3. VOLATILITY TRANSITIONS IN EUROPEAN STOCK MARKETS: A CLUSTERING-BASED APPROACH

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Abstract

This paper investigates the dynamics of stock market volatilities across fifteen European countries using advanced clustering techniques and transition matrix analysis. The study leverages daily data from MSCI national indices covering the period from January 2008 to June 2024. We use a GARCH(1,1) model on daily log-returns to estimate volatilities. The objective is to analyze similar volatility dynamics across national indices. Therefore, we use a set of clustering algorithms that rely on the employment of Dynamic Time Warping (DTW) in combination with k-means, agglomerative clustering, Gaussian mixture and spectral analysis to identify clusters on a monthly basis. The optimal configuration of clusters is decided repetitively each month using metrics such as the Silhouette Score, Davies-Bouldin Index, Calinski-Harabasz Index, and Cluster Size Standard Deviation. Transition matrices are computed to capture the probabilities of transitioning between clusters over time, both for all countries collectively and for each country individually. The analysis includes the computation of stationary distributions and expected times in clusters, providing insights into the stability and long-term behavior of market volatilities. Our findings highlight significant differences in volatility patterns across countries, with implications for investors, policymakers, and financial analysts.

Keyword: Stock Market Volatility, GARCH Model, Dynamic Time Warping (DTW), Clustering Analysis

JEL Classification: C61, C38, C58

1. Introduction

The analysis of stock market volatility lies at the core of financial research as it provides insights into market behavior, risk management, and economic stability. As a gauge for risk, volatility impacts investment decisions, portfolio management, and financial policy formulation. This study offers a new perspective on how stock market volatility shows similarities in various European

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countries by means of clustering techniques and transition matrix analysis. The central research question explores how volatility patterns in European stock market indices evolve over time, focusing on their key characteristics and stability.

By addressing this question, we aim to reveal the underlying structures and transitions in market volatilities, providing a comprehensive view of their dynamics. This study makes several contributions to existing literature. First, it makes use of advanced clustering techniques, such as Dynamic Time Warping (DTW) and Gaussian Mixture Models (GMM), in analyzing similar behavior for volatilities across national indices. Previous studies have mainly relied on simpler methods or limited datasets. By incorporating multiple clustering methodologies and robust evaluation metrics, we provide a more detailed and nuanced understanding of volatility patterns. Additionally, this research introduces the novel use of transition matrices to track the movement of indices between different volatility clusters over time, a new approach in financial market analysis.

Our analysis uses daily data from MSCI national indices for fifteen European countries, with a time sample covering the period from January 2008 to June 2024. The countries included are Austria, Belgium, Czech Republic, Germany, Spain, France, United Kingdom, Greece, Hungary, Ireland, Italy, Netherlands, Poland, Romania, and Sweden. Volatilities were computed by applying a GARCH(1,1) model on log-returns at the daily frequency.

To examine the similarity of dynamics for these volatilities, we used several clustering techniques that combine DTW with k-means, agglomerative clustering, GMM, and spectral analysis. We evaluated each technique monthly by means of metrics such as the Silhouette Score, Davies-Bouldin Index, Calinski-Harabasz Index, and Cluster Size Standard Deviation. We designed an aggregated ranking system that helps us to automatically choose the best-performing model each month. Transition matrices were then calculated for each national index to capture the probabilities of moving between clusters. This helped us to obtain insights into the stability and evolution of volatility patterns.

The results of this analysis have implications that could be valuable to investors, policymakers, and financial analysts. As such, firstly, investors can enhance portfolio management and risk assessment by understanding the stability and dynamics of similar volatility behaviors, which helps investment decisions. Secondly, policymakers can use these insights to formulate strategies that foster financial stability and mitigate systemic risks. Lastly, financial analysts can benefit from the revealed temporal dynamics and transition probabilities to formulate analysis for potential future trends.

The study continues with a section that reviews the related literature, a section that describes the data and the methodological elements employed in this analysis, a section of results and some concluding remarks.

2. Literature review

Several studies explored the nature of volatility in financial markets, with seminal contributions made by (Engle, 1982) who introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, and (Bollerslev, 1986) who extended it to the Generalized ARCH (GARCH) model. These mathematical models became standard frameworks for analysing and forecasting financial market volatility. Additionally, the seminal work of (Poon and Granger, 2003) realized a comprehensive review of volatility forecasting methods and issues and evoked methodologies and modelling strategies that are still relevant today. From a different perspective, given its behavioral nature, market volatility has significant implications for the general field of behavioral finance, as described by Barberis and Thaler (2003). While many methods have been developed for forecasting market trends and stock market volatility, a universally applicable model remains elusive. Recent research has shifted towards identifying predictive models that effectively manage non-linearities, noise, and uncertainty, as even minor improvements in accuracy can lead to substantial investment gains (Janková and Dostál, 2022; Onalan, 2022; Bang and Ryu, 2023).

Clustering techniques have been widely applied in finance to identify patterns and group similar entities. Early applications include sectoral classification of stocks and identifying similar trading behaviours. More recently, clustering has been employed to analyze volatility patterns. Relevant studies for the analysis developed in this paper are Cont (2007) and Cai, Le-Khac and Kechadi (2016). They applied clustering to financial time series and provided a framework that reveals its usefulness in uncovering hidden structures and relationships within financial data.

We chose the Dynamic Time Warping (DTW) methodology due to its strength in measuring similarities between temporal sequences. Introduced in the context of speech recognition by (Berndt and Clifford, 1994), DTW was also employed in several applications in finance. Papers by Han (2024) and Majumdar and Laha (2020) demonstrated the efficacy of this method in aligning and clustering financial time series, with the notable result that it can provide more accurate and meaningful comparisons than traditional distance measures.

Gaussian Mixture Models (GMM) are widely used for clustering and density estimation. In the financial domain, GMM has been employed to model return distributions and to cluster financial assets. Studies by McLachlan and Peel (2004) and Fraley and Raftery (2002) have provided comprehensive reviews of GMM applications, emphasizing their flexibility and robustness in capturing the complexities of financial data distributions.

Another method that we included in our battery of clustering techniques is spectral clustering. This method uses the eigenvalues of similarity matrices, and it was applied in various fields, including finance, for the detection of similarities. The method was introduced by Ng, Jordan and Weiss (2002), highlighting its effectiveness in identifying clusters with complex structures. In financial applications, spectral clustering was used to segment markets and identify co-movements between assets, as demonstrated by studies such as those by Li and Tian (2017) and Cortés *et al.* (2024).

To ensure a valid grouping of volatility, we designed an algorithm to evaluate the clustering results and decided to use metrics such as the Silhouette Score, Davies-Bouldin Index, and Calinski-Harabasz Index. Rousseeuw (1987) introduced the Silhouette Score as a measure of how similar an object is to its own cluster compared to other clusters. Davies and Bouldin (1979) proposed an index based on cluster dispersion and separation, while Caliński and Harabasz (1974) developed an index that considers the variance ratio within and between clusters.

The next step of our analysis consisted of designing and evaluating transition matrices. These concepts were widely used in finance to model and analyze the dynamics of states or regimes. Credit rating transitions, for example, are often modelled using transition matrices, as explored by Jarrow *et al.* (1997). In the context of volatility, transition matrices can capture the probabilities of shifting between different volatility regimes. Studies by Hamilton (1989) on Markov switching models and more recent work by Ang and Timmermann (2012) on regime-switching volatility provides a theoretical foundation for our approach.

The works on financial market dynamics, especially on capital markets, emphasise the role of pattern recognition, volatility modelling, and cluster formation in understanding stock market behaviour. In this regard, Huang, Hsu and Wang (2007) focus on pattern recognition in financial databases, highlighting the importance of identifying recurring patterns to predict market movements. Their study underscores the utility of advanced algorithms in discerning meaningful trends amidst vast datasets, which can significantly enhance predictive accuracy. Albu *et al.* (2015) characterize the dynamics of asymmetric volatility across European stock markets using a GARCH-based modelling method combined with the Markov Switching approach. It demonstrates how volatility asymmetry influences sentiment indexes through the MIDAS methodology, establishing a clear relationship between these variables. On the other side, Lupu, Hurduzeu and Nicolae (2016) perform a comparative analysis using GARCH models to estimate conditional volatilities for US and European markets. It reveals that benchmark stock indices exhibit higher risks compared to sustainability indices, with these volatility differences providing significant explanatory power for economic sentiment indices.

In the same field, Sandoval Junior (2017) examines the formation and evolution of clusters within networks of financial market indices. His work illustrates how financial markets can be understood as complex networks where indices form clusters based on their co-movement patterns. This clustering behaviour is crucial for identifying systemic risks and understanding market interdependencies. Coroneo, Jackson and Owyang (2020) explore international stock comovements using endogenous clusters. Their study applies a novel clustering methodology to reveal how stocks from different countries group together based on their return characteristics. This approach allows for a deeper understanding of global market integration and the factors driving these comovements. More recently, Trivedi *et al.* (2021) investigated volatility spillovers, cross-market correlations, and co-movements among European Union stock markets. Their empirical case study showed how interconnectedness among markets can lead to synchronized volatility, affecting regional financial stability.

The studies mentioned above provide a comprehensive view of the integration of pattern recognition, volatility modelling, and clustering techniques and offer a robust framework for understanding and predicting market behaviour in an increasingly complex financial landscape. They also argue for continuing research in the field.

Our paper contributes to the literature by integrating advanced clustering techniques with transition matrix analysis to explore the dynamics of stock market volatilities. By employing multiple clustering methodologies and robust evaluation metrics, we provide a comprehensive framework for understanding volatility patterns across different markets. Furthermore, our use of transition matrices to capture the temporal evolution of clusters offers a novel perspective on market stability and dynamics. This approach enhances the existing body of research by providing deeper insights into the factors influencing market behaviour and volatility.

The literature on volatility modelling, clustering techniques, and transition matrix analysis provides a rich foundation for this study. By building on these established methods and integrating them in a novel way, our research offers significant contributions to the understanding of stock market volatilities and their implications for various stakeholders. Future research can build on this work by exploring additional factors and extending the analysis to other markets and financial instruments.

3. Data and Methodology

3.1 Data Collection and Preprocessing

Our data consists of daily values covering the period January 2008 to June 2024 for the MSCI national indices spanning the following countries (daily closing prices of stock market indices): Austria, Belgium, Czech Republic, Germany, Spain, France, United Kingdom, Greece, Hungary, Ireland, Italy, Netherlands, Poland, Romania, and Sweden.

Our analysis focuses on the similarities of daily volatilities at the European level and their tendency to change within monthly intervals. We used a GARCH(1,1) model to extract these volatilities for each time series of log-returns and we computed some statistical properties. We notice very similar levels of central moments with the exception of the Romanian market, where all moments tend to have larger values.

Figure 1 displays the distributions for daily log returns, and Table 1 presents the statistical proprieties of daily volatilities computed with a GARCH (1,1) model for MSCI national indices.

Figure 1. Distributions for daily log-returns computed for the national MSCI indices

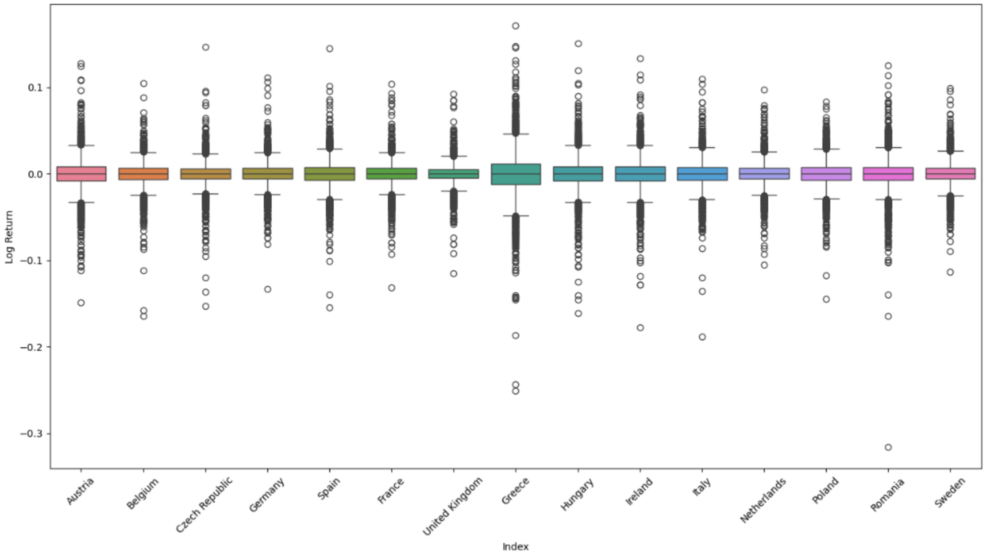


Table 1. Statistical properties of daily volatilities computed with a GARCH (1,1) model for MSCI national indices

	mean	std	min	25%	50%	75%	max	skewness	kurtosis
Austria	0.25	0.12	0.14	0.18	0.21	0.27	1.09	3.03	12.36
Belgium	0.20	0.09	0.10	0.14	0.17	0.21	1.02	3.67	19.74
Czech Republic	0.19	0.10	0.10	0.14	0.16	0.21	1.19	4.35	28.09
Germany	0.19	0.09	0.09	0.14	0.17	0.22	0.79	2.71	10.48
Spain	0.22	0.10	0.11	0.16	0.19	0.25	0.92	2.75	11.01
France	0.19	0.09	0.09	0.14	0.17	0.22	0.83	2.76	10.96
United Kingdom	0.16	0.08	0.09	0.12	0.14	0.18	0.78	3.26	14.90
Greece	0.38	0.16	0.18	0.27	0.33	0.43	1.41	2.12	6.07
Hungary	0.24	0.12	0.13	0.18	0.21	0.27	1.22	3.38	15.93
Ireland	0.25	0.12	0.13	0.18	0.21	0.27	1.25	3.17	14.60
Italy	0.23	0.10	0.11	0.17	0.20	0.25	1.11	2.78	12.24
Netherlands	0.19	0.08	0.09	0.14	0.17	0.22	0.87	2.70	11.72
Poland	0.21	0.09	0.12	0.16	0.19	0.24	0.92	2.52	9.61
Romania	4.19	0.13	0.45	4.16	4.18	4.21	4.65	-14.87	343.38
Sweden	0.20	0.09	0.10	0.14	0.17	0.22	0.73	2.30	6.80

As previously mentioned, the preprocessing steps involved a computation of the log returns for each index using the formula:

$$r_t = ln(\frac{P_t}{P_{t-1}})$$

where P_t is the closing price at time t.

This was followed by the employment of a GARCH(1,1) model to estimate the daily volatilities. The GARCH model is defined as:

$$r_t = \mu + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where σ_t^2 is the conditional variance, ω, α_1, and β_1 are parameters of the model. The conditional volatilities were annualized by multiplying by $\sqrt{252}$.

To analyze the dynamics of volatilities, we performed clustering using various methodologies on a monthly basis. We used several clustering techniques in a repetitive algorithm to extract the best clustering architecture.

K-means with Dynamic Time Warping (DTW)

We initialized the TimeSeriesKMeans model with DTW as the metric and performed clustering:

$$argmin_C \sum_{i}^{k} \sum_{x_j \in C_i} DTW(x_j, \mu_i)$$

where C is the set of clusters, μ_i is the mean of cluster i, and DTW is the Dynamic Time Warping distance.

Agglomerative Clustering with DTW

We computed the DTW distance matrix and performed hierarchical clustering using average linkage:

$$d(i,j) = \frac{1}{(|C_i||C_j|)} \sum_{x \in C_i, y \in C_j} DTW(x,y)$$

where $d(i,j)$ is the distance between clusters C_i and C_j.

Gaussian Mixture Model (GMM)

We fit a Gaussian Mixture Model to the data:

$$p(x) = \sum_{i=1}^{k} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i)$$

where π_i is the mixing coefficient, and $\mathcal{N}(x|\mu_i, \Sigma_i)$ is the Gaussian distribution with mean μ_i and covariance Σ_i.

Spectral Clustering with DTW

We computed the DTW distance matrix and performed spectral clustering using the affinity matrix:

$$A_{ij} = \exp^{-\gamma DTW(x_i,x_j)^2}$$

where A is the affinity matrix and γ is a scaling parameter.

3.2 Clustering Evaluation

The selection of the best clustering model is a critical step in our analysis of the dynamics of volatilities across multiple European stock market indices. This process is repeated on a monthly basis to ensure that the clusters accurately reflect the changing market conditions. The methodology involves a ranking system to determine the best model for each month.

The clustering performance was evaluated using the following metrics:

Silhouette Score

$$s(i) = \frac{b(i) - a(i)}{max\{a(i), b(i)\}}$$

where $a(i)$ is the mean intra-cluster distance and $b(i)$ is the mean nearest-cluster distance for sample i.

Davies-Bouldin Index

$$DB = \frac{1}{k}\sum_{i=1}^{k} max_{j \neq i}\left(\frac{\sigma_i + \sigma_j}{d(c_i, c_j)}\right)$$

where σ_i is the average distance of all samples in cluster i to the centroid of cluster i, and $d(c_i, c_j)$ is the distance between centroids c_i and c_j.

Calinski-Harabasz Index

$$CH = \frac{(N - k)}{(k - 1)} \frac{\sum_{i=1}^{k} |C_i| \left|\left|\mu_i - \mu\right|\right|^2}{\sum_{i=1}^{k} \sum_{x_j \in C_i} \left|\left|x_j - \mu_i\right|\right|^2}$$

where N is the total number of samples, k is the number of clusters, μ is the overall mean of the data, and μ_i is the mean of cluster i.

Cluster Size Standard Deviation

This metric evaluates the balance of cluster sizes by computing the standard deviation of the number of samples in each cluster.

3.3 Ranking and extraction of the best model

The process of ranking the clustering models is the main step in our methodology to ensure the best model is selected for each month. This involves a systematic comparison of the performance metrics obtained from various clustering techniques. Once the clustering labels are generated for each technique, we evaluate them using the four distinct metrics mentioned above: Silhouette Score, Davies-Bouldin Index, Calinski-Harabasz Index, and Cluster Size Standard Deviation. These metrics provide a comprehensive view of the clustering quality, capturing aspects such as cohesion, separation, and balance of cluster sizes.

To begin with, we rank the clustering techniques based on each metric individually. For the Silhouette Score and Calinski-Harabasz Index, higher values indicate better clustering performance, so we assign higher ranks to models with higher scores. Conversely, for the Davies-Bouldin Index and Cluster Size Standard Deviation, lower values indicate better performance, as they suggest tighter, more well-separated clusters and more balanced cluster sizes, respectively. Therefore, models with lower scores in these metrics receive higher ranks. This ranking is performed by assigning a rank order to each model's performance on each metric, creating a ranked list for every metric.

Once we have the ranks for each metric, we calculate the average rank for each clustering technique across all metrics. This involves summing the ranks obtained by each model across the four metrics and then dividing by the number of metrics.

The average rank provides a single consolidated score that reflects the overall performance of the clustering technique, balancing all evaluated aspects. This approach ensures that no single metric unduly influences the selection process, giving a fair chance to all models based on their comprehensive performance.

The model with the highest average rank is then selected as the best clustering model for that month. This model is deemed to provide the most balanced and accurate clustering of the volatility data for the given period. The rationale behind using the average rank is to mitigate the impact of any single metric that may be out of the norm and to ensure that the selected model performs consistently well across multiple dimensions of clustering quality. By using this aggregate ranking method, we aim to capture the overall effectiveness of each clustering technique in a balanced manner.

After determining the best model for the month, we record the clustering labels generated by this model for further analysis. These labels represent the monthly clustering of volatilities, reflecting the grouping of stock market indices based on their volatility patterns. By consistently applying this method each month, we can track the dynamics of these clusters over time, providing valuable insights into the stability and transitions of market volatilities.

This multi-faceted approach to model selection ensures robustness in our clustering analysis. It allows us to adapt to changing market conditions, capturing the most relevant patterns and trends in volatility. This methodology not only enhances the accuracy of our analysis but also provides a reliable foundation for subsequent evaluations and interpretations of the clustering results.

3.4 Transition Matrix

The computation of the transition matrix is a critical component of our methodology, providing insights into the dynamics and stability of the volatility clusters over time. This matrix captures the probabilities of transitioning from one cluster to another between consecutive months, offering a comprehensive view of the temporal evolution of market volatilities. The transition matrix is computed both for all countries collectively and for each country individually, allowing for a detailed analysis at both aggregate and granular levels.

For the overall transition matrix, we start by extracting the cluster labels for each country for every month. These labels, which were determined during the clustering phase, are used to track the movement of indices between different clusters over time. We initialize a square matrix where the number of rows and columns corresponds to the number of clusters. This matrix will eventually contain the transition probabilities from one cluster to another. We then iterate through each pair of consecutive months, counting the number of transitions from each cluster to every other cluster. This involves checking the cluster label for each country in one month and noting the cluster it transitions to in the following month.

The transition matrix T is computed as:

$$T_{ij} = \frac{Number\ of\ transitions\ from\ cluster\ i\ to\ cluster\ j}{Total\ number\ of\ transitions\ from\ cluster\ i}$$

Once we have counted all the transitions, we normalize the matrix to obtain probabilities. Each element of the matrix (T_{ij}) represents the probability of transitioning from cluster i to cluster j. This normalization is done by dividing each entry by the total number of transitions originating from the cluster corresponding to that row. The resulting matrix provides a comprehensive view of how clusters evolve over time across all countries, highlighting common patterns and potential interdependencies in market behavior.

For the transition matrix of each individual country, the process is similar but is performed separately for each country’s data. We extract the cluster labels for each country, creating a unique transition matrix for each. By initializing a square matrix for each country and counting the transitions between clusters month by month, we capture the specific dynamics of each market. This allows us to observe how the volatility patterns within a single country’s indices move between clusters over time. The normalization process is applied in the same way, ensuring that the transition probabilities accurately reflect the movement within each country's volatility clusters.

The separate computation for each country's transition matrix provides a more detailed perspective, revealing the unique characteristics and behaviors of different markets. For instance, while one country's indices might show a high probability of remaining in the same cluster, indicating stability, another country might exhibit frequent transitions between clusters, suggesting higher volatility or sensitivity to external factors. By comparing these individual transition matrices, we can identify country-specific trends and factors influencing market volatility.

Visualizing these transition matrices through heatmaps further enhances our understanding. The heatmaps display the transition probabilities in a color-coded format, making it easy to identify patterns and anomalies.

For the overall transition matrix, the heatmap shows the aggregated behavior of all countries, providing a macro-level view of volatility dynamics. For individual countries, the heatmaps reveal detailed transitions specific to each market, offering a micro-level analysis. These visualizations are essential for interpreting complex data and drawing meaningful conclusions about the stability and evolution of market volatilities.

4. Results

In our analysis, we examined the transition matrices extracted for each country to gain deeper insights into the dynamics of the volatility clusters over time. This involved several steps, including computing stationary distributions and expected times in clusters and visualizing these results through various plots.

Firstly, we extracted the transition matrices for each country, which capture the probabilities of moving from one cluster to another between consecutive months. These matrices serve as the basis for computing the stationary distributions and expected times in clusters. To begin, we computed the stationary distribution for each country's transition matrix.

The stationary distribution represents the long-term probabilities of being in each cluster, assuming the system continues to transition according to the observed probabilities. This was achieved by finding the eigenvector corresponding to the eigenvalue of one for the transpose of the transition matrix. The stationary vector was then normalized to ensure that the probabilities sum to one.

The results of the stationary distribution calculations were stored and subsequently visualized. We created a bar plot comparing the stationary distributions across different countries. Each bar in the plot represents the probability of a stock market index being in a specific cluster in the long run. This visualization clearly compares the dominant clusters in each country, highlighting the differences and similarities in the long-term behavior of market volatilities.

Figure **2** illustrates the stationary distributions of volatility clusters across different countries. Each bar represents the long-term probability of a stock market index being in a specific cluster.

This comparison highlights the dominant clusters in each country, providing insights into the long-term behavior and stability of market volatilities. Notably, we observe that some countries exhibit a high probability of indices remaining in certain clusters, indicating stable volatility patterns, while others show more evenly distributed probabilities across clusters, suggesting more variable market conditions.

Next, we computed the expected time that indices spend in each cluster before transitioning to another cluster. This metric provides insights into each cluster's stability and duration. The expected time in clusters was calculated using the fundamental matrix derived from the transition matrix, specifically focusing on the transient states. By inverting the identity matrix minus the submatrix of the transition matrix (excluding absorbing states), we obtained the fundamental matrix, which was then used to compute the expected times.

Figure 2. Comparison of Stationary Distributions across Countries

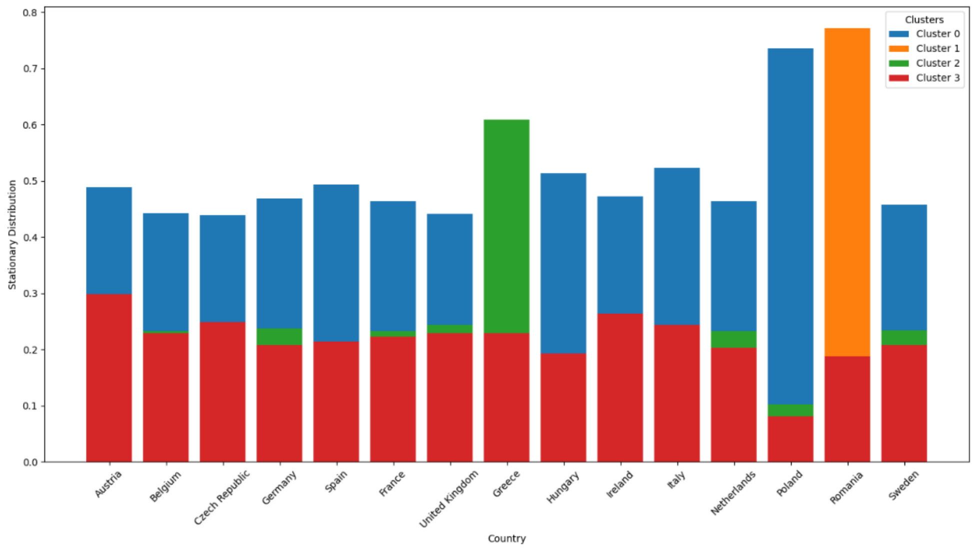
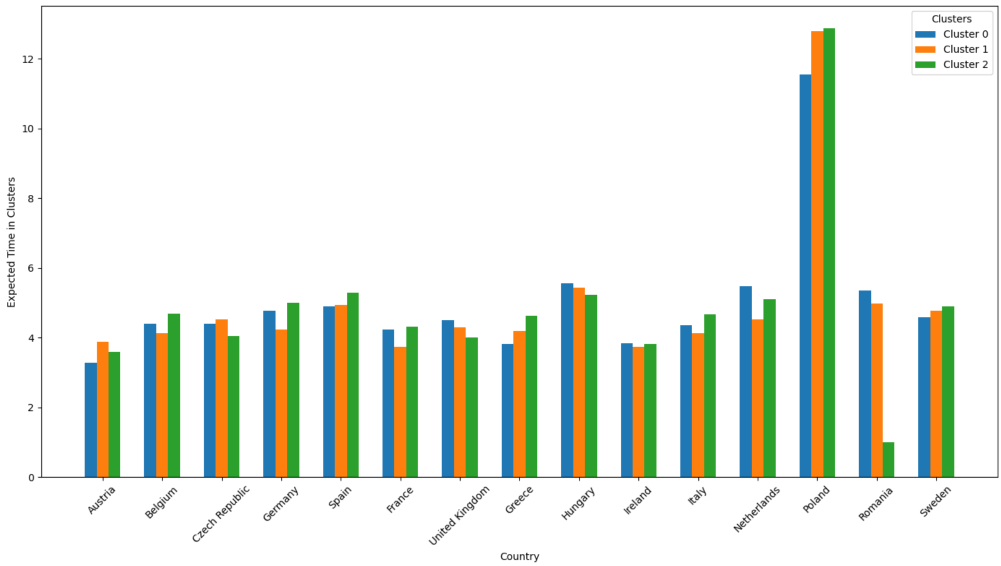


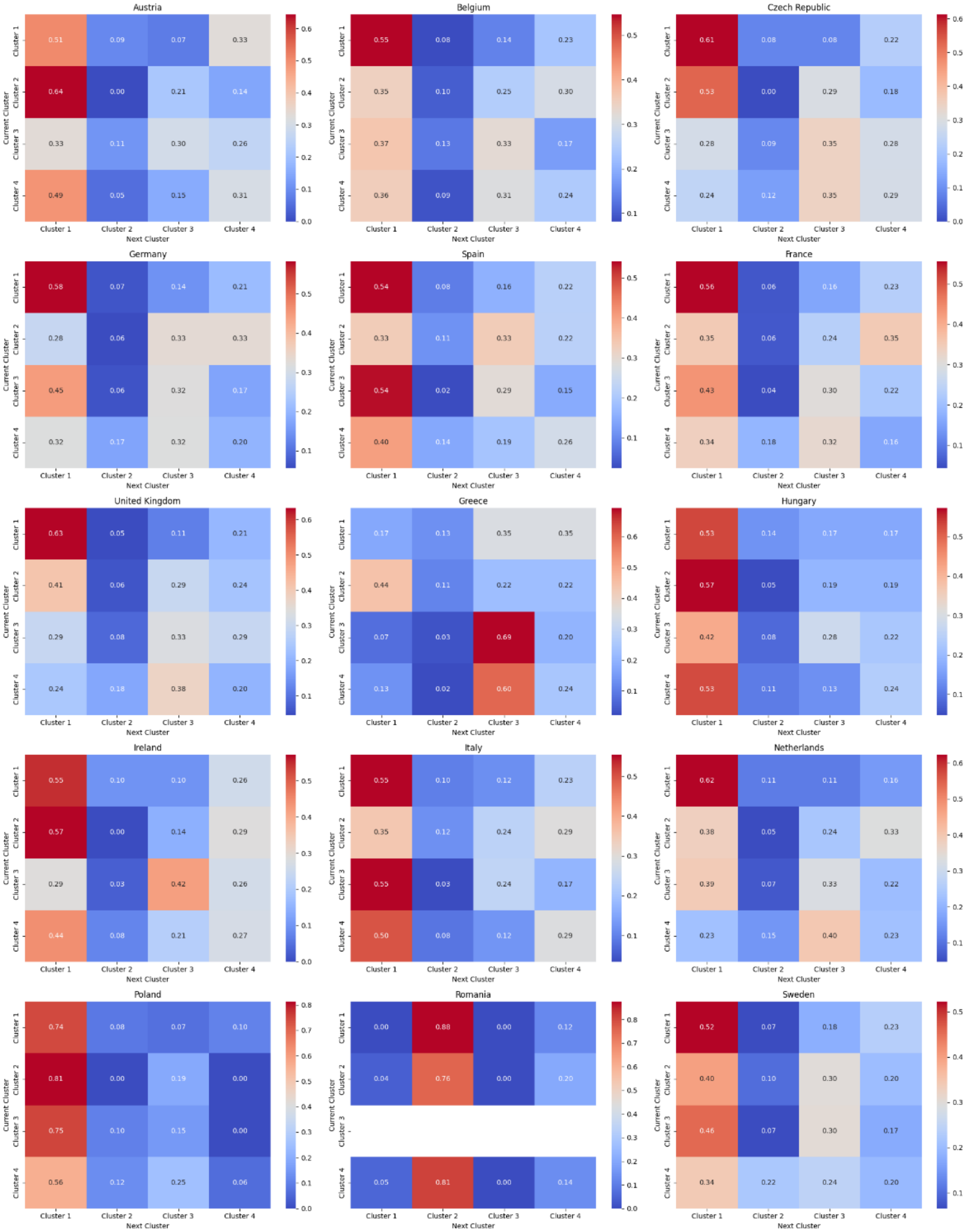
Figure **3** presents the expected time that indices spend in each volatility cluster before transitioning to another cluster.

Figure 3. Comparison of Expected Time in Clusters across Countries



This metric offers insights into the stability and duration of each cluster. The bar plot compares these expected times across different countries, showing how long indices typically remain within each cluster.

Figure 4. Multiple heatmaps – results of clustering for volatilities using the best clustering method



Countries with longer expected times in certain clusters indicate more persistent volatility patterns, whereas shorter expected times suggest more frequent transitions and higher market volatility.

The expected times were visualized using a bar plot, where each bar represents the expected number of time periods an index spends in a particular cluster for each country. This plot allows us to compare the stability of clusters across different countries, identifying which clusters tend to be more persistent and which are more transient.

To provide a comprehensive overview of the transition dynamics, we also created heat maps of the transition matrices for each country. These heatmaps visually represent the probabilities of moving between clusters, with the colour intensity indicating the strength of the transition probabilities.

By examining these heatmaps, we can identify patterns and trends in the transition behavior of each country's stock market indices, offering valuable insights into the volatility dynamics.

Figure 4 contains multiple heatmaps representing the transition matrices for each country. Each heatmap displays the probabilities of transitioning between clusters, with colour intensity indicating the strength of these probabilities.

This visualization helps identify patterns and trends in the transition behavior of each country's stock market indices. By examining these heat maps, we can gain valuable insights into the volatility dynamics and how they differ from one country to another.

The detailed analysis of stationary distributions, expected times in clusters, and the transition matrices themselves contributes to a robust understanding of the temporal dynamics and stability of volatility clusters. These results not only highlight the differences in market behaviour across various European countries but also provide a foundation for further investigation into the factors influencing these dynamics. Through this comprehensive approach, we aim to shed light on the complex interplay between market volatilities and their underlying drivers.

To exemplify we produce here the chart with the dynamics of clustering transition for Germany across the whole sample.

Figure 5 shows the time series plot of the cluster transitions for Germany over the sample period. This plot tracks the dynamics of volatility clusters, illustrating how the volatility patterns of German stock market indices have evolved over time. The plot highlights periods of stability and transition, providing a clear visual representation of the changes in volatility behaviour.

Figure 5. Evolution of clusters for the volatility of Germany

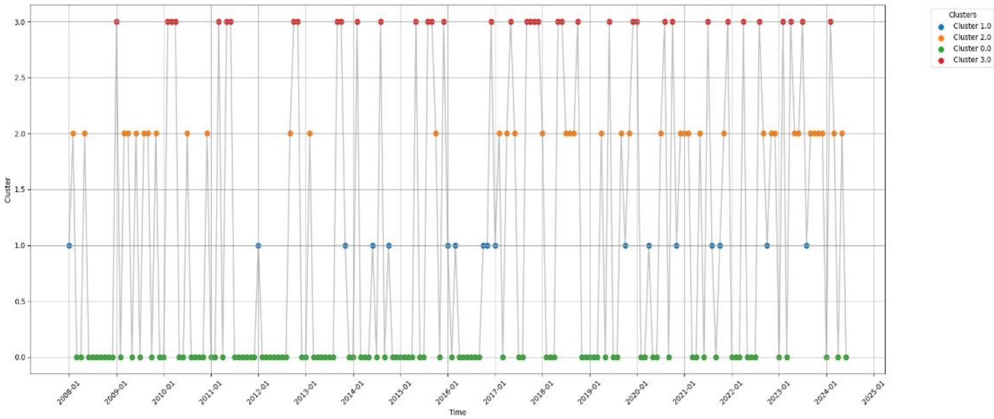
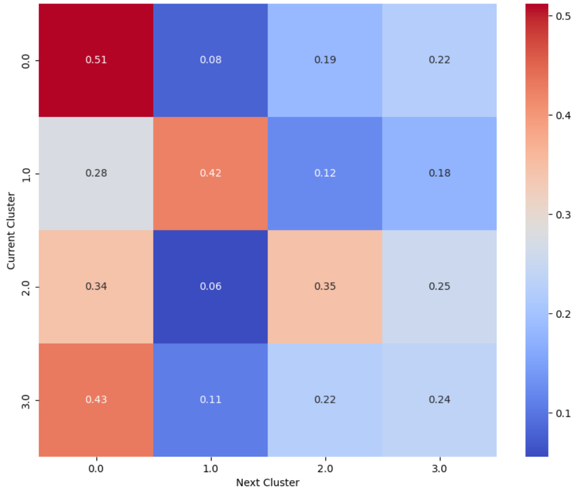


Figure **6** presents the aggregated transition probabilities of clusters across all countries. This figure provides a comprehensive view of the overall cluster dynamics at the European level, showing the likelihood of indices moving from one cluster to another. By aggregating data from all countries, this visualization offers insights into common patterns and interdependencies in market behaviour across the continent.

Figure 6. Transition probabilities of clusters across all countries



5. Concluding Remarks

This study comprehensively analyzed the dynamics of stock market volatility across various European countries by employing advanced clustering techniques and transition matrix computations. The primary objective was to understand the temporal evolution of volatility patterns and their stability over time. By leveraging daily data from MSCI national indices spanning from January 2008 to June 2024, we provided a robust framework for analyzing volatility clusters and their transitions.

Our methodology involved the use of multiple clustering algorithms, including K-means with Dynamic Time Warping (DTW), Agglomerative Clustering with DTW, Gaussian Mixture Model (GMM), and Spectral Clustering with DTW. Each algorithm was evaluated monthly using a set of comprehensive metrics: Silhouette Score, Davies-Bouldin Index, Calinski-Harabasz Index, and Cluster Size Standard Deviation. The best-performing model for each month was selected based on an aggregated ranking system, ensuring a balanced and accurate representation of the volatility data.

The transition matrices were computed for all countries collectively and for each country individually. These matrices captured the probabilities of transitioning from one cluster to another, offering insights into the clusters' stability and dynamics. We further analyzed the stationary distributions and expected times in clusters to understand their long-term behavior and persistence.

Several key findings emerged from our analysis. Firstly, the stationary distributions revealed significant differences in the long-term volatility patterns across different countries. Some countries exhibited high probabilities of remaining in certain clusters, indicating stable volatility patterns, while others showed more evenly distributed probabilities, suggesting variable market conditions.

Secondly, the expected times in clusters highlighted the stability of certain clusters in specific countries. Countries with longer expected times in particular clusters indicated more persistent volatility patterns, whereas those with shorter expected times suggested higher volatility and more frequent transitions.

This study's main contribution lies in its detailed and methodical approach to understanding the dynamics of stock market volatility. By integrating multiple clustering techniques and robust evaluation metrics, we provided a comprehensive framework for analyzing and comparing volatility patterns across different countries. This approach not only enhances the accuracy of volatility analysis but also offers valuable insights into the temporal evolution and stability of market behaviours.

For investors, policymakers, and financial analysts, the findings of this study are of significant importance. Understanding the stability and dynamics of volatility clusters can help in making informed investment decisions, developing risk management strategies, and formulating financial policies. The insights into long-term volatility patterns and transition probabilities can guide stakeholders in assessing market risks and opportunities more effectively.

Despite the robustness of our methodology, several limitations must be considered. First, the reliance on historical data implies that the findings may not fully capture future market behaviors, especially in the face of unprecedented economic events. Second, the choice of clustering algorithms and evaluation metrics, while comprehensive, may still overlook other potentially relevant aspects of market dynamics.

Future research could build on this study by incorporating additional factors such as macroeconomic indicators, geopolitical events, and market sentiment data to enrich the analysis. Moreover, extending the analysis to include more recent data and other global markets could provide a more comprehensive understanding of volatility dynamics. Another avenue for future research is the exploration of machine learning techniques and more advanced clustering algorithms to capture complex market behaviours more accurately.

In conclusion, this study provided a thorough analysis of stock market volatilities using a robust methodological framework. The findings offer valuable insights into the stability and dynamics of volatility clusters, contributing significantly to the field of financial analysis and risk management. By addressing the limitations and exploring new research directions, future studies can further enhance our understanding of market volatilities and their implications for stakeholders.

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