5. PREDICTION BASED ON TIME SERIES. APPLICATIONS IN QUALITY CONTROL

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Abstract

In this paper we propose a prediction model based on time series involving EWMA type approach. After a brief historical sketch and a short presentation of the GLM - General Linear Model we construct the predictor which is an average exponentially weighted depending on previous and current values of the series.

The last paragraph is dedicated to an analogy with SPC - Statistical Process Control and possible applications are emphasized. Open theoretical problems are discussed also.

Keywords: time series, EWMA, GLM, SPC, predictor white noise, correction factor
JEL Classification: C32, C53

1. Preliminaries

Time series (or "chronological series" in the French literature) are considered by some authors – for example Stigler (1986, [28], pp. 22-30) – of the same origin as the regression and statistical correlation theories, for they offer the connection (dependence) between a measurable variable Y – for example, the volume of sales of a certain product in a given period of time – and the "discrete" time variable as such: sales per month, quarter, year, and so on.

The famous American qualitologist William Edward Deming (1900 - 1993) thinks that regression and correlation, as well as time series can fit the so-called curve fitting procedure (Deming, 1964). Deming justifies his statement with the fact that the respective sub-domains have as common work procedure the well-known ordinary least square method (Gauss – Legendre – Markov – see also Dictionary of General Statistics, *Dicționar de Statistică Generală,* Ed. Economică, Bucureşti, 2003).

The junction between the time series theory and the regression and correlation theory has been made by economists such as Clement Juglar (1819 - 1905), S. W. Jevons

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(1835 – 1883), W. Clair Mitchell (1874 - 1948), Ragnar Frisch (1894 - 1973), Johann Schumpeter (1883 - 1956) and others. Among their forerunners we can mention the Belgian mathematician Pierre François Verhulst (1804 - 1849) and the British demographers Benjamin Gompertz and William M. Makenam (details in Moineagu *et al.*, 1976).

In this respect, Verhulst managed, with the help of the "logistic curve" (which he had introduced in 1838), to develop an interesting forecast/prediction, on five-year periods, of the population of Belgium, his home country. The logistic curve is the following:

$Y_{(t)} = \dfrac{A}{B + C \cdot \exp(-Dt)}, \quad t \geq 0, \quad A,B,C,D > 0$, where variable Y represents the population growth in a certain geographic area.

He estimated that Belgium will be able to accommodate 9.500.000 inhabitants at most, depending on its economic-geographic conditions. Interesting is the fact that Verhulst's prediction has been surprisingly realistic: in 1994, the actual Belgian population was approximately 10,200,000 inhabitants (see also Dinu - Vodă, 2006, pp. 4-6).

If we refer to the modern approach to time series, there are introduced new concepts and working instruments – such as autoregression, predictors of different forms, generated by serial or temporal models, etc. – but it is difficult to establish a precise chronology, able to advance a specific disciplinary delimitation.

Other elements of actuality: the case of co-integrated series (Andrei, Bourbonnois, 2008), the introduction of time gaps, (Pecican, 2006) spectral analysis, (Harvey, 1993), the use of neuronal networks in time series forecast (Ucenic, Atsalakis, 2005), dealing with the short term prediction models (Albu, 2008), particularizing informatics solutions to treating time series (Vogelvang, 2005, Andrei T., 2008). Domains – apparently disparate – such as the statistical quality control, metrology, etc., also brought their contribution to the development of this theory.

It is also worth mentioning the new managerial system known as the Six Sigma Approach, where time series play an important role in the analysis of dependent data. They (time series) are used as prediction instruments – the prediction being considered by most authors in the field as essential in the process of knowledge (Deming, 1964, page 24) – see also our recent monograph, *The Six Sigma Approach. Interpretations, Controversies, Procedures*, Ed. Economică, Bucureşti, 2008 (in Romanian).

2. Introduction: Essential concepts of the time series theory

As it is well known (Box and Jenkins, 1970) a time series (chronological or "dynamic") represents a set of observations obtained through a measurement process of a random variable X, recorded in time and indexed by it.

Theoretically, these values can be recorded continuously but in reality, they are observed at equal time intervals and indexed in order to be able to work with the sample vector of volume (n): $x_t = (x_{t1}, x_{t2}, \dots x_{tn})$.

Due to some theoretical reasons, we assume that time series are stationary, that is they are generated by a process or a phenomenon in a state of statistical equilibrium - similar to the one defined in the statistical control of fabrication processes (see Montgomery, 1996, or Petrescu-Vodă, 2004).

Statistical equilibrium (the state of statistical control) is expressed through the variability of the studied characteristic, in its natural variability interval: if variable X is normally distributed N(μ, σ^2), this interval is calculated based on the three sigma principle: $\mu \pm 3\sigma$, in the interval of length $\mu + 3\sigma - (\mu - 3\sigma) = 6\sigma$ being found approximately 99.73% of the values of X. Here, $\mathrm{E}(x) = \mu$ and $\mathrm{Var}(x) = \sigma^2$ (also see document SR ISO 8258-C1/1999 "Shewhart control chart" (ISO official standard).

The following assumptions are made for a time series Xt:

1. The mean and variance of the series are stable (homogenous) in time, $\mathrm{E}(x_t) = \mu_x$, $\mathrm{Var}(x_t) = \sigma_x^2$ for all time moments (t) considered;
2. The covariance of any two terms of the series does not depend but of the absolute difference of time between these terms, that means we have $\mathrm{Cov}(x_t, x_{t+k}) = \mathrm{C}_x(\mathrm{k})$, for every t, in the initial conditions $\mathrm{Cov}(x_t, x_{t+k}) = \mathrm{C}_x(\mathrm{k})$.

Then, the autocorrelation function is defined as:

$$\rho_x(\mathrm{k}) = \mathrm{C}_x(\mathrm{k}) / \sigma_x^2, \quad \mathrm{k} > 0, \quad \rho_x(0) = 1 \tag{2}$$

If we denote $\bar{x} = \sum_1^n x_t / \mathrm{n}$ (empirical mean) respectively $\tilde{\mathrm{s}}^2 = \sum_1^n (x_t - \bar{x})^2 / \mathrm{n}$ (empirical variance) then the autocovariance coefficient is:

$$\mathrm{C}_x(\mathrm{k}) = \sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x}) / \mathrm{n} \tag{3}$$

Having the autocorrelation function $\rho_x(\mathrm{k})$ of a stationary time series, we also have the necessary information to build a so-called linear-predictor with the help of the ordinary least square method, using a given finite number of previous values: $x_{t-1}, x_{t-2}, \dots, x_{t-n}$. The predictor's coefficients will be determined by the correlations among variables, that is the variables $\rho_x(1), \rho_x(2), \dots \rho_x(\mathrm{k})$. The prediction equation has the form:

$$x_t = \mathrm{A}_{\mathrm{k},1} \cdot x_{t-1} + \mathrm{A}_{\mathrm{k},2} \cdot x_{t-2} + \dots + \mathrm{A}_{\mathrm{k},\mathrm{k}} \cdot x_{t-k} + e_{\mathrm{k},t} \tag{4}$$

where $e_{k,t}$ is the error variable associated to this linear predictor in variables x_{t-j}, $j = 1, 2, \dots, \mathrm{k}$.

3. The General Linear Model (GLM)

The simplest stationary series is in fact a set of random variables $\{a_t\}$, independent and identically distributed. Building a time series as a linear combination of past and present values of $\{a_t\}$ leads to the autocorrelation phenomenon.

GLM is a mathematical instrument used in a wide spectrum of domains – from ANOVA (see Mărgăritescu, 1981) to the mechanical durability analysis (Dorin and others, 1994). The time series $\{x_t\}$ has the following form:

$$x_t = B_0 \cdot a_t + B_1 \cdot a_{t-1} + \ldots = \sum_{i=0}^{\infty} B_i \cdot a_{t-i} \tag{5}$$

where values a_i are not directly measured, being considered a so-called "white noise" (Chow, 1985, pp. 185-186).

GLM is adequate to a prediction operation since it is relatively easy to memorize analytically. Thus, being given observations for x_t $(t \leq n)$, then any future value $x_{n+k}\,(k > 0)$ could be written:

$$x_{n+k} = \left(a_{n+k} + B_1 \cdot a_{n+k-1} + \ldots + B_{k-1} \cdot a_{n+1}\right) + \left(B_k \cdot a_n + B_{k+1} \cdot a_{n-1}\right) \tag{6}$$

The first pair of brackets contains the components of x_{n+k} depending on future (unknown) terms of a_t, t > 0, while the second one contains the previous (known) terms of a_t, for $t \leq n$.

4. The EWMA predictor

Among the particular models there are the ones which use the so-called exponentially weighted moving average. The idea was introduced in the 50s (20th century) in the context of the statistical control of processes (see Lowry and others, 1992). Based on EWMA we can construct charts for monitoring the average of a certain process, using whether individual values, or samples of at least n=2 observations.

The exponentially weighted moving average is defined as:

$$z_i = \lambda\, x_i + (1-\lambda)\, z_{i-i}$$

where $0 < \lambda < 1$ is a constant and the initial value – necessary when i=1 – is $z_0 = \mu_0$. In most of the cases, this initial value is considered to be the average of some previous preliminary values, meaning $z_0 = \overline{x}$. Replacing z_{i-1} in (7), we have:

$$z_i = \lambda\, x_i + (1-\lambda)\left[\lambda\, x_{i-1} + (1-\lambda)\, z_{i-2}\right] \tag{8}$$

Or $$z_i = \lambda\, x_i + (1-\lambda) x_{i-1} + (1-\lambda)^2 \cdot z_{i-2} \tag{9}$$

By recurrence, we find immediately (for z_{i-j}, j = 2, 3, ...):

$$z_i = \lambda \cdot \sum_{j=0}^{i-1} (1-\lambda)^j\, x_{i-j} + (1-\lambda) \cdot z_0 \tag{10}$$

The weights of $\lambda(1-\lambda)^j$ decrease geometrically, their sum approaches 1, since

$$\lambda \cdot \sum_{j=0}^{i-1}(1-\lambda)^j = \lambda\left[\frac{1-(1-\lambda)^j}{1-(1-\lambda)}\right] = 1-(1-\lambda)^i \qquad (11)$$

Thus, the EWMA value is a weighted average of all previous values and of the current one, which is insensible to the deviation of data.

This fact – the EWMA robustness towards "normality hypothesis violation" as Eugen Mãrgãritescu expressed (1921 – 1996) see (Margaritescu, 1981) – makes the procedure appropriate for the direct study of individual values. The econometric calculus often uses monthly or quarterly data that are represented by a single number, so the normality of the data is difficult to be verified, and perhaps most of the times it doesn't happen.

In the EWMA methodology, the independence of values x_i and the homogeneity/ homoscedasticity of the variances of these variables are sufficient. Then, the variance of predictor z_i is

$$\sigma_{z_i}^2 = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)\right]^{2i}$$

where $\mathrm{Var}(x_i) = \sigma^2$, $\forall\, i$.

Thus, the process monitoring chart is based on the three sigma principle; some authors (such as Prabhu – Runger, 1997) recommend values for λ between 0.25 and 0.4. The following elements are taken into account:

1. The accepted upper variability limit:

$$\mathrm{UVL} = \mu_0 + 3\sigma\left\{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\right\}^{1/2} \qquad (13)$$

2. The central line: $\mathrm{LC} = \mu_0$

3. The accepted inferior variability limit:

$$\mathrm{LVI} = \mu_0 + 3\sigma\left\{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\right\}^{1/2} \qquad (14)$$

Some conclusions are needed:

a) In the EWMA model, the control limits are variable, and depending on the rank (i) of the sample, unlike the classical case of Shewhart control charts (where n is usually fixed and the same for each sample).

b) In the classical case of Shewhart charts the limits are straight lines; in the EWMA situation, these limits tend to be linear if the value under the radical approaches 1.

5. A new prediction model

Time series allow the construction of prediction models even in the situation in which the values x_t are not stationary.

Then, we can investigate the character (perhaps stationary) of the first degree differences, meaning the quantities $d_t = x_t - x_{t-1}$ based on which an autoregressive model can be built

$$\mathrm{d}_t = x_t - x_{t-1} = \varepsilon_t - \theta\,\varepsilon_{t-1}, \quad 0 < \theta \le 1 \tag{15}$$

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{t-1}, \ldots$ is the set that generates the white noise with variance σ_ε^2.

So we have

$$x_t = x_{t-1} + \varepsilon_t - \theta\,\varepsilon_{t-1} \tag{16}$$

If the value x_t degree can be written as a sum of the fixed average (μ) of the process and an error factor e_t, then

$$x_t = \mu + e_t \tag{17}$$

Since because $\mathrm{E}(x_t) = \mu$, we deduce that $E(e_t) = 0$.

In many real cases, the errors e_t are not always independent – this is the case of ANOVA model (see Isaic-Maniu and Vodă, 2006) due to uncontrollable factors that can appear in certain periods of time. In this case, e_t error can be written as (see Bârsan-Pipu, 1999):

$$e_t = \varepsilon_t - \theta\,\varepsilon_{t-1} \tag{18}$$

Through recurrence we find

$$e_{t-1} = \varepsilon_{t-1} - \theta\,\varepsilon_{t-2} \tag{19}$$

$$\varepsilon_{t-2} = \varepsilon_{t-2} - \theta\,\varepsilon_{t-3} \tag{20}$$

It follows that e_t will be correlated with e_{t-1}, as both contain the random variable ε_{t-1}, but e_t will not be correlated to e_{t-2}, since the ε values they include are not common, they are independent.

Thus, the time series model has the form

$$x_t = \mu + \varepsilon_t - \theta\,\varepsilon_{t-1} \tag{21}$$

where μ is a systematic part, and the other terms represent the white noise.

We can thus make a prediction for the nth moment, using the equation (16):

$$x_{n+1} = x_n + \varepsilon_{n+1} - \theta\,\varepsilon_n \tag{22}$$

Although the value of ε_{n+1} at moment (n+1) is unknown, we do know that $\mathrm{E}(\varepsilon_{n+1}) = 0$, and that it is a random variable uncorrelated with the other data we have.

So the best value for ε_{n+1} is zero and consequently the best prediction for x_{n+1} is

$$\hat{x}_{n+1} = x_n - \theta\, \varepsilon_n \tag{23}$$

Replacing x_n (from 23) in the previous relationship we find

$$x_{n+1} - \hat{x}_{n+1} = \varepsilon_{n+1} \tag{24}$$

That is ε_{n+1} is precisely the difference between the value of (x) at moment (n+1) and its prediction $\hat{x}$ - so

$$\varepsilon_n = x_n - \hat{x}_n \tag{25}$$

As a consequence, the prediction equation at moment (n) will be

$$\hat{x}_{n+1} = (1-\theta)x_n + \theta\,\hat{x}_n \tag{26}$$

At the moment (n-1) we will have

$$\hat{x}_n = (1-\theta)x_{n-1} + \theta\,\hat{x}_{n-1} \tag{27}$$

and at (n-2):

$$\hat{x}_{n-2} = (1-\theta)x_{n-1} + \theta\,\hat{x}_{n-1} \tag{28}$$

Multiplying above (starting with moment n-1) with $\theta, \theta^2, \quad \theta^3 \ldots$ we deduce that

$$\begin{aligned} \hat{x}_{n+1} &= (1-\theta)x_n + \theta\,\hat{x}_n \\ \theta\,\hat{x}_n &= (1-\theta)\theta\, x_{n-1} + \theta^2\,\hat{x}_{n-1} \\ \theta^2\hat{x}_{n+1} &= (1-\theta)\theta^2\, x_{n-2} + \theta^3\,\hat{x}_{n-2} \end{aligned} \tag{29}$$

After summing these relations we obtain

$$\hat{x}_{n+1} = (1-\theta)[x_n + \theta\, x_{n-1} + \theta^2\, x_{n-2} + \ldots] \tag{30}$$

This shows that the prediction is an exponentially weighted moving average (EWMA) of the previous and current data in the series.

6. A connection to the statistical control of processes

Between some economic phenomena – such as the exchange rate evolution in conditions o relative stability (excluding speculative jumps or unfavorable contexts as severe economic crises), the evolution of unemployment, or of labor force absorption on a certain market and the behavior in time of a manufacturing process, some connections can be made for mathematical modeling: obviously, under certain hypotheses that allow for the necessary flexibility for ulterior calculus.

Thus, in econometrics as well as in SPC (statistical process control) we start with an initial state, we deal with an actual phenomenon/process that at a certain moment (t) generates a measurable event/feature. For example, a certain budget deficit, an accepted “value” of unemployment or a certain average value (quarterly for instance) of the euro/leu, dollar/euro relation, etc.

These processes are submitted to some disturbing factors (some, random) that lead to the output value for the studied characteristic which may be different from its target value (T).

There are two strategies/approaches regarding the attitude in these situations:

1. The phenomenon/process is left to evolve freely, without any outside intervention, in the hope that due to internal or external conjectural causes it will self-adjust.
2. Depending on the phenomenon/process evolution relative to the set target value, the direct corrective intervention can be applied, through specific means.

In economics, self-adjustment is more plausible than in the sphere of manufacturing. Thus, the exchange rate can sometimes be "naturally" stabilized, without interventions by the National Bank, only through economic mechanisms specific to the respective market (supposedly free and competitive).

However, an "automatic statistical control" is not universally applicable from the initial moment of the process until the final output without external corrective interventions, due to several reasons (usually of technical and economic nature as well as managerial).

Thus, for the mathematical modeling in such cases we need realistic hypotheses concerning the development of that specific process – whether it is from the economic or social domain or from technological ones.

We assume that given phenomenon/process generates at a (t) moment a characteristic y_t for which T is its objective or target value.

If the process was also developed previously to moment (t) – so it was not initiated in that specific moment – than we can suspect that there were deviations from T (even negligible) and the analyst's purpose is to find a function – called SPC "controller" (control - in the cybernetic sense), of control in the cybernetic sense, and not of verification (or inspection) so that the standard deviation of these deviation remains at a minimum level.

The deviations $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ determine the x_t level at which the characteristic is found at moment (t), that is

$$\mathrm{x}_t = \mathrm{f}(\varepsilon_t, \varepsilon_{t-1}, \ldots).$$

The intervention in the process must be made at moment (t) with $x_t = \mathrm{x}_t - \mathrm{x}_{t-1}$ meaning the increment between the two levels of the characteristic, at moments (t) and (t-1).

If no corrective intervention is made in the process at moment (t+1) than the output value y_{t+1} will deviate from the target value T with (y_{t+1} - T), the latter representing the perturbation at moment (t+1).

If we assume the process is modeled by a auto-regressive time series, that is

$$\mathrm{y}_t - \mathrm{y}_{t-1} = \varepsilon_t - \theta\, \varepsilon_{t-1} \qquad (31)$$

the best estimator for this perturbation factor (y_{t+1} - T) at a (t) moment is $\left(\hat{y}_{t+1} - T\right)$,where $\hat{y}_{t+1}$ is the optimal prediction for EWMA given by relation (30).

There is also an open problem that consists in quantifying the intervention element that is determining the value that must be effectively applied to x_t , so that the output elements y_{t+1} do not suffer modifications.

Consequently, this compensatory corrective factor is meant to cancel the estimated perturbing factor $\hat{y}_{t+1} - T$, so we must have

$$x_t = \left(\hat{y}_{t+1} - T\right) \cdot A \tag{32}$$

where A is the corrective factor.

Assuming that we must perform this intervention, the ε_{t+1} deviation at output will be

$$\varepsilon_{t+1} = \left(y_{t+1} - T\right) - \left(\hat{y}_{t+1} - T\right) = y_{t+1} - \hat{y}_{t+1} \tag{33}$$

Analogously, $x_{t-1} = \left(\hat{y}_t - T\right) \cdot A$ so $x_t - x_{t-1} = \left(\hat{y}_{t+1} - \hat{y}_t\right) \cdot A$, and since $\hat{y}_{t+1} = y_{t+1} - \varepsilon_{t+1}$ we can write

$$x_t - x_{t-1} = \left[\left(y_{t+1} - y_t\right) - \left(\varepsilon_{t+1} - \varepsilon_t\right)\right] \cdot A \tag{34}$$

Conclusions

The results achieved in connection with the accomplished research prove the statistical methods' completeness aspects, their flexibility, adaptability and transference from a certain domain into other.

Also, the possibility of using the extension of EWMA (Exponentially Weighted Moving Average) method from the control of manufacturing technological procedures towards the control and observing the procedures in the most extensive sense was confirmed.

The EWMA procedure includes a practical main advantage having the characteristic of being "immune" at the values' non-gaussian nature, being sensitive only at the values' independence and variances' homoscedasticity.

Another advantage is due to the opportunity of operating individual values (for example, the exchange rates).

The main inconvenience of this method is caused by the character of economic and social processes, where the deciders' involvement capability is restricted, if compared to the opportunities existing in manufacturing procedures.

Finally, this procedure allows for some alternative predictions for the trend methods, which are quite exposed in the case of pace failures.

References

Aldrich, J. (1999), "Determinacy in the linear model: Gauss to Bose and Koopman". *International Statistical Review*, 67(2): 211 - 219.

Albu, L. (2006), *Un model dinamic de estimare a tendințelor pe termen lung ale PIB-ului potențial*. MPRA Paper 3708.

Albu, L. (2008), "A Model to Estimate the Composite Index of Economic Activity in Romania"*, Romanian Journal of Economic Forecasting*, 9(2): 44 -50.

Andrei, T. And R. Bourbounais, (2008), "*Econometrie"*, Bucureşti: Editura Economică.

Andrei, T. Stancu, St., *et al.* (2008), "*Introduction in Econometrics",* Bucureşti: Ed. Economică.

Baron, T., Tövissi, L. *et al.* (1988), "*Quality and Reliability* , *Practical Manual",* vol. I, Bucureşti : Editura Tehnică, (in Romanian).

Bârsan-Pipu, N. (1999), "*Statistical process control and adjustment: concepts and methods",* Proceedings of the 13th Scientific Session on Mathematics, Transilvania University, Braşov, pp. 84 - 95.

Bârsan-Pipu, N., Al. Isaic-Maniu, and V., Gh. Vodă, (2002), "Monitorizarea proceselor de fabricație cu fişe de control CUSUM şi EWMA". *Studii şi Cercetări de Calcul Economic şi Cibernetică Economică*, vol. XXXVI, nr. 1, pp. 13 - 27.

Box, G., E., P. and G. Jenkins, (1970), "*Time Series - Forecasting and Control",* Holden Day, San Francisco.

Chow, G., C. (1983), "*Econometrics",* New York: McGraw-Hill Book Co..

Deming, W., E. (1964), "*Statistical Adjustment of Data",* (Student Edition), New York: Dover Publication Inc.

Dinu, Gh. and V. Gh. Vodă, (2006), "*Diagnoza economică în controlul statistic al calității",* Bucureşti: Ed. Universitară.

Dobrescu, E. (2003), "*Tranziția în România. Abordări econometrice"*, Bucureşti: Ed. Economică.

Dorin, Al., C., Isaic-Maniu, Al. and Vodă, V., Gh. (1994), "*Probleme statistice ale fiabilității",* Bucureşti: Editura Economică.

Dornbusch, R., Fischer, S. and Stortz, H. (2002), "*Macroeconomics"*, 8th Edition. , New York: McGraw-Hill Book Co. / Irwin.

Granger, C. W. and P. Newbold, (1997), "*Forecasting Economic Time Series",* New York: Academic Press.

Harvez, A.C. (1993), "*Time Series Models"*, New York: Howester.

Isaic-Maniu, Al. and V., Gh. Vodă, (2006), "*Proiectarea statistică a experimentelor. Fundamente şi studii de caz*" , Bucureşti: Editura Economică.

Lowry, C. A., W. H. Woodall, and C. W. Champ, (1992), "EWMA Charts", *Technometrics*. 34(1): 46 - 53.

Mărgăritescu, E. (1981), "*Metoda comparației multiple",* Bucureşti: Ed. Academiei R. S. Romania, Colecția "Probleme actuale de statistică".

Mihoc, Gh. and Ursianu, V. (1962), "*Matematici aplicate în statistică",* Bucureşti: Ed. Academiei Republicii Populare Române.

Moineagu, C., I. Negură and V. Ursianu, (1976), "*Statistica*", Bucureşti: Editura Ştiințifică şi Enciclopedică, Bucureşti.

Montgomery, D. C. (1996), “*Introduction to Statistical Quality Control”,* (Third Edition), New York : John Wiley and Sons Inc.

Petrescu, E. and V. Gh. Vodă, (2004), *Fişe de control de proces. Teorie şi studii de caz*. Bucureşti: Ed. Economică.

Păuna, B., I. Ghizdeanu, C. Scutaru, et. al. (2008), “The ‘Dobrescu Macromodel’ of the Romanian Market Economy – 2005*”*, *Romanian Journal of Economic Forecasting*, 9(3): 129 - 131.

Pecican, E., S. (2006), “*Econometrie*“, Bucureşti : Ed. C. H. Beck.

Prabhu, S., S. and Runger, G., C. (1997), “Designing a multivariate EWMA control chart*”, Journal of Quality Technology*, 29(1): 8 - 15.

Stigler, S., M. (1986), “*The History of Statistics*: *The Measurement of Uncertainty before 1900”,* Belknap Press, Cambridge, Mass, USA.

Stigler, S., M. (1989), “Galton`s account of the invention of correlation*”, Statistical Science*, 4(2): 73 - 79.

Ucenic, C. and G. Atsalakis, (2005), *Time series prediction of the Greek manufacturing index for the basic metals sector using neuro-fuzzy approach.* Proceedings of the International Conference on the Management, pp. 131 - 136, Greece.

Vodă, V. Gh. (1980), “*Noi modele statistice în studiul durabilităţii produselor*”, Bucureşti: Ed. Academiei R. S. România.

Vogelvang, B. (2005), “*Econometrics* - *Theory and Applications with E-Views”*, Pearson Education, Ltd, Harlow.