

# THE OPTIMAL HEDGING RATIO FOR NON-FERROUS METALS

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## Abstract

*The increased volatility that characterized the markets during the last years emphasized the need for hedging. Given their industrial usage, the non-ferrous metals have a great importance for the economic activity. The volatility and unpredictability of metals prices create risks for an important number of companies and for the economy. The existence of basis risk leads to the need for the optimal hedge ratio estimation. Our paper estimates the optimal hedging ratio in the case of the non-ferrous metals traded on the London Metals Exchange using three methods: the ordinary least squares regression, the error-correction model, and the auto regressive distributed lag model. It also provides an in-sample and an out-of-sample comparison between these three models. The results show that the optimal hedge ratio and hedging effectiveness increase with the hedging horizon, converging to 1 for long tenors. Our findings also show that the more complex models provide a greater in-sample hedging effectiveness, but for the out-of-sample analysis the increase in performance is not significant.*

**Keywords:** hedging, optimal hedging ratio, risk management, OLS, error-correction model

**JEL Classification:** G13, G15, G32

## 1. Introduction

The increased volatility that characterized the markets during the last years emphasized the need for hedging. The basic principle of hedging is to combine a risk generating spot position with a contrary position in a futures contract or in another highly correlated asset. If the correlation between the spot and futures price would be perfect, then the naive one-to-one hedge ratio would lead to a variance in the hedged portfolio equal to zero. However, the correlation between the two prices is not perfect in reality (the basis risk) and the naïve hedging ratio is not the one that minimizes the hedged portfolio's variance. In this context, we need to estimate the optimal hedging

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ratio (OHR), which appears in the existing literature as being risk-minimizing or utility-maximizing.

In order to derive the OHR, the models that focus on the risk-minimizing objective use different risk measures to be minimized, such as: variance [Johnson (1960), Ederington (1979), Myers and Thompson (1989)], the mean-Gini coefficient [Lien and Luo (1993), Shalit (1995)], the generalized semivariance [Lien and Tse (2000)] or the mean generalized semivariance [Chen *et al.*, 2001]. The utility-maximizing hedge ratio is derived by using specific functions of return and risk, which are discussed in Cecchetti *et al.*, 1988), Kolb and Okunev (1993), Hsin *et al.*, 1994), Bessembinder and Lemmon (2002) or in Cotter and Hanly (2012).

The models used for OHR estimation range from simple to highly complex ones: ordinary least squares regression [Ederington (1979), Benet (1992)], error correction models [Ghosh (1995), Lien (1996)] or the conditional heteroskedastic methods [ARCH and GARCH: Cecchetti *et al.*, 1988), Baillie and Myers (1991), Kroner and Sultan (1993), Floros and Vougas (2004)]. Chen *et al.*, 2004) proposed a version of the error-correction models, the auto-regressive distributed lag model (ARDL), based on the simultaneous equations models considered by Hsiao (1997) and Pesaran (1997), obtaining a joint estimation of the short-run and long-run hedging ratio. Lee and Yoder (2007) and Alizadeh *et al.*, 2008) estimated time-varying hedge ratios using a Markov regime switching model for markets such as corn and nickel, and crude oil, respectively. Power *et al.*, 2013) estimated the OHR through a non-parametric copula GARCH model.

Numerous studies compared the performance of the OHR estimated through several model and the results are mixed. For example, Juhl *et al.*, 2012) show that the OLS regression and the error correction model lead to similar results when spot and futures prices are cointegrated, while Hsu *et al.*, 2008) find that copula-based GARCH models perform better than OLS and other types of GARCH methods. Some studies take into account the impact of hedging horizon length on the optimal hedging ratio and hedging effectiveness [Ederington (1979), Geppert (1995), Chen *et al.*, 2004), Juhl *et al.*, 2012) and Armeanu *et al.*, 2013)].

Our paper analyzes the optimal hedging ratio for the non-ferrous metals traded on the London Metals Exchange (LME), providing also a comparison between the hedging effectiveness of three different models: the OLS, the error correction model (ECM) and the auto regressive distributed model (ARDL), developed by Chen *et al.*, 2004). The six metals (aluminum, copper, lead, nickel, tin and zinc) were chosen for their importance in the world economy, given by their industrial usage. Also, LME represents one of the most liquid commodity exchanges at global level. In addition, there are very few studies that focus on the non-ferrous metals market. Dewally and Mariott (2008) estimated OHR for the period 1998-2006 using two models: OLS and ARDL and with hedging horizons length up to 8 weeks. Dinica (2013) made an in-sample comparison of the three discussed models for the period 2000-2012. Our study improves the research by adding an analysis of the non-ferrous metals prices and basis behavior, by adding an out-of-sample comparison between the three models and by expanding the data sample with one and a half years.

The remainder of the paper is organized as follows. The second section provides a description of the methodology used and presents the database. In the third section

the empirical results obtained by estimating the OHR using the above models are discussed, while in the last section the conclusions are given.

## II. Methodology

The first step of the methodology consists in discussing the spot prices evolutions of the six analyzed metals, together with some descriptive statistics, such as the mean, median, minimum, maximum or standard deviation. As mentioned above, the main reason for OHR estimation is the imperfect correlation between spot and futures prices. Thus, we further analyze the evolution of the basis (the difference between the futures and spot prices), also providing the basis descriptive statistics.

The next step of the methodology consists in testing the stationarity and cointegration of data series. As Cotter and Hanly (2006) mentioned, non-stationary data usage in estimations can lead to spurious results. In addition, Juhl *et al.*, 2012) explained that the proper specification of the model is dependent on the behavior of the time series. Given that the series are not unit root processes, a simple regression on levels or levels or price changes can be applied. However, if the series are unit root processes, but are not cointegrated, a regression on price changes can be appropriate. Finally, in the case that time series are both unit root processes and cointegrated, an error-correction term can be included into the regression. For testing the unit root hypothesis, the augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test were applied. In order to test for cointegration between the spot and futures prices we used the Johansen cointegration test.

Starting from Pesaran (1997) bivariate model, one can obtain different models to estimate the optimal hedging ratio. The same derivation method was used by Lee *et al.*, 2009). The Pesaran (1997) bivariate model illustrates the following:

$$\Delta S_t = \alpha(1 - \phi) - (1 - \phi)S_{t-1} + \gamma F_{t-1} + u_t \quad (1)$$

$$\Delta F_t = \alpha(1 - \rho) - (1 - \rho)F_{t-1} + \delta S_{t-1} + v_t \quad (2)$$

assuming that

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim iid(0, \Sigma), \quad \Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{u,v} \\ \sigma_{u,v} & \sigma_v^2 \end{pmatrix}$$

where:  $\sigma_{u,v}$  is the covariance between  $u_t$  and  $v_t$ , and  $\sigma_u^2$  and  $\sigma_v^2$  are the variances of  $u_t$  and  $v_t$ .

The simplest way to estimate the optimal hedge ratio is to run the OLS (ordinary least squares) model, where  $\beta$  is the estimation of  $h^*$ . Assuming that both spot and futures prices follow a random walk, we can set that  $\phi = \rho = 1$  and  $\gamma = \delta = 0$  in the above bivariate model. In this case, the equations can be written as follows:

$$\Delta S_t = u_t \quad (3)$$

$$\Delta F_t = v_t \quad (4)$$

Taking into consideration the assumption that  $u_t$  and  $v_t$  are jointly normally distributed, we have:

$$u_t = \left( \frac{\sigma_{u,v}}{\sigma_v^2} \right) v_t + \varepsilon_t \quad (5)$$

where:  $\sigma_{u,v} / \sigma_v^2$  represents the regression coefficient of  $u_t$  on  $v_t$ , and  $\varepsilon_t$  is distributed independently of  $v_t$ . Based on this, the OLS model can be estimated.

$$\Delta S_t = \alpha + \beta \Delta F_t + \varepsilon_t \quad (\text{OLS}) \quad (6)$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

The estimation of the minimum variance hedging ratio is given by  $\beta$  and equals  $\sigma_{u,v} / \sigma_v^2$  under the normal joint condition.

The OLS model specification assumes the absence of autocorrelation and heteroskedasticity in the first differences. Also, the above relation is used to estimate the short-run hedge ratio, representing a short-run relation between the two variables.

Another model used to estimate the optimal hedging ratio is the error-correction model (ECM). Scutaru (2011) shows that the ECM is used mainly in short-term forecast because the long-term adjustment to equilibrium is relatively slow. The long-run relation between spot and futures price is represented by the following equation:

$$S_t = \alpha + bF_t + \varepsilon_t \quad (7)$$

Assuming that the series are cointegrated and the spot price and the futures price are unit-root processes, Pesaran (1997) concludes that it must be either  $|\phi| < 1, \rho = 1, \gamma \neq 0, \delta = 0$  or  $|\rho| < 1, \phi = 1, \delta \neq 0, \gamma = 0$  in the above bivariate model. In this case, the equations (1) and (2) can be written as follows:

$$\Delta S_t = \alpha(1 - \phi) - (1 - \phi)S_{t-1} + \gamma F_{t-1} + u_t \quad (8)$$

$$\Delta F_t = v_t \quad (9)$$

If  $u_t$  and  $v_t$  are jointly normally distributed, the equation (10) holds. Having  $u_t = \beta v_t + \varepsilon_t$ , equations (6) and (7) can be written as follows:

$$\Delta S_t = \alpha + \lambda \varepsilon_{t-1} + \beta \Delta F_t + e_t \quad (\text{ECM}) \quad (10)$$

$$e_t \sim N(0, \sigma^2)$$

where:  $\varepsilon_{t-1} = S_{t-1} - (\hat{a} + \hat{b}F_{t-1})$ , representing the lagged error term of the long-run relation (7). The coefficient  $\beta$  is the hedging ratio estimated using the error-correction model, which will be denoted by ECM from this point forward in our analysis.

Chou *et al.*, 1996), Floros and Vougas (2004) and Degiannakis and Floros (2010), following the method proposed by Engle and Granger (1987), estimated the optimal hedging ratio using an error correction model with lagged values of the differences in spot and futures prices.

$$\Delta S_t = \alpha + \lambda \varepsilon_{t-1} + \beta \Delta F_t + \sum_{i=1}^n \theta_i \Delta F_{t-i} + \sum_{j=1}^m \phi_j \Delta S_{t-j} + e_t \quad (11)$$

The optimal lag lengths of spot and futures differences  $m$  and  $n$  are decided by iterating for each lag until the autocorrelation of the residuals is eliminated. In our study, we used an error correction model without lagged differences because the residuals were not autocorrelated.

Chen *et al.*, 2004) propose a version of the error-correction models, based on the simultaneous equations models considered by Hsiao (1997) and Pesaran (1997), obtaining a joint estimation of the short-run and long-run hedging ratio. This model is also called the autoregressive distributed lag (ARDL) cointegration model.

Pesaran (1997) argues that the existence of a long-run relation between the spot and futures price does not depend on whether the futures price is integrated of order 1. If there is a long-run relationship between the two prices, then it must be either  $|\phi| < 1, \gamma \neq 0, \delta = 0$  or  $|\rho| < 1, \delta \neq 0, \gamma = 0$ . In this case, the bivariate model becomes:

$$\Delta S_t = \alpha(1 - \phi) - (1 - \phi)S_{t-1} + \gamma F_{t-1} + u_t \quad (12)$$

$$\Delta F_t = \alpha(1 - \rho) - (1 - \rho)F_{t-1} + v_t \quad (13)$$

If  $u_t$  and  $v_t$  are jointly normally distributed, the equation (10) holds. Having  $u_t = \beta v_t + \varepsilon_t$ , equations (12) and (13) can be written as follows:

$$\Delta S_t = \alpha_1 + \alpha_2 S_{t-1} + \alpha_3 F_{t-1} + \beta \Delta F_t + \varepsilon_t \quad (13)$$

The model incorporates both short and long-run relations and the short-run hedge ratio is given by  $\beta$ , while the long-run hedge ratio is given by  $-\alpha_3/\alpha_2$ . This equation will be denoted by ARDL from this point forward in our analysis.

Further, the different models are compared based on the hedging effectiveness of the estimated OHR. For the in-sample analysis, the hedging effectiveness is given by the adjusted  $R^2$  statistic. For comparing the out-of-sample models, we need to calculate the hedging effectiveness (HE) indicator, given by:

$$HE = 1 - \frac{[Var(\Delta V)_h]}{Var(\Delta S_t)} \quad (14)$$

The hedging effectiveness indicator shows how much variance of the unhedged portfolio is eliminated through hedging. The models having the greatest values of this indicator will be considered as the most effective for the hedging purpose.

In the existing literature, the relation between hedging horizon and hedging ratio it is also analyzed, namely the determination coefficient. In order to test for the impact of the length of the hedging horizon on the optimal hedge ratio and on the hedging effectiveness, two regressions are used, the endogenous term being the hedging ratios estimated above, namely the adjusted  $R^2$  obtained, while the exogenous term is the length of the hedging horizon, expressed in weeks. More specifically, the regressions used are:

$$\beta_i = a + bT_i + e \quad (15)$$

$$Adjusted R^2_i = a + bT_i + e \quad (16)$$

where:  $T_i$  is the hedging horizon, expressed in weeks.

The database used for the analysis is represented by the daily cash and futures prices of the non-ferrous metals traded on the London Metals Exchange (LME) during the period April 3, 2000-September 30, 2013. For each metal (aluminum, copper, lead, nickel, tin and zinc) and for each type of price (cash or futures) there are 3405 observations. The futures price is represented by the nearest-to-maturity contract price, while for the cash price the LME official settlement price is used, both expressed

in USD/ton. Compared with other studies, our dataset has the longest range (13.5 years) and is the most recent.

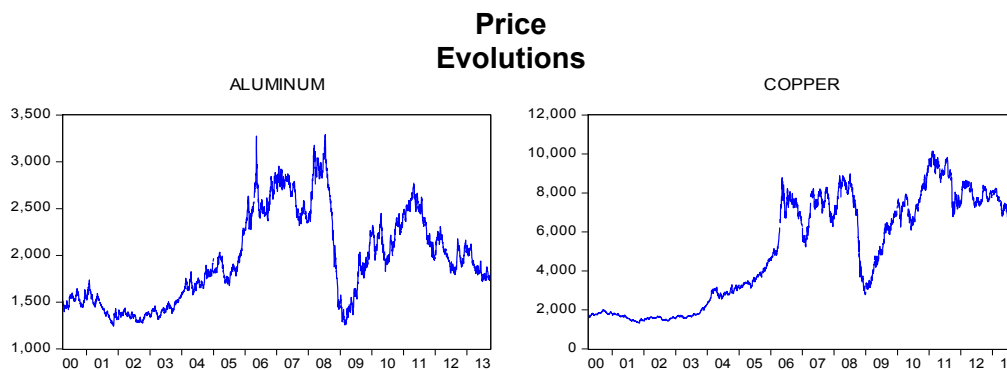
Also, in order to compute the optimal hedge ratio for different hedging horizons we matched the data frequency with the hedging horizon. For example, in order to compute the 1 week hedging ratio we used weekly data and for computing the 1 day hedging ratio we used daily data. By this methodology, we avoid the problems associated with data overlapping, like the existence of autocorrelated error terms in the regression. A detailed description of this issue can be found in Chen *et al.*, 2004). The sample size of our study allowed us to use non-overlapped data in order to compute the hedging ratio for 13 different hedging horizons, from 1 day to 12 weeks. In order to compute a hedging ratio for one metal and for one hedging horizon length, a regression using a specific model was estimated. Having 6 metals and 13 hedging horizons, 78 hedging ratios were estimated for each analyzed model.

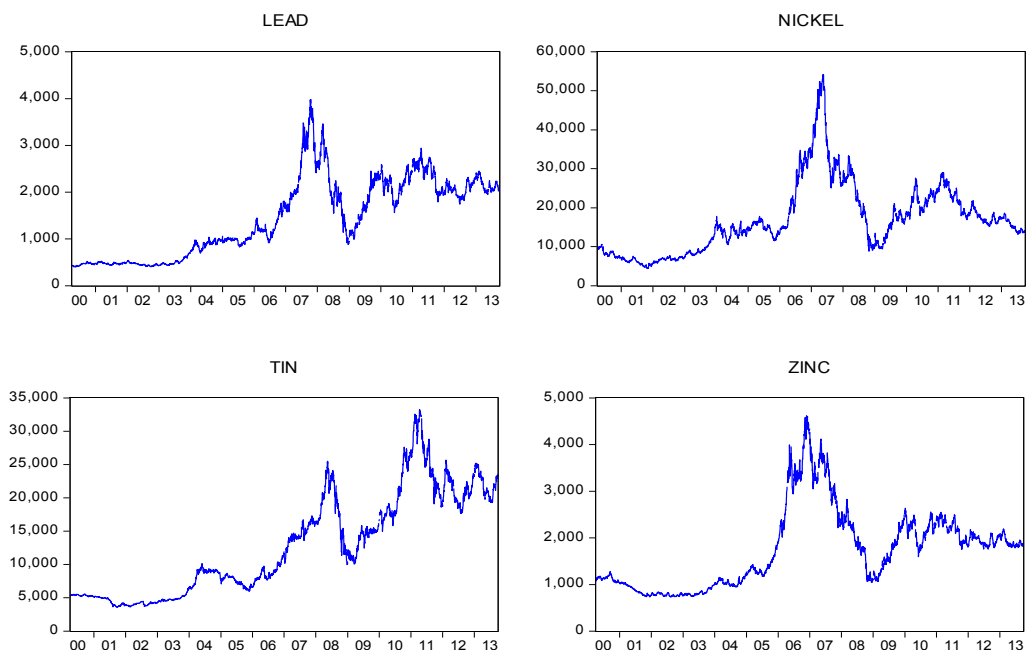
In order to run the out-of-sample hedging effectiveness analysis, the database was split into two parts. The first half was used to re-estimate the hedging ratios using the same methodology as above, while the second half was used to compute the variances of the portfolios obtained by hedging with the estimated hedging ratios. We run the out-of-sample analysis only for aluminum and copper, the most representative metals in our sample.

### III. Empirical results

As earlier mentioned, the first step of the methodology consists in discussing the spot price evolutions of the six analyzed metals. Figure 1 depicts the evolutions of the analyzed metals' prices during the considered period.

Figure 1





Source: London Metals Exchange.

As one can notice, the prices of the six analyzed metals show a volatile evolution during the period 2000-2013. The first period of the sample, between 2000 and 2003 is characterized by the smallest price changes, without a definite trend. However, starting in 2004, as a result of better economic conditions and as commodities were considered a new asset class and included into financial portfolios, the prices saw a generalized surge at an increased pace. The prices of the six analyzed metals peaked around 2007. During the financial crisis, the prices sharply declined as a result of the economic activity contraction. However, starting with 2009, the prices rose again, as the economy showed signs of recovery. After the financial crisis, two of the six analyzed metals registered new highs: copper and tin, while the other four metals recovered only partly the decline in prices.

The data presented in Table 1 emphasize even more the volatile behavior of the six metals price evolution during the analyzed period. Thus, the ratios of the maximum to the minimum prices range between 2.65 in the case of aluminum and 12.26 in the case of nickel and the swings between the minimum and maximum prices were made in periods of just a few years. The smallest standard deviation is that of aluminum, the least volatile metal in the sample. The greatest standard deviation is that of nickel, of around 9102 USD/ton, but the highest ratio of the standard deviation to the mean throughout the sample, which is a better approximation for volatility, appears in the case of tin: 59.25%.

Table 1

**Descriptive Statistics – Spot Prices**

	Aluminum	Copper	Lead	Nickel	Tin	Zinc
Mean	1,956.90	5,085.93	1,449.72	16,903.12	12,808.58	1,744.41
Median	1,873.50	5,290.00	1,301.00	15,450.00	10,600.00	1,778.00
Minimum	1,243.00	1,319.00	399.00	4,420.00	3,595.00	725.50
Maximum	3,291.50	10,148.00	3,980.00	54,200.00	33,255.00	4,619.50
Maximum to minimum price ratio	2.65	7.69	9.97	12.26	9.25	6.37
Standard deviation	485.55	2,750.77	835.04	9,102.53	7,589.67	851.51

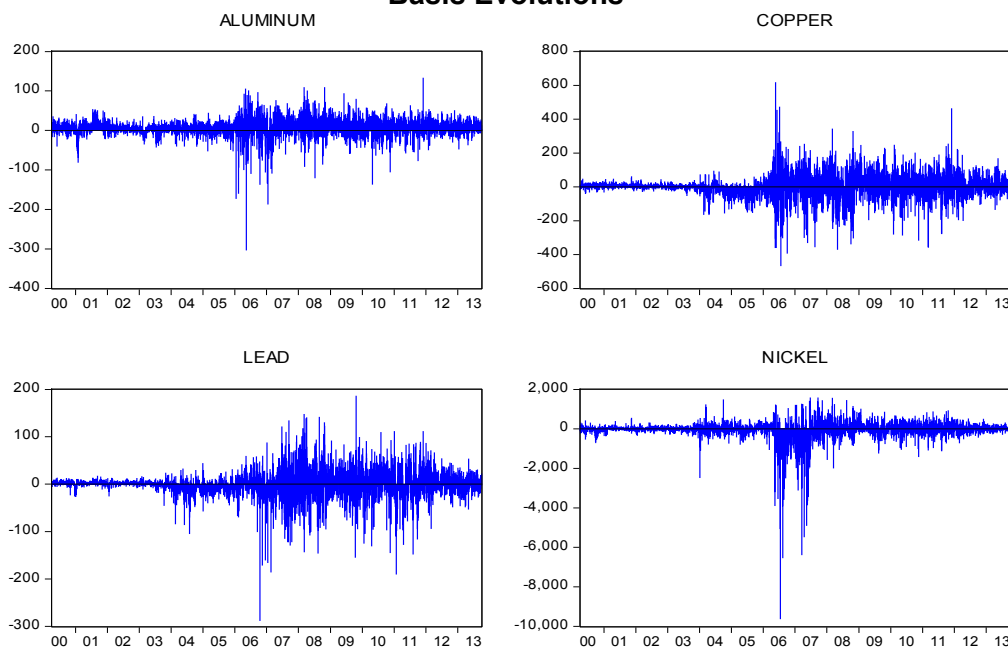
Source: Authors' calculations.

The analysis of price evolutions during the sample and the descriptive statistics outline that the prices of the nonferrous metals traded at LME are highly volatile and unpredictable, emphasizing the need for hedging the risks arising from this behavior.

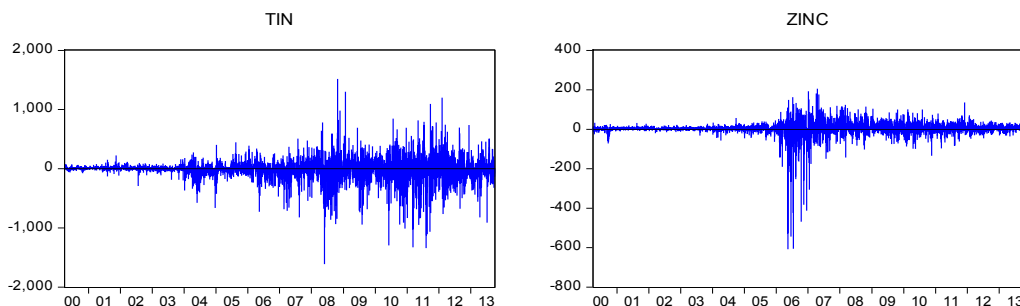
In addition, we further discuss the evolution of the basis (the difference between the spot and futures prices), depicted in Figure 2. As one may see, in all cases the basis significantly increased during the analyzed period and it was characterized by a volatile and unpredictable behavior.

Figure 2

**Basis Evolutions**







Source: Authors' calculations.

Table 2 presents the main descriptive statistics of the basis. The average value of the basis is positive in the case of three metals (aluminum, copper and zinc) and negative in the case of the rest. The median values are positive for all metals. However, both the mean and median values are not significantly different as compared to zero in all cases. The basis is characterized by extreme values, the maximum amplitude (the difference between the maximum and the minimum value) being much higher than the basis average. The basis amplitude is significant even compared with the mean price, ranging from 21.35% of the mean value in the case of copper to 66.41% in the case of nickel. The basis is also characterized by a large standard deviation, in all cases higher than the basis average or median value.

Table 2

**Descriptive Statistics – Basis**

	Aluminum	Copper	Lead	Nickel	Tin	Zinc
<b>Mean</b>	6.53	1.66	-0.36	-53.94	-14.79	3.03
<b>Median</b>	7.00	2.50	1.00	2.00	3.50	4.75
<b>Minimum</b>	-303.50	-467.00	-288.00	-9,630.00	-1,610.00	-607.00
<b>Maximum</b>	133.75	619.00	186.50	1,595.00	1,515.00	205.00
<b>Amplitude</b>	437.25	1,086.00	474.50	11,225.00	3,125.00	812.00
<b>Standard deviation</b>	24.07	73.28	29.96	520.27	204.53	40.66

Source: Authors' calculations.

The above analysis of prices and basis evolutions emphasizes the volatile and unpredictable behavior of prices as a rationale for hedging and the volatile nature of the basis as an argument for the optimal hedge ratio estimation.

The next step of the methodology consists in testing other two characteristics of the time series: stationarity and cointegration. In order to test the unit root hypothesis the augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test and the Kwiatkovski, Phillips, Schmidt and Shin (KPSS) test were applied and for testing the cointegration the Johansen cointegration test was used.

The results are given in Table 3. The unit-root test results show that all the prices of the six analyzed metals are unit root processes and are integrated of order 1, the first differences being stationay. The Johansen test provides evidence that cash prices and futures prices series are co-integrated in the case of each metal.

Table 3

**Stationarity Tests**

Metal	Series type	ADF test		PP test		KPSS test
		t stat	p value	Adj. t stat	p value	LM - Stat.
Aluminum	Cash	-1.880	0.665	-1.834	0.688	0.856
	First Difference	-59.567	0.000	-59.579	0.000	0.055
Copper	Cash	-1.886	0.661	-2.137	0.524	0.461
	First Difference	-61.928	0.000	-61.850	0.000	0.060
Lead	Cash	-2.414	0.372	-2.450	0.354	0.469
	First Difference	-59.663	0.000	-59.647	0.000	0.043
Nickel	Cash	-1.502	0.829	-1.621	0.785	0.833
	First Difference	-57.197	0.000	-57.264	0.000	0.070
Tin	Cash	-2.718	0.229	-2.742	0.219	0.310
	First Difference	-58.877	0.000	-58.874	0.000	0.045
Zinc	Cash	-1.660	0.769	-1.653	0.772	0.759
	First Difference	-58.177	0.000	-58.180	0.000	0.081
Critical values for ADF test: 1%: -3.432; 5%: -2.862; 10%: -2.567						
Critical values for PP test: 1%: -3.961; 5%: -3.411; 10%: -3.127						
Critical values for KPSS test: 1%: 0.216; 5%: 0.146; 10%: 0.119						

These results suggest that the regressions should be estimated on the basis of the first differences between prices in order to avoid spurious results, and that by adding an error-correction term to the model specification can lead to better performance.

Table 4

**Johansen Cointegration Test**

Metal	No cointegrating vector	At most one
Aluminum	331.31	3.47
Copper	453.99	1.68
Lead	349.48	2.61
Nickel	232.60	2.87
Tin	498.86	2.54
Zinc	543.36	0.85
Critical values: None - 1%: 20.04; 5%: 15.41; At most one: 1%: 6.65; 5%: 3.76		

Source: Authors' calculations.

The paper goal is to derive the short-run and the long-run hedging ratio by applying the models described in the 'Methodology' section for the non-ferrous metals market during the analyzed period, to compare the three models' hedging effectiveness both in-sample and out-of-sample and to quantify the impact of the hedging horizon on the optimal hedging ratio and on the hedging effectiveness. By applying the OLS model on the analyzed database, we obtained the results presented in Table 5.

Table 5

Optimal Hedging Ratio Estimated with the OLS Model

		Aluminum	Copper	Lead	Nickel	Tin	Zinc
1D	Hedge ratio	0.449*	0.506*	0.565*	0.376*	0.588*	0.411*
	Adjusted $R^2$	0.236	0.282	0.321	0.197	0.329	0.258
1W	Hedge ratio	0.847*	0.898*	0.900*	0.848*	0.871*	0.823*
	Adjusted $R^2$	0.766	0.845	0.845	0.718	0.811	0.745
2W	Hedge ratio	0.910*	0.941*	0.945*	0.929*	0.919*	0.863*
	Adjusted $R^2$	0.867	0.918	0.911	0.874	0.916	0.848
3W	Hedge ratio	0.942*	0.942*	0.989	0.937**	0.950*	0.941*
	Adjusted $R^2$	0.914	0.951	0.941	0.816	0.926	0.926
4W	Hedge ratio	0.961**	0.958*	0.989	1.026	0.951*	0.928**
	Adjusted $R^2$	0.943	0.964	0.963	0.921	0.959	0.863
5W	Hedge ratio	0.966***	0.991	0.986	1.039****	0.974***	1.019
	Adjusted $R^2$	0.953	0.967	0.975	0.926	0.968	0.965
6W	Hedge ratio	0.961***	0.984	0.969***	1.054*	1.004	0.921*
	Adjusted $R^2$	0.944	0.982	0.969	0.970	0.981	0.922
7W	Hedge ratio	0.995	0.978**	0.996	1.027	0.999	0.951
	Adjusted $R^2$	0.972	0.989	0.972	0.930	0.970	0.856
8W	Hedge ratio	0.994	0.968***	0.989	1.102*	1.021	0.935*
	Adjusted $R^2$	0.971	0.967	0.977	0.967	0.981	0.974
9W	Hedge ratio	0.967****	0.974****	0.999	1.120*	0.978	0.991
	Adjusted $R^2$	0.964	0.981	0.972	0.944	0.980	0.957
10W	Hedge ratio	0.995	1.014	0.993	1.068*	0.982	1.051*
	Adjusted $R^2$	0.977	0.983	0.983	0.975	0.980	0.986
11W	Hedge ratio	0.980	1.001	0.985	0.961	0.987	0.974
	Adjusted $R^2$	0.978	0.991	0.976	0.948	0.989	0.848
12W	Hedge ratio	0.990	1.004	0.976	1.097*	1.043	0.996
	Adjusted $R^2$	0.985	0.984	0.972	0.982	0.992*	0.969

Different of 1: \* - significance at 1% level; \*\* - significance at 5% level; \*\*\* - significance at 10% level; \*\*\*\* - significance at 15% level

Source: Authors' calculations.

The optimal hedging ratio derived by the OLS model is significantly lower than the naive one-to-one hedging ratio for the short hedging horizons. The optimal hedge ratio is less than 1 at the 1% significance level for the hedging horizons up to 2 weeks for all analyzed metals. Also, the results show that for 3 and 4-week hedging horizons generally the optimal hedging ratios are significantly different from 1. Starting with the 5-week hedging horizon, the significances tend to be mixed, showing a convergence of the optimal hedge ratio to the unit value in the long run. Out of 78 calculated hedge

ratios, 33 are different by 1 at the 1% significance level, 4 at the 5% level, 5 at the 10% significance level and 3 at the 15% significance level.

By applying the ECM and the ARDL models, we obtained the results in Tables 6 and 7.

**Table 6**

**Optimal Hedging Ratio Estimated with the ECM Model**

		Aluminum	Copper	Lead	Nickel	Tin	Zinc
1D	Hedge ratio	0.508*	0.565*	0.564*	0.449*	0.571*	0.484*
	Adjusted $R^2$	0.654	0.712	0.692	0.487	0.720	0.609
1W	Hedge ratio	0.892*	0.918*	0.918*	0.886*	0.889*	0.884*
	Adjusted $R^2$	0.871	0.921	0.920	0.831	0.907	0.868
2W	Hedge ratio	0.932*	0.938*	0.940*	0.969**	0.928*	0.913*
	Adjusted $R^2$	0.933	0.960	0.958	0.930	0.960	0.905
3W	Hedge ratio	0.983	0.980***	0.969**	1.016	0.984	0.953**
	Adjusted $R^2$	0.951	0.974	0.969	0.903	0.962	0.959
4W	Hedge ratio	0.991	0.966*	0.997	1.027****	0.965*	0.972****
	Adjusted $R^2$	0.973	0.982	0.982	0.956	0.980	0.924
5W	Hedge ratio	0.975***	0.976**	0.980**	1.028***	0.952*	0.986
	Adjusted $R^2$	0.976	0.983	0.986	0.967	0.984	0.987
6W	Hedge ratio	1.008	0.997	0.984	1.030**	1.015****	0.953*
	Adjusted $R^2$	0.971	0.989	0.982	0.981	0.990	0.960
7W	Hedge ratio	0.987	0.987***	0.972**	1.047**	0.977**	1.021
	Adjusted $R^2$	0.986	0.995	0.989	0.971	0.986	0.925
8W	Hedge ratio	1.014	0.995	1.005	1.090*	1.020***	0.998
	Adjusted $R^2$	0.986	0.985	0.989	0.983	0.990	0.985
9W	Hedge ratio	1.003	0.994	1.014	1.111*	1.005	0.976
	Adjusted $R^2$	0.974	0.990	0.986	0.963	0.988	0.976
10W	Hedge ratio	0.997	1.013	0.994	1.014	0.963*	1.044**
	Adjusted $R^2$	0.988	0.991	0.993	0.987	0.992	0.992
11W	Hedge ratio	1.013	1.001	1.003	1.063**	0.986	1.072*
	Adjusted $R^2$	0.988	0.996	0.991	0.974	0.994	0.932
12W	Hedge ratio	1.001	1.008	1.003	1.048*	1.024**	1.010
	Adjusted $R^2$	0.992	0.993	0.989	0.989	0.996	0.988

Different of 1: \* - significance at 1% level; \*\* - significance at 5% level;  
 \*\*\* - significance at 10% level; \*\*\*\* - significance at 15% level

Source: Authors' calculations.

Table 7

Optimal Hedging Ratio Estimated with the ARDL Model

	Aluminum			Copper			Lead		
	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$
1D	0.509*	1.000	0.654	0.565*	0.999	0.711	0.564*	1.000	0.692
1W	0.893*	1.003	0.871	0.918*	1.000	0.921	0.918*	1.001	0.920
2W	0.933*	1.003	0.933	0.938*	1.000	0.960	0.940*	1.001	0.958
3W	0.983	1.002	0.951	0.980***	0.999	0.973	0.969**	1.000	0.968
4W	0.992	1.003	0.972	0.966*	0.998	0.982	0.997	0.998	0.982
5W	0.976***	1.003	0.975	0.976**	1.003	0.983	0.980***	1.006	0.986
6W	1.007	1.001	0.971	0.996	0.998	0.989	0.984	1.001	0.982
7W	0.988	0.995	0.986	0.987***	0.994	0.994	0.972**	0.996	0.988
8W	1.013	1.001	0.985	0.995	0.998	0.985	1.004	1.000	0.988
9W	1.002	1.010	0.974	0.994	1.000	0.990	1.014	1.004	0.986
10W	0.997	1.006	0.988	1.013	1.005	0.991	0.995	1.007	0.993
11W	1.012	1.001	0.988	1.001	0.996	0.996	1.003	0.995	0.990
12W	1.001	0.999	0.992	1.007	0.996	0.992	1.003	0.997	0.989
	Nickel			Tin			Zinc		
	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$	$\beta$	$-\alpha_2/\alpha_1$	Adj $R^2$
1D	0.450*	1.016	0.487	0.571*	1.002	0.720	0.485*	1.005	0.609
1W	0.887*	1.017	0.831	0.889*	1.003	0.907	0.885*	1.003	0.868
2W	0.969**	1.014	0.930	0.928*	1.003	0.960	0.913*	1.007	0.905
3W	1.016	1.024	0.902	0.984	1.002	0.962	0.954*	1.005	0.959
4W	1.026****	1.016	0.956	0.965*	1.001	0.980	0.973	1.005	0.923
5W	1.028****	1.023	0.966	0.953*	0.999	0.984	0.986	1.008	0.987
6W	1.031**	1.020	0.981	1.015****	1.001	0.990	0.954**	1.007	0.960
7W	1.046**	0.999	0.971	0.975**	1.000	0.986	1.021	0.995	0.924
8W	1.088*	1.022	0.982	1.020***	0.999	0.990	0.998	1.003	0.984
9W	1.109*	1.044	0.962	1.004	0.997	0.988	0.978	1.019	0.976
10W	1.014	1.026	0.987	0.963*	0.999	0.992	1.043*	1.013	0.992
11W	1.061**	1.002	0.973	0.986	1.003	0.994	1.071***	1.009	0.931
12W	1.046**	1.023	0.989	1.023**	1.001	0.996	1.009	0.995	0.988

Different of 1: \* - significance at 1% level; \*\* - significance at 5% level; \*\*\* - significance at 10% level; \*\*\*\* - significance at 15% level

Source: Authors' calculations.

As in the case of the first model, the optimal hedging ratios derived by the ECM and the ARDL models are significantly lower than the naive hedging ratio for the short hedging horizons, up to 2 weeks, at the 1% significance level. However, starting with the 3 weeks hedging horizon, the significance of the differences tend to be mixed, showing a convergence of the optimal hedge ratio to the unit value in the long run; which results are similar with those obtained by applying the OLS regression.

Out of 78 calculated hedging ratios using the ECM, 26 are different from 1 at the 1% significance level, 12 at the 5% significance level, 5 at the 10% significance level and 3 at the 15% significance level.

In addition, the long-run hedge ratio derived with the ARDL model is not different from the unit value. Concerning the short-run hedging ratio, out of 78 calculated ratios, 25

are different from 1 at the 1% significance level, 11 at the 5% significance level, 6 at the 10% significance level and 3 at the 15% significance level.

Table 8

**Relation between Hedging Horizon and Hedging Ratio - the Adjusted  $R^2$**

		Hedging horizon - $\beta$	Hedging horizon - adjusted $R^2$
OLS	b	0.0241	0.0310
	$R^2$	0.3922	0.3769
ECM	b	0.0220	0.0155
	$R^2$	0.3872	0.3761
ARDL	b	0.0219	0.0155
	$R^2$	0.3859	0.3746

Source: Authors' calculations.

The next step of the methodology consisted in analyzing the relation between the hedging horizons and the hedging ratio, namely the determination coefficient. The results are synthesized in Table 8. In all cases, the coefficients of the hedging horizon length are positive and strongly significant, showing that the optimal hedge ratio and the hedging effectiveness increase with the hedging horizon.

The in-sample analysis of the three models shows that the optimal hedge ratio estimated by the ECM and the ARDL model is significantly higher than the one estimated by the OLS regression (at the 1% significance level). Also, the hedging ratio estimated with the ECM is not significantly different as compared with that estimated with the ARDL model.

All the adjusted coefficients of determination are higher in the case of ECM and ARDL, showing a better in-sample hedging effectiveness achieved by applying more advanced models, as compared to OLS. The results are consistent with other results in literature.

However, a more important comparison is made through the out-of-sample analysis. In order to run the out-of-sample analysis, we split the database into two parts. The first half (the first 6 years and 9 months) was used to estimate the optimal hedge ratio using the three models. The second half was used to calculate the hedging effectiveness of each model by the formula discussed in the methodology. The out-of-sample analysis was performed for aluminum and copper, the most representative metals in our sample. Aluminum and copper accounted for more than 63% of the LME's volume of futures contracts in 2013. Also, these metals have the highest correlation with the prices of the other four metals. The results are synthesized in Table 9.

Table 9

**Out-of-sample Hedging Effectiveness Comparison**

	Aluminum			Copper		
	HE OLS	HE ECM	HE ARDL	HE OLS	HE ECM	HE ARDL
1D	24.02%	24.28%	25.25%	28.43%	30.43%	30.43%
1W	80.43%	80.48%	80.48%	83.79%	83.87%	83.87%
2W	86.47%	86.88%	86.90%	89.96%	90.52%	90.52%

	Aluminum			Copper		
	HE OLS	HE ECM	HE ARDL	HE OLS	HE ECM	HE ARDL
3W	93.07%	92.95%	93.01%	94.54%	95.23%	95.24%
4W	93.26%	93.63%	93.69%	93.74%	94.91%	94.90%
5W	96.35%	96.40%	96.49%	97.75%	97.65%	97.65%
6W	97.11%	97.11%	97.17%	97.27%	97.40%	97.40%
7W	96.20%	96.15%	96.19%	96.07%	96.12%	96.12%
8W	95.65%	96.60%	96.59%	93.61%	96.72%	96.93%
9W	98.64%	98.72%	98.72%	98.47%	98.66%	98.66%
10W	98.31%	98.33%	98.29%	98.46%	98.35%	98.34%
11W	99.02%	98.98%	99.01%	99.12%	99.15%	99.17%
12W	97.64%	98.19%	98.34%	98.22%	98.23%	98.31%

Source: Authors' calculations.

The hedging effectiveness increases in the case of all three models with the hedging horizon, showing once again that the most effective hedging is that for longer tenors. Generally, the hedging effectiveness indicators of the ECM and ARDL models are higher than those of the OLS regression, but the differences are not statistically significant. Thus, although still proven, the superiority of the more advanced models is starting to fade in the case of the out-of-sample analysis. However, this result cannot be statistically significant and further research on the topic is necessary.

#### IV. Conclusions

The increased volatility that characterized the markets in the last years emphasized the need for hedging. The basic principle of hedging is to combine a risk-generating spot position with a contrary position in a futures contract or in another highly correlated asset. When spot and futures prices are perfectly correlated, the naive one-to-one hedge ratio leads to a perfect hedging, the price changes in spot being offset by the price changes in the futures contract. However, the difference between spot and futures prices is not constant over time, causing a basis risk. In this case, it is necessary to estimate the optimal hedging ratio that minimizes the hedged portfolio variance. Our paper estimates the optimal hedging ratio in the case of the non-ferrous metals traded at the London Metals Exchange using three methods: the ordinary least squares regression, the error-correction model, and the auto regressive distributed lag model.

The first step of the methodology consisted in discussing the spot prices evolutions of the six analyzed metals. The analysis of price evolutions during the sample and the descriptive statistics outline that the prices of the nonferrous metals traded at LME are highly volatile and unpredictable, emphasizing the need for hedging the risks arising from this behavior. In addition, we further discussed the evolution of the basis (the difference between the spot and futures prices), showing that the metals market is also characterized by an important basis risk. Thus, the analysis of prices and basis evolutions emphasizes the volatile and unpredictable behavior of prices as a rationale for hedging and the volatile nature of the basis as an argument for the optimal hedge ratio estimation in the case of the metals market.

The next step of the methodology consists in testing two other characteristics of the time series: stationarity and cointegration. The results show that all the prices of the six analyzed metals are unit root processes, integrated of order 1 and cointegrated.

Then, we estimated the OHR using the above-mentioned three models. The results show that both the optimal hedge ratio and hedging effectiveness increase with the hedging horizon, converging to 1 for long tenors. Although the long term hedge ratio is not significantly different from the unit value, for short tenors the OHR is significantly lower than 1. The in-sample analysis shows that the more advanced models provide a better hedging effectiveness (shown by the fact that all the adjusted coefficients of determination are higher in the case of ECM and ARDL models). However, the superiority of the more advanced models is starting to fade in the case of the out-of-sample analysis, but this result cannot be statistically significant and further research on the topic is necessary.

The paper provides similar results with other papers in the literature for what it concerns the fact that both the OHR and hedging effectiveness increase with the hedging horizon and that the long-term hedge ratio is converging to 1. It also contributes to literature by providing the first out-of-sample comparison between the three models for the metals market.

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