# 15 FORECASTING ROMANIAN GDP USING A SMALL DSGE MODEL*

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## Abstract

*In this study I apply a simple DSGE model to forecast the quarterly Romanian GDP. The forecast is based on the posterior distribution of the model parameters resulted from the Bayesian estimation. The forecast for the 2006-2007 period shows that the realized GDP is within the confidence interval of the forecast when the shock uncertainty is also included. The projection for the 2007-2010 period indicates an average growth rate of almost 6%.*

**Keywords:** forecasting methods, DSGE models, Bayesian methods, real business cycles.

**JEL Classification**: E60, C68.

## 1. Introduction

The paradigm that nowadays dominates macroeconomics is that of the dynamic stochastic general equilibrium models (DSGE). The DSGE approach originates from the real business cycles model (RBC, henceforth), build for the first time by Kydland and Prescott (1982), and then extended through the consideration of both other types of shocks besides the productivity ones (like monetary, inflationary, fiscal ones, etc.) and also through the introduction of different types of imperfections and rigidities.

In the present study I investigate a DSGE model, namely Hansen's (1985) model, for Romanian economy for the 2000–2007 period. This paper has a twofold purpose. First I estimate the model on Romanian data and, second of all, I use this model to forecast the quarterly GDP series.

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This model was introduced by Hansen (1985) as an extension of the standard RCB model. The main feature of the model is to introduce of the indivisible character of the work effort. This small change was introduced in order to improve the predictions of the model in explaining the cyclical behavior of the labor market variables, like the higher variation than in the real data of the hours worked.

The paper is organized as follows: in the second section I present the building blocks of the model, and the log-linearized version; in the third section I estimate the model and discuss the impulse response functions; in the fourth section, I use the estimation results to forecast the quarterly GDP; the last section concludes and draws some possible developments.

## 2. The model

The model I study is that of Hansen (1985), one of the most representative real business cycle models. In this section I present the model as outlined by Uhlig (1995).

The model consists of a finite number of representative agents characterized by an infinite life. Each agent maximizes the expected lifetime utility. In each period the agent optimally chooses the consumption, investments and the labor effort, under the constraints given by its income.

In terms of the social planner, the problem is to maximize the total utility of the agent given by:

$$\max E_0\left[\sum_{t=0}^{\infty}\beta^t\,\frac{C_t^{1-\eta}-1}{1-\eta}-AN_t\right] \tag{1}$$

where: β is the discount factor, $C_t$ is the consumption, η is the relative risk aversion coefficient, $N_t$ is the number of hours worked, and A is a parameter characterizing the utility function.

The maximization problem is done under several constraints. The first constraint is the equilibrium condition for the goods market, namely:

$$C_t + I_t = Y_t \tag{2}$$

In each period the agent faces another budgetary constraint:

$$K_t = I_t + *(1-\delta)K_{t-1} \tag{3}$$

The next constraint is given by the production function. I assume a Cobb-Douglas production function:

$$Y_t = Z_t * K_{t-1}^{\alpha} * N_t^{1-\alpha} \tag{4}$$

where: Nt is the time resource, Zt is the total factor productivity and α is the elasticity of the production with respect to capital. The production function is characterized by constant returns to scale.

The last constraint is given by the specification of the TFP dynamics through the equation:

$$\log Z_t = (1-\rho) * \log \overline{Z_t} + \rho * \log Z_{t-1} + \varepsilon_t \tag{5}$$

The parameter ρ is the technological progress constraint. The error term in the equation above is a white noise process, representing the innovation in the technological progress.

I use the Lagrangian approach to derive the necessary conditions. They are given by the first order derivatives of the objective function with respect to $K_t$, $C_t$, $N_t$ and $\lambda_t$. By eliminating $\lambda_t$, we get the following three equations:

$$A = C_t^{-\eta} * (1-\alpha) * \frac{Y_t}{N_t} \tag{6}$$

$$1 = \beta * E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\eta} * R_{t+1}\right] \tag{7}$$

$$R_t = \alpha * \frac{Y_t}{K_{t-1}} + 1 - \delta \tag{8}$$

We can derive now the steady state solution by fixing the variables with respect to time. The next equations describe the steady state of the model:

$$\overline{R} = \frac{1}{\beta} \tag{9}$$

$$\frac{\overline{Y}}{\overline{K}} = (\overline{R} + \delta - 1)/\alpha \tag{10}$$

$$\overline{K} = \left(\frac{\overline{Y}}{\overline{K}}\overline{Z}\right)^{\frac{1}{1-\alpha}}\overline{N} \tag{11}$$

$$\overline{I} = \delta\overline{K} \tag{12}$$

$$\overline{C} = \overline{Y} - \delta\overline{K} \tag{13}$$

The linearized system is given by the following equations:

$$c_t * \overline{C} + i_t * \overline{I} = y_t * \overline{Y} \tag{14}$$

$$i_t * \frac{\overline{I}}{\overline{K}} + (1-\delta) * k_{t-1} = k_t \tag{15}$$

$$y_t = \alpha * k_{t-1} + (1-\alpha) * l_t + z_t \tag{16}$$

$$z_t = \rho * z_{t-1} + \varepsilon_t \tag{17}$$

$$0 = (-\eta) * c_t + y_t - 1 \tag{18}$$

$$0 = (-\eta) * (c_t - c_{t+1}) + R_{t+1} \tag{19}$$

$$R_t = \alpha * \beta * \frac{\overline{Y}}{\overline{K}}(y_t - k_{t-1}) \tag{20}$$

## 3. Data and Estimation of the Model

In this section, I estimate the linearized model given by equations (14) to (20), and discuss the impulse response functions to the productivity shocks. As the model is estimated in a log-linearized form, the variables being deviations from the trend, it is necessary to use the observable time series as deviations from trend.

As the model contains a single type of shock, namely the productivity shock, in the estimation I use a single observable time series, namely the GDP series, (see Mancini (2007) for details). The GDP series is the constant 1995 prices series. The series was logged, deseasonalized through Census X12 in Eviews and then filtered using the Hodrick-Prescott filter. The obtained series stands for the deviations of GDP from its trend.

The set of parameters to be estimated is given by $\{\alpha, \beta, \delta, \rho, \eta, \sigma_a\}$. Parameters α, β and δ are calibrated using the results in Caraiani (2007) and Dobrescu (2006).

The capital share is estimated at 0.35. Although Caraiani (2007) suggested an α=0.4, Dobrescu (2006) argued for a lower α, namely α=0.4. I decided to use this latter result. For β, I use the results in Caraiani (2007), namely β=0.99. The quarterly depreciation rate is computed by Caraiani (2007) at δ=2.4%.

The other parameters in the model, namely $\{\rho, \eta, \sigma_a\}$, are estimated using Dynare package. Details for the mathematical background in Dynare can be found in Juillard (2007). A short description of the Bayesian techniques used in Dynare can be found in Mancini (2007). A more detailed description of the estimation of DSGE models using the Bayesian approach is described in An and Schorfheide (2007).

**Table 1**

**Bayesian estimation results**

| Parameters | Prior Mean | Posterior Mean | Confidence Interval | Confidence Interval | Prior Distribution | Standard deviation |
|---|---|---|---|---|---|---|
| ρ | 0.7 | 0.74 | 0.46 | 0.99 | Beta | 0.2 |
| η | 1.00 | 1.10 | 0.71 | 1.45 | Normal | 0.25 |
| $\sigma_a$ | 0.02 | 0.0049 | 0.0036 | 0.0063 | Inverted Gamma | Infinite |

*Source: Own computations.*

We notice that the shocks persistence, namely ρ=0.74, is less strong than the usual estimates for the technological shocks persistence in the USA or other G7 economies. This implies that the effects of productivity shocks manifest in Romania for a shorter period of time.

The γ coefficient from the utility function is estimated at a value close to the prior mean, suggesting a rather moderate risk aversion.

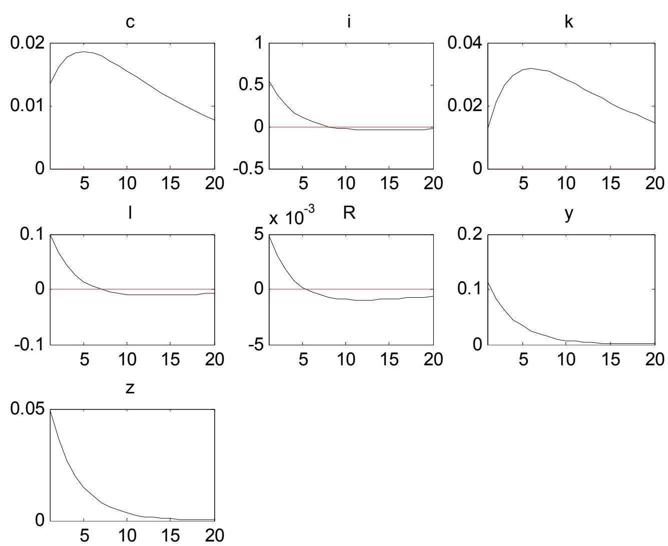
I discuss now the results of the estimation with respect to the impulse response functions. We saw already that the model is based on a single type of exogenous shock, namely the productivity shock. The impulse response functions were computed using the average posterior means for the parameters.

A positive productivity shock has significant effects on TFP for almost 16 quarters, which is for four years. The impact on GDP is similar in shape, and it lasts for approximately 12 quarters. The persistence difference between the two responses comes from the lower than usual ρ value.

The impulse response functions of capital and consumption have a particular shape, namely a hump-shaped response. The peak is reached after three quarters for consumption and after five quarters for capital stock. The impact on them lasts in the medium run and in the long run.

**Figure 1**

**Impulse Response Functions to Productivity Shocks**




*Source: Own computations.*

## 4. Forecasting GDP

In this section, I continue to apply the DSGE model to Romania by making several forecasts, both relatively to the real GDP dynamic during 2006 quarter 1 and 2007 quarter 2 in section 4.1, and in the medium run, for the 2007 quarter 3 to 2010 quarter 4 period, in the section 4.2.

The application of DSGE models to make forecasts has been used relatively recently. Some of the most significant contributions in this field were done by Smets and Wouters (2004), Del Negro and Schorfheide (2004), Dib, Gammoudi and Moran (2005), Kilponen and Ripatti (2006). We can also notice the contribution of Liu and Gupta (2006) who applied the same model as in this paper to forecast in the medium run the South African Economy.

In a natural extension of their contribution to the DSGE modeling, which they applied mostly to the Euro Area data, Smets and Wouters (2004) showed that a new Keynesian (NK, henceforth) model with rich rigidities can be used to forecast macroeconomic variables. They showed that the estimated DSGE model has good out-of-sample performances, better that those of the unstructured VARs, and even those of the Bayesian VARs (BVAR, henceforth).

Del Negro and Schorfheide (2004) extended Smets and Wouters work. They used the foundations given by BVAR models in order to introduce into them priors derived from the estimated DSGE models. They showed that, even for a simple NK model, using priors derived from a DSGE model led to excellent performances relative to both unstructured VARs and BVARs.

Dib, Gammoudi and Moran (2005) used a NK model for a closed economy characterized by rigid prices, and applied the model for Canadian economy. They showed that using this model for forecasting the main macroeconomic variables allowed for good results as compared to unstructured VARs, especially as the forecast horizon increased.

A good example of practical regular use of the DSGE approach to forecasting exercises is that of the Bank of Finland (BOF). BOF introduced the regular use a DSGE model, called AINO, in the official forecast of the central bank. The model proved to offer good results, as it was capable to reproduce the tendencies of the Finnish economy both in the short and in the medium run (the forecasts are done for an eight to ten quarters horizon), see Kilponen şi Ripatti (2006).

### *4.1. An Analysis of Forecasting Performance*

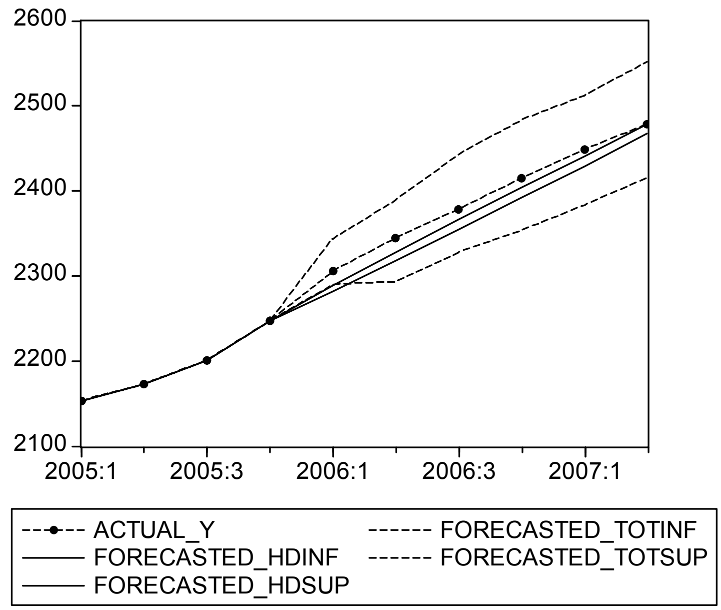
I realize now a forecasting exercise, first of all for the period between 2006 quarter 1 and 2007 quarter 2. The sample chosen allows for comparison with the actual GDP. Figure 2 presents the results of the forecast for six quarters ahead. As previously discussed, the forecasting of quarterly GDP is done for the GDP in 1995 constant prices.

The forecast from the posterior distribution allows the computation of two confidence intervals, namely, a first interval given by (HPDInf, HPDSup), which represents the 90% confidence interval given by the uncertainty relative to the value of parameters, and a second interval given by (HPDTotalInf, HPDTotalSup), which gives the 90% confidence interval related to the uncertainty relative to both parameters and shocks.

Figure 2 shows that for the 2006 quarter 1–2007 quarter 2 period, the actual GDP is outside the first confidence interval, while it is within the second confidence interval. We can also notice that the actual GDP tend to converge towards the area given by the first confidence interval, which suggests that the model does better for medium run forecasts horizons.

**Figure 2**

**Forecasting GDP for the 2006:01-2007:02 period**




*Source: Own computations.*

A second way to evaluate the model is to construct a filtered GDP series with the help of the structural model and compare it with the actual GDP series. The filtered GDP series is based on the posterior distribution of the endogenous GDP variable, and it is based only on previous period information. The difference between filtered and actual series represents the one step ahead forecast error.

Figure 3 shows the figure of the two series. We notice that the structural model is able to provide a good representation of actual GDP, with the exception of some quarters in 2004, a year in which the GDP growth was well above its potential, (see Annex 2 for the one step ahead forecast error).

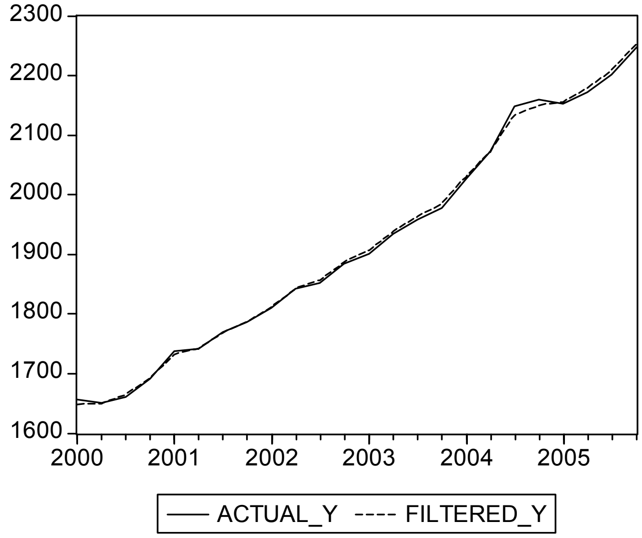
### *4.2. Forecasting GDP for 2007-2010*

This section continues the analysis of the forecasting capacity of the estimated DSGE model by forecasting the GDP for the period between 2007 and 2010. Figure 4 presents the results of the forecast. Here, the 90% confidence interval is given by the uncertainty associated with both parameters and shocks.

In terms of annual growth of GDP, the model forecasts an average growth rate of about 6% for the studied period between 2007 and 2010.

**Figure 3**

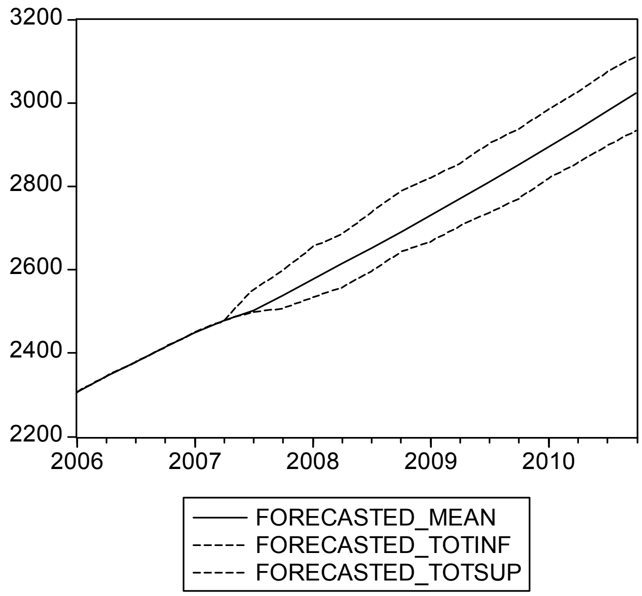
**Filtered GDP versus Actual GDP 2000:01-2005:04**




*Source: Own computations.*

**Figure 4**

**Forecasting GDP for the 2006:01-2007:02 period**




*Source: Own computations.*

## 5. Conclusion

In this study I applied a standard DSGE model, namely the real business cycles model, in Hansen (1985), in order to forecast quarterly GDP. The forecast is based on the posterior distribution of the parameters of the model resulted from the Bayesian estimation.

The results of the estimation show a lower than usual value of persistence of the technological shocks which implies that the effects of productivity shocks last less than usual. The risk aversion appears to be moderate.

In a first stage, I analyzed the forecasting performance of the DSGE model relative to the realized GDP for the period between 2006 quarter 1 and 2007 quarter 2. The forecast for the 2006-2007 period shows that the realized GDP is within the confidence interval of the forecast when the shock uncertainty is also included. This suggests that while the model can have a good performance for the trend of the growth, its overall performance it subject to the level of temporary productivity shocks. This is also underlined by the fact that for 2004, the model has higher one-step-ahead forecast errors, due to the temporary positive productivity shock which led to a temporary higher growth rate of GDP.

The second forecasting exercise was done for the 2007–2010 period. The projection indicates a growth rate of annual GDP of almost 6% which is in line with other estimates.

This paper showed that we can forecast Romania's GDP using a DSGE model. Future studies should extend the forecasting to the other essential macroeconomic variables, such as the inflation rate, or the exchange rate, by considering more complex NK models.

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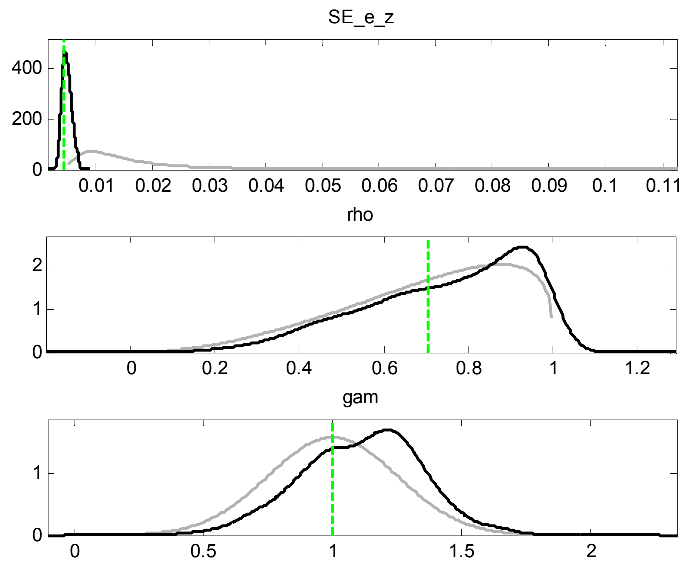
**Annex 1**

## Bayesian Estimation Results




**Annex 2**

## One Step Ahead Forecast Error