1. CATASTROPHE REINSURANCE PRICING -MODIFICATION OF DYNAMIC ASSET-LIABILITY MANAGEMENT

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Abstract

This study emphasizes the catastrophic reinsurance pricing and its sensitivity based on the asset-liability management (ALM) model. For this purpose, the instantaneous interest rate elastic stochastic ALM model of asset liability valuation is modified. Further, taking the earthquake disaster loss in China as an example, the rates of the catastrophe reinsurance are simulated by Monte Carlo method and the sensitivities of asset liability ratio, trigger level, debt structure and basis risk of the catastrophe reinsurance pricing are studied. This paper provides a validation study on the modification of the ALM model, and a quantitative reference regarding the rates of catastrophe reinsurance for the reinsurance company to deal with huge catastrophe losses such as earthquake or hurricane.

Keywords: catastrophe reinsurance, catastrophe bonds, asset-liability management, Monte-Carlo simulation

JEL Classification Codes: C1, C4

1. Introduction

Enormous losses are caused by catastrophic risk, which have a serious impact on the economic development of countries. It is imperative for people have catastrophic insurance to play an important role in undertaking the catastrophic risk. Natural catastrophes have brought many problems to the insurance and reinsurance companies. One of the most serious problems is the short-term risk of insolvency for insurance and reinsurance company due to the sudden and huge losses of catastrophic risk. For example, the losses of Hurricane Andrew that occurred in 1992 was about US$ 30 billion and more than 60 financial companies fell into bankruptcy (Muermann, 2008). Therefore, it is of importance for

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insurance and reinsurance companies to secure the catastrophe losses. For the insurance companies, it is an inevitable requirement to develop new financial derivatives to transfer insurance risk to reinsurance market. The effect of the following factors on market equilibrium are advanced disaster-resistant technologies used by insureds, alternative financial innovations employed by insurers, and various disaster policies implemented by governments, respectively (Wu, 2020). For the reinsurance companies, with the aim to avoid falling into insolvency, it is required to reserve adequate solvency which is strictly dependent on the reinsurance pricing*, i.e*. debt structure, asset-liability ratio, trigger level and so on. Since the early 1990s, innovative financial instruments such as catastrophe bonds, catastrophe options and futures have been developed to deal with catastrophe risks. Recently, asset and liability management (ALM) has gotten more and more interests on CAT reinsurance pricing (Lee and Yu, 2007; Pan and Xiao, 2017; Nowak and Romaniuk, 2018).

The theory of asset-liability management began with Markowitz (1952). Emphasis was placed on asset liability management until Sharpe and Tint (1990) optimized ALM by the portfolio selection techniques in the framework of static mean variance (M-V). The early management of assets and liabilities is mostly based on static research methods and models, such as duration, immunity, Modern Portfolio Theory (MPT), option pricing theory (OPT), etc. The key research focuses on two aspects: CAT reinsurances (Yu *et al*. (2021) studied the premium rate selection of special crop insurance market with the participation of catastrophic insurance. Perrakis and Boloorforoosh (2018) evaluates a government-sponsored Excess-Of-Loss (XOL). Catastrophe (CAT) reinsurance contract using the financial option approach with extreme risk. Roux (2018) attempts to study how individuals respond to the availability of an insurance that would safeguard their interests if a climate change catastrophe occurred. Lehtonen (2017) analyses the materials that the reinsurance company Munich Re has distributed to stakeholders and asks how climate change is objectified by the reinsurance industry. How are weather-related catastrophes made into a financial risk and opportunity? Burke et al., (2016) explore the design and implementation of portfolio risk analysis on both multi-core and many-core computing platforms. Chang et al., (2010) model claim arrival and loss uncertainties jointly in a doubly-binomial framework to price an Asian-style catastrophe (CAT) option with a non-traded underlying loss index using the no-arbitrage martingale pricing methodology. Froot and O'Connell (2008) model the equilibrium price and quantity of risk transfer between firms and financial intermediaries. Major (2004) presents general formulas for gradients of risk measures including VaR (Value at Risk) and TvaR (Tail Value at Risk). Christensen and Schmidli (2000) deals with the problem of pricing a financial product relying on an index of reported claims from catastrophe insurance. Lowe and Stanard (1997) describes the dynamic financial analysis model currently being used by a property catastrophe reinsurer to manage its business.) and CAT bonds Nowak and Romaniuk (2018) price the catastrophe bonds with a generalized payoff structure, assuming that the bondholder's payoff depends on an underlying asset driven by a stochastic form and is described by the multi-factor Cox-Ingersoll-Ross model. Ma and Ma (2013) propose a mixed approximation method to find the numerical solution for the price of catastrophe risk bonds. Jarrow (2010) provides a simple closed form solution for valuing Cat bonds. The pricing methodology is based on the reduced form models used to price credit derivatives. Härdle and Cabrera (2010) derive the price of a hypothetical modelled-index loss (zero) coupon CAT bond for earthquakes, which is based on the compound doubly stochastic Poisson pricing methodology. Unger (2010) focuses on the development of a numerical PDE approach to price components of a Bermudan-style callable catastrophe (CAT) bond. Egami and Young (2008) present a method for pricing structured CAT bonds based on utility indifference pricing. Vaugirard (2003) develops an arbitrage approach to valuing insurance-linked securities, which accounts for catastrophic events and interest rate randomness, notwithstanding a framework of non-traded underlyings. Cox and Pedersen (2000) briefly discuss the theory of equilibrium pricing and its relationship to the standard arbitrage-free valuation framework. Equilibrium pricing theory is used to develop a pricing method based on a model of the term structure of interest rates and a probability structure for the catastrophe risk. This pricing methodology can be used to assess the default spread on catastrophe risk bonds relative to traditional defaultable securities). CAT bond is one of the most important financial securities related to insurance. It provides counterparty risk to reinsurance companies and transfers catastrophe risk to capital market. In equilibrium, low risks are transferred through reinsurance, while medium and high risks are transferred through partial and full securitization respectively. The increase of loss scale improves the trigger risk level of selective securitization. As a result, catastrophe exposures characterized by lower probability and higher severity are more likely to be retained or reinsured than securitized (Subramanian and Wang, 2018).

Since the Basel agreement for the global insurance industry became the new solvency framework for insurance companies, more attention has been paid to the dynamic research on the solvency of financial institutions. The dynamic research methods of solvency, such as dynamic financial analysis (DFA) and stochastic programming model, are increasingly applied in the management of assets and liabilities of insurance companies. Lowe (1997), Burkett *et al*., (2001) and Musulin (2001) studied the application of (DFA Dynamic Financial Analysis) model and stochastic process on the solvency of financial companies. Zijia Wang *et al*. (2021) aims to fill this gap in the literature by quantitatively assessing the impact of the choice of income process on some finite-time and infinite-time ruin quantities. To carry this analysis, we consider a generalized Sparre Andersen risk model with a random income process which renews at claim instants. Gabriele Torri *et al*. (2022) propose a model to study the stability of non-life insurance sector in presence of catastrophic events. Lee and Yu (2007) adopted Monte Carlo method based on stochastic programming model to study the impact of catastrophe securitization on catastrophe reinsurance contract pricing, and analysed relevant risk factors. One of the most prominent risks of ALM is the interest rate risk. However, in Lee's work (2007), the interest rate elasticity was input as a constant for numerical simulation, this simplification was not suitable for the practical circumstance. In addition, the catastrophe reinsurance pricing model under the ALM perspective of Lee and Yu (2007) has not been validated according to the actual CAT losses.

Currently, the Solvency II[4]proposal has become the framework of insurance and reinsurance companies in China, which is also known as China Risk Oriented Solvency System (C-ROSS). More than three years after the implementation of C-ROSS, the overall risk management capability of the insurance companies has been significantly enhanced. In the insurance companies, the optimization strategy of insurance assets allocation is established under the C-ROSS. Obviously, preventing catastrophe disasters and decreasing their negative influence have become a crucial task for both the government and scientists of China, especially facing huge CAT losses, destructive earthquakes have caused great damage to both property and human life. For example, a magnitude 8.0 earthquake occurred in Wenchuan area of Southwest China in 2008, which is the most serious natural disaster in more than 30 years. It is estimated that the economic loss caused by Wenchuan earthquake exceed 140 billion US dollars. According to the statistics data of 1952-2018 in China, the

[4] *Solvency II is the updated set of regulatory requirements for insurance firms that operate in the European Union.*

average annual earthquake loss is ￥8.31 Million. The earthquake losses in China are very representative among huge CAT losses, which cannot only be used for the validation of ALM by Lee and Yu (2007), but also for providing guidance of reserving solvency for reinsurance companies facing huge catastrophe losses.

Therefore, it is of great significance to study catastrophe pricing based on earthquake loss in China, and also to provide a validation study on the solvency scale for the reinsurance companies facing huge catastrophe losses. For this reason, the catastrophe reinsurance pricing model of Lee and Yu (2007) is applied and modified in this paper to investigate the catastrophe reinsurance pricing in the light of earthquake loss data in China. In order to make the results of simulation closer to the practical circumstance, the definition of instantaneous interest rate elasticity is introduced as the derivative of stochastic interest rate depicted by CIR model. According to the earthquake losses in China, the sensitivity of parameters to on-line rate (POL) is further studied.

This research contents of this paper are as follows. Section 2 introduces the catastrophic reinsurance contrast valuation model. Section 3 describes the numerical results and discussion. Section 4 elaborates the conclusions of the research.

2. The Contrast Valuation Model of Catastrophic Reinsurance

2.1 Dynamics Model of the Asset

In the light of the investigation and research of Lee and Yu (2007), considering the influence of random interest rate, the asset dependence value of reinsurance company can be expressed by the following equation:

$$\frac{dC_t}{C_t} = \alpha_c\, dt + \beta_c dr_t + \gamma_c dW_{C,t} \tag{1}$$

where C_t is the asset value of the reinsurance company at time *t*. r_t is the instantaneous interest rate at time *t*; α_c is drift due to credit risk; β_c is the instantaneous interest rate elasticity of reinsurance company assets; $W_{C,t}$ is Brownian motion, also known as Wiener process that means the reinsurer credit risk on the asset values. γ_c is the waviness in the credit risk. The definition of β_c is introduced as follows:

$$\beta_c = \frac{\Delta r_t}{\Delta C} \times \frac{C}{r_t} \tag{2}$$

To avoid the negative interest rates, the Cox-Ingersoll-Ross model (Cox, 1985) is used describe the instantaneous interest rate, as shown in Equation (2). The CIR model follo the stochastic differential equation as an extension of the Vasicek model and is a type "one factor model" describing instantaneous interest rate movements driven by only c source of market risk.

$$dr_t = e(f - r_t)dt + g\sqrt{r_t}dZ_t$$

where *e* is measurement of the mean-reverting force; *f* is the long-run mean of the interest rate; g is the volatility parameter of the interest rate; and Z_t is a Wiener process independent of $W_{C,t}$. Based on Equations (1) and (2), we can establish the model of the asset dynamics of reinsurance companies as bellow:

$$\frac{dC_t}{C_t} = (\alpha_c + \beta_c ef - \beta_c er_t)\, dt + \beta_c g\sqrt{r_t} dZ_t + \gamma_c dW_{C,t} \tag{3}$$

Adopting the risk neutral pricing method, the dynamics pricing of the financial derivative can be defined as follows:

$$dr_t = e^*(f^* - r_t)dt + g\sqrt{r_t} dZ_t^* \tag{4}$$

where e^*, f^*, Z_t^* can be respectively defined as follows:

$$e^* = e + \lambda_r, \quad f^* = \frac{ef}{e + \lambda_r}, \quad dZ_t^* = dZ_t + \frac{\lambda_r \sqrt{r_t}}{g}\, dt$$

The term λ_r is the pricing at the market of interest rate risk，which is invariant in Cox (1985); Z_t^* is the Wiener process of Z_t based on risk neutral measure. Risk neutral measures are expressed in *Q* with respect to *P* (Physical measure). Risk can be offset by asset dynamics of reinsurance companies

$$\frac{dC_t}{C_t} = r_t\, dt + \beta_c g\sqrt{r_t} dZ_t^* + \gamma_c dW_{C,t}^* \tag{5}$$

where $W_{C,t}^*$ is a Wiener process of risk neutral measurement, which has nothing to do with Z_t^*.

2.2 Dynamics Model of the Liability

The liability refers to the current worth of coming claims after relating to non-catastrophic policies, and the liability worth of reinsurers, expressed as D_t, can be calculated as follows:

$$\frac{dD_t}{D_t} = (r_t + \alpha_D)\, dt + \beta_D dr_t + \gamma_D dW_{D,t} \tag{6}$$

where α_D is the risk premium for the small impact, β_D is described as the transient interest rate elasticity in liabilities model. The connotation of β_D is similar to β_c and is also lead into the simulation as below:

$$\beta_D = \frac{\Delta r_t}{\Delta D} \times \frac{D}{r_t}$$

Based on the Cox-Ingersoll-Ross model described above, the liability dynamics model of reinsurers can be represented with respect to risk neutral measure as below:

$$\frac{dD_t}{D_t} = r_t\, dt + \beta_D g\sqrt{r_t} dZ_t^* + \gamma_D dW_{D,t}^* \tag{7}$$

2.3 The CAT Loss Dynamics Model

This kind of catastrophe risks is not only widespread, but also difficult to predict, causing huge economic losses. The CAT loss dynamics model in this paper is the process of underwriting catastrophe loss at time *t*, which can be represented as below:

$$L_T = \sum_{j=1}^{N(t)} l_t \tag{8}$$

For purpose of estimating the impact of the basic risk on reinsurance appraised price, the dynamic description of the comprehensive catastrophe index, $\mathrm{L_{T,index}}$, is as follows:

$$L_{T,index} = \sum_{j=1}^{N(t)} l_{t,index} \tag{9}$$

where $N(t)$ is the process for the loss number at time *t*. Term l_t refers to the quantity of loss due to the jth catastrophe in a specific period covered by the reinsurance contract; $l_{t,index}$ refers to the number of losses in the comprehensive loss index.

There are many natural disasters in China, such as earthquakes and floods, and the catastrophic risks have a broad geographical distribution, diverse types, high frequency of occurrence, and huge losses. Based on China's earthquake loss statistics from 1952 to 2018, the numerical results show that the l_t and $l_{t,index}$ are characterized by lognormal distribution, which can be described as follows:

$$l_t = \frac{1}{\sqrt{2\pi} \times 2.39542547t} e^{\frac{-(\ln t - 12.8057559)^2}{2\times 2.39542547^2}} \tag{10}$$

$$l_{t,index} = \frac{1}{\sqrt{2\pi} \times 2.39542547t} e^{\frac{-(\ln t - 12.8057559)^2}{2\times 2.39542547^2}} \tag{11}$$

c_t and $c_{t,index}$ are assumed to be mutually independent and the same lognormal distribution, and μ_c (μ_{index}) and $\sigma_c^2(\sigma_{index}^2)$ denote their logarithmic means and variance respectively. $N(t)$ is the loss number process of earthquake in mainland China from 1952 to 2018, which obeys the negative binomial distribution as below:

$$N(t) = \frac{(k+9)!}{[k!\,9!]} \times 0.5445^8 \times 0.4555^k \tag{12}$$

2.4 Catastrophic Reinsurance Valuation

In this section, we examine catastrophe reinsurance valuations under various claim scenarios. Therefore, the reinsurance contract without default risk is evaluated first, then the reinsurance contract with default risk is evaluated, and then evaluates the reinsurance contract considering CAT bond issuance and underlying respectively is evaluated.

No Default Risk

First of all, we study the scenario of no default risk, that is, in the case of no risk of default, the compensation amount of P_T in the reinsurer's contract can be represented as below:

$$P_T = \begin{cases} A-B & L_T \geq A \\ L_T - B & A > L_T \geq B \\ 0 & otherwise \end{cases} \tag{13}$$

where P_T is the total amount of indemnity in the reinsurance contract during the underwriting period, T; L_T represents catastrophic aggregate losses during T; B denotes the additional level in the reinsurer's contract arrangement; A represents the upper limit level of the huge disaster losses that the reinsurer will recoup.

With Default Risk

Second, this section studies the pricing of reinsurance contracts when default risks are taken into account, that is to say, at the cost of default risk, the compensation amount of reinsurance contract during T period， P_{dT} can be represented as bellow:

$$P_{dT} = \begin{cases} A-B & L_T \geq A \ and \ C_T \geq D_T + A - B \\ L_T - B & A > L_T \geq B \ and \ C_T \geq D_T + L_T - B \\ \frac{(A-B)C_T}{D_T+A-B} & L_T \geq A \ and \ C_T < D_T + A - B \\ \frac{(L_T-B)C_T}{D_T+L_T-B} & A > L_T \geq B \ and \ C_T < D_T + L_T - B \\ 0 & otherwise \end{cases} \tag{14}$$

where C_T and D_T are the assets and liabilities of reinsurers respectively. In this case, it is distinctly seen that the risk of default decreases the reinsurance contracts value. In the payment of the contracts, the financial situation plays an important role.

With Default Risk and CAT Bonds

This section mainly studies catastrophe reinsurance pricing under catastrophe bond and default risk scenarios. Assume that a reinsurance company provides catastrophe reinsurance and issues catastrophe bonds, transferring risk to the capital markets. The catastrophe bonds are regarded as a capital infusion, which can amplify the ability of reinsurers to undertake more reinsurance coverage. We also allow for insurers' risk transfer choices to be observable to capital market investors in this modified model. The yield of the CAT bond, $P_{CAT,T}$, can be represented as below at maturity as below:

$$P_{CAT,T} = \begin{cases} H_{CAT} & L^* \leq K \\ \pi_p \times H_{CAT} & L^* > K \end{cases} \tag{15}$$

where H_{CAT} is the face value of CAT bonds; L^* is the underlying disaster losses, which can be L_T or $L_{index,T}$; K is the trigger level coefficient representing the provision of CAT bond; π_p ($0<\pi_p<1$) is the proportional coefficient that represents the portion of the principal paid to CAT bondholders. Let $H_{CAT,T}$ denote the subsequent beliefs of capital markets regarding an insurer's type given that it has chosen securitization. It is assumed that the amount exempted by catastrophe bondholders, θ, is represented as below:

$$\theta(C^*) = H_{CAT} - P_{CAT,T} \tag{16}$$

Under these circumstances, the claim amount of the reinsurance contract in the underwriting period, P_{bT} can be represented as below:

$$P_{bT} = \begin{cases} A - B & L_T \geq A \text{ and } C_T + \theta \geq D_T + A - B \\ L_T - B & A > L_T \geq B \text{ and } C_T + \theta \geq D_T + L_T - B \\ \frac{(A-B)(C_T+\theta)}{D_T+A-B} & L_T \geq A \text{ and } C_T + \theta < D_T + A - B \\ \frac{(L_T-B)(C_T+\theta)}{D_T+L_T-B} & A > L_T \geq B \text{ and } C_T + \theta < D_T + L_T - B \\ 0 & otherwise \end{cases} \quad (17)$$

A and B represent the upper limit and additional level stipulated in the reinsurance contract, respectively. In these scenarios, the equilibrium is characterized by a single trigger where insurers with risks below the trigger choose either reinsurance or retention, whereas insurers with risks above the trigger choose securitization.

Basic Risk

Earthquakes are a catastrophic risk, in the event of loss is often immeasurable. There is a fundamental risk when most studies of catastrophe bond payments only refer to the risk loss index of major disasters and do not refer to the actual seismic losses of reinsurers in major disasters. The basic risk degree can be expressed by the coefficient of correlation between catastrophe loss of single reinsurer and comprehensive loss index (loss correlation coefficient, ρ_l). The lower the ρ_l, the basic risk is getting higher. When ρ_l=1, there is no basic risk.

2.5 The Premium Rate on Line (POL)

In the light of Lee's work (2007), the insurance premium rate, or the premium rate on line (POL), can be calculated or estimated as bellow:

$$POL = \frac{1}{A-B} \times E_0^Q \left[e^{-\int_0^T r_s ds} \times PO_T \right] \quad (18)$$

where *POL* is the rate per RMB insured by catastrophe reinsurance; E_0^Q is the expected value on the release date for pricing action *Q*; and PO_T = P_T, P_{dT}, or P_{bT}, which is the compensation amount of the reinsurers under the above-mentioned alternative circumstances.

3. Numerical Simulation and Results Discussion

According to the reinsurance pricing model in different scenarios, The Monte Carlo approach is adopted to estimate the reinsurance contract POL under the substitution condition in this paper. The interest rate parameters for the Cox et al. (1985) model are set to be the estimates reported in Duan and Simonato (1999). Following Pennacchi et al. (2014), we assume that the initial spot interest rate r is equal to 2%.Disaster loss data are based on China's earthquake loss data (1952-2018). For numerical simulation, the dynamics parameters of asset-liability management and the earthquake loss parameters are set in Table1. Both the trigger level (K) and the liability (D) of reinsurer are CNY 10 billion. The ratio of reinsurer's asset to liability (C/D) is 1.0. The ratio (B/D) of the additional level of the reinsurance contract to the liability of reinsurer is assigned to 0.2. Meanwhile, the ratio of the upper limit level (A/D) is assigned to 1.0. The sensitivity of parameters is further investigated by numerical simulation. The reinsurers' coverage is supposed to last a year, and the numerical simulation is based on 20,000 paths per week (Lee and Yu, 2002, 2007).

Table 1. Dynamic Parameters of Assets and Liabilities and Catastrophe Loss

Symbol	Parameters	Values
C	Assets of reinsurer	C/D=1.1-1.5
γ_c	Credit risk volatility	5%
D	Liabilities of reinsurer	1.0e+10
γ_D	Credit risk volatility	5%
r	Initial transient interest rate	2%
ρ_{CD}	Correlation coefficient of asset liability credit risk of reinsurance company	0.2
e	Magnitude of average reducing force	0.2
f	Long-term average of interest rate	5%
g	Interest rate fluctuation	10%
λ_r	Market price of interest rate risk	-0.01
K	Triggering levels	1.0e+10
π_P	Principal ratio of payment when debt relief is triggered	0.5
B	Additional standard of a reinsurer	B/D=0.2
A	Maximum amount of indemnity under reinsurance contract	A/D=1.0
σ_c	Standard deviation of insurers earthquake loss logarithm	2.39542547
σ_{index}	The standard deviation of the logarithm of earthquake loss of comprehensive loss index	2.39542547
μ_c	Average value of earthquake loss logarithm of the insurer	12.8057559
μ_{index}	The average value of the logarithm of earthquake loss of comprehensive loss index	12.8057559
ρ_l	Correlation coefficient between reinsurer's CAT losses logarithm and the composite index	$0.1 \le \rho_l \le 1$

3.1 Sensitivity of the Reinsurer's Original Capital Position (C/D) on POLs

In order to research the sensitivity of reinsurer's original capital position (C/D) to non-default free and default risk POLs, five situations with different C/D values (1.1-1.5) were simulated. Table 2 shows the simulation results of non-default POLs and default risk POLs. One may see that the amount of non-default insurance is higher than the corresponding amount of default risk insurance, indicating that the reinsurance contract is more valuable without considering the default risk.

It is clear from Table 2 that the default risk POLs increase with the increase of C/D, showing that the original capital position (C/D) of the reinsurer plays a significant role in the default risk POLs. For the case of (r, σi) = (8, 2.395425), POL increases from 0.47158 to 0.63560 with the increase in C/D from 1.1 to 1.5. In brief, the more superior the reinsurance company's original capital position is, the higher the value of the default risk rate and the value of the reinsurance contract is.

When the asset liability ratio C/D and earthquake loss variance σi remain unchanged, the fair pricing rate POL of earthquake reinsurance increases with the increase in earthquake frequency coefficient. For example, for the case of C/D=1.1 and σi = 2.395425, POL increases from 0.453944 to 0.47158 with the inrease in r from 7 to 8.

When the asset liability ratio C/D and earthquake frequency coefficient remain unchanged, the fair pricing rate POL of earthquake reinsurance considering default risk decreases with the increase in earthquake loss variance. For the case of C/D = 1.2 and r = 8, the default risky POL decreases from 0.56255 to 0.51369 with the increase of σi from 1 to 2.395425.

Table 2. Sensitivity of C/D Non Default Risk POL and Default Risk POL

r σ_i	Non default POL	Default risky POL				
		C/D=1.1	1.2	1.3	1.4	1.5
7,1	0.78187	0.49716	0.54192	0.58595	0.62901	0.67079
7,2.395425	0.71277	0.45394	0.49496	0.53455	0.57326	0.61147
7,3	0.69422	0.44172	0.48173	0.52043	0.55829	0.59514
8,1	0.81488	0.51598	**0.56255**	0.60818	0.65287	0.69740
8,2.395425	0.74209	0.47158	**0.51369**	0.55608	0.59582	0.63560
8,3	0.72273	0.45893	0.49981	0.54062	0.57963	0.61874
9,1	0.84061	0.52941	0.57798	0.62464	0.67206	0.71715
9,2.395425	0.76630	0.48513	0.52921	0.57179	0.61465	0.65515
9,3	0.74561	0.47233	0.51483	0.55690	0.59752	0.63739

3.2 Sensitivity of the Trigger Level (K) on POLs

In this section, the sensitivity of trigger level (*K*) on default risk POLs is studied. For this purpose, three situations are simulated with input parameters *K* of CNY 8 billion, 10 billion, 12 billion respectively. The simulation results of POLs are shown in Table 3 under the assumption of CAT bond issuance. The results indicate that the default risk POLs in Table 3 is higher than those in Table 2. This suggests that the issuance of CAT bonds is conducive to hedging default risk and improving the value of reinsurance contracts.

The study also exhibits that the risk of default POLs increases with the decrease in *K*. For example, when *K* decreases from CNY 12 billion to CNY 8 billion, the default risk POLs increase from 0.57701 to 0.58924. The lower the trigger level is, the higher the risk of default POLs is. In addition, it is clear from Table 3 that the default risk POLs increase with the increase in the initial capital position (C/D) of the reinsurer. For example, when C/D increases from 1.1 to 1.5, the risk of default POLs increases from 0.58924 to 0.75629.

Table 3 Triggering Level Sensitivity of CAT Bond Default Risk POLs

r σ_i	POL considering issuing catastrophe bonds				
K=8.0E+9	C/D=1.1	1.2	1.3	1.4	1.5
7,1	0.58924	0.63321	0.67508	0.71671	0.75629
7,2.395425	0.53246	0.57169	0.61063	0.64792	0.68395
7,3	0.51724	0.55562	0.59261	0.62977	0.66442
8,1	0.6146	0.65997	0.70467	0.74799	0.78986
8,2.395425	0.55496	0.59662	0.63671	0.6753	0.71317
8,3	0.53894	0.57919	0.61779	0.65637	0.69265

r σ_i	POL considering issuing catastrophe bonds				
K=8.0E+9	C/D=1.1	1.2	1.3	1.4	1.5
9,1	0.63399	0.68102	0.72664	0.77185	0.81545
9,2.395425	0.57373	0.6168	0.65773	0.69868	0.73797
9,3	0.55632	0.59827	0.63822	0.67732	0.71619
K=1.0E+10	C/D=1.1	1.2	1.3	1.4	1.5
7,1	0.5825	0.6268	0.66995	0.71144	0.75084
7,2.395425	0.52844	0.56787	0.60597	0.64441	0.67981
7,3	0.51326	0.55183	0.58929	0.62587	0.66148
8,1	0.60878	0.65413	0.69863	0.74219	0.78449
8,2.395425	0.55084	0.59209	0.63269	0.6708	0.71002
8,3	0.53472	0.57457	0.61351	0.65263	0.68934
9,1	0.62822	0.67513	0.72135	0.76701	0.81091
9,2.395425	0.56911	0.61177	0.654	0.69455	0.73374
9,3	0.55252	0.59425	0.6351	0.67441	0.71254
K=1.2E+10	C/D=1.1	1.2	1.3	1.4	1.5
7,1	0.57701	0.62135	0.66407	0.70646	0.74501
7,2.395425	0.52362	0.56352	0.60284	0.64039	0.67694
7,3	0.51002	0.54838	0.58607	0.62278	0.65816
8,1	0.60315	0.64838	0.69323	0.73741	0.77948
8,2.395425	0.54635	0.58804	0.62858	0.6675	0.70632
8,3	0.53121	0.57139	0.61115	0.64912	0.68632
9,1	0.62284	0.66997	0.7157	0.76163	0.80562
9,2.395425	0.56486	0.60778	0.65026	0.6912	0.7303
9,3	0.5489	0.59041	0.63122	0.67111	0.70933

3.3 Sensitivity of Basis Risk on POLs

In Table 4, the influence of basic risk on the fair pricing rate of earthquake reinsurance when issuing earthquake catastrophe bonds is further considered. Basic risk refers to the risk of insufficient hedging caused by inconsistent changes between hedging instruments and risk subjects. When considering the earthquake reinsurance fair pricing rate, the higher the basic risk correlation coefficient, the higher the earthquake reinsurance fair pricing rate. For the case of ($\mathbf{r}, \boldsymbol{\sigma_i}, \boldsymbol{\sigma_{index}}$) = (8, 2.395425, 2.395425), K = 800000 and C/D = 1.1, POL increases from 0.55234 to 0.56097 with the increase of ρ_l from 0.3 to 0.8. It shows that the higher the basic risk correlation is, the higher the fair pricing rate of reinsurance and the value of reinsurance contract is.

Comparison of Table 4 and Table 3. under the same asset liability ratio C/D and earthquake loss variance, the fair pricing rate POL of earthquake reinsurance increases with the increase in earthquake frequency coefficient. Under the same of the asset liability ratio C/D and earthquake frequency coefficient, the fair pricing rate POL of earthquake reinsurance decreases with the increase of earthquake loss variance. In addition, it is clear from Table 4 that the default risk POLs increase with the increase of the initial capital position (C/D) of the reinsurer.

Table 4. Basic Risk Sensitivity of CAT Bond Default Risk POLs

K	800000			1000000			1200000		
ρ_l	0.3	0.5	0.8	0.3	0.5	0.8	0.3	0.5	0.8
r σ_i σ_{index}	C/D=1.1								
7,1,1	0.5875	0.5892	0.5929	0.5814	0.5832	0.5869	0.5757	0.5768	0.5805

K	800000			1000000			1200000		
ρ_l	0.3	0.5	0.8	0.3	0.5	0.8	0.3	0.5	0.8
$\mathbf{r}\ \sigma_i\ \sigma_{index}$	C/D=1.1								
7,2.395425 2.395425	0.5298	0.5327	0.5387	0.5252	0.5279	0.5339	0.5212	0.5239	0.5296
7,3,3	0.5142	0.5172	0.5231	0.5098	0.5133	0.5191	0.5063	0.5097	0.5152
8,1,1	0.6122	0.6143	0.6184	0.6067	0.6083	0.6113	0.6014	0.6027	0.6061
8,2.395425 2.395425	0.5523	0.5556	0.5610	0.5477	0.5505	0.5560	0.5436	0.5464	0.5522
8,3,3	0.5365	0.5388	0.5446	0.5309	0.5349	0.5407	0.5280	0.5311	0.5367
9,1,1	0.6327	0.6343	0.6368	0.6268	0.6285	0.6313	0.6219	0.6227	0.6257
9,2.395425 2.395425	0.5706	0.5739	0.5786	0.5662	0.5694	0.5747	0.5630	0.5655	0.5707
9,3,3	0.5536	0.5565	0.5623	0.5491	0.5524	0.5577	0.5458	0.5489	0.5546
$\mathbf{r}\ \sigma_i\ \sigma_{index}$	C/D=1.3								
7,1,1	0.6730	0.6756	0.6799	0.6678	0.6698	0.6741	0.6620	0.6646	0.6683
7,2.395425 2.395425	0.6072	0.6108	0.6171	0.6033	0.6061	0.6125	0.5987	0.6029	0.6086
7,3,3	0.5902	0.5935	0.5997	0.5864	0.5892	0.5962	0.5828	0.5861	0.5926
8,1,1	0.7024	0.7046	0.7096	0.6963	0.6987	0.7025	0.6910	0.6936	0.6975
8,2.395425 2.395425	0.6346	0.6374	0.6422	0.6294	0.6320	0.6378	0.6252	0.6287	0.6346
8,3,3	0.6150	0.6180	0.6244	0.6107	0.6138	0.6203	0.6066	0.6110	0.6170
9,1,1	0.7250	0.7269	0.7304	0.7199	0.7218	0.7252	0.7147	0.7161	0.7197
9,2.395425 2.395425	0.6550	0.6574	0.6635	0.6511	0.6537	0.6595	0.6470	0.6499	0.6556
9,3,3	0.6355	0.6384	0.6441	0.6311	0.6349	0.6405	0.6279	0.6308	0.6378
$\mathbf{r}\ \sigma_i\ \sigma_{index}$	C/D=1.5								
7,1,1	0.7537	0.7561	0.7616	0.7481	0.7514	0.7564	0.7429	0.7455	0.7517
7,2.395425 2.395425	0.6803	0.6845	0.6906	0.6766	0.6807	0.6874	0.6729	0.6770	0.6835
7,3,3	0.6615	0.6649	0.6721	0.6580	0.6609	0.6685	0.6546	0.6580	0.6653
8,1,1	0.7872	0.7901	0.7942	0.7813	0.7843	0.7893	0.7771	0.7795	0.7846
8,2.395425 2.395425	0.7103	0.7133	0.7199	0.7059	0.7097	0.7159	0.7029	0.7060	0.7131
8,3,3	0.6895	0.6930	0.7003	0.6857	0.6892	0.6966	0.6827	0.6855	0.6934
9,1,1	0.8140	0.8156	0.8194	0.8089	0.8103	0.8151	0.8037	0.8061	0.8105
9,2.395425 2.395425	0.7343	0.7380	0.7441	0.7307	0.7343	0.7401	0.7265	0.7310	0.7372
9,3,3	0.7134	0.7167	0.7225	0.7090	0.7132	0.7191	0.7061	0.7097	0.7159

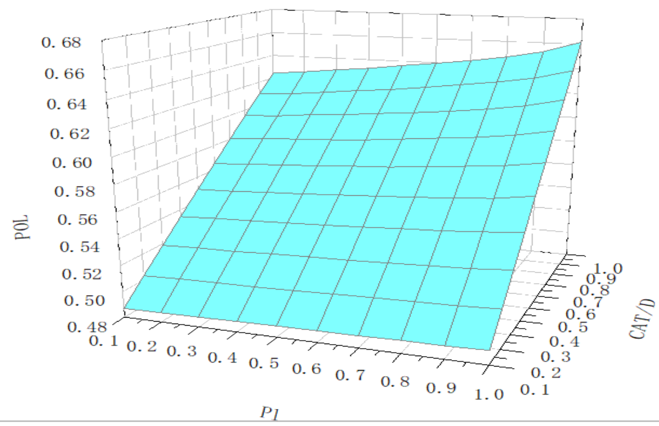
3.4 Sensitivity of CAT Debt Structure Of Reinsurer (CAT/D) and Loss Correlation Coefficient (ρ_l) to POLs

In this section, the sensitivity of CAT debt structure of the reinsurer (CAT/D) and loss correlation coefficient (ρ_l) on the POLs are investigated. Firstly, the effect of basic risk on the POLs is investigated. The basic risk degree can be expressed by the loss correlation coefficient ρ_l.. The higher the ρ_l is, the lower basic risk will be. When ρ_l=1, basic risk does not occur. As Figure 1 shows, it is quite obvious that the POLs of default risky increase with the increase of ρ_l, especially under the circumstance of high CAT debt structure.

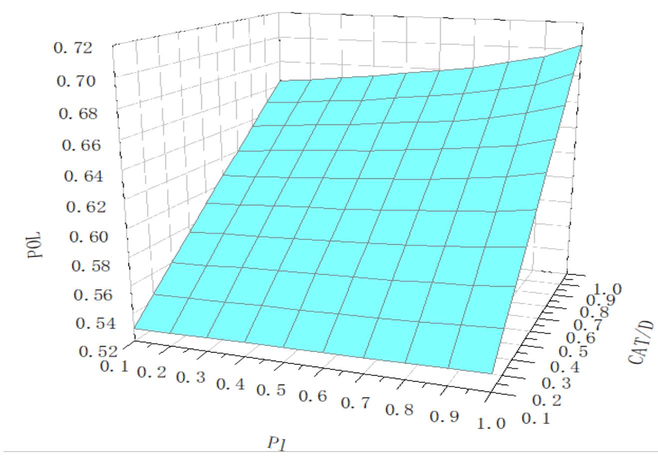
It is also obvious that the risk of default POLs increases with the increase of CAT/*D*, which is in good agreement with the research results of Lee and Yu (2007). The higher the CAT debt structure (CAT/D) is, that is, the more the CAT bonds are issued to transfer risks to the capital market, the higher the contract value of the default risk POLs and the reinsurer is. Finally, compared with results of three cases in Fig.1, it is concluded that in the case of issuing CAT bonds and consideration of basis risk, the default risk POLs increase with the increase in the initial capital position (C/D) of the reinsurer.

Moreover, the curves in Figure1 in this paper are smoother and more comprehensive than those in the research of Lee and Yu (2007). This is mainly attributed to the definition formulas of instantaneous interest rate elasticity introduced in the catastrophe reinsurance pricing model. In Lee's work, the interest rate elasticity was input as a constant in numerical simulation.

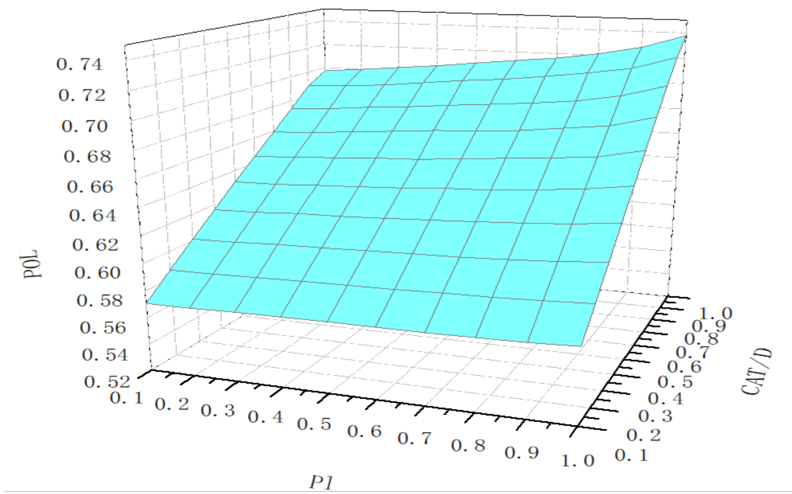
Figure 1. Sensitivity of CAT Debt Structure of Reinsurer (CAT/D) and Loss Correlation Coefficient (ρ_C) to POLs a: C/D=1.1, b: C/D=1.2, c: C/D=1.3



(a)



(b)



(c)

In this paper, the elasticity of instantaneous interest rate was defined as the derivative of stochastic interest rate depicted by CIR model. The definition of interest rate elasticity gives a more realistic description, and makes the results of simulation closer to the practical circumstance.

4. Conclusions and Prospects

This essay primarily focuses on the pricing of catastrophe reinsurance according to the earthquake loss in China as well as provides guidance on the reserving solvency for the reinsurance companies facing huge catastrophe losses. For this purpose, the model of the asset-liability dynamics is proposed and the rates of the catastrophe reinsurance are evaluated on the basis of the earthquake loss in mainland China. The Monte Carlo method was adopted for the numerical simulation and the sensitivity of *K*, *C/D*, *CAT/D* and ρ_c on the POL parameters was examined. The research results reflect that the trigger level, asset/liability ratio and debt structure have significant impacts on the POLs, and the sensitivities of *C/D, K, CAT/D,* ρ_c on the POL are examined. The results demonstrate that the higher the *C/D*, CAT/*D*, ρ_c are and the lower the *K* is, the higher the reinsurer's contract and the ROL is. Compared to the research results of Lee and Yu (2007) and Lo *et.al*. (2021), the simulation results in this paper are closer to the practical circumstance, which is mainly attributed to the introduction of instantaneous interest rate elasticity.

This research provides a modification of the ALM model by introducing the definition of interest rate elasticity and the research of parameter sensitivity on POL. It also makes a validation study on the modification of the ALM model in the light of the earthquake loss data of China as a representative of huge CAT loss, and provides a quantitative reference on the rates of catastrophe reinsurance for the reinsurance company to deal with huge catastrophe losses such as those determined by earthquake or hurricane.

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