1. NEW EVIDENCE ON THE INFORMATION CONTENT OF IMPLIED VOLATILITY OF S&P 500: MODEL-FREE VERSUS MODEL-BASED

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Abstract

This paper provides new evidence to compare the information content of model-free implied volatility (MFIV) and model-based volatility for forecasting future volatility of the S&P 500. We choose Black and Scholes (BS) implied volatility as our model-based volatility and VIX as our measure of MFIV. By using non-overlapping monthly samples from January 2004 to June 2019, we find that both BS implied volatility and MFIV are informationally efficient and subsume information contained in the historical realized volatility for forecasting future volatility. This is the first study show that BS implied volatility and MFIV contain the same information and there is no winner for forecasting future volatility. This implied that a forecast model could include both BS implied volatility and MFIV

Keywords: Implied volatility, VIX, Realized volatility, Information, Volatility forecasts, Volatility models

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1. Introduction

Forecasting volatility is an important task for financial market participants. Since volatility plays an important role in derivative pricing, risk management, portfolio management and

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asset allocation, it has attracted the attention of many scholars and industry practitioners in the last couple of decades. Researchers have been using different models and methods to forecast future volatility. Forecasting is mainly based on two types of information; historical information and implied information based on the prices of traded options. The trades in the option market are based on both historical information and the expectation of future information, thus option prices reflect investors' expectations of future volatility. If the option market is informationally efficient and the option pricing models that are used to "translate" implied volatility are correct, the implied volatility should subsume all of the historical information, therefore implied volatility should provide better forecasting of future volatility than historical information. However, empirical studies have not reached any unified conclusions on whether implied volatility contains all historical information (see, among others, Christensen and Prabhala, 1998; Jiang and Tian, 2005; Becker, Clements and White, 2007; Kambouroudis, McMillan and Tsakou, 2016).

In practice, the most common pricing model used to calculate implied volatility is the Black and Scholes model (also known as the BS model). The BS model is based on several assumptions that may not hold in practice; thus, implied volatility derived from the BS model may be biased. Britten-Jones and Neuberger (2000) proposed a new method to calculate implied volatility, namely model-free implied volatility (MFIV). One advantage of MFIV is that it does not rely on any option pricing models, and is thus immune to model specification errors.

In the literature, the examination of information content of BS implied volatility and MFIV show conflicting results. Jiang and Tian (2005) claim that MFIV subsumes all of the information content of BS implied volatility for forecasting the future volatility of the S&P 500 index. In contrast, Biktimirov and Wang (2017) find that BS implied volatility dominates MFIV for forecasting future the volatility of the S&P 500 index. A similar conclusion was supported by Muzziolio (2010) on the Germany stock market index and Cheng and Fung (2012) on the Hong Kong stock market index. Theoretically, MFIV is based on the price of options with an infinite range of continuous strike prices of the underlying assets. However, in practice, the options that trade on the market have a limited number of strike prices. Researchers have therefore adopted different methods to calculate their MFIV (Jiang and Tian, 2015; Muzzilolio, 2010; Cheng and Fung, 2012). In 2003, the Chicago Board Options Exchange (CBOE) followed the concept of MFIV and calculated volatility index (VIX), which was previously calculated based on the BS model. Since VIX is actively traded in the market, Biktimirov and Wang (2017) utilized the VIX index to measure MFIV.

There might be two reasons for the conflicting results relating to the information content of BS implied volatility and MFIV. One is the use of different methods of MFIV mentioned above. The other one is that different criteria have been used to select a small number of options to calculate BS implied volatility. Such artificial judgment for setting the criteria may yield different conclusions. This study contributes to the literature by providing new evidence on the information content of BS implied volatility and MFIV for forecasting the future volatility of the S&P 500 index. This is the first study to show that BS implied volatility and MFIV contain the same information and that neither has any advantage over the other for forecasting future volatility. One feature of this study is that it tries to avoid the use of artificial judgment to calculate BS implied volatility and MFIV. Thus, we choose the VIX index as our proxy for MFIV, since VIX is actively traded in the market and widely accepted as a measure of MFIV. Regarding the calculation of BS implied volatility, we do not rely on the selected options, instead we use all of the options that were traded on the market and compute the implied volatility surface. Details are explained in the methodology section. Since VIX utilizes a large number of options, and our measure of BS implied volatility is also based on a large number of options, we hypothesize that BS implied volatility and MFIV should contain the same information for forecasting future volatility.

Using non-overlapping monthly samples over a period from January 2004 to June 2019, we show that both BS implied volatility and MFIV are informationally efficient and subsume information contained in the historically realized volatility for forecasting future volatility. BS implied volatility and MFIV contain the same information and there is no winner for forecasting for future volatility. However, both BS implied volatility and MFIV are biased forecasts of future volatility.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 introduces data and methodology. Section 4 provides the empirical results and robustness check, and Section 5 concludes.

2. Literature Review

There are two widely used methods to predict the volatility of financial assets: 1) Time-series forecasting method. In this method, future volatility forecasting is based on either historical realized volatility or Generalized Autoregressive Conditional Heteroscedasticity (GARCH) type models. 2) Implied volatility method. In this method, future volatility forecasting is based on either the BS model implied volatility or model-free implied volatility.

Black and Scholes (1973) provide a European, non-dividend option model, which allows us to calculate the implied volatility from the price of an option. Based on the BS model, Day and Lewis (1992) examine the information content of implied volatility from S&P 100 index call options relative to the GARCH family models. Their results show that implied volatility contains incremental information relative to the GARCH family models. Canina and Figlewski (1993) found that implied volatility from S&P 100 index call options has weak correlation with future volatility. Christensen and Prabhala (1998) claim that previous studies are affected by overlapping samples and mismatching maturities between the option and the volatility forecast horizon. Adopting non-overlapping monthly samples, Christensen and Prabhala (1998) find that the implied volatility of the S&P 100 index outperforms historical volatility for forecasting future volatility. Fung (2007) also adopts the non-overlapping monthly sampling method and applies it to the Hang Seng Index (HSI) options. His results suggested that the predictive power of implied volatility outperforms historical realized volatility.

Britten-Jones and Neuberger (2000) first proposed the concept of MFIV, which initiated a new research stream in the field of implied volatility. Unlike model-based implied volatility, the MFIV is not dependent on any option pricing model, but is instead derived from no-arbitrage conditions. It is calculated by utilizing option prices from a set of options with various strike prices. Britten-Jones and Neuberger (2000) derive the MFIV under diffusion assumption. Jiang and Tian (2005) extend Britten-Jones and Neuberger's (2000) model by incorporating the jump process and developing a simple method for implementing the MFIV in practice[2]. In addition, Jiang and Tian (2005) examine the forecasting ability and information content of the MFIV of S&P 500 index options. Their results suggested that MFIV

[2] *Britten-Jones and Neuberger's (2000) model requires options with an infinite range of continuous strike prices of the underlying assets.*

subsumes all of the information contained in the BS implied volatility and historical realized volatility, and MFIV is a more efficient estimator for future volatility as compared to the others.

CBOE launched the CBOE Volatility Index, which has traded as VIX since the 1990s. Researchers have used the information of VIX and examined its predictive power. Fleming, Osdiek and Whaley (1995) show that VIX (now VOX) contains information about future volatility, but the forecast is biased. Blair, Poon and Taylor (2001) find that VOX provides more accurate forecasts than the GARCH-type models. In 2003, CBOE adopted the MFIV method to calculate VIX, which was previously calculated using the BS model. Becker, Clements and White (2007) find that VIX does not contain additional information for forecasting future volatility relative to the GARCH model. Goit and Laurent (2007) examined the information content of CBOE VIX and VOX when the historical volatility can be decomposed into jump and continuous components. They find that the implied volatility subsumes other volatility information, and that even the jump and continuous components of historical volatility are considered.

The comparison between BS implied volatility and MFIV is performed using option prices of individual stocks. Using 149 stocks in the US, Taylor, Yadav and Zhang (2010) find that at-the-money BS implied volatility outperforms MFIV in forecasting future volatility. They claim that the underperformance of MFIV might be due to the illiquidity of out-of-the-money options of individual stocks.

In terms of studies beyond the US market, Muzzioli (2010) constructs MFIV for the DAX-index options and shows that BS implied volatility is a better predictor for future volatility than MFIV. Frijns, Tallau and Tourani-Rad (2010) constructed VIX for Australia and find that implied volatility outperformed the GARCH model for the forecasting of future volatility. Cheng and Fung (2012) show that BS implied volatility subsumes the information content of MFIV in the Hong Kong stock market. Kambouroudis, McMillan and Tsakou (2016) examine the information content of implied volatility. They use three US and six European volatility indices and show that implied volatility on its own performs worse than alternative forecasts in predicting future volatility. A combination of implied volatility, GARCH model and historical realized volatility could therefore improve forecasts. Pati, Barai and Rajib (2017) examine the information content of implied volatility in three Asia-Pacific stock markets, namely those in India, Australia and Hong Kong. They find that implied volatility is a biased forecast of future volatility but contains additional information beyond the information contained in the GARCH family model forecasts. Biktimirov and Wang (2017) compare the efficacy of BS implied volatility with MFIV in forecasting future volatility in 13 stock market indices across North America, Europe and Asia. They find that both BS implied volatility and MFIV improve the forecasts of the GARCH model. However, BS implied volatility dominates MFIV for forecasting future volatility.

3. Data and Methodology

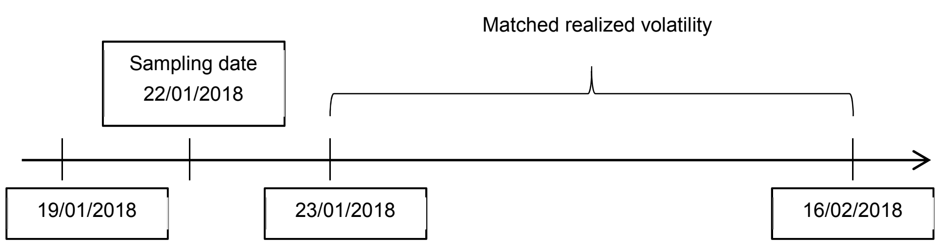
3.1. Data and Sampling Procedure

This study uses daily values of the S&P 500 index, S&P 500 options and VIX in the US market for the period between January 2004 and June 2019. We choose the starting date as January 2004 because CBOE adopted the MFIV method to calculate VIX toward the end of 2003. Option prices are collected from OptionMetrics. S&P 500 index and VIX values are collected from Bloomberg. The risk-free rates are interpolated from the zero curve surfaces available in the OptionMetrics database.

VIX is the volatility expectation of the S&P 500 index for the subsequent 30 calendar days. Thus, we choose the volatility of the next 30 calendar days as the forecasting horizon in this study. Following Christensen and Prabhala (1998), Jiang and Tian (2005) and Fung (2007), we use monthly non-overlapping samples. This means that we have exactly one MFIV, one model-based implied volatility and one realized volatility covering each time period (30 days) in our sample. As claimed by Christensen and Prabhala (1998), overlapping samples may lead to serial correlation and overstate the t-statistics of the coefficients. According to CBOE, options of the S&P 500 index are European options and expire on the third Friday of the expiration month. In each month, we collect option information on the next trading day immediately after the expiration date (the third Friday of the month), we only use options that will expire on the Friday of the subsequent month to calculate our model-based implied volatility, this ensures that the implied volatility is for the next 30 days. Figure 1 illustrates an example of the sampling procedure. For the month of January of 2018, the options expire on January 19th, 2018. We collect option information and the value of VIX on January 22nd, 2018, which is the Monday immediately after the third Friday of January of 2018. We only use options that expire on February 16th, 2018. The matched realized volatility is the standard deviation over the period from January 23rd, 2018 to February 16th, 2018.

Figure 1

Timeline for the Sampling Procedure



3.2. Volatility Measures

Model-free implied volatility is proxied by VIX. It is derived from out-of-the-money call and put options centered around an at-the-money strike price. The detailed calculation method can be found in the CBOE *VIX White Paper* (2019). Since the BS model is widely used in practice and academic research, we use BS implied volatility as the measure of our model-based implied volatility. For a given price of call option or put option, BS implied volatility is calculated by numerical methods using the BS formula. We construct the implied volatility surface for maturity of 30 (calendar) days from the available option prices. Given that the options trading on the market have a limited number of strike prices, a curve-fitting method is used to construct the implied volatility surface. Quadratic spines are applied to fit a smooth curve to the BS implied volatilities[3]. Given the implied volatility surface for maturity of 30 days, we choose at-the-money[4] implied volatility as our measure of BS implied volatility.

[3] *We also tried cubic spines, but these do not affect the conclusions of this study.*
[4] *We define at-the-money when the strike price equals to the current price.*

Realized volatility is measured by the annualized standard deviation of daily returns during the option's remaining term:

$$RV = \sqrt{\frac{252}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2}. \quad (1)$$

where: n is the number of trading days remaining until the option maturity date, r is the daily return and $\bar{r}$ is the average return. We assume that there are 252 trading days in a year in order to compute the annualized volatility.

3.3. Descriptive Statistics

Table 1 shows the descriptive statistics of realized volatility (RV), natural logarithm of realized volatility (lnRV), BS implied volatility (BSIV), natural logarithm of BS implied volatility (lnBSIV), VIX index (VIX) and natural logarithm of index (lnVIX). On average, BS implied volatility and VIX are higher than realized volatility and VIX shows the highest value. This indicates that BS implied volatility and VIX are likely to be biased forecasts for the realized volatility. However, the biasedness of the volatility forecasts will be tested via regression analysis. From the skewness and kurtosis statistics in Table 1, the logarithm of volatility measures is closer to a normal distribution than the actual level of volatility. Thus, we use the logarithm of volatility measures to perform regression analysis in this study. Regressions based on the log form of volatility measures are statistically better specified than those based on the level of volatility.

Table 1

Summary Statistics of Volatility Measures

Variable	Mean	Medium	Max.	Min.	Std. Dev.	Skew.	Kurt.	N
RV	14.32	11.62	77.04	3.48	9.78	3.28	18.23	186
BSIV	16.93	14.31	58.08	8.73	8.03	2.39	9.79	186
VIX	18.25	15.25	64.70	9.43	8.94	2.46	10.21	186
lnRV	2.51	2.45	4.34	1.25	0.51	0.71	3.93	185
lnBSIV	2.75	2.66	4.06	2.17	0.37	1.14	4.21	186
lnVIX	2.82	2.72	4.17	2.24	0.38	1.18	4.31	186

Table 2 shows the correlation matrix of volatility measures. Both BS implied volatility and VIX are positively correlated with the realized volatility. BS implied volatility and VIX are highly correlated, with a correlation coefficient of more than 99%. This indicates that BS implied volatility and VIX might contain the same information.

Table 2

Correlation Matrix of Volatility Measures

	lnRV	lnBSIV	lnVIX
lnRV	1.0000		
lnBSIV	0.7709	1.0000	
lnVIX	0.7720	0.9945	1.0000

3.4. Econometrics Modelling

Following the studies of Christensen and Prabhala (1998) and Jiang and Tian (2005), we use univariate and encompassing regressions to analyze the information content of volatility forecasts. In a univariate regression analysis, the realized volatility is regressed against a single volatility forecast. This focuses on analyzing the predictive power and information content of one volatility forecast. In the encompassing regression analysis, the realized volatility is regressed against two or more volatility forecasts. This focuses on analyzing the relative importance of competing volatility forecasts and whether one volatility forecast could subsume all the information contained in others. The univariate and encompassing regressions are restricted versions or full versions of the following specification:

$$lnRV_t = \alpha + \beta_1 lnRV_{t-1} + \beta_2 lnBSIV_t + \beta_3 lnVIX_t + \varepsilon_t. \quad (2)$$

where: the subscript *t* represents the month of observation, $BSIV_t$ and VIX_t are the values of BS implied volatility and VIX in the sampling day of month *t* as mentioned in Section 3.1, RV_t is the matched realized volatility as mentioned in Section 3.1.

If each volatility forecast contains independent information for predicting future volatility, the coefficient of each volatility forecast should be significantly different from zero. If one volatility forecast, for example, VIX, is informationally efficient, the coefficient of VIX should be significantly different from zero, while the coefficients of the other volatility forecasts should be insignificant. If none of the volatility forecasts contain information for predicting future volatility, all the coefficients should be insignificant.

There is one particular case that needs special treatment. If *lnBSIV* and *VIX* are statistically significant when they are individually included in the univariate regression, this indicates that both of them, individually, have predicting power for future volatility. However, as shown in Table 2, *lnBSIV* and *VIX* are highly correlated, this could cause a multicollinearity problem and the coefficients of both volatility forecasts will then become insignificant in an encompassing regression. To solve this problem, we follow the method from Chung ***et al***. (2011). *lnVIX* is regressed against *lnBSIV*, we then take the residual μ_t^{lnVIX} from the regression and replace $lnVIX_t$ with μ_t^{lnVIX} in equation (2). This enables us to investigate whether VIX provides incremental information for forecasting future volatility than BS implied volatility. Similarly, *lnBSIV* is regressed against *lnVIX*, and we take the residual μ_t^{lnBSIV} from the regression and replace $lnBSIV_t$ with μ_t^{lnBSIV} in equation (2). This enables us to investigate whether BS implied volatility provides incremental information for forecasting future volatility than VIX.

4. Empirical Results

4.1. Univariate and Encompassing Regression Analysis

Table 3 shows the results of univariate regressions. The Dubin-Wastson (DW) statistic is not significantly different from 2 for any of the regressions, indicating no autocorrelation among regression residuals. If the volatility forecast does not contain information about future volatility, the coefficient should be zero. In all three regressions, the coefficients of volatility forecasts are statistically significant. This indicates that lagged realized volatility, BS implied volatility and VIX contain important information for predicting future volatility.

Furthermore, if the volatility forecast is unbiased, the coefficient of volatility forecast should not be different from 1 and the intercept should not be different from zero. The χ2 test examines the joint hypothesis $H_0\colon \alpha = 0\ and\ \beta_i = 1$ (*i = 1, 2 or 3*). From the χ2 test statistics shown in Table 3, all null hypotheses are rejected at 1% significance level. The results indicate that all volatility forecasts are biased estimators for predicting future volatility. Among those three regressions, the regressions with *lnBSIV* and *lnVIX* have a similar adjusted R^2 of around 59%, this is higher than the regression with *lnRV$_{t-1}$*. This provides evidence that BS implied volatility and VIX explain the variations of future realized volatility better than lagged realized volatility.

Table 3

Univariate Regressions

	(1)	(2)	(3)
	lnRV	lnRV	lnRV
lnRV$_{t-1}$	0.663***		
	(11.14)		
lnBSIV		1.059***	
		(16.24)	
lnVIX			1.041***
			(16.41)
Intercept	0.846***	-0.399**	-0.424**
	(5.59)	(-2.20)	(-2.34)
N	186	186	186
Adj. R2	0.436	0.592	0.594
DW	2.289	1.888	1.912
χ2 test	31.990***	98.162***	167.196***

*Note: Regressions are estimated by OLS with heteroskedasticity robust standard error. t statistics in parentheses. DW indicates Durbin-Watson statistic. χ2 test is for the joint hypothesis $H_0\colon Intercept = 0\ and\ Coefficient_i = 1$ (i = lnRVt-1, lnBSIV or lnVIX). *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.*

Tables 4 shows the results of encompassing regressions. Columns (1) and (2) analyze the information efficiency of the BS implied volatility and VIX relative to lagged realized volatility. The coefficients of *lnBSIV* and *VIX* are still significant but the coefficients of lnRV$_{t-1}$ become insignificant. This indicates that BS implied volatility or VIX is informationally efficient. Furthermore, if our implied volatility measures subsume the information contained in the lagged realized volatility, the coefficient implied volatilities should not be different from 1 and the coefficients of lagged realized volatility should not be different from 0. The χ2 test examines the joint hypothesis $H_0\colon \beta_1 = 0\ and\ \beta_i = 1$ (*i = 2 or 3*). From the χ2 test statistics shown in the table, none of the null hypotheses can be rejected at 10% significance level. This indicates that BS implied volatility or VIX subsume the information contained in the lagged realized volatility.

Due to the high correlation between lnBSIV and VIX, we cannot include both lnBSIV and VIX in the same regression. Instead, we use the method mentioned in Section 3.3 to test whether BS implied volatility and VIX are complementary for forecasting future volatility. Columns (3) and (4) in Table 4 report the results. The coefficient of μ_t^{lnVIX} is insignificant, indicating that VIX does not provide incremental information for forecasting future volatility than BS implied volatility. The coefficient of μ_t^{lnBSIV} is insignificant, indicating that BS implied volatility does not provide incremental information for forecasting future volatility than VIX. In addition, Adjusted R^2 in all four regressions are almost the same. Those results indicate

that VIX and BS implied volatility contain the same information.
Summarizing the analysis above, we conclude that both BS implied volatility and VIX are biased forecasts of future volatility. BS implied volatility and VIX are informationally efficient and subsume information contained in the historical realized volatility for forecasting future volatility. BS implied volatility and VIX contain the same information and there is no winner for forecasting future volatility. This conclusion is different from previous studies. Jiang and Tian (2005) claim that MFIV subsumes all the information content of BS implied volatility for forecasting the future volatility of the S&P 500 index. In contrast, Biktimirov and Wang (2017) find that BS implied volatility dominates MFIV for forecasting the future volatility of the S&P 500 index.

Table 4

Encompassing Regressions

	(1)	(2)	(3)	(4)
	lnRV	lnRV	lnRV	lnRV
$\ln RV_{t-1}$	0.018	0.015	0.010	0.010
	(0.20)	(0.16)	(0.11)	(0.11)
lnBSIV	1.038***		1.048***	
	(9.61)		(9.66)	
lnVIX		1.025***		1.030***
		(9.60)		(9.64)
μ_t^{lnBSIV}				0.384
				(0.64)
μ_t^{lnVIX}			0.655	
			(1.10)	
Intercept	-0.387**	-0.414**	-0.393**	-0.417**
	(-2.20)	(-2.36)	(-2.22)	(-2.37)
N	186	186	186	186
Adj. R2	0.590	0.592	0.590	0.590
DW	1.910	1.928	1.917	1.917
χ2 test	0.836	0.425		

Note: Regressions are estimated by OLS with heteroskedasticity robust standard error. t statistics in parentheses. DW indicates Durbin-Watson statistic. χ2 test is for the joint hypothesis $H_0: Coefficient_{lnRVt-1} = 0 \; and \; Coefficient_i = 1$ *(i = lnBSIV or lnIVX). *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.*

4.2. Robustness Check

4.2.1. Efficiency of the Forecasts with an Alternative Proxy for Historical Volatility

In this section, we examine whether our conclusion holds when a different proxy for historical volatility is used. Compared to lagged realized volatility, the GARCH model can capture the dynamic and cluttering feature of historical volatilities, and contains richer information of historical volatility. There is evidence that implied volatility (both model-free and model-based) does not subsume the information content of GARCH model forecasts, and the information content of implied volatility and GARCH model forecasts are complementary (Cheng and Fung, 2011; Kambouroudis, McMillan and Tsakou, 2016). Thus, we use a GARCH (1,1) model forecast as the measure of historical volatility in this section. The GARCH model developed by Engle (1982) and Bellerslev (1986) involves a joint estimation of return process equation (3) and the conditional variation equation (4):

$$r_t = \mu_t + e_t, \tag{3}$$

$$h_t^2 = \omega + \alpha e_{t-1}^2 + \beta h_{t-1}^2. \tag{4}$$

where: r_t is daily return of the S&P 500 index, μ_t is the constant mean and $e_t = h_t\, z_t$ is the innovation term with $z_t \sim N(0,1)$.

At the sampling day in each month (as mentioned in Section 3.1), we estimate the GARCH (1,1) model using the past 1,000 daily observations and forecast the out-of-sample volatility for the next 19 trading days, which is the number of trading days until the maturity of our sampling options. The GARCH model volatility forecast (*GARCHV*) is the annualized average volatility of those 19 trading days forecasts and is defined as follows:

$$GARCHV = \sqrt{\frac{252}{T-1}\sum_{t=1}^{T} h_t^2}. \tag{5}$$

where: T is 19 in this study and we assume 252 trading days in a year. Similar to other volatility measures in this study, we take the natural logarithm of *GARCHV*, expressed as *lnGARCHV*. Table 5 shows the univariate and encompassing regression results with *lnGARCHV*. Column (1) shows that the GARCH model volatility forecast contains information for predicting future volatility.

Table 5

Regression Analysis with GARCH Model Volatility Forecasts

	(1)	(2)	(3)	(4)	(5)
	lnRV	lnRV	lnRV	lnRV	lnRV
lnGARCHV	0.831***	-0.074	-0.070	-0.080	-0.080
	(6.29)	(-0.68)	(-0.67)	(-0.73)	(-0.73)
lnBSIV		1.126***		1.132***	
		(11.70)		(11.61)	
lnVIX			1.103***		1.112***
			(12.24)		(11.80)
μ_t^{lnBSIV}					0.442
					(0.72)
μ_t^{lnVIX}				0.681	
				(1.18)	
Intercept	0.252	-0.382*	-0.409**	-0.381*	-0.407**
	(0.71)	(-1.96)	(-2.11)	(-1.95)	(-2.10)
N	186	186	186	186	186
Adj. R2	0.409	0.591	0.592	0.591	0.591
DW	1.551	1.892	1.915	1.910	1.910
χ2 test (a)	52.668***				
χ2 test (b)		1.976	1.402		

Note: Regressions are estimated by OLS with heteroskedasticity robust standard error. t statistics in parentheses. DW indicates Durbin-Watson statistic. χ2 test (a) is for the joint hypothesis $H_0\!: Intercept = 0\ and\ Coefficient_{lnGARCHV} = 1$ *. χ2 test (b) is for the joint hypothesis* $H_0\!: Coefficient_{lnGARCHV} = 0\ and\ Coefficient_i = 1$ *(i = lnBSIV or lnIV). *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.*

The result of the χ2 test (a) shows that the GARCH model volatility forecast is a biased estimator for predicting future volatility. The regression results in columns (2)-(5) and χ2 test (b) yield same conclusion as shown in Section 4.1.

4.2.2. IV Regressions

Christensen and Prabhala's (1998) claim that the volatility forecast measures may contain measurement errors due to the possible nonsynchronous observations of option quotes and index levels in the dataset, or the misspecification error of the BS model. This is known as the error in variables (EIV) problem. One way to solve the EIV problem is by using a IV regression. Both Christensen and Prabhala (1998) and Jiang and Tian (2005) found substantial differences between the estimation results from OLS regressions and IV regressions. These results suggest the presence of the EIV problem.

Following Jiang and Tian (2005), we adopt IV regressions in this section. We use lagged BS implied volatility as instrumental variables for the BS implied volatility, and similarly lagged VIX as instrumental variables for VIX. When both BS implied volatility and VIX are present in the regression, both lagged BS implied volatility and lagged VIX are used as instrumental variables. Table 6 shows the IV regression results and only the second stage is reported[5]. The results of the regressions and χ2 test (a) in columns (1) and (2) yield the same conclusion as our univariance regression analysis in Section 4.1. The results of the regressions and χ2 test (b) in columns (3) and (4) yield the same conclusion as our univariance regression analysis in Section 4.1. We therefore show that BS implied volatility and VIX are informationally efficient and subsume information contained in lagged realized volatility in columns (1)-(4).

Table 6

IV Regressions

	(1)	(2)	(3)	(4)	(5)
	lnRV	lnRV	lnRV	lnRV	lnRV
$\ln RV_{t-1}$			0.147	0.121	0.118
			(0.84)	(0.63)	(0.52)
lnBSIV	1.023***		0.831***		-0.189
	(13.89)		(3.35)		(-0.06)
lnVIX		1.014***		0.856***	1.047
		(13.87)		(3.14)	(0.30)
Intercept	-0.301	-0.348	-0.141	-0.206	-0.217
	(-1.45)	(-1.65)	(-0.47)	(-0.62)	(-0.50)
N	186	186	186	186	186
Adj. R2	0.591	0.593	0.584	0.587	0.585
DW	1.858	1.888	2.019	2.011	2.010
χ2 test (a)	98.339***	169.112***			
χ2 test (b)			0.886	0.488	

*Note: Regressions are estimated by two-stage least squares with heteroskedasticity robust standard error. Only the second stage is reported. t statistics in parentheses. DW indicates Durbin-Watson statistic. χ2 test (a) is for the joint hypothesis $H_0: Intercept = 0 \; and \; Coefficient_i = 1$ (i = lnBSIV or lnIV). χ2 test (b) is for the joint hypothesis $H_0: Coefficient_{lnRVt-1} = 0 \; and \; Coefficient_i = 1$ (i = lnBSIV or lnIV). *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.*

[5] *The results of first stage estimations are available on request.*

Once both BS implied volatility and VIX are included as independent variables, both of them become insignificant due to very high correlation between them, and the result shows that there is no winner between these two implied volatilities in forecasting future volatility.

5. Conclusion

This paper provides new evidence to compare the information content of MFIV and model-based volatility for forecasting future volatility of S&P 500. One feature of this study is that it tries to avoid artificial judgment when calculating BS implied volatility and MFIV. Thus, we choose the VIX index as our proxy of MFIV since VIX is actively traded in the market and widely accepted as a measure of MFIV. Regarding the calculation of BS implied volatility, we do not rely on selected options; instead, we use all the options that were traded on the market and compute the implied volatility surface. Since VIX uses a large number of options and our measure of BS implied volatility is also based on a large number of options, we hypothesize that BS implied volatility and MFIV should contain the same information for forecasting future volatility.

By using non-overlapping monthly samples from January 2004 to June 2019, we find that both BS implied volatility and VIX are biased forecasts of future volatility. BS implied volatility and VIX are informationally efficient and subsume information contained in the historically realized volatility for forecasting future volatility. BS implied volatility and VIX contain the same information and there is no winner for forecasting future volatility. This conclusion is different from previous studies. Jiang and Tian (2005) claim that MFIV subsumes all of the information content of BS implied volatility for forecasting the future volatility of the S&P 500 index. In contrast, Biktimirov and Wang (2017) find that BS implied volatility dominates MFIV for forecasting the future volatility of the S&P 500 index.

Our results also have implications for practitioners. We show that both BS implied volatility and MFIV subsume information content of historical realized volatility and contain useful information about future volatility. Thus, there is no need to use historical volatility as an input for a forecasting model once we have implied volatilities. Furthermore, since there is no winner for forecasting for future volatility between BS implied volatility and MFIV, a forecast model could include both BS implied volatility and MFIV. In addition, we also show the usefulness of utilizing information on all the options that are traded on the market.

It is unclear whether our conclusion will hold for different forecasting horizons, and this will require different VIX indices to be examined. Since practitioners use several modified versions of the BS model, future studies could try to apply modified versions of the BS model and test whether our results are sensitive to different option pricing models.

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