5 A MODEL TO FORECAST THE EVOLUTION OF THE STRUCTURE OF A SYSTEM OF ECONOMIC INDICATORS

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Abstract

The problem of forecasting the economic systems' activity over a large time horizon is the main concern of both the researchers in the economic field and the system managers. In most cases, its solving is reduced to simulating the structure of the economic indicators which render the system activity. Usually, a certain mode of structuring the indicators is supposed and the functional and statistical correlations among the indicators is considered as previously known (according to certain tolerances).

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In other words, the structure of the simulated system of indicators is supposed to be known, in accordance with the tolerance limits. We use this structure in order to achieve the factorial analysis, according to several working hypotheses, then lay out several additional objectives for the whole system – up to the level of factors that influence the economic process (eventually, on the basis of percentage quotation).

We shall analyze several possibilities for simulating the structure of an economic indicator system.

We mention that: *a)* in economic practice the decision-makers have to fulfill this objective; *b)* each decision-maker has an own *"algorithm"*, based on his gained experience; *c)* the amount of information he uses is inversely proportional to the forecasting horizon; *d)* the indicators' aggregation level is approximately proportional to the forecasting horizon; *e)* the methodology of calculating the indicators lies at the basis of this activity.

The diversity of the ways to solve the problem of forecasting the economic indicator structure by the decision-makers requires to work out some simulating procedural models, which include a large amount (Stoica, Andreica, Sãndulescu, 1989). Consequently, the automation of the procedure to simulate the economic indicator structure supposes to work out an all-inclusive model that associates a procedural chain to each variant of the possible way to solve the problem. However, this model

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will not exclude the presence of the human factor during its solving, because the statement of the working hypotheses, the logical validation of the partial and final results and their reasoning coverage belong exclusively to the decision-makers.

By ensuring the decision-makers' access to the process of finding the solutions to the problem we shall succeed in getting the results to be applied in practice.

Further on, we shall analyze which are the possibilities to simulate the level and the structure of the economic indicators of a holding, for a given time horizon.

Thus, the problem to be solved can be stated as follows: let us consider the system of indicators at moment zero (basic year) $S_0 = \{I_k^0\},\ k = \overline{1,n}$ and we want to find their evolution for holding and enterprises up to the moment: T: $S_t = \{I_k^t\},\ t = \overline{1,T}$; $k = \overline{1,n}$.

Ways to solve the problem and the additional information required

The working hypotheses: *a)* there are relationships of totally or partially functional dependence among the economic indicators, but an independent approach to some of them is also possible; *b)* the indicator evolution can be simulated:

- on the basis of the trend;
- in correlation (dependence) with other already estimated indicators;
- on the basis of rates (indexes) or exogenously given trajectories.

The simulation algorithm will have a finite number of steps.

We denote by "step" (iteration) a finite number of indicators (if, at each step, an indicator evolution is determined for the whole time horizon) or a finite number of time units into which the **T** forecasting horizon is divided (if, at each step, the structure of the whole indicator system is determined for the considered time subdivision). We shall approach the problem related to the first mentioned aspect. Essentially, the algorithm content will be the following one:

Step 1

We choose one of the economic indicators (I_k) and we determine its evolution, independently of the other ones, for $\overline{1,T}$ time horizon or in correlation with the basic year level of the other indicators.

Acting variants:

1) One gives (in a determinist mode) the yearly growth mean rate of the indicator $I_k : \overline{R}_k$ (evolution without taking into consideration the trend).

The procedure of finding the indicator evolution is:

$$I_k^t = I_k^0\left(1+\overline{R}_k\right)^t,\quad t = \overline{1,T}$$

2) One generates the yearly growth mean rate, uniformly inside an interval $\overline{R}_k$. (interval limits are given). The procedure at point 1 is applied.

3) One gives (in a determinist mode) the indicator level at the end of the forecasting horizon, I_k^T .

The procedure of finding the indicator evolution is:

$$\overline{R}_k = \sqrt[T]{I_k^T / I_k^0} - 1 \Rightarrow I_k^t = I_k^0 \left(1 + \overline{R}_k\right)^t, \quad t = \overline{1, T-1}$$

or:

$$\bar{i}_k = \sqrt[T]{I_k^T / I_k^0} \Rightarrow I_k^t = \left(\bar{i}_k\right)^t I_k^0, \quad t = \overline{1, T-1}.$$

4) One generates the indicator level at the end of the forecasting horizon, I_k^T , inside an interval (interval limits are given).

The computing procedure is similar to that at point 3.

5) One gives the function analytical expression, according to which the indicator evolves (the time function). In this case, the procedure according to which the indicator evolution is determined is relatively simple: $I_k^t = f(t)$, $t = \overline{1, T}$.

6) One gives the functional analytical expression, according to which the yearly growth mean rate is developed (or the yearly growth index):

$$R_k^t = f(t) \text{ sau } i_k^t = g(t).$$

The procedure of finding the indicator evolution is:

$$I_k^t = I_k^{t-1}\left(1 + R_k^t\right)$$

or

$$I_k^t = I_k^{t-1} \cdot i_k^t, \quad t = \overline{1, T} .$$

7) One gives the functions for defining the field where I_k^t, R_k^t or i_k^t are generated. The calculating procedure is: we generate the searched parameters for each *t* moment and then we use the calculating relationships at point 6, for R_k^t and i_k^t .

8) One gives the evolution of the I_k indicator for a previous time and the indicator level is estimated for each moment of the time horizon, considering that the evolution trend will be maintained.

The computing procedures of the indicator evolution on the basis of the trend include: adjusting the time series with the aid of one of the analytical methods and extrapolating the indicator evolution. Thus, we shall get: I_k^t , $t = \overline{1, T}$.

9) One gives the previous evolution of the yearly growth rate (index). By a sequence of procedures similar to those at point 8, we shall get R_k^t (or i_k^t) and then, in order to find the evolution of the indicator, I_k^t, the computing procedure from point 6 will be applied.

Step 2÷n

Having found the evolution of the indicator, I_k, we shall estimate in the next step the evolution of another indicator, I_f, in correlation with the first one or independently. The simulation model we present does not select automatically the sequence of the indicator simulation (being procedural), this fact becomes the decision-makers' task. However, according to the decision-makers' options, the procedural chain to meet their complaints will be presented.

For this reason, we shall not present a set of rules (instructions) referring to the hierarchy of the system of indicators, and the correlations to be observed by the decision-makers in order to state the working hypotheses or the selection of decision-making variants.

All these rules are supposed to be known by the users. The functional and statistical relationships among the indicators are mentioned, provided that the parameters of the statistical relationships is exogenously determined or by using a simulating procedure presented at step 1.

Coming again to the decision-makers' acting variants for steps 2-n, we shall present several possibilities to simulate the other indicators of the system. We shall consider that the indicator I_f is a function of a single other indicator (already determined) or of more indicators (with an evolution known from previous steps), in the case we passed beyond step 2.

1) Let us suppose that I_f is a function of the indicator I_k, whose evolution is known from step 1. We shall have the following situations:

1.1. $$I_f^T = \gamma_k^T \cdot I_k^T \quad \text{or} \quad I_f^t = \gamma_k^t \cdot I_k^t,$$

where the coefficient γ_k^T, or γ_k^t, respectively, may represent a structure indicator and it is determined by one of the variants of step 1.

1.2 $$I_f^t = \gamma_k^t \cdot I_k^{t-1}$$

Such a situation has a reduced frequency in reality, but allows the decision-makers to verify some working hypotheses. The coefficient will be determined according to one of the variants of step 1.

2) We suppose that I_f is found according to several indicators.

We shall have the following situations:

2.1. $$I_f^T = \sum_{k \neq f} I_k^T \quad \text{or} \quad I_f^t = \sum_{k \neq f} (1 + R_k)^t I_k^0$$

or $$I_f^t = \sum_{k \neq f} (i)^t I_k^0 \quad \text{or} \quad I_f^t = \sum_{k \neq f} i_k^t I_k^{t-1}$$

Such a dependence is frequently noticed, when the structure of the indicator I_f is a tree-like one and I_k represents the first decomposition level indicators or the primary indicators (which cannot be decomposed) from all the decomposition levels of a tree-like structure, being necessary to be expressed in the same measuring unit: unit value.

2.2. $I_f^T = \sum_{k \neq f} \gamma_k^T I_k^T$ or $I_f^t = \sum_{k \neq f} \gamma_k^t I_k^t$.

This situation includes the previous one (for $\gamma_k = 1$), but the previous remark is valid also for the indicators expressed in physical units.

Finding of the indicator I_f can be done – in addition to those presented at step 1, for estimating γ_k - also by a Rosen type algorithm, where the functional will be quadratic.

$$[\min]S = (I_f^t - \sum_{k \neq f} \gamma_k^t I_k^t)^2 , \qquad t = \overline{0, T-1}$$

and the linear constraints:

$$\gamma_k^{\min} \leq \gamma_k^t \leq \gamma_k^{\max}$$

where: $\gamma_k^{\min}$, $\gamma_k^{\max}$ represent the interval limits, where γ_k^t is estimated to take values.

This algorithm will be applied as follows:

Step 1: One calculates γ_k^0 with the aid of the basic year data:

$$[\min]S = (I^0 - \sum_{k \neq f} \gamma_k^0 I_k^0)^2$$

$$\gamma_k^{\min} \leq \gamma_k^0 \leq \gamma_k^{\max}$$

Then, we shall get: $I_k^1 = \gamma_k^0 I_k^0$, $k \neq f$.

Further on, either we shall consider γ_k^0 to be constant or we shall pass to a new step:

Step 2: $[\min]S = (I_f^1 - \sum_{k \neq f} \gamma_k^1 I_k^1)^2$

$$\gamma_k^{\min} \leq \gamma_k^1 \leq \gamma_k^{\max}$$

getting γ_k^1 and so on (up to step ***T***).

If the evolution of indicators I_k is known for a previous time period, then the coefficients γ_k can be estimated with the aid of the least squares method.

2.3 $I_f^t = \sum_{k \neq f} \gamma_k^t I_f^{t-1}$ or $I_f = \sum_{k \neq f} \overline{\gamma}_k I_k^{t-1}$

where: $\overline{\gamma}_k$ represents a dynamic mean coefficient.

Both γ_k^t and ºi $\overline{\gamma}_k$ can be determined by a fit procedure, similar to those at step 1. Additionally, $\overline{\gamma}_k$ can be determined by one of the methods presented in sub-paragraph 2.2, if we know the necessary data.

If we shall notice the way in which the forecasted levels are reflected into the economic efficiency field, then we shall also find the economic efficiency indicators of holdings and enterprises.

A peculiarity will be met in the case of the efficiency indicators calculated as a ratio of the indicators of results to the indicators of effort (resources consumption). The fact that their level depends, generally, on the level of two indicators, these will have to be previously known or at least their growth rates (indexes).

If we know the level (that can be standardized) of the efficiency indicator or its rate, then we shall have to know the level or the rate of one of the two indicators, according to which the efficiency indicator is calculated. For instance, if we know the net production growth rate (RPN) and that of the net labor productivity (RW), we can find the growth rate of the working personnel number:

$$RN = \frac{RPN - RW}{1 + RW}$$

As we already know the indicator evolution at holding level, this will have to be laid out on enterprise.

In other words, the indicator evolution for the subordinated enterprises must be determined.

Finding the enterprise indicators level

1) Thus, we shall consider as known the holding indicator level and this will have to be equal to the sum of the enterprise indicators (which are going to be found).

Consequently, $I_k^T = \sum_j I_{kj}^T$ or $I_k^t = \sum_j I_{kj}^t$. One yields: I_k^T (or I_k^t) ºi I_{kj}^0, and I_{kj}^T (or I_{kj}^t) will be obtained by using the step 1 fit procedure, making the remark that after totalizing the indicators I_{kj}, we compare it with I_k, and their ratio is balanced, by redistributing the differences upon values I_{kj}, according to certain criteria (usually – proportional to their value).

2) If we know the previous evolutions of I_{kj} and I_k, we can find, on the basis of the trend, the value I_{kj}^t, by solving the functional:

$$[\min] \sum_t (I_k^{t+1} - \sum_j \beta_j I_{kj}^t)^2$$

where: β_j is a parameter expressing the statistical dependence of the indicator I_{kj} as against I_k.

Finally, we shall get β_j^1. Consequently, $I_{kj}^1 = \beta_j^1 I_{kj}^0$.

Further on, we shall consider β_j^1 as being constant $(I_{kj}^t = \beta_j I_{kj}^{t-1})$, or according to values I_{kj}^1 or to other statistical data, a new quadratic function will be build, and so on.

The solving of this procedure to estimate β_j can be made with the aid of the least squares method or using Mierlea's algorithm (Mierlea, 1984). The author starts from an initial solution, which he improves after a given number of iterations or according to some admissible error limits.

This algorithm is based on a modified gradient method. Therefore, if we know the trend, β_j and I_{kj} will be obtained according to one of the above-mentioned procedures.

In the case when the indicator I_k depends upon indicator I_{fj} :

$$I_k^T = \sum_j \gamma_{kj}^T I_{fj}^T \quad \text{or} \quad I_k^t = \sum_j \gamma_{kj}^t I_{fj}^t$$

then: γ_{kj} and I_{fj} will be ascertained, according to one of the mentioned methods and finally, to make up the balance between the results and the indicator level I_k .

Remark. If the decision-maker would like to correct the level of certain holding indicators after simulating the enterprise indicators evolution, then it is recommended that after simulating each indicator this be laid out on enterprises and only if the decision-maker accepts the results, he may pass to other indicators (another step).

All these variants of solving our problem imply to work out an all-inclusive global simulation model to facilitate the "input" into the indicators system and to ensure the possibility of obtaining certain indicators, as a consequence of their correlation or, to transform them into "causes" from "effects", with the possibility of setting the level and the structure of the other indicators.

As a matter of fact, an estimated calculation of the procedural chains number existing in a procedural model of simulating the indicators system can be: $m^n P_{(n)}$, where: n = the number of indicators, and m = the number of independent simulation variants of their level.

In the economic practice the things are more simple, as the interdependences among the indicators makes the simulation of a part of indicators evolution be sufficient ($n' \ll n$), the others being determined as consequences of the already estimated indicators level.

Certainly, the role of the "factorial" indicators and of the "resulting" indicators can alternate, the decision-makers having the "privilege" to state the working hypotheses.

Nevertheless, the presented algorithm is not the single one possible to solve the given problem. Another variant of the problem solving is the "step by step" simulation of the indicator system.

Consequently, the evolution of each indicator is specified only for the immediate time moment $(t \rightarrow t+1)$, and the working hypotheses stay valid only for the respective period of problem solving.

Practically, in this way we have to solve, for each time moment t, a problem equivalent to the given one, provided that the time interval necessary to solve the problem is a unit (year, quarter). Such a problem solving is fit especially when the time unit taken into consideration is under a year or when T, for which the simulation is made, is of at most five years.

Additionally, for the reduced time horizons it is much easier to correlate the value indicators with the physical ones, as the manufacturing list of the holding's subordinated units is known in a rather high proportion. For large time horizons, the simulation of the production physical indicators is more difficult, namely, this will include large groups of products or basic products, but with a high share in the respective units activity. We outline the fact that the aggregated level of the physical indicators is closely related to the time horizon size for which the simulation is made.

We have previously mentioned that from a functional point of view a series of dependences occur among the system indicators, which can be easily identified.

However, when structurally analyzing the indicator system we shall notice that it includes certain synthetic indicators which are found in the functional correlation, but not in the structural one, such as the global production and the working personnel number. The synthetic indicators can cover tree-like structures of other indicators.

The structural analysis of the system of indicators leads us to the conclusion that in order to simulate the evolution of the holding's production one of the used synthetic indicators is the global production, which "covers" the other value indicators into a tree-like structure.

The value indicators integrated into its structure are related to the higher decomposition level indicators by means of certain structure (weight) coefficients. To know these coefficients, at a certain moment of the forecasting horizon, means to solve the problem almost entirely, remaining only to settle the level of any single economic indicator within the tree-like structure.

Thus, after getting the exogenous variables (the weight coefficients) within the synthetic indicator's structure (variables either generated or considered equal to the previous period ones), in order to settle the level of all the indicators of the tree-like structure it suffices to know the level of a single value indicator, regardless of its decomposition level.

Particular case. If we consider as constant the indicator structure on the whole time horizon, it is enough to simulate the level of a single economic indicator for the given (time) period, the rest going to be determined in correlation with this indicator.

Finding the level of the other indicators will be carried on as follows: for lower level indicators, according to the relation $I_f = \gamma I_k$ (where I_f is the estimated level, I_k is the previously settled indicator level and ***ã*** is the weight coefficient, by means of which the direct relation between the two indicators is made) or $I_f = (1-\gamma)I_k$.

The value indicators of a higher decomposition level will be determined according to the relations $I'_f = \gamma^{-1} I_k$ or $I'_f = (1-\gamma)I_k$, while the indicators placed on the same decomposition level is determined only after setting the indicators placed on a higher decomposition level or the indicators placed on a lower level, by totalizing or applying other decomposition operator.

The set of variants to explore an aggregated indicator structure or the indicators system depends upon its form.

Knuth and Tucker (1974) present a series of variants for several types of structures, each variant being possible to be a procedural chain.

Finally, we shall outline the fact that there is another algorithm of simulating the economic indicators derived from combining the two types of the presented algorithms.

The mixt algorithm implies to simulate the evolution of certain indicators for the whole forecasting horizon – in one iteration – and to "fix" them for the future iterations, while the other indicators will be simulated step by step.

This procedural model was experimented on a holding (Andreica, 2005). The time horizon over which the economic indicators evolution was simulated was five years.

With the aid of the model, different developing strategies of a holding have been simulated, with a view to identify a certain variant able to ensure a corresponding dynamics regarding the efficiency of using the production factors. The decision-makers have stated the working hypothesis in order to reduce the searching fields, regarding the simulation developing context. These hypotheses limit the model to the level of a procedural chain. Being a simulation model of "if…then" type, it does not cover the knowledge to validate the hypotheses and the conclusions, to direct the solution searching process. Its advantage consists in determining the decision-maker to participate effectively in working out the solution and its crystallization. The partially accepted solutions that the decision-makers assimilate while simulating represent the support of improving the acting rational strategy.

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