9. FORECASTING PRICES OF PRESALE HOUSES: A REAL OPTION APPROCACH

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Abstract

We are currently witnessing a boom in the real estate industry worldwide, and presale investing (or "Buying Off Plan", as it is known in Europe) is increasingly popular. This paper conducts a pricing model based on the real option method to assess presale houses. This paper takes into account the ded uctible margins (option premium) and the tax factor to capture the most important characteristics of real-world presale system in house trading markets. This paper finally proposes a closed-form solution for the price of presale house. Simulation results and sensitivity analyses are also examined. The proposed model would help investors to predict and determine the presale house investment.

Keywords: margin, tax, real estate, house trading markets, closed-form solution

JEL Classification: G11, G12, R32

1. Introduction

As far as Chinese people are concerned, you won't really have a large fortune until you have your own land (house). The price of lands in Taiwan becomes more and more expensive, even though the government enforces regulation (luxury tax) to restrain it. In Taiwan, the house price has undergone great surges over the two decades. As Figure 1 shows, the housing price index in Taipei, the capital city of Taiwan, increased by two times from 1998 to 2012. On the other hand, the index is obviously jagged over the period in other cities.

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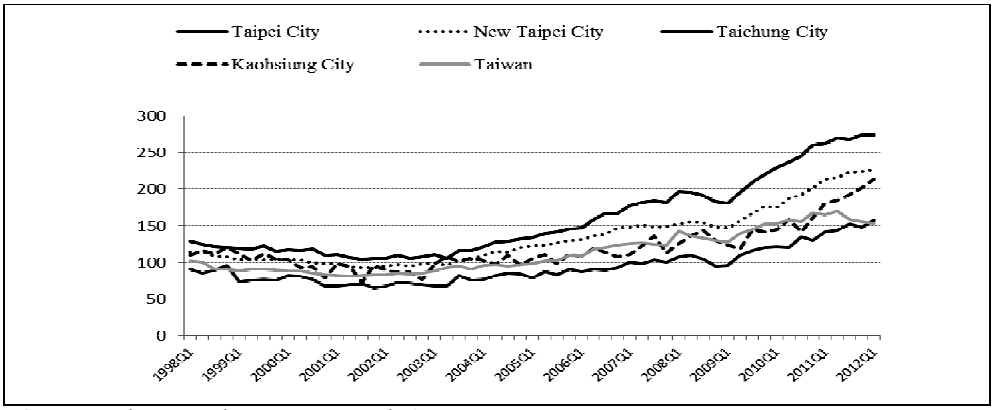
Figure 1

House Price Index in Taiwan, 1998-2012



Source: Taiwan Real Estate Research Center.

Real estate is not only a consumption commodity, but also an investment good. Some buy it to live in, others buy it to run business, and many others buy it to make profits. How would the real estate price level increase sharply? There are several explanations that we could account for the reasons. The first important reason is the land scarcity. The strong demand for real estate from foreign capital inflow and the growing population were accompanied by fixed supply. Another major factor that is widely believed is that speculative activities caused an irrational high price level. Not all of everyone can afford the high price of the real estate. The mortgage from the bank is one way to get a house, another way to have one's own house is the forward sales (presales) contracts. Since pre-sale market is commonly believed to aid the speculative activities, the phenomenon of presale market is easily observed around the world, especially popular in many Asian countries such as Taiwan, Hong Kong, Singapore, Korea and Mainland China.

Builders sell the properties after they obtain the building permits. For developers, this method can reduce the price of the price volatility and increase financial leverage. Once the developer receives the building permits, he can presell the property to the buyer with non-physical products then take the cash to progress the next construction phase with just little private money. Comparatively, real estate buyers just buy the rights to purchase the real estate in the future. The buyers can avoid the suddenly rise in the house price and only required a small percentage of the house price in cash as deposit and the remainder by mortgage. Therefore, this can alleviate their financial stress and lock the price on their expected level.

Under a presale contract, buyers make a series payments according to the completed percentage of construction. The builder receives the final payment when it transfers the property to the buyer. In addition, one feature of the presale mechanism is similar to the future or forward transaction. During the presale to completion period, there are at least three embedded options. The first is the option to wait (defer). Titman (1985) is the original to model the value of option to wait. The land developer can postpone

the development project until it makes sure of the future demand. The second is the option to abandon. Myers and Majd (1990) consider that a firm has an option to abandon the investment project at any time and gets the scrap value. However, they model the abandonment option without obtaining a closed-form solution, and they turn it to numerical analysis. The developer can cease constructing when the subsequent develop cost is higher than the sale price of property either the residual payment from the buyer. The third is a buyer's default option. The buyers can halt payment if the value of the property is lower than the remaining payments. Chan, Wang and Yang (2012) employ a two period game theoretical model to show that when a presale contract has a high down payment ratio, the value of the buyer's default option is approximately zero.

We now consider a scenario that a man is about to get married before the end of this year. He hopes that he could have his own house property before the wedding. Unfortunately, he cannot purchase a house right away because the policy of luxury tax is not clear now and the real estate market is unstable. He does not want to lose any money on the policy of a blast of the real estate speculation. In addition, it is rumored that the government won't intervene in the real estate market. Therefore, the housing prices keep raising and the restriction of high housing prices from the government would be an untruth. He is worried that if the housing prices sharply go up, he might not afford to pay such a price as well. For this reason, he is in a dilemma. Consequently, the compound option would be his best choice. He could buy a Call on Call right now. If the policy is not true or would not work very well, and the housing prices keep going up six months later, he could buy the other right of purchasing houses by the deadline of the compound option and purchases his house before the wedding. Thus, the price would not get out of control for him. However, the policy of a blast of the real estate speculation could by any possibility be implemented strictly and the house prices would be collapsed. He could buy a Put on Call and sell the house in fixed exercise price negotiated before by the expiration of the compound option and earn a fortune.

The Real Option Analysis (ROA) is first known as a decision support technique in the area of capital investments. The concept of real means adapted mathematical models used to evaluate financial options to more tangible investments. Previous research by using real option framework in real estate markets includes Quigg (1993), Williams (1993, 1997), Grenadier (1995, 1996), Capozza and Li (1994, 2001), Lai, Wang and Zhou (2004), Wang and Zhou (2006), Buttimer, Clark and Ott (2008), Bulan, Myer and Somerville (2009), Parthasarathy and Madhumathi (2010), Chan, Wang and Yang (2012).

A presale is allowed to sell a property either pre-built or under construction. In some regions, it is normally permitted only at a definite period subsequent construction commences. These agreements are used by builders or developers for the purpose of securing buyers in real estate projects; on the other hand, the buyers are willing to accept the presale due to the fear of the price of property might increase rapidly in the future that they can't afford the same property when it is completed. In most countries, the common practice is that the builder is obliged to complete a certain proportion of the project before the administration grants presale consent. A presale contract gives the buyers the right to make the payment base on the progress of the construction,

generally speaking, 15-30% of the house price should be a down payment, and the others are mortgaged.

As far as we know, the presale system in real estate was invented by Taiwan. The first presale contract was sold by Chang, Ke-Dong who was the chief executive officer of Huamei Construction and Development Company in 1969. After that, Yeh, Tiao-Hui is devoted to popularizing. This system prevails in Taiwan more than 30 years without any laws to stipulate the details. Finally, the Real Estate Brokerage Business Act was legislated by the government and included the pre-sale contract into the law formally in 1999. During this period, the presale system not only offered a new way to afford a house to the public but also stimulated the house market prosperity and creates new high repeatedly. Recently, the government has imposed a special tax on luxury goods and put real estate login in reality in practice for blasting real estate speculation. We now model the tax into account and incorporate the contingent claims valuation to determine the optimal pre-sale contract price.

This paper is distinguished by two key contributions. We introduce each of these ideas briefly below and then describe them in detail in subsequent sections. First, this paper makes a contribution to the presale real estate literature on real options and on policy establishment such as the luxury tax. Relative to the former, our study simultaneously takes into account the compound options (Geske, 1979) and tax factors (Buttimer *et al*., 2008). Furthermore, we also consider deducible margins (option premium) in our pricing model. To the best of our knowledge, this setting is the first in the presale real estate literature. The closed-form solution for the price of presale house is the second contribution. Instead of the two-stage game theory used in Chan, Wang, and Yang (2012), we apply the compound options approach and finally propose a closed-form solution. We can easily calculate the presale house price by entering the parameters into the derived model. Using our proposed model, the house buyer could calculate the remaining payment (Call on Call) when he/she accepts the contracts. Moreover, a speculator could also computes the Put on Call option value and buy this kind of options when he/she expects that a crash happens in the real estate markets. The third contribution is that we offer simulation and sensitivity analysis to support for the tax effects. The results indicate that the luxury tax could slightly carb speculators' speculative actions and let the house price increase up slowly.

The organization of this study is as follows. The next section briefly reviews the real option in real estate literature. Section 3 explains the assumptions and models the pre-sale contract. Section 4 provides a numerical result. The last section concludes.

2. Literature Review

2.1 Review of Real Option Approach in Real Estate

There is a large amount of literature on the real option analysis which is used on real estate pricing; however, researches on presale contract are relatively few. Arnott and Lewis (1979) construct a model and examine the economics of the transition of land from rural to urban use. The result shows that the use of land takes place when the ratio of the land to the value of the post development property is equal to the ratio of

the expected growth rate of rents to the interest rate. Brennan and Schwartz (1985) introduce a real option framework by evaluating natural resource investments and show that the net present value analysis is lack of flexibility. Capozza and Helsley (1990) consider that development to convert land from agricultural to urban use is economically irreversible and show that uncertainty delays the conversion of land from agricultural to urban use. Capozza and Li (1994, 2001) show that the ability to vary the level of capital increases hurdle rents and delays development decisions. They also offer an empirical test of land as a real option to show that investment spending accelerates when the investment rate increase, especially when the elasticity of substitution between capital and other factors are high. Furthermore, they analyze land development decisions with variable capital intensity and present that projects are optimally delay exceed the point when the net present value becomes nonnegative even under certainty.

McFarlane (1999) analyzes the effects of different taxes on housing investment and constructs a model which is similar to the Arnott and Lewis (1979) model to show how the direction and magnitude of tax effects depend on the presence of agricultural rents and type of urban growth rent. Anderson (1993) proposes an optimal timing model of the effects of use-value property assessment on land development decisions and subsequent land use. Titman (1985) constructs a model and demonstrates that uncertainty increases the expected future value of vacant land. Quigg (1993) is the first to value the option premium for waiting to develop land in Seattle and find that the premium is about 6 percent of the land price. Bulan *et al*. (2009) show that the increase of uncertainties such as idiosyncratic and systematic risks cause developers delay investment projects of real estate. Moel and Tufano (2002) empirically investigate mine opening and shouting decisions. They explore a sample of 285 mines to document that real option analysis is a useful approach for decision making. Furthermore, Parthasarathy and Madhumathi (2010) provide a case study analysis in India to examine the applications of real options analysis in valuing real estate. Motivated in parts by the work of Quigg (1993) and Chan, Wang, and Yang (2012), we apply the real option approach to evaluate the presale house price.

2.2 Taxation and Compound Option

Scholes (1976) extends the Black and Scholes (1973) model and incorporates the effect of taxes in the pricing model. Scholes (1976) model also shows that higher tax rates result in higher hedge ratios and lower option prices, ceteris paribus. Feldstein (1977) documents that investors would like to place large amount of their money on non-land capital because land taxes diminish land values. Jou and Lee (2008) use a real option framework to examine the tax effect on land values and development in a competitive real estate market. The results document that tax rates have positive effect on development while land values decrease by increasing tax rates.

Geske (1977, 1979) is the first to price an option on an option, or compound options. Lajeri-Chaherli (2002) then uses the martingale approach to value compound options. However, the above mentioned solutions are only two folded. Lee, Yeh, and Chen (2008) propose a generalized pricing formula to make sequential compound option more elastic and powerful. The compound option theory is widely used not only in pricing financial derivatives but also in real option analysis. In this paper, we try to

employ the concept of compound option theory (Chan, Wang, and Yang, 2012) into pricing presale house. Furthermore, we consider the deducible option premium and the luxury tax into the pricing model.

3. Model Construction

3.1 The Settings

We assume that the home buyer purchases a house at time T_0 for an exogenously determined price S_0. The builder can either sell the house to the buyer at the construction completion time T at the market price $S(T)$ or presell houses to buyers at time T_0 using an option contract. If the house buyer accepts the presale contracts, he would pay an option premium to the builder at time T_0, and the buyer has the right to buy the house but no obligation to do so. When the builder receives the option premium, he can invest in the following development and reduce the debt finance. There is a unique feature in the housing trading market, that is, the deposit (option premium) can deduct from the property price when you pay for the house. We now assume the house price process is a geometric Brownian Motion (G.B.M) given by the following:

$$dS(t) = (\mu - \delta)S(t)dt + \sigma S(t)dW^P \tag{1}$$

where the growth rate of $S(t)$ has a constant expected growth rate μ and δ is the forfeiture charge for default, σ is the instantaneous volatility of $S(t)$, W^P is a standard Wiener process under P measure.

By using Girsanov Theorem, we can transform the P to Q measure under risk neutral; that is:

$$dS(t) = (r - \delta)S(t)dt + \sigma S(t)dW^Q \tag{2}$$

where: $dW^Q = dW^P + \left(\frac{\mu - r}{\sigma}\right)dt$ is an equivalent martingale measure.

The $\left(\frac{\mu - r}{\sigma}\right)$ is the risk premium and r is risk free rate. Here, we assume the government taxes the house trading at a property tax rate τ.

Therefore, the price of the house $S(t)$ becomes $S(t)(1-\tau)$

$$dS(t) = (r - \delta)S(t)(1-\tau)dt + \sigma S(t)dW^Q \tag{3}$$

Now, we price the presale option in the next section.

3.2 Pricing Presale Options

The presale option is different from a financial call option, the premium can deduct from the purchase price when it is exercised or forfeited. X is the pre-negotiated

purchase price, and we take it as an exogenous variable. Suppose that the $V(T)$ denotes the price of an European presale option at time T, which satisfies the following equation:

$$Max\{S(T)-(X-V(T)),0\} \tag{4}$$

The above equation has an intuitive interpretation: the price of the option equals the payoff to the house buyer with the house price minus strike price deduct from option premium.

$$V(T_0)=e^{-rT}E^{Q}Max\{S(T)-(X-V(T)),0\} \tag{5}$$

The solution to the above equation is given by:

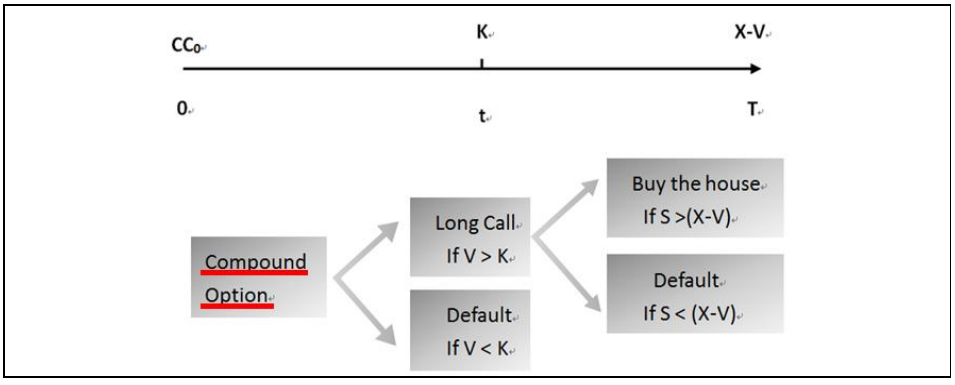
$$V(T_0)=\frac{S_0e^{-(r\tau+\delta-\delta\tau)T}N(d_1)-e^{-rT}XN(d_2)}{(1-e^{-rT})N(d_2)} \tag{6}$$

where: $d_1=\dfrac{\ln(S(T_0)-\ln(X-V(T_0)))+\left((r-\delta)(1-\tau)+\dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$, $d_2=d_1-\sigma\sqrt{T}$, and $N(\cdot)$ is the cumulative normal density function.

We now apply the equation into a compound option model. We assume that the buyer has two options to determine whether to buy or not. The first time to pay for the compound option is T_0 and the exercise price is K, if the presale option premium V is higher than K, the buyer exercises the option and has the right to own the next option. The second time to make the other decision is t, if the house price is lower than the strike price minus option premium, the default is occurred (do not exercise the option) and pays a forfeiture charge; otherwise, he accepts the contract and pays for the remaining payment (Call on Call). We then consider another scenario, if a speculator expects that a crash happens in the real estate market, he buys a Put on Call option and sells the house in higher price (comparative of former house price) and earns a fortune. The foregoing is presented in Figure 2.

Figure 2

The process of the Compound Option Strategy



We model this behavior by compound option theory as following equation:

$$CC_0 = e^{-rt} E^Q \{Max[PV_t[(Max(S_T - (X - V), 0) - K], 0]\} \tag{7}$$

The solution to the above equation is given by:

$$\begin{aligned} CC_0 = {} & S_0 e^{-(r\tau+\delta-\delta\tau)T} N_2(m, n; \rho) - (X - V) e^{-rT} N_2\left(m - \sigma\sqrt{t}, n - \sigma\sqrt{t}; \rho\right) \\ & - K e^{-rT} N_1\left(m - \sigma\sqrt{t}\right) \end{aligned} \tag{8}$$

where: $m = \dfrac{\ln\left(\dfrac{S_0 e^{-(r\tau+\delta)t}}{\varphi e^{-(r+\delta\tau)t}}\right) + \dfrac{\sigma^2}{2} t}{\sigma\sqrt{t}}$, $n = \dfrac{\ln\left(\dfrac{S_0 e^{-(r\tau+\delta)T}}{(X - V) e^{-(r+\delta\tau)T}}\right) + \dfrac{\sigma^2}{2} T}{\sigma\sqrt{T}}$, $\rho = \sqrt{\dfrac{t}{T}}$, $0 \le t \le T$, $N(\cdot)$ is the bivariate cumulative normal distribution. φ is the underlying asset price when it lets $V = K$ and φ is solving by the following equation :

$$\varphi^{-(r\tau+\delta-\delta\tau)(T-t)} N(g_1) - (X - V) e^{-r(T-t)} N(g_2) = K \tag{9}$$

where: $g_1 = \dfrac{\ln\left(\dfrac{S_t e^{-(r\tau+\delta)(T-t)}}{(X - V) e^{-(r+\delta\tau)(T-t)}}\right) + \dfrac{\sigma^2}{2}(T - t)}{\sigma\sqrt{t}}$, $g_2 = \dfrac{\ln\left(\dfrac{S_t e^{-(r\tau+\delta)(T-t)}}{(X - V) e^{-(r+\delta\tau)(T-t)}}\right) - \dfrac{\sigma^2}{2}(T - t)}{\sigma\sqrt{t}}$

We set $S_t = \varphi$, and solve for the parameter φ. Similarly, we can also have the solution for the compound option Put on Call at time $T = 0$ as following equation.

$$\begin{aligned} PC_0 = {} & -S_0 e^{-(r\tau+\delta-\delta\tau)T} N_2(-m, n; -\rho) + (X - V) e^{-rT} N_2\left(-m + \sigma\sqrt{t}, n - \sigma\sqrt{t}; -\rho\right) \\ & + K e^{-rt} N_1\left(-m + \sigma\sqrt{t}\right) \end{aligned} \tag{10}$$

We now both have the Call on Call and Put on Call closed-form solutions; meanwhile, we have some numerical results in the next section.

4. Numerical Results

4.1 Simulation Results

This paper first conducts a based case for comparison purpose. We normalize the completed house price at time zero, S_0 as 100. The volatility of the housing price, σ, is set as 23.68%, which is calculated by the SinYi housing price index in the last decade. We assume that the property tax rate, τ, is 6%. The forfeiture charge is used to prevent the buyer's default if the buyer does not exercise the option (buy the house or sell the house at the fixed price negotiating before). We set the forfeiture charge as 1% of the house price. K is the exercise price of the compound option; we set it to be 20. X is the exercise price of the buyer's option to purchase the house, we expect it grow at risk-free rate, $X = S_0 e^{rT} = 101.39$. T is the time to the house completion; we assume that the house completes in one year. t is the time to the compound option

expires, we assume it to be 0.25 year. r is the risk-free rate. We use the one year time deposit interest rate by the Bank of Taiwan, and we take it for 1.38%. Instead of the Monte Carlo simulation used in Lupu (2006) and Anghelache, Cozmanca and Radu (2011), this paper uses parameters of the based case and the derived model to complete the simulation.

We now show results of the base case and other numerical results in Table 1 and Table 2. The value of Call on Call (CC) and Put on Call (C) with various levels of the main parameters is in rational scope. The volatility of real estate σ takes the values of 15%, 23.68% and 30% which are in the range of reported results for global public real estate market. We take the risk free rate r from 1.38% to 5%, which are below the highest interest rate (Egypt, 9.25%) in the world. The deed tax in Taiwan is 6%, so we take the value of τ for 6%, 15% and 30%.

Table 1

Simulation of the Value of the Compound Option Premium to σ , τ , δ and r

Panel A: $\sigma = 15\%$							
δ	$r(\%)$	**CC**	**PC**	**CC**	**PC**	**CC**	**PC**
		$\tau = 6\%$		$\tau = 15\%$		$\tau = 30\%$	
0.01	1.38	0.862	6.188	0.828	6.298	0.774	6.490
	2.5	2.650	2.948	2.164	3.516	1.585	4.406
	5	27.870	0.000	20.606	0.001	9.598	0.276
0.02	1.38	0.297	8.760	0.316	8.630	0.351	8.408
	2.5	0.784	6.384	0.745	6.520	0.685	6.742
	5	9.898	0.248	6.481	0.800	3.302	2.273
0.05	1.38	0.023	13.331	0.026	12.496	0.035	12.239
	2.5	0.036	12.126	0.044	11.806	0.065	11.236
	5	0.304	8.514	0.304	8.514	0.304	8.514
Panel B: $\sigma = 23.68\%$							
δ	$r(\%)$	**CC**	**PC**	**CC**	**PC**	**CC**	**PC**
		$\tau = 6\%$		$\tau = 15\%$		$\tau = 30\%$	
0.01	1.38	6.619	2.168	6.498	2.223	6.305	2.316
	2.5	10.126	1.019	9.311	1.211	8.160	1.543
	5	28.136	0.006	21.903	0.049	14.575	0.357
0.02	1.38	4.157	3.659	4.267	3.574	4.457	3.430
	2.5	6.001	2.432	5.869	2.502	5.656	2.618
	5	14.763	0.341	12.491	0.578	9.672	1.074
0.05	1.38	1.098	7.332	1.303	6.965	1.704	6.326
	2.5	1.665	6.345	1.849	6.074	2.194	5.607
	5	3.641	3.951	3.641	3.951	3.641	3.951

Panel C: $\sigma = 30\%$							
δ	$r(\%)$	CC	PC	CC	PC	CC	PC
		$\tau = 6\%$		$\tau = 15\%$		$\tau = 30\%$	
0.01	1.38	12.205	1.180	12.051	1.213	11.800	1.267
	2.5	15.847	0.603	14.981	0.706	13.705	0.889
	5	29.487	0.026	24.650	0.092	18.917	0.317
0.02	1.38	8.846	2.105	9.005	2.049	9.279	1.956
	2.5	11.117	1.402	10.949	1.443	10.676	1.513
	5	19.073	0.307	17.131	0.449	14.509	0.734
0.05	1.38	3.852	4.770	4.240	4.490	4.963	4.004
	2.5	4.798	4.072	5.114	3.871	5.685	3.527
	5	7.580	2.495	7.580	2.495	7.580	2.495

Note: σ is the instantaneous volatility rate, τ is the tax rate, δ is the forfeiture charge and r is the risk free rate. The based case is also used for other parameters.

In Table 1, we can easily find that the option premiums for the base case are 6.619 (CC) and 2.168 (PC). When the volatility of real estate market σ upraises to 30%, the value of CC and PC are 12.205 and 1.180, the CC conforms to traditional option theory, but the PC is in the opposite direction. One reason for the phenomenon may be accounted as follows. Besides, the risk free rate increases to 5%, the option premium for CC and PC become 28.136 and 0.006. The PC is nearly nil; it indicates higher interest rate lead lower PC value but higher CC. We assume that the maximum tax rate is 30%, for the sake of the government may impose a heavy tax rate in order to blast the real estate speculation. As it shown in Table 2, when τ increase from 6%, 15% to 30%, the price of CC decreases from 6.619, 6.498 to 6.305. The tax rate increases twice but the price of the option varies only about 1.8% to 3%. Therefore, the tax rate may have a small influence in option premium.

Table 2

Simulation of the Value of the Compound Option Premium to σ , K , T and t

Panel A: $\sigma = 15\%$							
T	t	CC	PC	CC	PC	CC	PC
		$K = 15$		$K = 20$		$K = 30$	
0.6	0.1	0.292	4.876	0.025	9.602	0.000	19.563
	0.25	1.007	5.560	0.306	9.841	0.014	19.515
	0.5	1.945	6.446	0.872	10.338	0.117	19.515
1	0.1	1.410	1.783	0.222	5.589	0.001	15.354
	0.25	2.402	2.745	0.862	6.188	0.058	15.349
	0.5	3.545	3.836	1.842	7.098	0.370	15.558
1.5	0.1	3.929	0.278	1.051	2.393	0.010	11.339
	0.25	4.709	1.027	2.039	3.341	0.192	11.459
	0.5	5.697	1.964	3.177	4.410	0.739	11.903

Panel B: $\sigma = 23.68\%$							
T	t	**CC**	**PC**	**CC**	**PC**	**CC**	**PC**
		$K = 15$		$K = 20$		$K = 30$	
0.6	0.1	4.486	1.128	1.913	3.548	0.176	11.797
	0.25	5.894	2.505	3.494	5.088	0.988	12.548
	0.5	7.649	4.208	5.344	6.869	2.333	13.790
1	0.1	9.456	0.054	5.254	0.844	0.841	6.417
	0.25	10.182	0.748	6.619	2.168	2.213	7.728
	0.5	11.396	1.911	8.290	3.771	3.986	9.397
1.5	0.1	14.660	0.000	9.760	0.055	2.501	2.782
	0.25	14.839	0.110	10.468	0.721	4.128	4.348
	0.5	15.527	0.747	11.696	1.882	5.979	6.096
Panel C: $\sigma = 30\%$							
T	t	**CC**	**PC**	**CC**	**PC**	**CC**	**PC**
		$K = 15$		$K = 20$		$K = 30$	
0.6	0.1	9.334	0.375	5.563	1.597	1.337	7.357
	0.25	10.589	1.599	7.411	3.404	3.199	9.158
	0.5	12.472	3.431	9.679	5.603	5.493	11.349
1	0.1	15.942	0.000	11.162	0.178	3.877	2.880
	0.25	16.361	0.353	12.205	1.180	5.924	4.867
	0.5	17.436	1.376	13.858	2.764	8.279	7.117
1.5	0.1	21.832	0.000	16.829	0.000	7.690	0.817
	0.25	21.886	0.002	17.194	0.293	9.293	2.358
	0.5	22.408	0.473	18.231	1.261	11.342	4.303

Note: K is exercise price of the compound option, T is the time to house completion and t is the time to compound option expiration. The based case is also used for other parameters.

Table 2 shows the results of the variation of exercise price K and the time to compound option expiration t and the time to the completion of construction T. It reveals that when exercise price is high ($K = 30$) and volatility is as low as 15%, the Call on Call is nearly worthless and the Put on Call has small effect by the extension of time to the house complete. Furthermore, we find that within the same period of t to T, 0.5 year, the earlier the compound option is expired, the less the option premium is. For instance, for the $t = 0.1$ to $T = 0.6$ and $t = 0.5$ to $T = 1$, the duration of the expiration to completion are the same, nevertheless, the earlier expired day $t = 0.1$ has lower option premium from any level of σ and then $t = 0.5$.

4.2 Sensitivity Analyses

Figure 3 (A) displays results for varying levels of house price volatility. That is true according to the finance literature; we know the value of a call option increases with the uprising volatility. When the future price of the house is more volatile, the call option is more valuable. In contrast, the put option is decreasing with the volatility increase (under r = 1.38% and 2.5%).

In Figure 3 (B) , we vary the forfeiture charge from zero to 0.2. It shows that the price of call option is decreasing when the forfeiture charge increase. When the forfeiture is more than 8%, the Call on Call is nearly worthless. This is because the higher the

forfeiture charge would lead the call option buyer fear of default and has less desire to buy it. On the other hand, the Put on Call is opposite, this may due to the lower house price but with the relative higher exercise price, the buyer of Put on Call is willing to pay more to buy it.

Figure 3 (C) shows that an increase in the risk-free rate implies that a presale call option is more precious to the home builder, and this is because the expected present value of purchase price is lower after discounting at the higher risk-free rate. Therefore, home builder isn't willing to launch a presale contract when the risk free-rate increases.

In Figure 3 (D), we vary the compound option exercise price. As we know, the call option is cheaper when the exercise price increases; on the contrary, the put option is more valuable. That means if the exercise price is too high, the probability of the presale option exceeds the exercise price is low and the call option is unattractive.

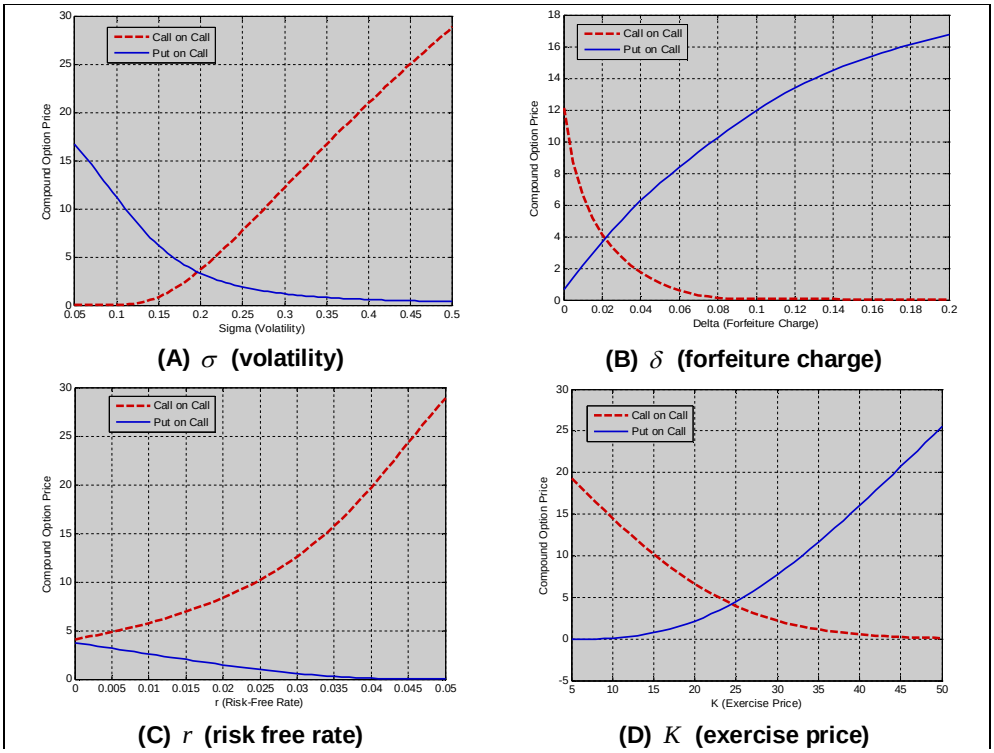
The impact of the time to complete T and time to the compound option maturity t are present in Figure 3 (E) and (F). Longer time period t makes a call option more valuable because of the time value, in other words, the home buyer is willing to pay more money to extend the compound option expiration. On the other hand, the call price increases as the time to complete T is longer. This is because the longer period offers the home buyer more choices whether to buy or abandon.

In Figure 3 (G) , we vary the purchase price X . It is obvious when the purchase price X increases, the exercise price $(X-V)$ is higher, and the call option becomes worthless but the put option is opposite.

In Figure 3 (H), we vary the property tax-rate between zero to 50%. As it shows that the property tax-rate has slightly effect on both call and put option. In Taiwan, the purpose of luxury tax is to establish a more balanced real estate markets, reduce the negative perception of the public, and maintain fairness of tax. The results indicate that the luxury tax could slightly carb speculators' speculative actions and let the house price increase up slowly. It has slightly act's effect on the real estate markets. Parts of the duty-free articles are easy to become tax planning spaces, but have no reduction of the tax act performance in danger. For the high-consumption product markets, luxury tax is indeed to eliminate the public negative perception and can increase the tax burden on high income earners and make the tax acts fairer.

Figure 3

Sensitivity Analyses

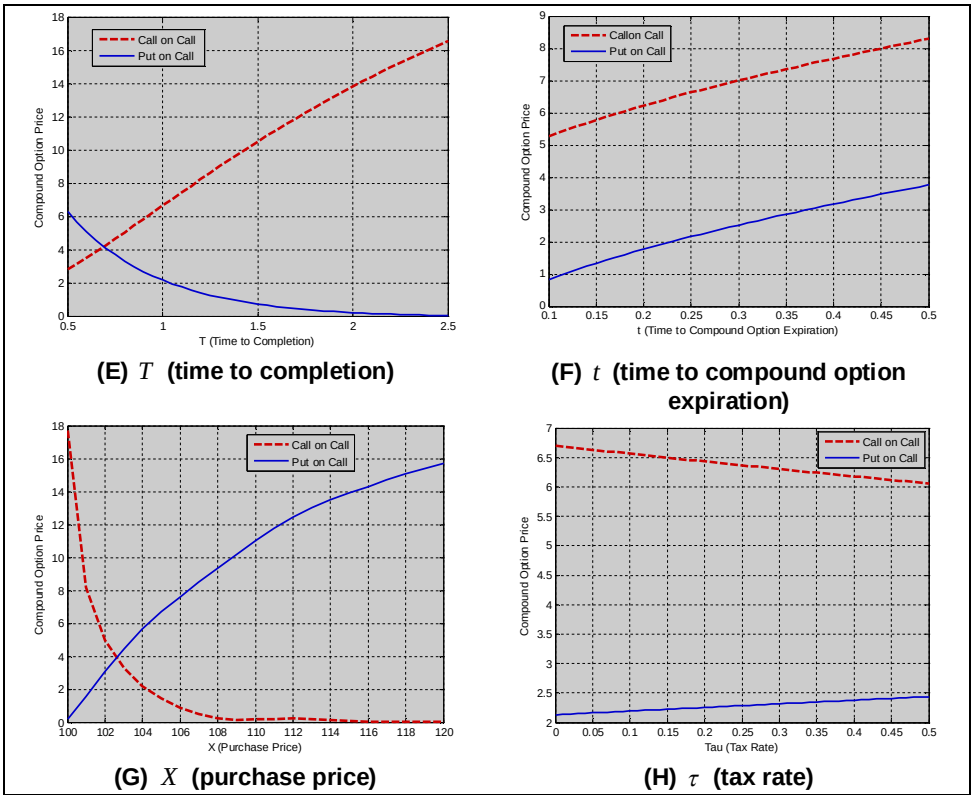


(A) σ (volatility) **(B) δ (forfeiture charge)**

(C) r (risk free rate) **(D) K (exercise price)**

Figure 3 (Continued)

Sensitivity Analyses



(E) T **(time to completion)**

(F) t **(time to compound option expiration)**

(G) X **(purchase price)**

(H) τ **(tax rate)**

5. Conclusion

In this paper, we incorporate several unique feature of presale house system, e.g. the practice of applying the option premium as a reduction to the purchase price (exercise price) and make it more flexible as a compound option. We provide a model to compute the presale option premium in order to let the real estate market more efficient. The home buyer may refer to the computing price to judge whether to buy or not. We present many numerical results about some crucial elements which affects the option price. It may be useful to the house consumer to make a decision. With the sensitivity analysis, the results show that the luxury tax has effect on the presale house price. The introduction of the luxury tax truly let the presale house increase up slowly. For the high-consumption product markets, luxury tax is indeed to eliminate the public negative perception and can increase the tax burden on high income earners and make the tax acts fairer.

In the real world, we may encounter the burst of the real estate bubbles, thus the follow-up could incorporate the jump process into the model to deal with the crash. The prices of real estate to be registered is carried out from August 1st, 2012 in Taiwan at once, disclosure of information about real estate transactions helps promote transparency in the market and curb runaway house prices. Beginning in August, land administration agents, real estate buyers and real estate brokers must register the value of property transactions within 30 days of closing a deal. After this implementation of this new system, we incorporate the genuine trade prices into the model to compute the implied volatility in real estate market to know if the market is rational. We may compile a volatility index in real estate market to be a new financial derivative like volatility index (VIX).

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