5. TESTING FOR NONLINEARITY IN UNEMPLOYMENT RATES VIA DELAY VECTOR VARIANCE

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Abstract

We discuss the application of a new test for nonlinearity for economic time series. We apply the test for several monthly unemployment series from the developed economies. We find nonlinearities in the unemployment for most of the European economies, but not for US, UK or Japan.

Keywords: nonlinearity, surrogate data, rank test, time reversal, unemployment.

JEL Classification: C22, C50, E24.

I. Introduction

The nonlinear features of economic time series are widely studied and various models that deal with them were proposed. However, as pointed by Barnett et al. (1997), based on a large scale comparison of different tests for nonlinearity and chaos, the research on nonlinearities in unemployment rates is very limited.

Among the first to uncover nonlinearities in unemployment rates were Brock and Sayers (1988) for the case of US unemployment and Frank *et al.* (1993) for the case of Canadian unemployment series. Later evidence pointed also to asymmetries in the US unemployment data that can also be linked to nonlinearities, see Rothman (1991) or Montgomery et al. (1998).

More recently, Panagiotidis and Pelloni (2007) tested for nonlinearity in US and Canadian unemployment rates at both aggregate and sectoral level using univariate and multivariate tests. They found evidence of nonlinearities for US sectoral unemployment rates but not for the aggregate ones, while they also found mixed results for Canada.

Several models that deal with the nonlinearities in unemployment were proposed. Altissimo and Violante (2001) studied the dynamics of output and unemployment in US using a nonlinear-VAR model. Caporale and Gil-Alana (2007) proposed a

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nonlinear model that can explain both the asymmetry and the long memory in unemployment.

The possible nonlinearities in unemployment rates are important as they can be tied to the two essential and much documented features of hysteresis and asymmetry. Usually, the asymmetry is modeled using Markov-switching models, as such a feature basically implies the presence of nonlinearities. In the context of testing for unit root against a smooth transition process model, following Kapetanios et al. (2003), a link has also been established between nonlinearity and hysteresis, see Gustavsson and Österholm (2006) or Franchi and Ordóñez (2008) for some results.

We propose in this paper the use of a newly developed test for nonlinearity which is based on the concept of surrogate data. We contribute in several directions. First of all, we implement the test on economic time series, one of the first applications of this test in economics to the best of our knowledge, see also the recent work by Addo et al. (2013a, 2013b, 2013c). Second, we also discuss and implement the surrogate data approach. Although widely used in physics, few applications in economics used surrogate generated data so far. We equally discuss new evidence on the presence on nonlinearities in unemployment data.

The paper is organized as follows. The second section details the methodology used throughout the paper. We implement the test and analyze the results in the third section. Conclusion and possible implications are discussed in the last section.

II. Methodology

We present and discuss in this section the two main ingredients of our approach, namely the Delay Vector Variance (DVV, hereafter) test and the surrogate data concept.

II.1. The Delay Vector Variance Test for Nonlinearity

The DVV test for nonlinearity was proposed by Gautama *et al*. (2004). They proposed a technique which has some relation with two older tests for nonlinearity, the false nearest neighbors as well as the δ-ε method due to Kaplan (1994). Other applications of the method can be found in Gautama *et al*. (2003), where the test is applied to brain's electrical activity, as well as in Mandic *et al.* (2008) where the test is implemented on a range of machine learning problems.

We follow the presentations of the test in Gautama *et al. (2004)* and Mandic *et al*. (2008). The common background for DVV and other related method is the use of the time delay embedding in a phase space through the so-called delay vectors, *DVs* hereafter, *x(k)* that are characterized by an embedding dimension and a time lag τ. We can write:

$$x(k) = [x_{k-m\tau}, \ldots, x_{k-\tau}] \quad (1)$$

For every DV *x(k)* there is also a corresponding target, the next sample x(k).

II.1.1. The δ-ε Kaplan Test

We first present a more known method *δ-ε* due to Kaplan (1994), following the perspective in Mandic *et al*. (2008).

- The $\delta_{i,j}$ Euclidean distances between the DVs x(i) and y(j) are derived. Using the L2 norm, Kaplan computed the distance between corresponding targets by $\varepsilon_{i,j}$.
- An average of *ε* values is computed, conditional on *δ*: $\varepsilon(r) = \bar{\varepsilon}_{j,k}$ (2)

  for $r \le \delta_{j,k} < r + \Delta r$, with *r* the width of bins by which the *ε*'s were averaged.
- One computes $E = \lim_{r \to 0} \varepsilon(r)$ which is the smallest value for $\varepsilon(r)$ and it can be understood as a measure for predictability of the time series.
- Based on the value of E, a left-tailed nonlinearity test can be constructed based on surrogate data.

II.1.2. The DVV test

The DVV test is summarized below, following Mandic et al. (2008):

The mean, denoted by μ_d, and the standard deviation, denoted by σ^2, are computed over all pairwise Euclidean distances between *DVs*, for $\|x(i) - y(j)\|$ for all $i \ne j$.

Sets Ω_k are generated by grouping all *DVs* that lie within a certain neighborhood of *x(k)*, given by the distance r_d. Each $\Omega_k(r_d)$ set is generated according to: $\Omega_k = \{x_i \| \|x(k) - x(i)\| \le \tau_d\}$. (3)

For each set $\Omega_k(r_d)$ one computes the variance of the corresponding targets, namely $\sigma_k^2(r_d)$. Based on them a target variance can be computed which is given by:

$$\sigma^{*2}(r_d) = \frac{\frac{1}{N}\sum_{k=1}^{N} \sigma_k^2(r_d)}{\sigma_x^2} \tag{4}$$

Finally, the so called DVV plots, which graph the target variance as a function of the standardized distance $(r_d - \mu_c)/\sigma_d$, can be computed.

II.1.3. Alternative nonlinear tests

In the context of surrogate data approach to testing for nonlinearity, see Schreiber and Schmitz (1997), two other tests for nonlinearity were proposed, the third-order autocovariance approach (C3, hereafter) and the reversibility due to time reversal (REV, hereafter).

The C3, combined with surrogate data method, leads to a two-tailed test for nonlinearity. The C3 approach leads to the following metric:

$$t^{C3}(\tau) = \langle x_k x_{k-\tau} x_{k-2\tau} \rangle \tag{5}$$

By definition, the reversibility characterizes time series for which the probability properties are invariant with respect to time reversal. Schreiber and Schmitz (1997) proposed a metric based on this concept which, in combination with the surrogate approach, leads to a two-tailed test for nonlinearity. They proposed a metric which is given below:

$$t^{REV}(\tau) = \left\langle (x_k - x_{k-\tau})^3 \right\rangle \tag{6}$$

II.1.4. Hypothesis Testing

For each test, the null hypothesis is that of a linear process. We follow the approach of Mandic *et al*. (2008, p. 1146) who, based on the contribution by Theiler and Pritchard (1996), proposed the use of nonparametric rank based tests since the probability functions of the proposed metrics are not known.

The procedure consists of the following steps. From an original series, 99 surrogate series are created. Test statistics for the original t_0 as well as for the 99 surrogate series are constructed and the series $\{t_0, t_{s,i}\}$ is sorted so that the rank r of t_o is determined. The following rules can be followed:

- the right-tail test rejects the null hypothesis if the rank $r > 90$;
- the left-tail test reject the null in case $r \leq 10$;
- for the two-tailed test, the null is rejected if $r > 95$ or $r < 5$.

II.2. Surrogate Data

Surrogate data are intrinsically tied to the testing for nonlinearity as they were initially proposed as a way to test for nonlinearities by Theiler et al. (1992). Since then many extensions were proposed to the initial approach, see Schreiber and Schmitz (2000) for a comprehensive review. However, this approach remained outside the mainstream applied economics, see Kugiumtzis (2002) for a general presentation within the context of application to economics, as well as Kugiumtzis (1999) and Kugiumtzis (2008) for related research. We discuss in the following paragraphs the main types of surrogate data proposed in the literature as well as what we will use.

The standard approach in surrogate data testing involves the use of phase randomization (Fourier transformed data, FT hereafter), since, as Maiwald et al. (2008) argue, the Gaussian processes do not posses information in the Fourier transform but in their mean and auto covariance which are preserved under this transformation. The FT surrogate data are used with the null hypothesis that the time series in cause is a stationary Gaussian process.

An algorithm that performs better and it is currently widely used is the Amplitude Adjusted Fourier Transform (AAFT, hereafter). A newer algorithm is the IAAFT algorithm, due to Schreiber and Schmitz (1996), which is an improvement of the AAFT. This extension corrects one the of the main problems of AAFT method which consist in the fact that the amplitude spectrum of the surrogates is flatter than the one of the original series. Both AAFT and IAAFT are used with the null hypothesis of a linear Gaussian process transformed through a nonlinear invertible function.

Following Gautama et al. (2004) we use the IAAFT transform and a few alternatives based on it. Three different types of surrogate data are used, two of which are based on the modern IAAFT, one that leads to surrogate with identical signal distributions as the original series and we call this surrogate 1st type surrogate data, and one that leads to identical amplitude spectra and we call it 2nd type surrogate data. Finally, we also use a phase randomized type of surrogate data, the 3rd type used throughout this paper.

III. Estimation and Results

III.1. Data

We use monthly unemployment rates from several developed economies, namely Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands, Portugal, Sweden and UK. The data sample is between January 1983 and October 2011. For the case of Germany we considered only the sample corresponding to the aftermath of the reunification, starting with January 1991.

We use the logistic transformation following Wallace (1987) and Panagiotidis and Pelloni (2007). The formula is given by:

$$\log\left(\frac{u_t}{1-u_t}\right) \text{where } 0 < u_t < 1 \qquad (7)$$

Then, we implement the log difference following the standard approach in the literature. Before applying the tests, the data was pre-whitened using an AR(p) processes.

III.2. Results

We apply on the DVV test[2] as well for comparison reasons, the C3 and REV tests. We follow thus previous applications of the DVV test that compared its performance with that of C3 an REV approaches. Equally, the C3 and REV approaches are also seen as natural in the context of surrogate data approach, see Schreiber and Schmitz (1997).

On sensitive issue in economics which was not solved until now, is the choice of both the embedding dimension and the time lag. Two techniques are normally used, the mutual information for the time delay and the false neighbors approach for the embedding dimension. Others - see Zbilut (2005) - consider that in order to reveal the whole complexity of economic time series, an embedding dimension of 10 should be used. For the time delay, the same author suggests a value of 1.

We run the tests by considering different values for the embedding dimension and time delay. We vary the embedding dimension from 2 to 5 for the DVV test. In the case of the two other tests, C3 and REV, since they are dependent on the time lag, we

[2] *The analysis was performed using the Matlab code associated to the original paper by Gautama et al. (2004).*

run them on time lags from 1 to 5. The results are presented in Annex 1, Tables 1 to 9; the figures in each table stand for the results of the *rank tests*.

The DVV test leads to similar results for most of the European unemployment rates, in most of the cases with little differences as regards the embedding dimension used. The DVV tests indicates the presence of nonlinearities for Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands and Portugal. The DVV test does not reject the null of linearity for US and Japan. For some embedding dimension and some of the types of surrogate data, there are a few cases of rejection of linearity for Sweden and UK, however, in most of the cases, the linearity hypothesis cannot be rejected.

In case of C3 and REV approaches, there is much more mixed evidences. The findings are sensitive to both the type of surrogate data used as well as the time delay. However, the findings are in line with the results in Gautama *et al.* (2004) who found - for a series of different classes of signals, from ARMA and ARCH models to sunspots series, chaotic and Henon type ones - that DVV correctly rejected the null of linearity, while REV and C3 approaches led to wrong results in many cases.

IV. Conclusion

Although the research on chaos in economics reached a limit in the sense that no contribution was able up to now to test for the presence of chaos "conditioned on an economic model", as Barnett and Serletis (2000) and Barnett (2006) suggested, the research on nonlinearities in economic processes continues to draw attention and this is the case of unemployment dynamics too.

The initial research on the chaotic and nonlinear features of unemployment pointed to nonlinearities in US unemployment, see Brock and Sayers (1988) for an early example, however, more recent research, see Panagiotidis and Pelloni (2007) contradicted this evidence for the case of US. The findings here point to additional evidence of lack of nonlinearities in the US unemployment rates, however we found that the null of linearity was rejected for most of the European economies included in sample.

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Annex 1. Tables for DVV, C3 and REV Tests

Table 1

Results for the rank tests for DVV and 1st type surrogate data

	m=2	m=3	m=4	m=5
Belgium	**100**	**100**	**100**	**100**
Denmark	**100**	**100**	**100**	**100**
Germany	**96**	**95**	**95**	**92**
Ireland	**100**	**100**	**100**	**100**
France	**97**	88	**96**	**97**
Italy	64	**100**	**100**	**100**
Netherlands	**100**	**100**	**100**	**100**
Portugal	**100**	**100**	**100**	**100**
Sweden	52	70	54	50
United Kingdom	17	**96**	30	55
United States	63	22	15	9
Japan	54	26	42	4

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 2

Results for the rank tests for DVV and 2nd type surrogate data

	m=2	m=3	m=4	m=5
Belgium	**100**	**100**	**100**	**100**
Denmark	**100**	**100**	**100**	**100**
Germany	**99**	**95**	**95**	90
Ireland	**100**	**100**	**100**	**100**
France	**100**	**92**	**97**	**94**
Italy	67	**100**	**100**	**99**
Netherlands	**100**	**100**	**100**	**100**
Portugal	**100**	**100**	**100**	**100**
Sweden	56	74	56	60
United Kingdom	50	**96**	30	61
United States	55	30	5	23
Japan	17	24	39	7

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 3

Results for the rank tests for DVV and 3rd type surrogate data

	m=2	m=3	m=4	m=5
Belgium	**100**	**100**	**100**	**100**
Denmark	**100**	**100**	**100**	**100**
Germany	**100**	**98**	**99**	**95**
Ireland	**100**	**100**	**100**	**100**

	m=2	m=3	m=4	m=5
France	**100**	**100**	**99**	**99**
Italy	**99**	**100**	**100**	**100**
Netherlands	**100**	**99**	**100**	**100**
Portugal	**100**	**100**	**100**	**100**
Sweden	67	**100**	**98**	78
United Kingdom	**94**	**94**	82	87
United States	86	28	20	43
Japan	63	54	46	11

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 4

Results for the rank tests for C3 and 1st type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	60	83	95	42	43
Denmark	15	**1**	**100**	22	55
Germany	88	87	27	27	74
Ireland	90	56	38	10	42
France	69	17	35	**96**	59
Italy	**1**	20	44	53	**5**
Netherlands	41	91	**1**	36	10
Portugal	**1**	11	45	55	33
Sweden	38	18	30	87	7
United Kingdom	53	57	**1**	36	72
United States	16	**1**	21	81	**1**
Japan	17	25	35	53	51

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 5

Results for the rank tests for C3 and 2nd type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	64	85	93	39	44
Denmark	21	**1**	**100**	28	56
Germany	30	86	12	28	83
Ireland	93	54	23	11	57
France	57	26	32	95	61
Italy	**1**	22	47	57	**4**
Netherlands	35	82	**3**	41	10
Portugal	**1**	9	53	48	29
Sweden	49	22	33	80	8
United Kingdom	23	56	**1**	28	71
United States	26	**1**	28	71	**1**
Japan	20	25	29	51	53

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 6

Results for the rank tests for C3 and 3rd type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	60	92	93	30	52
Denmark	27	**1**	**100**	30	51
Germany	26	89	20	25	72
Ireland	92	50	36	6	47
France	55	17	31	**97**	64
Italy	**1**	24	46	69	**2**
Netherlands	40	81	**2**	41	8
Portugal	**1**	17	58	54	41
Sweden	33	19	35	78	10
United Kingdom	87	44	**1**	35	75
United States	20	**3**	22	85	**1**
Japan	19	23	40	51	42

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 7

Results for the rank tests for REV and 1st type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	14	19	**4**	8	**3**
Denmark	71	51	**98**	**99**	77
Germany	52	65	59	20	56
Ireland	84	62	51	63	61
France	80	60	37	70	80
Italy	**3**	85	39	**1**	77
Netherlands	44	51	8	7	23
Portugal	**99**	10	88	7	**5**
Sweden	46	64	54	22	87
United Kingdom	25	57	65	27	74
United States	**4**	75	**1**	42	16
Japan	56	52	68	35	8

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 8

Results for the rank tests for REV and 2nd type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	15	21	**2**	**5**	6
Denmark	70	40	95	**98**	76
Germany	54	44	55	20	72
Ireland	76	46	51	64	67
France	69	59	34	66	78
Italy	**2**	85	30	**5**	77
Netherlands	41	32	10	8	18
Portugal	**100**	**100**	93	6	10
Sweden	39	72	44	37	82
United Kingdom	25	53	65	40	61
United States	7	65	40	61	75
Japan	67	60	72	34	9

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.

Table 9

Results for the rank tests for REV and 3rd type surrogate data

	τ=1	τ=2	τ=3	τ=4	τ=5
Belgium	9	16	**1**	**3**	**3**
Denmark	77	46	**100**	**100**	85
Germany	39	57	61	20	57
Ireland	88	52	58	66	64
France	76	54	32	76	82
Italy	**1**	94	26	**1**	**96**
Netherlands	80	32	**1**	**2**	10
Portugal	**100**	**100**	**98**	**1**	**1**
Sweden	32	80	43	23	92
United Kingdom	17	57	83	23	78
United States	**1**	74	**1**	50	11
Japan	53	61	73	26	**4**

Note: Significant rejections of the null hypothesis at the level of 0.1 are bolded.