The generalized specification for the conditional variance is given by:

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{p} \left(\alpha_i + \gamma_i N_{t-i}\right)\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad \left(\omega_0 > 0 \text{ and } \alpha_i, \beta_j, \gamma_i \geq 0\right)$$

$$\left.\begin{array}{l} \varepsilon_{t-i} < 0 \text{ ise, } 1 \ (\textit{bad news}) \\ \varepsilon_{t-i} \geq 0 \text{ ise, } 0 \ (\textit{good news}) \end{array}\right\} = N_{t-i}$$

where σ_t^2 is the conditional variance at time t, α_i is the coefficient for the ARCH process, N_{t-i} is asymmetric effects of good and bad news on conditional volatility and β is the coefficient for the GARCH process. In addition if $\gamma_i \neq 0$ news impact is asymmetric and $\gamma_i > 0$ leverage effect exists (Brooks, 2008: 406).

EGARCH Model: The EGARCH or Exponential GARCH model was proposed by Nelson (1991). The EGARCH model is given by:

$$\ln\left(\sigma_t^2\right) = \omega_0 + \sum_{i=1}^{p} \alpha_i \frac{\left|\varepsilon_{t-i}\right| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

Note that the left-hand side is the log of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be non-negative. The presence of leverage effects can be tested by the hypothesis that. In the equation γ_i represent leverage effects which accounts for the asymmetry of the model. If $\gamma_i < 0$ it indicates leverage effect exist and if $\gamma_i \neq 0$ impact is asymmetric. The meaning of leverage effect bad news increase volatility.

APGARCH Model: The Generalized Asymmetric Power ARCH (APGARCH) model, which was introduced by Ding, Granger and Engle (1993), is presented in the following framework:

$$\sigma_t^{\delta} = \omega_0 + \sum_{i=1}^{p} \alpha_i (\left|\varepsilon_{t-i}\right| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}$$

$$\left(\omega_0 > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ \delta \geq 0 \text{ and } \left|\gamma_i\right| \leq 1\right)$$

where ω_0 is a constant parameter, α_i and β_j are the standard ARCH and GARCH parameters, γ_i is the leverage parameter and δ is the parameter for the power term. A positive (resp. negative) value of the γ_i means that past negative (resp. positive) shocks have a deeper impact on current conditional volatility than past positive (resp. negative) shocks. In the APGARCH model, the power parameter δ of the standard deviation can be estimated rather than imposed, and the optional γ_i parameters are added to capture asymmetry.

The model imposes a Box and Cox (1964) transformation in the conditional standard deviation process and the asymmetric absolute innovations. In the APGARCH model, good news ($\varepsilon_{t-i} > 0$) and bad news ($\varepsilon_{t-i} < 0$) have different predictability for future volatility, because the conditional variance depends not only on the magnitude but also on the sign of ε_t.

Failure to capture fat-tails property of high-frequency financial time series has led to the use of non-normal distributions to better model excessive third and fourth moments. To accommodate this, rather than to use Normal (Gaussian) distribution the Student-*t* distribution and Generalized Error Distribution (GED) used to employ GARCH-type models (Mittnik et al. 2002: 98). Bollerslev (1987) tried to capture the high degree of leptokurtosis that is presented in high frequency data and proposed the Student-*t* distribution in order to produce an unconditional distribution with fat tails.

3. Data and Empirical Results

The section shows the empirical results of models. The VIX returns are analyzed. The characteristics of the data are presented in the first subsection. The second subsection shows the estimated results of asymmetric GARCH-type model specifications and the corresponding qualification tests.

3.1. Data

In this study, we used daily VIX returns for the period September 26th, 2012 – September 27th, 2017. The VIX returns are calculated by log return $r_t = \ln(p_t / p_{t-1})$ of the closing values. The data used in the study is obtained from the Yahoo Finance. Table 1 presents the descriptive statistics for VIX return series (RVIX).

Table 1

Descriptive statistics

	RVIX
Mean	-0.000475
Minimum	-0.299831
Maximum	0.401011
Standard Deviation	0.075429
Skewness	0.745042
Excess Kurtosis	4.026095
Jarque-Bera (p-value)	965.26 (0.000)
ADF-Test (N, 0)*	-37.49039
PP-Test (N, 0)*	-45.27840
ARCH-LM (p-value)	54.01 (0.000)

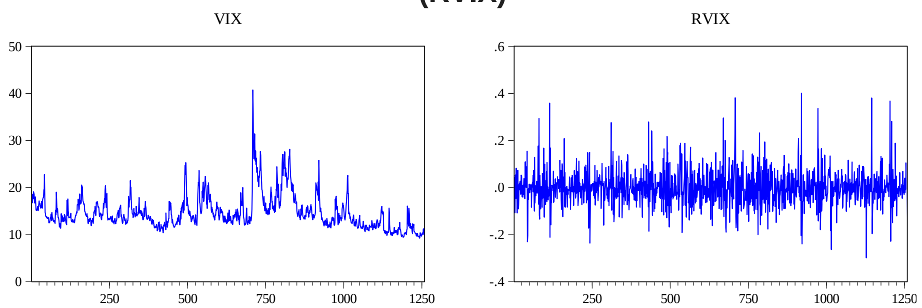
*Notes: * (N, 0) indicates that there is no constant and no trend in the regression model with lag=0.*

According to descriptive statistics, volatility, as measured by standard deviation is high (0.0745042). It is not surprising that this series exhibit asymmetric and leptokurtic (fat tails) properties. The VIX return series have positive skewness, and the excess kurtosis exceeds zero indicating fat tails and leptokurtic distribution. Thus, the VIX returns are not normally distributed. Additionally, by Jarque-Bera statistic and corresponding *p*-value, we reject the null hypothesis that returns are well approximated by the normal distribution. For this reason, in this study we used the Student-*t* distribution and GED distribution, which takes into account fat tail problem. ARCH-LM statistics highlight the existence of conditional heteroskedastic ARCH effect. The VIX return series are subjected to two unit root tests to determine whether stationary I(0). The Augmented-Dickey–Fuller (ADF) and Phillips–Peron (PP) test statistics reject the hypothesis of a unit root at the 1% level of confidence. MacKinnon critical value at the 1% confidence level is -2.57.

As well as descriptive statistics, examining the VIX closing value and return series (RVIX) graphs in Figure 1 shows the volatility clustering in several periods. Volatility clustering which means that there are periods of large absolute changes tend to cluster together followed by periods of relatively small absolute changes.

Figure 1

Daily CBOE Volatility Index Series (VIX) and Log-Return Series (RVIX)



3.2. Estimation Results

In this subsection, the TGARCH, EGARCH and APGARCH models are estimated for VIX return series under Student-*t* and GED distributions. The standard of model selection is based on in-sample diagnosis including Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Hannan-Quinn criterion (HQC), log-likelihood (LL) values, and Ljung-Box Q and Q^2 statistics on standardized and squared standardized residuals respectively. Under every distribution, the model which has the lowest AIC and SIC or highest LL values and passes the Q-test simultaneously is adopted.

Table 2 presents the results of this estimation procedure and from this table one can see that all of the ARCH and GARCH coefficients are statistically significant at the 1% confidence level. Further, β is close to 1 but significantly different from 1 for all models, which indicates a high degree of volatility persistence. β values suggesting that there are substantial memory effects. Furthermore, all models are stationary in the sense that stationary coefficients[2] are lower than 1.

Table 2

Asymmetric GARCH-Type Model Estimation Results

	TGARCH (1,1)	EGARCH (1,1)	APGARCH (1,1)
μ	-0.001529 [-0.9484^b]	0.000168 [0.1010^b]	-0.000301 [-0.1870^b]
ω	0.000757 [4.59691]	-0.498651 [-4.60646]	0.009957 [2.36946^a]
α	0.335758 [4.75128]	0.072726 [1.96258^a]	0.141330 [7.49857]
β	0.718076 [16.2605]	0.918876 [50.8638]	0.821489 [25.9540]
γ	-0.373928 [-5.14374]	0.292190 [8.69130]	-0.999999 [-4.80000]
δ	-	-	0.833803 [6.25461]

[2] *For TGARCH model* $(\alpha+\beta+k\gamma)<1$, *for EGARCH model* $\beta<1$ *and for APGARCH model* $\alpha_i E(|z|-\gamma_i z)^{\delta}+\beta_j<1$.

	TGARCH (1,1)	EGARCH (1,1)	APGARCH (1,1)
LL	1,628.06	**1,637.12**	1,634.58
AIC	-2.5808	**-2.5953**	-2.5896
SIC	-2.5563	**-2.5707**	-2.5610
HQC	-2.5716	**-2.5860**	-2.5789
Q(10)	24.477 (0.006)	26.408 (0.003)	26.320 (0.003)
Q^2(10)	4.8311 (0.902)	3.9096 (0.951)	3.9334 (0.950)
ARCH-LM	0.503458 (0.4780)	0.106238 (0.7445)	0.041699 (0.8382)

a denotes 5% significance level, b denotes not significant; z-statistics of corresponding tests in brackets. LL is the value of the maximized log-likelihood, AIC-Akaike Information Criterion, SIC-Schwarz Information Criterion and Hannan-Quinn criterion (HQC). Q(10) and Q^2(10) are the Ljung-Box statistics for remaining serial correlation in the standardized and squared standardized residuals respectively using 10 lags with p-values in parenthesis. ARCH-LM denotes the ARCH test statistic with lag 1.

The asymmetric volatility models include a leverage term (γ) which allows positive and negative shocks of equal magnitude to elicit an unequal response from the market. Table 3 presents details of this leverage term and reveals that for all models fitted; the estimated coefficient was negative (for EGARCH positive but according to the EGARCH model, the coefficient is interpreted in opposite direction) and statistically significant. This means that past positive shocks lead to higher subsequent volatility than past negative shocks (asymmetry in the conditional variance).

From Table 2, the evidence of long memory process could be also found in the results of the model estimation because the power term (δ) of APGARCH model is 0.833803. The estimated power term was significantly different from two. This means that, the optimal power term was some value other than unity or two which would seem to support the use of a model which allows the power term to be estimated.

The results given in Table 2 show that the all models succeed in taking into account all the dynamical structure exhibited by the returns and volatility of the returns as the Ljung-Box statistics for up to

10 lags on the standardized residuals (Q) significant at the 5% level and the squared standardized residuals (Q^2) non-significant at the 5% level for VIX return series. Also, there is no evidence of remaining ARCH effects according to the ARCH-LM test statistic with lag 1.

In summary, ranking by AIC, SIC, HQC and LL favors the EGARCH (1,1) Student-*t* specification in VIX return series. To conserve space, the results of the models with other distributions declined to present, but they are available upon request.

4. Conclusion

The VIX is based on S&P 500 data. The VIX can be used as a predictor for S&P 500 returns, stock market volatility, economic activity, financial instability, financial crises etc. Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry. The long-memory properties of this index have been investigated in numerous empirical studies that have provided mixed results.

The purpose of this study is to examine the comparative performance of asymmetric volatility models (TGARCH, EGARCH and APGARCH) under Student-*t* and GED distributions by using daily returns of CBOE Volatility Index (VIX). The results of models highlight that in the presence of asymmetric responses to innovations in the market, the EGARCH (1,1) Student-*t* model which accommodates the kurtosis of VIX return series is preferred. The estimation results indicate that strong leverage effects are present in VIX returns. Further, in VIX return series the volatility persistence is higher. Thus, shocks in the VIX return series have substantial memory effects.

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