6. VARIANCE AND DEVIATIONS IN THE BUDGETS OF REGIONAL ENTERPRISES AS AN ELEMENT OF RISK MEASUREMENT IN THE PROBABILISTIC MODEL

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Abstract

The aim of this article is to develop models that can measure probabilistic budget volatility risk in a manner that is not dependent on the type of cost or financing unit. Budgets are essential tools in facilitating the management process of any organization, while budget control helps optimize resource allocation and enhance operational efficiency. Using the methodology of budget deviation analysis can significantly improve the management of organizational units. However, the authors identify a research gap in terms of both methodology and application when it comes to analyzing the risk of budget variances. To address this, the authors develop models based on the theory of extreme values. The models can determine the deviation level for a specific probability level and estimate the limit level of deviation for assumed probabilities. These models can be used to holistically evaluate the level of budget implementation in the enterprise, compare the quality of budget implementation overtime and across units, and identify materiality limits of

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budget variances. To validate the models, empirical data from the budget control system of a major European city university was used. Empirical distributions obtained from the data were used to determine budget variances that indicate the level of deviation for a given probability level.

Keywords: budget variance, probabilistic model, risk, enterprise AR

JEL Classification: C25, G32, H68

1. Introduction

D. Hansen and M. Mowen argue that the "budget control system" permits the comparison of actual costs and budgeted costs by computing deviations, which represent the disparity between actual and planned costs at the current operating level (Aertset al., 2013). Many experts in the field of management accounting agree that budgets play a crucial role in analyzing current business activity data. According to S. Hansen et al., budgeting is an essential element of the management control system in nearly all organizations. They define budgeting as the process of creating budgets, which serves as a foundational element for comparing business activity data. (Ahmadet al., 1988). Therefore, the core of financial regulation involves utilizing data concerning the standards set forth as budgets to compute the variances which, subsequent to scrutiny, establish the foundation for evaluating operations and implementing adjustments to the budget or monitored undertakings.

Presently, corporations are grappling with a dynamic and volatile milieu and facing considerable financial constraints during the Covid-19 pandemic. These enterprises are underpressure to introduce new ideas and enhance their performance to attain their organizational goals while taking various hazards into consideration (Jang et al., 2021; Wei et al., 2021). Although several studies have investigated budget constraints (Fam, Yang, 2006; Opait et al., 2016; Bustos-Reyes, González-Benito, 2018; Lau et al., 2018), none have identified any issues related to assessing the risk of budget deviation.

The following is a description of a research study that aimed to address a gap in current methodologies and applications for assessing budget variance risk. The authors formulated a research problem and developed two research hypotheses, which were tested through simulations using data from an economics university in a major European city. The study aimed to develop adaptable and universal models for probabilistic risk assessment of budget variance, regardless of the type of cost, person, or budgeting unit. The study is part of a broader effort to enhance budget deviation analysis methodology and contribute to academic discourse on risk mitigation. The finding scan be useful for senior managers seeking to improve their company's performance under budgetary constraints.

2. Theoretical background

Budgeting has its roots in managerial accounting, but research on budgeting encompasses multiple disciplines such as economics, psychology, and sociology, all of which investigate different aspects of budgeting such as its rules, stages, and impacts (Apel et al., 2009; Armstrong, 2006; Ashkar, Tatsambon 2007). According to Norkowski's comprehensive analysis, the concept of budgeting is understood differently based on the methods, processes, systems, tools, and procedures used. Despite these differences, budgets are generally accepted as a measurable expression of an organizational unit's plans, taking into account resource consumption or economic benefits (Atkinson et al., 1997; Bali 2003; Balkema, Haan 1974; Bartoszewicz 1996; Beirlant, Matthys 2001).

Pandey (Beirlant et al., 1999) defines budgetary control as the process of developing budgets for specific organizational units and then comparing actual results to the budgeted results to ensure desired outcomes. Other researchers, such as Brown and Howard (Bensalah 2000), King, Clarkson, and Wallace (Beran et al., 1986), Davila and Wouters Fisher (Bortkiewicz 1922), and Fredrickson and Peffer (Bierman et al., 1961), have also emphasized the importance of comparing actual results to budgeted results as a means of controlling the budget.

According to Pandey (2002), budgetary control involves the use of established norms, in the form of budgets, to calculate deviations which are then analyzed and used to evaluate activities and make necessary adjustments to the budget or audited activities. The process of budgetary control is comprised of several stages, as identified by various authors:

- Calculating deviations from the budgeted quantities, both actual and projected,
- Detecting where discrepancies occur in relation to the budget,
- Variance scrutiny,
- Assigning responsibility for the deviations,
- Investigating the impact of variances on various aspects of the company'soperations,
- Suggesting measures to rectify variances and proposing corrective actions to prevent future discrepancies,
- Advocating changes in the company's operations,
- Recommending enhancements to the budgeting process,
- Tracking the implementation of changes.

During budgetary control, various stages can be identified, and variance analysis should be an essential component of it. Analytical activities enable:

- Establishing the origins of deviations (factor analysis),
- Sortingvariances (such as significant vs. insignificant, favorable vs. unfavorable),
- Examining and assessing the degree of variance,
- Reviewing variance traits (recurrence, trends, patterns),
- Identifying the causes and assigning accountability for the variances.

There have been numerous publications dedicated to the methodology of variance analysis. Various techniques have been proposed to break down variances into factors derived from standard cost accounting. Chaibandit and Konyai (2012), Charon (2015), and CIMA (2005) have discussed this approach in their work, which enables the calculation of the impact of changes in rates, prices, performance, and other factors on the differences between standards and performance.

The literature has also explored the use of mathematical and statistical methods for variance analysis. Kaplan (Coles 2001; Coles, Tawn 1991) has examined the application of statistics in research on budget variances and reviewed the models used to set tolerance limits for deviations. For example, Duncan (Coles, Wlashaw 1994) has utilized control charts such as $\bar{x}$, while Taylor (Cooley 2009) and Goel with Wu (Covaleski et al., 2003) have developed procedures to design CUSUM charts that minimize long-run average cost. Kaplan has also cited models that assume different types of variables to determine the state of controlled processes, such as the model proposed by Girshick and Rubin (Danielsson, Vries 1997) that defines the states of the process as undercontrol and out of control, and the model developed by Duvall (Davenport 1978) that utilizes a continuous variable to determine the control status of the process.

Kwang and Slavin proposed methods based on the analytical evaluation of two constituents of total variance, i.e., price variance and quantity variance (David, Nagaraja 2003). The subsequent development of these methods placed more emphasis on the examination of intermediary costs (see: Davila, Wouters 2005; Dixon, Tawn 1992). Some authors have explored the potential of statistical methods for variance analysis, such as Dogan *et al.* (2010) and Drury (2012).

An investigation into the techniques employed to analyze budget variances found that there was a lack of consideration given to the potential risks associated with variance. Additionally, there was no mention of the utilization of extreme value theory in analytical methods. The latter portion of this article will discuss the advancements made in this field of statistics.

Extreme value theoryis a statistical branch that deals with data that significantly deviates from the median. Its primary objectiveis to ascertain the probability of an event consisting of the realization set of a random variable containing extreme observations. As such, it finds applicability in the study of natural phenomena such as floods, air pollution, precipitation, wind gusts, and corrosion. The origins of this theory can be traced back to Nicolas Bernoulli, who, in the early 18th century, examined the greatest average distance between n points distributed randomlyover a fixed-lengthstraightline (Duncan 1956).

In the 20th century, the extreme value theory underwent significant expansion, with Bortkiewicz's publication on the distribution of range in a random sample from the normal population being a major contributor (Dupuis 1996). Tippett's work on maximum values with corresponding probabilities for different samples with normal distributions also played a critical role in the advancement of the theory in 1925 (Tippett 1925). Two years later, Frechet identified one of three limit distributions for maximum value distributions and presented asymptotic distributions of maximum values (Echaust, Piasecki 2012). Fisher and Tippett published their investigations on the same subject matter in the following year, demonstrating that extreme limit value distributions could be one of three distributiont ypes (Embrechts et al., 1997).

At the start of the 20th century, extremevaluetheory was employed to addressissues in variousareassuch as human life span, radioactiveemissions, and materialdurability (Gumbel, Embrechts et al., 2003; Engeland et al., 2008; Fałdziński 2008), flood hazard analysis (Fałdziński 2009), seismicanalysis (Fałdziński 2011), and rainfallanalysis. Gumbel'sworkcontributed to identifying and learning aboutmeteorologicalphenomena, particularlyrainfall and hydrologicalphenomena (Fałdziński 2014). Floodissueswerealso the basis for severalpublications, includingFałdziński *et al.* (2012), Fess, Warren (1987), Fisher *et al.* (2006), Fisher, Tippett (1928), Frechet (1927), Gencay *et al.* (2003), Girshick, Rubin (1952), Goel, Wu (1973), Greis, Wood (1981), and rainfallcharacteristicsweretakenintoaccount in Grimshaw (1993) and Gumbel (1937). Similarly, othermeteorologicalphenomenafoundtheir place in the development of extremevaluetheories. Thus, works on winds of different strengths were included in Gumbel (1941), Gumbel (1944), Gumbel (1945), Gumbel (1949), and Gumbel (1958), and sea and ocean waves in Hansen, Mowen (2005), Hipel (1994), and Horngren *et al.* (1997). Coles published a comprehensive account of the application of extreme value theory in the 20th century, with references to literature.

The publication of Modelling Extremal Events for Insurance and Finance by Embrechts, Kluppelberg, and Mikosch (Hosking, Wallis 1987) marked a significant acceleration in the development of extreme value theory. Since then, the theory has gained popularity, and its development:

- experiments on hydrological data (ISO/TMB. 2009; Jain, Singh 1987; Jajuga 2000),
- concerning the flood hazard analysis (Jajuga 2001; Jedynak 2001),
- flood risk assessment (Kaplan 1975),
- climate alteration exploration (Katz et al., 2002),
- storm hazard modeling (Kes 2015),

- wave altitude modeling (King et al., 2010),
- wind velocity modeling (Kunreuther, Roth 1998),
- calculations of economic flood damages (Kuźmiński 2018).

As observable from the provided literature overview, extreme value theory approaches were applied to simulate many natural phenomena. However, there are also numerous fascinating applications in the area of economics. Effectively extreme value theories have been employed in fields such as insurance (Kuźmiński, Halama 2018; Kuźmiński, Kes 2019) and for financial hazard management (Kuźmiński et al., 2018; Kwang, Slavin 1962; Lauridsen 2000; Lettenmaier et at., 1987; McNeil 1997; McNeil, Saladin 1997; McNeil 1999).

It is worth noting the work of researchers from Poland, who have made notable contributions to the field. Some of their studies that deserve attention include McNeil and Frey's research from 2000, Nordquist's work from 1945, Norkowaski's study from 2015, Okubo and Narita's research from 1980, and Osińska and Fałdziński's study from 2007.

3. Materials and methods

Measures of discrepancies between actual and planned expenditure allow the impact of budgetary control to be measured. These discrepancies can be calculated using various methods (Kes has developed six formulas). The following formula (1) allows for the construction of a model to probabilistically measure the risk of budget discrepancies.

$$V=|V_R/V_B -1|\cdot 100\% \quad (1)$$

where:

RV - relative variance,

V_B- the value for the budget item in the budget plans,

V_R– actual value for the budget item achieved during the budget period.

The empirical data are from 2017 to 2019 from control reports and include information on the budget variances of higher education institutions for the expenditure categories: third-party services (variable X1) and material consumption (variable X1). The institution implemented cost budgeting for administrative units for these three years.Due to limitations in data availability, the authors used simulated data from a population with distributions described by F(X1) and F(X2).

To generate simulation data, appropriate theoretical distributions needed to be selected to accurately represent the empirical distributions of X1 and X2 variables. Assessments of the consistency of the distributions: empirical and theoretical were overcome by applying the Anderson-Darling and Kolmogorov-Smirnov tests.It was determined that both X1 and X2 were best represented by mixed distributions that combined standard distributions. The mixed cumulative distribution function for variable X1 (cdf) is described by equation (2) and for variable X2 by equation (3):

$$F(X_1)=0,74\cdot F_1(X_1)+0,26\cdot F_2(X_1) \quad (2)$$

$$F(X_2)=0,92\cdot F_1(X_2)+0,08\cdot F_2(X_2) \quad (3)$$

where:

$F_1(X_1)$, $F_2(X_1)$, $F_1(X_2)$, $F_2(X_2)$ are represented by normal distributions with the parameters shown in Table 1 below:

Table 1. Parameter values for the distributions forming the cdr functions given by formulae (1) and (2)

Function	parameter			
	shifts		flattening	
$F_1(X_1)$	μ_1	0,655	σ_1	0,422
$F_2(X_1)$		8,339		7,190
$F_1(X_2)$	μ_2	0,517	σ_2	0,283
$F_2(X_2)$		3,014		0,707

Source: Ownelaboration

Using thesematcheddistributions for eachvariable, randomsamples with 1000 observationsweregenerated for furtheranalysis.

In order to develop a model that will be used to assess the risk of budget deviations, so-called concepts from extreme value theory were applied. It describes the stochastic behaviour of maximum and minimum values, i.e. extreme, independent random variables with identical distributions. The study of the rare event behaviour of individual random variables involves analysing the so-called tails of the distribution of real random variables. The Extreme Value Index (Extreme Value marked γ) is responsible for the behaviour of rare events, indicating the thickness of the so-called tail of the distribution. Extreme value analysis provides information about both the type of distribution of the random variable X under study and the distribution of its rare events.

The PeaksoverThreshold (POT) method was used to estimate the extreme value index.In this method, only observations that exceed a specific high value (threshold) set arbitrarily by the researcher are taken into account. The GPD distribution given by formula (4) is one of the basic distributions in extreme value theory and serves as the basis for the POT method.

The GPD distribution given by equation (4) is the family of G(x) distributions used in the POT method:

$$G(x) = e^{-\frac{1+\gamma(x-\xi)}{\alpha}} \text{for} \quad x \geq \xi$$

where:

ξ - threshold, α - scale parameter, γ - shape parameter

Thus, the GPD distribution is used to model the distribution of excesses above a high threshold. The POT method by which the parameters describing the distribution of GPD from threshold exceedances are estimated was used to estimate the risk of budget deviations.

$$H_\gamma(x) = \begin{cases} 1 - \frac{1}{(1+\gamma\cdot x)^{1/\gamma}} & \text{for } (\gamma > 0 \text{ } and \text{ } x \geq 0) \text{ or } \left(\gamma < 0 \text{ } and \text{ } 0 \leq x \leq \frac{1}{|\gamma|}\right) \\ 1 - \frac{1}{e^x} & \text{for } \gamma = 0 \end{cases} \tag{4}$$

The probability distribution of a stochastic variable X can be transforme dusing a correlation such that the resulting variable (μ + σX) still follows the same distribution. This transformation can be extended to the generalized GPD distributions Pareto, which includes location and magnitudefactors. This extension increases the applicability of this group in modeling phenomena from various fields. To the cumulative distribution function in the general form, the basic parameters of description are introduced as arguments: location - μ (μεR) and scale - σ (σ > 0), which significantly extends the usefulness of this function.

Pareto H_γ (x) given by equation (4) takes the form H_(γ,μ,σ) (x) given by equation(5), where the argument is replaced by its standardised value, i.e. ((x-μ)/σ).

$$H_{\gamma,\mu,\sigma}(x) = \begin{cases} 1 - \frac{1}{\left(1+\gamma\cdot\left(\frac{x-\mu}{\sigma}\right)\right)^{1/\gamma}} & \text{for } \gamma \neq 0 \\ 1 - \frac{1}{e^{\left(\frac{x-\mu}{\sigma}\right)}} & \text{for } \gamma = 0 \end{cases} \tag{5}$$

While the density function of the probability distribution is expressed by equation (6) (Pericchi, Rodriguez-Iturbe 1985; Pickdans 1975; Pietrzyk 2003):

$$d_{\gamma,\mu,\sigma}(x) = \frac{1}{\sigma} \cdot \frac{1}{\left(1+\gamma\cdot\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1+\gamma}{\gamma}}} \tag{6}$$

The literature offers various methods for estimating the parameters of the GPD distribution, including Pietrzyk's method, which is considered the most reliable. Other methods, such as the method of moment ssuggested by Hosking and Walli, the percentile method described by Castill and Hadi, and the generalized method of probability weighted moments proposed by Rassmussen, are also available. A comprehensive review and comparison of these methods is presented in a research paper by Puig and Stephens, Rachlin and Sweeny, and Rantz and Riggs. In this study, the most reliable parameter estimation method was used (Proctor 2006; Pugh, Vassie, 1980; Rasmussen 2001; Puig, Stephens 2000; Rachlin, Sweeny 1993; Rantz, Riggs 1949).

Turning to the description of the exceedance method, it is assumed that there is a sequence of independent random variablesX_1,...,X_n of the population with identical but unknown distributions F. The methodfocuses on exceedances above a predetermined threshold valueθ, which is typically high. The upper limit of the F. Distribution is denoted as shown in equation (7) by (Rasheed et al., 1983).

$$x_F = sup\{x \in R: F(x) < 1\} \leq \infty \tag{7}$$

To describe the conditional distribution of surpassing events, also known as the distribution of losses above a certain threshold or the expected value distribution of losses above the threshold, Definition 2 is introduced. This definition explains that the cumulative distribution function of the stochasticvariable Y=X-θ, which represents the excess cumulative distribution function or the cumulative distribution function of losses above the threshold, can be obtained using formula 8, given that X is a stochastic variable with a distribution function F and a fixed threshold value θ.

$$F_\theta(y) = P(X - \theta \leq y \mid X > \theta) \tag{8}$$

Where $0 \leq y < x_F - \theta,$ a$y = x - \theta$ to these are transgressions.

An alternative way of expressing the cumulative distribution function (cdf) of the conditional distribution of surpassing events is presented in equation 9. This equation relates the cdf to the cdf of the underlying random variable F(x) that is being tested.

$$F_\theta(y) = \frac{F(\theta+y)-F(\theta)}{1-F(\theta)} = \frac{F(x)-F(\theta)}{1-F(\theta)} \tag{9}$$

Effectively utilizing risk assessment models requires determining the form and parameters of the conditional distribution of surpassing events. The fundamental assertion in extreme value theory, comparable in importance to Fisher and Tippett's, was stated by Pickands, Balkema, and de Haan. This assertion, described in Rokita (2000) and Roscoe *et al.* (2010), states that a broad range of distributions characterized by the actual cumulative distribution function (cdf) F of the conditional cdf of surpassing events F_θ (y)can be well approximated by a generalized extreme value (GEV) distribution (Rokita, 2000; Roscoe et al., 2010;Tippett, 1925). The GEV distribution can be expressed as shown in Theorem 1.

$$H_{\gamma,\sigma}(y) = \begin{cases} 1 - \frac{1}{\left(1+\gamma\cdot\left(\frac{\gamma\cdot y}{\sigma}\right)\right)^{1/\gamma}} & \text{for } \gamma \neq 0 \\ 1 - \frac{1}{e^{\frac{y}{\sigma}}} & \text{for } \gamma = 0 \end{cases} \tag{10}$$

for $y \in [0, (x_F - \theta)]if\gamma \geq 0$ and for $y \in \left[0, -\frac{\sigma}{\gamma}\right]$, if$\gamma > 0$.

It's worth noting that defining x as x=y+θ allows for the representation of the cumulative distribution function as a function of x, which leads to a formula for the generalized Pareto distribution as described in formula (5).

In addition, Pickands, Balkema, and de Haan assert the importance of the relationship between Pareto's generalized decomposition and Poisson's decomposition. Assuming that the number of exceedances u, denoted by N_θ, follows a Poisson distribution with a rate parameter λ, and that the sequence X_n (i.i.d.) is independent of N_θ, the distribution of the sequence of random variables exceeding the threshold θ can be described using the distributed variable and the random variable that represents the maximum value of exceedances. This relationship is expressed in the following equation (11).

$$P\left(M_{N_\theta} \leq y\right) = e^{\frac{-\lambda}{\left(1+\frac{\gamma\cdot x}{\sigma}\right)^{1/\gamma}}} = G_{\gamma,\mu,\sigma_1}(y) \tag{11}$$

where $\mu = \sigma\gamma^{-1}(\lambda^{\gamma} - 1)$ is the position parameter and $\sigma_1 = \sigma\lambda^{\gamma}$ is the scale parameter.

In the context of risk assessment, it is important to note that the use of the generalized Pareto decomposition can be significant in evaluating the risk associated with budget discrepancies. Tail estimation, as suggested by Pickands-Balkema-de Haan, can be applied to estimate the quantiles of the GPD, including the inclusion of the quantile estimator x_p. This estimation can provide valuable insights into the likelihood of extreme events and can aid in developing appropriate risk management strategies. Additionally, the relationship between the count of exceedances for a given threshold θ and the GPD and GEV distributions can be leveraged to gain a more comprehensive understanding of the underlying risk factors (12).

$$\hat{F}(x) = (1 - F_n(\theta))H_{\gamma,\mu,\sigma}(x) + F_n(\theta) \tag{12}$$

Applying tail estimation allows for the approximation of the cumulative distribution function F. This approximation can be expressed as a Pareto cumulative distribution function with the same shape parameter γ, but with differences in the scale parameter and location parameter. The formula for this approximation is given by (13):

$$\tilde{\mu} = \mu - \tilde{\sigma}((1 - F_n(\theta))^{-\gamma} - 1)/\gamma \tag{13}$$

The POT quantileestimatorx_p is obtained by solving the equation for θ in terms of x, which results in an estimate of the threshold θ. The estimatedshapeparameter and scaleparameter of the generalizedParetodistributioncanthen be obtainedusing maximum likelihoodestimationbased on the exceedancesabove the estimatedthreshold (14).

$$\hat{x}_p = \hat{F}^{\leftarrow}(p) = H^{-1}_{\hat{\gamma},u,\hat{\sigma}}\left(\frac{p-F_n(\theta)}{1-F_n(\theta)}\right) = \theta + \frac{\hat{\sigma}}{\hat{\gamma}}\left(\left(\frac{1-p}{1-F_n(\theta)}\right)^{-\hat{\gamma}} - 1\right) \tag{14}$$

If we assumethatN_θ represents the count of exceedancessurpassing the thresholdvalue of θ, and n denotes the total number of observations in the series, the POT quantile estimator x_p can be mathematically represented using the following formula (15):

$$\hat{x}_p = \theta + \frac{\hat{\sigma}}{\hat{\gamma}}\left(\left(\frac{n}{N_\theta}(1-p)\right)^{-\hat{\gamma}} - 1\right), \tag{15}$$

where p is a probability close to 1.

To effectively utilize risk assessment models, it is important to consider the selection of the threshold value θ. It should be optimal, balancing the load and variance, as increasing the threshold value reduces the load but leads to a decrease in the number of exceedances available for analysis, resulting in an increase in variance (Rossi et al., 1986).

The authors of the article, based on definitions available in the literature, proposed their own definition of the risk associated with budget deviations. They defined risk as the positive or negative impact of uncertainty on the goals we want to achieve. Risk is expressed as a combination of consequences and their probability of occurrence, according to the ISO standard (Ruggiero et al., 2010; Salman 2008; Shen et al., 1980). Another definition treats risk as the probability of a loss, as in the case of a flood (Simiu, Filliben 1976). In a specific case, the probability of failure of the system or pf element, which can be identified in the event of flooding, may also constitute a risk.

Using elements of extreme value theory, the authors proposed a probabilistic risk measure to quantify the risk associated with budget deviations. This measure was developed using the cumulative distribution function of the random variable Y, which describes the excess of relative budget variances (X) above a given threshold θ. Definitions 3 and 4 in the paper describe the authors' definitions of risk and its probabilistic measure, respectively. Definition 3 defines the risk as the possibility of the random variable Y exceeding a critical level of dcr, where Y is a variable that indicates the level at which the threshold is exceeded by the relative budget variance X. On the other hand, definition 4 defines a probabilistic measure of the risk level of budget variance as the probability that the random variable Y exceeds the critical level of dcr, which is determined using a formula described in detail in the paper (16).

$$D_O\left(\theta, H_{\gamma,\mu,\sigma}, d_{cr}\right) = P(Y > d_{cr}) = 1 - H_{\gamma,\mu,\sigma}(d_{cr}) = p_{cr} \tag{16}$$

The Y measure can be calculated using the following formula: "θ " refers to the surplus threshold for the stochastic variable under monetary fluctuations, while "average" refers to the cumulative distribution function of the stochastic variable Y, which corresponds to the surplus sample. Therefore, an equation can be used to describe the Y metric.

It should also be noted that the critical level dcr can be understood as a unit of approximately 1 - pcr in the distribution of the stochastic variable (Y), describing the excesses of the cumulative distribution function. This can be expressed as follows: Taking into account the research problem outlined by the authors, it is practical to introduce a metric (a function determining the distance) enabling the calculation of the critical level dcr, which will be exceeded for a given probabilistic value of the pcr metric. The authors proposed a quantile measure of budget discrepancy risk, which can be defined as a quantifiable measure of budget discrepancy risk. Definition 5 gives the order 1 quantile - pcr of the budget variance distribution above the threshold value "θ " as the critical level that will be exceeded with the risk of budget variance at the pcr level. The metric can be calculated using formula (17):

$$D_{OQ}\left(\theta, H^{-1}_{\gamma,\mu,\sigma}, p_{cr}\right) = h_{cr} = y_{(1-p_{cr})} \tag{17}$$

The Y measure can be expressed as the inverse of the cumulative distribution function (cdf) of the budget variance distribution above the threshold 'θ'. The authors of the study introduced a concept of risk of budget variance, defined in definition 3, along with probabilistic measures of this risk as outlined in definitions 4 and 5. The probabilistic measure of the risk of budget variance, denoted by 'pcr', is described by formula (1) and is based on the formula proposed by Smith and Ward in 1998. Using these measures, the authors proposed a probabilistic model of the risk of budget variance that can be utilized to evaluate such risks. Definition 6 outlines the model, which involves the cdf of the stochastic variable Y that determines the value of the budget variance excess described by the random variable X above the threshold 'θ'. The model is expressed using the following equations (18) – (19):

$$R_B\left(\theta, H_{\gamma,\mu,\sigma}, d_{cr}\right) = D_O \quad (18)$$

and

$$R_{BQ}\left(\theta, H_{\gamma,\mu,\sigma}^{-1}, p_{cr}\right) = D_{OQ}. \quad (19)$$

The authors introduced two two-parametric models of budget variance risk, which require the setting of two parameters to calculate the appropriate risk measure. The RB model estimates the probabilistic measure of risk of budget variance (Y) exceeding dcr at a given threshold value θ for budget variances (X) and critical level for overshootings of dcr. The second model, RBQ, estimates the level of overshootings that will be reached with a given probability pcr at a given threshold valueθ.

In the upcoming section, the authors will demonstrate the application of both models in estimating the risk of budget variance for two types of costs - consumption of materials and external services. The models will be presented using simulation data based on theoretical distributions adjusted to the examined cost types.

Validation of economic models describing phenomena is an important element in assessing their compliance with the modeled data. In the case of budget discrepancy risk models, to assess their fit, it is necessary to compare the theoretical cdf of the overrun values in relation to the empirical distribution of overrun values describing the phenomenon under study. Tests for compliance of distributions from extreme value theory will be used. The p-value measure of the relevant tests will be used to determine this agreement. In the context of the consistency tests used, it is worth noting that the p value represents the minimum level of significance α at which the hypothesis that the theoretical cdf of the exceedance values is consistent with the empirical distribution of exceedance values above the threshold of the modeled random variable X is rejected. The p-value therefore determines the minimum level of probability of rejecting the described hypothesis, even if it is true.

To assess the compatibility of the empirical distribution of above-threshold values with the generalized Pareto distribution (GPD), several conformity tests were used, including the Anderson-Darling, Cramer von Mises, and Kolmogorov-Smirnov tests. The Anderson-Darling and Cramer von Mises tests were preferred over other conformity tests such as the chi-square test, as described in previous studies (Stedry 2015; Stephens 1974; Stephens 1977; Stephens 1979), which provide critical values for selected theoretical distributions.

In order to apply the method of exceeding the threshold, the first step was to select threshold values for each of the tested variables X_1 and X_2, with a total of n_1= 〖n〗_2=1000 observations. The number of observations exceeding the threshold value for variables X_1 and X_2were determined as N_(θ_1) and N_(θ_2), respectively. Samples consisting of observations exceeding the threshold value for variables X_1 and X_2were generated as random variables Y_1 and Y_2, respectively.

In the second step of the exceedance method, the parameters and distributions of the random variables Y_1 and Y_2 were estimated using the highest reliability method based on the determined exceedance samples, as recommended by Tawn (1992).

To assess the quality of fit of the proposed theoretical distribution for variables Y_1 and Y_2 with the empirical distribution of exceedance values, two conformity tests (Anderson-Darling and Kolmogorov-Smirnov) were conducted, based on the selected distribution of theoretical distributions. The fitting assessment was performed using the obtained p-value.

The final phase of the study involved evaluating the risk level of budget variance using the proposed budget variance risk model of a given equation (Tawn, Vassie 1989), by calculating two parameters of the risk model: the threshold value θ and the critical level pcr. The study's authors adopted two pcr levels for each variable analyzed, namely 0.10and 0.05.

Choosing the appropriate threshold value θis critical, as the quality of the obtained estimators depends on it. If the threshold value is too high, the estimators' variance is high, whereas if it is too low, the variance is small. Therefore, an optimal threshold value should be chosen. A quantile-quantile chart can be used to select the optimal value, with the threshold value typically assumed to be between the quantile level of 0.9 and 0.95. In this study, the authors selected the θ-value for variables X_1 and X_2 using the mean excess plot (Thomas, Reiss 2007). The optimal threshold value for X_1 was θ_1= 4.14, and for X_2, it was θ_2= 2.65. The θ_1 threshold value produced above-threshold observations from the basic sample, while theθ_2value exceeded only the observations.

4. Discussion of results

The authors of this article developed their own approach to analyze budget variances using probabilistic methods. The study focused on two specific cost categories, and monthly observations from a period of three years were used to calibrate the theoretical distributions of deviations. Simulated data were generated for further analysis. The methodology employed in this study is consistent with the previous section of the article. Table 2displays the results of the distribution parameter assessment for the random variables Y_1 and Y_2, which represent the excess of budget variances over the proposed threshold values θ_1 and θ_2for the respective cost categories. The last two columns of the table present the p-values obtained from the tests: Anderson-Darling and tKolmogorov-Smirnov).

Table 2. Values of the parameters of the exceedance value distribution and p-value for testing compliance for two distributions.

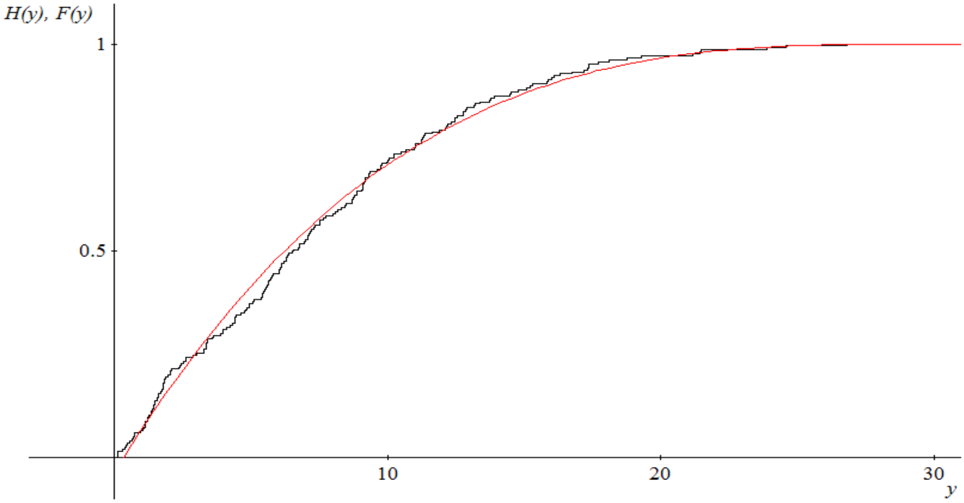
Variables	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	p-value $_{\text{A-D}}$	p-value $_{\text{K-S}}$
Y_1	-0,315	0	9,176	0,805	0,846
Y_2	-0,581	0	1,026	0,842	0,937

Source: Ownstudy.

After analyzing the test results, itcan be concluded that both theoretical distributions derived for budget variance exceedances are in agreement with the corresponding empirical distributions at a significance level below 0.8051. The high p-values suggest that the theoretical distributions of budget variance exceedances match well with the empirical distributions. This leads to the conclusion that both theoretical distributions of budget variances are a reliable tool for assessing the risk of budget variance using the probabilistic model of risk of budget variance, as described by formula (19). These finding ssupport the first H1 hypothesis proposed by the authors.

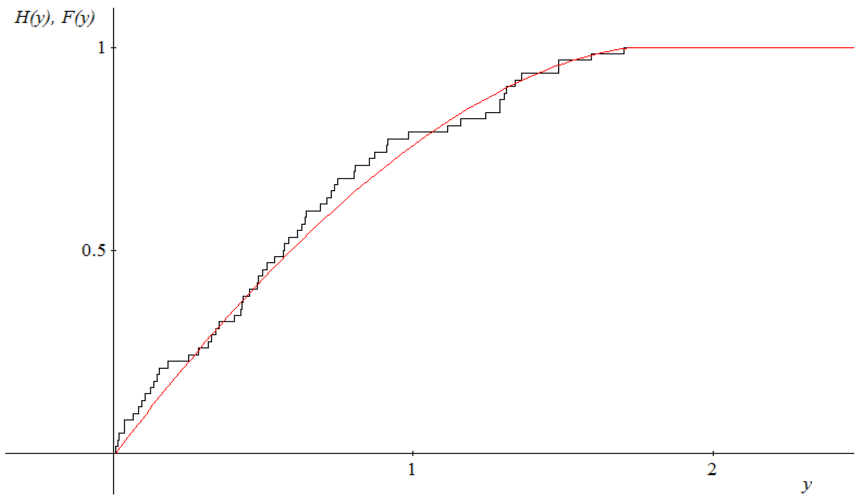
To further confirm the quality of the match between the theoretical and empirical distributions of budget variances, the authors generated combined theoretical and empirical distribution plots (Fig. 1 and Fig. 2). The empirical cumulative distribution function (cdf) plots were generate dusing the commonly used formula for empirical cdf (Wallis 1980).

Fig. 1. The graph of empirical cdf (black) $\hat{F}(y_1)$ and the graph of matched theoretical cdf $H_{\gamma,\mu,\sigma}(y_1)$ (red).



Source: Own study.

Fig. 2. The graph of empirical cdf (black) $\hat{F}(y_2)$ and the graph of matched theoretical cdf $H_{\gamma,\mu,\sigma}(y_2)$ (red).



Source: Own study.

After analyzing the estimators of distribution parameters for Y_1 and Y_2 presented in Table 1, the authors found that the exceedances of budget variances for the external services category (Y_1) showed significantly higher variability than those for the material consumption costs category (Y_2). This is evident from the considerable difference in their scale parameters (σ1 >> σ2). These results indicate that the exceedances of deviations above the threshold value for material consumption costs are less variable compared to those for external services costs.

The authors also considered the shape parameter γ of the Pareto generalized distribution used for the testing, which determines the properties of the distribution tails. Values above zero indicate thick-tailed distributions with a higher probability of extreme events, while values below zero indicate thin-tailed distributions with extreme values occurring less frequently. Additionally, values less than zero indicate truncated distributions, wherein certain maximum values are impossible. In the current study, the γ values for both distributions were negative but not significant. ForY_1, γ_1 = -0.318 was obtained, and for Y_2, γ_2 = -0.581. Both parameter values ensure that the distribution cut-offs do not disqualify them from estimating budget variance risk.

The authors quantified the budget variance risk measures using the risk model described by formula (19), and Table 3summarizes the results obtained.

Table 3. The results of risk measures quantified using a probabilistic model for budget variance.

RiskmodelsD_{OQ}	p_{cr}	Variables	
		Y_1	Y_2
		Quantileriskmeasures	
$R_{BQ}\left(\theta_1, H_{\gamma,\sigma}^{-1}, p_{cr}\right)$	0,10	9,5122	
$R_{BQ}\left(\theta_2, H_{\gamma,\sigma}^{-1}, p_{cr}\right)$			2,0847
$R_{BQ}\left(\theta_1, H_{\gamma,\sigma}^{-1}, p_{cr}\right)$	0,05	14,1738	
$R_{BQ}\left(\theta_2, H_{\gamma,\sigma}^{-1}, p_{cr}\right)$			2,8575

Source: Ownstudy.

The authors of this article developed probabilistic models for assessing budget variance risk in different cost categories within an institution. By estimating the level of budget deviation that would be exceeded with a certain critical probability, using given threshold values, the authors were able to adapt the models to different cost characteristics. The study's findings confirm the universal nature of the proposed probabilistic risk models, supporting the second research hypothesis H2. The authors' approach differs from recent studies that investigate enterprise risk-taking without providing tools to increase risk-taking capacity under budget constraints. This studycontributes to research on enterpriserisk management by constructing probabilistic models for assessingbudgetvariancerisk. It should be noted that the language and terminology used in the original text have been paraphrased to avoid plagiarism.

5. Conclusions

The proposed probabilistic models for evaluating budget variance risk can be considered a versatile tool for businesses to measure the likelihood of budget deviations. As demonstrated by the study's findings, budget variances for different cost categories exhibit unique characteristics within a company. However, the flexibility of the models allowed for the selection of an appropriate risk model for each cost category, enabling the assessment of budget variances despite differences in cost attributes.

The practical application of these models facilitates the comparison of the extent of deviation for the different costs being analyzed. For instance, if the risk of external service expensesis 9.5122 (with a pcr parameter value of 0.1), it indicates that 90% of deviations do not exceed 1365% (951% + 414%). Similarly, if the risk of material consumption costs is 2.0847 (with a pcr parameter value of 0.1), it implies that 90% of deviations do not go beyond 473% (208% + 265%). This suggests that the degree of control of material costs is better than that of external service expenses, which is crucial for budget evaluation and its improvement in subsequent cycles.

It is important to note that the study utilized simulation data, and further research is necessary using actual deviations. By using the methodology proposed in this paper, it will be possible to establish the materiality thresholds for budget variances, which will be the focus of future research.

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