

# 7 THE DIFFUSION OF PRODUCT INNOVATIONS AND MARKET STRUCTURE

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## Abstract

In this paper I analyze the diffusion of a product innovation that was recently made available for licensed purchase within an industry with identical firms producing the same good. The main assumptions are a decreasing yet always positive incentive to adopt the innovation, and an extremely high cost of immediate adoption, but which decreases over time passed since the innovation has become available. The resulting equilibrium in the industry is a gradual adoption of the innovation rather than an immediate one, with each firm having an optimal time of adoption. In the long-run equilibrium, as the number of firms in the industry becomes very large, it is also shown that the incentive to innovate does not disappear. However, as the number of firms in the industry increases each firm is shown to have an incentive to adopt earlier. The assumptions here, as well as the results of this model, match the results of recent studies in the empirical literature.

**Keywords:** product innovation, long-run equilibrium, model

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## Introduction

The main assumption about the adoption of a product innovation in the literature is that it happens immediately after the period of monopoly allowed to the innovating firm. However, evidence shows that such a process is more gradual and that diffusion indeed happens when it comes to product innovations, as well as to process innovations. Some of the reasonable explanations for why this happens are the very high costs of adjustment of immediate adoption, as well as the decrease in the profits from adoption with the number of firms that have already adopted. In other words, firms weigh the costs and benefits of adoption in every period after the license has been made available and decide the optimal time to adopt.

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In a recent empirical study, Mulligan and Llinares (2003) show that the incentive to adopt a quality-enhancing innovation by skiing areas decreases with the number of direct competitors that have already adopted the innovation. Therefore, a diffusion process is found to occur even when talking about product innovations, not just when considering cost-reducing innovations. The latter case was discussed by Reinganum (1981b), who showed that the only equilibrium in a concentrated industry producing a homogeneous good was one where firms adopted the process innovation sequentially. In this paper, I will consider the adoption of a product innovation along with the effect of market structure on the resulting equilibrium. Although the assumption regarding the magnitude of early adjustment costs is maintained, the effect of a product innovation on the total present value of profits works through different channels rather than a cost-reducing process innovation.

Following Klepper (1996), the main assumption here is that when a firm adopts a product innovation, it can attract more buyers who have a preference for that product, and it can sell the good for a higher price than the price of the standard good. This mainly happens because the introduction of the product innovation creates a new demand, since the innovation is a new product, and this demand usually represents a submarket of the original market, but to which only those firms who have purchased the license have access. Therefore, it can be conceived that initially the number of firms willing to enter this submarket is smaller than the total number of firms given the relatively high costs of adoption and adjustment that this process would require. These firms will then make profits higher than in the original market, at least for a while, until all the firms eventually adopt the innovation.

It is shown in this paper that even in the case of a product innovation, the equilibrium outcome is a diffusion process rather than the simultaneous adoption by all the firms in the market. Furthermore, all firms have a positive incentive to adopt the innovation, albeit a decreasing one with the number of adopters. It is also shown that the larger the size of the original market will be, the earlier each firm will decide to adopt the innovation, and that as the number of firms becomes very large the incentives driving the diffusion process do not disappear.

### **The Model**

Let us consider an industry with  $n$  identical firms producing and selling a homogeneous good. These firms are originally in a Cournot-Nash equilibrium, producing a non-negative output and making non-negative profits. When faced with a linear inverse demand,  $p(Q) = a_1 - b_1Q$  and having constant marginal cost  $c_1$ , each firm will decide to produce  $q_1^e$  and will make profits  $\Pi_1^e$ , given by the equations below:

$$q_1^e(n) = \frac{a_1 - c_1}{b_1(n+1)} \quad (1) \quad \text{and} \quad \Pi_1^e(n) = \frac{(a_1 - c_1)^2}{b_1(n+1)^2} \quad (2)$$

At time  $t = 0$  a product innovation becomes available to be licensed by any firm in the market. This innovation, if adopted, provides access to the firm to a submarket, where the inverse demand is given by  $p(Q) = a_2 - b_2Q$ . We can safely assume that since this is a submarket of the original market,  $a_2 = a_1$  and  $b_2 < b_1$ , thus emphasizing the

upward pressure on price which allows only a small number of new buyers to benefit from the innovation. The Cournot-Nash equilibrium output of a firm in this submarket will be a function of the number of adopters,  $m = n$ :

$$q_2^e(m) = \frac{a_2 - c_2}{b_2(m+1)} \quad (3)$$

However, it is often also the case that the marginal cost of producing the new product is higher than the marginal cost of the standard good,  $c_2 > c_1$ , but simply assuming that  $a_2 - c_2 > 0$  ensures that there are important incentives to adopt the innovation. With these conditions, we can conclude that the equilibrium price in the submarket is higher than that for the standard good, while the equilibrium output is lower (see Figure 1). We further assume that the firm maintains the same level of output of the standard good, which is not affected by the number of adopters of the innovation. Therefore, a positive profit in the submarket represents the main incentive for the firms to adopt.

In addition, the firm is also able to sell this new product to a fraction  $\alpha$  of its current customers at the new price. Therefore, the increase in profits to firm  $i$  if it adopts the innovation, gross of adjustment costs, can be written as a function of  $n$  and  $m$ :

$$\Pi_2^e(n, m) = [\alpha q_1^e(n) + q_2^e(m)][p_2^e(m) - c_2] \quad (4)$$

This profit is non-negative for all values of  $m$ , and a strictly decreasing function of  $m$  and  $n$ , thus illustrating the decreasing incentive to adopt an innovation as the number of adopters grows. However, the incentive to adopt never disappears.

Let the function  $y(t)$ , defined for all non-negative values of  $t$ , represent the present value of all adoption and adjustment costs that a firm adopting the innovation at time  $t$  has to incur. We assume that this is a convex and decreasing function of adoption time, but that after a certain time it starts to increase, thus emphasizing that cost-saving from postponing adoption time cannot continue indefinitely. These two conditions can be expressed as follows:

$$y'(t) < 0, \quad y''(t) > 0 \text{ for all } t \in [0, \infty) \text{ and } \lim_{t \rightarrow \infty} y'(t) > 0 \quad (5)$$

Also, we would like to include a condition on  $y(t)$ , which specifies that immediate adoption is too costly, except for the first adopter:

$$y'(0) = -\Pi_2^e(n, 1) \quad (6)$$

A final condition on  $y(t)$  ensures that the objective function in the maximization problem that will follow is strictly concave for all values of  $t$ :

$$y''(t) > \Pi_2^e(n, 1) r e^{-rt} \quad (7)$$

#### Equilibrium profile of adoption dates

The present value of all costs and benefits to firm  $i$  when adopting the innovation at time  $\tau_i$  can be written as a function of the adoption dates of all firms as follows:

$$V_i(\tau_1, K, \tau_n) = \sum_{m=0}^n \int_{\tau_m}^{\tau_{m+1}} \Pi_1^e(n) e^{-\gamma t} dt + \sum_{m=i}^n \int_{\tau_m}^{\tau_{m+1}} \Pi_2^e(n, m) e^{-\gamma t} dt - y(\tau_i) \quad (8)$$

where  $\tau_0 = \tau_1 = \tau_2 = \dots = \tau_{i-1} = \tau_i = \tau_{i+1} = \dots = \tau_{n-1} = \tau_n$  and  $\tau_{n+1} = \infty$ . Since all firms are

identical, the exact ordering is not relevant. Furthermore, this weak ordering of adoption dates does not eliminate the possibility that more than one or even all firms adopt at the same time. It remains to investigate what profile of adoption dates constitutes an equilibrium under this setup.

According to the above specification, the present value of the profits of the firm  $i$  increases if the present value of adoption and adjustment costs are counterweighted by the total benefits represented by the profits in the new submarket. Each firm will choose its optimal adoption date,  $\tau_i^*$ , so that the above function may be maximized. Given the condition (7) on  $y(t)$ ,  $V_i(\cdot)$  is strictly concave, and so the first order conditions are necessary and sufficient for finding the maximizing values of its arguments. Using Leibniz's rule, we obtain:

$$\frac{\partial V_i}{\partial \tau_i} = 0 \Leftrightarrow -\Pi_2^e(n, i) e^{-\gamma \tau_i^*} - y'(\tau_i^*) = 0 \quad (9)$$

for all  $i = 1, \dots, n$ . Given that  $V_i(\cdot)$  is strictly concave and continuous in all its arguments, these maxima exist and are unique. Evaluating (9) for  $i = 1$  and  $\tau_1 = 0$  we obtain:

$$\left. \frac{\partial V_1}{\partial \tau_1} \right|_{\tau_1=0} = -\Pi_2^e(n, 1) - y'(0) \quad (10)$$

But given (6), we conclude that  $\left. \frac{\partial V_1}{\partial \tau_1} \right|_{\tau_1=0} \geq 0$  which means that  $\tau_1^* = 0$ . In addition,

$\lim_{\tau_n \rightarrow \infty} \frac{\partial V_n}{\partial \tau_n} = \lim_{\tau_n \rightarrow \infty} -y'(\tau_n) < 0$  using (5), and so  $\tau_n^* < \infty$ . In order to establish that indeed  $\tau_i^*$  is between  $\tau_{i-1}^*$  and  $\tau_{i+1}^*$ , we will evaluate (9) at each of these two values.

$$\begin{aligned} \left. \frac{\partial V_i}{\partial \tau_i} \right|_{\tau_i=\tau_{i-1}^*} &= -\Pi_2^e(n, i) e^{-\gamma \tau_{i-1}^*} - y'(\tau_{i-1}^*) = -\Pi_2^e(n, i) e^{-\gamma \tau_{i-1}^*} + \Pi_2^e(n, i-1) e^{-\gamma \tau_{i-1}^*} \\ &= e^{-\gamma \tau_{i-1}^*} [\Pi_2^e(n, i-1) - \Pi_2^e(n, i)] > 0 \quad (11) \end{aligned}$$

The above statement is true, since the exponential function is always positive and the function  $\Pi_2^e(n, m)$  is strictly decreasing in its second argument. Since  $V_i$  is concave, (11) shows that  $\tau_{i-1}^* < \tau_i^*$ , with a strict inequality sign due to the strictly decreasing submarket profit function in the number of adopters. A similar argument shows that  $\tau_i^* < \tau_{i+1}^*$ . Since in this analysis  $i$  is arbitrary, it follows that the ordering of all optimal adoption dates is strict:

$$0 = \tau_1^* < \tau_2^* < \dots < \tau_{i-1}^* < \tau_i^* < \tau_{i+1}^* < \dots < \tau_{n-1}^* < \tau_n^* < \infty \quad (12)$$

In order to show that the above profile of optimal adoption dates is a Nash Equilibrium, we must first define what we mean by a Nash Equilibrium in this context.

**Definition:** A profile of adoption dates,  $\tau = (\tau_1, \dots, \tau_n)$  is a Nash Equilibrium if  $V_i(\tau) = V_i(\tau_1, \dots, \tau_{i-1}, \tau_i^*, \tau_{i+1}, \dots, \tau_n)$  for all  $\tau_i^* \in [0, \infty)$  and  $i = 1, \dots, n$ .

To show that  $\tau^*$  from (12) is a Nash Equilibrium we need to reconsider the maximization problem of the firm  $i$ . In finding the optimal adoption date for firm  $i$ , we

have assumed that all other firms do not change their adoption dates as a result of changing  $\tau_i$ . Also, the maximizing value of  $\hat{\delta}_i$  did not depend on the adoption dates of other firms. Therefore,  $\tau_i^*$  from (9) was found for any given values of the other firms' adoption dates. This also applies in the case when all other firms adopt according to  $\tau^*$ , leading us to conclude that the optimal adoption date for firm  $i$  is also the Nash Equilibrium adoption date for that firm. And since this is valid for any  $i = 1, \dots, n$ , we can conclude that  $\hat{\delta}^*$  is a Nash Equilibrium profile as defined above.

However, since all firms are identical, there will be  $n!$  as many Nash Equilibria, since there are  $n!$  possible permutations of numbers from 1 to  $n$ , but where the optimal adoption dates are always strictly ordered.

Here it is interesting to comment on the economic reasons why an equilibrium where all firms adopt at the same time is not possible. First of all, if there is such an equilibrium, then it will be at a date when it is optimal for all firms to adopt the product innovation. Therefore, this date would be the maximizing value of  $\tau$  for the following objective function:

$$V_i(\tau) = \int_0^{\infty} \Pi_1^e(n) e^{-rt} dt + \int_{\tau}^{\infty} \Pi_2^e(n, n) e^{-rt} dt - y(\tau) \quad (13)$$

As above, the first order condition is necessary and sufficient to find the maximizing value,  $\tau^*$ , which will be uniquely determined due to the strict concavity of  $V_i$ :

$$\frac{\partial V_i}{\partial \tau} = 0 \Leftrightarrow -\Pi_2^e(n, n) e^{-r\tau^*} - y'(\tau^*) = 0 \quad (14)$$

In words,  $\hat{\delta}^*$  will be the earliest date at which the present value of extra profits earned by a firm by adopting the innovation covers just the present value of all adjustment costs. We know this date will be strictly greater than zero, since condition (6) specifies that immediate adoption is too costly for more than one firm. Furthermore, this date is likely to be a relatively late date since  $\Pi_2^e(n, n)$  is the smallest level of profit a firm can make in the new submarket. We can now consider the decision of firm  $i$  at this point, given that all other firms maintain their adoption dates at  $\tau^*$ . If firm  $i$  chooses to adopt at an earlier date than  $\tau^*$ , say  $\tau'$ , it will increase its present value of the total profit flows by  $\int_{\tau'}^{\tau^*} \Pi_2^e(n, 1) e^{-rt} dt > 0$ . Therefore, a profile of adoption dates when all firms adopt at the same time is not a Nash Equilibrium. Also, a similar argument shows that any more than one firm adopting at the same time is not a Nash Equilibrium.

#### The effect of the market structure on equilibrium adoption dates

To see the effect of the market structure as determined by the value of  $n$  on the equilibrium values of adoption dates, we should reconsider the first order conditions from the above maximization problem:

$$\frac{\partial V_i}{\partial \tau_i} = 0 \Leftrightarrow -\Pi_2^e(n, i) e^{-r\tau_i^*} - y'(\tau_i^*) = 0 \quad (9)$$

Totally differentiating (9) with respect to  $n$  we obtain:

$$-\frac{d\Pi_2^e(n,i)}{dn} \cdot e^{-\gamma\tau_i^*} + \Pi_2^e(n,i) \cdot e^{-\gamma\tau_i^*} r \cdot \frac{d\tau_i^*}{dn} - y''(\tau_i^*) \cdot \frac{d\tau_i^*}{dn} = 0$$

$$\frac{d\tau_i^*}{dn} \cdot [\Pi_2^e(n,i) \cdot e^{-\gamma\tau_i^*} - y''(\tau_i^*)] = \frac{d\Pi_2^e(n,i)}{dn} \cdot e^{-\gamma\tau_i^*} < 0$$

The term on the right side of the equal sign is negative because profits in the new submarket are decreasing in the size of the original market. Using condition (7), the sign of  $\frac{d\tau_i^*}{dn}$  is negative (the term in the brackets is positive due to the concavity of  $V_i$ ). Therefore, an increase in  $n$  will decrease the value of  $\tau_i^*$ , meaning that as the size of the original market increases, firms will choose to adopt the innovation earlier.

The case when  $n \rightarrow \infty$  also leads to the diffusion of the product innovation. The equilibrium adoption dates will be given by:

$$-q_2^e(i) [p_2^e(i) - c_2] e^{-\gamma\tau_i^*} - y'(\tau_i^*) = 0 \quad (15)$$

because  $\lim_{n \rightarrow \infty} \Pi_2^e(n, i) = \lim_{n \rightarrow \infty} [\alpha q_1^e(n) + q_2^e(i)] [p_2^e(i) - c_2] = q_2^e(i) [p_2^e(i) - c_2]$  and

$\lim_{n \rightarrow \infty} q_1^e(n) = \lim_{n \rightarrow \infty} \frac{a_1 - c_1}{b_1(n+1)} = 0$ . The same conditions as above apply here to

show that the firms will adopt the product innovation sequentially. The only difference is that as profits in the original market become normal, the incentive to adopt the innovation will be represented by only the positive profits made by adopting. However, as  $m \rightarrow n$ , profits in this submarket will also go to zero.

## Conclusions

Given the nature of incentives associated with a product innovation, it has been shown that a diffusion process of adoption is the only optimal result in a concentrated industry producing a homogeneous good. The main economic reasons for an immediate and/or a simultaneous adoption of the product innovation are that the present value of adjustment costs is very high early, and that the present value of profits from adoption is decreasing with the number of adopters. It has also been shown that as the number of firms in the market increases, each firm has an incentive to adopt the innovation earlier, and that as the number of firms becomes very large, the incentives driving the diffusion process do not disappear.

As an extension of this model, an analysis of the outcomes in the market when both a product and a process innovation are available would be interesting. One could look at the preference of the firms for either type of innovation and which type will be adopted sooner.

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