

## **6 INSTITUTIONAL STRUCTURES AS BENARD TAYLOR PROCESSES**

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### **Abstract**

Considering the epistemic and ontological sense of principles and functions and the way they contribute to the creation and evolution of institutional structures a model was developed in which the reaction-diffusion of mimes (Dawkins) in a human niche (Popper) is described as a Brusselator that presents far from equilibrium stabilities of Benard-Taylor type. These dynamic stabilities area associated with the formation and evolution of institutional structures leading to a new interpretation of Heraclit's 'panta rei' principle in visualizing human institutional history.

**Key words:** institutions, Benard-Taylor instabilities, reaction diffusion models, mimes dynamic

**JEL classification:** C3, C61, C62, D7, D87.

### **Epistemic sense and ontological sense**

Environment contains things that exist without the need to be experienced continuously by a conscious subject. This could be called objective ontology.

A statement may be established as true or false by objective fact. Other statements may have a state of truth value based on subjective opinions.

The ontology of perceptions is objective if statements on the existence of things are made about things that exist, regardless of our feelings about them, or, it is subjective if statements refer to truth values about the opinion of one or several persons.

In both cases one may have objective knowledge coming out of the associated sciences (e.g. social sciences as well as natural sciences).

### **Social reality and collective behaviour**

In a collectivity, individual beliefs, desires and decisions triggering actions, are correlated into collective ones.

General sets of social facts may have subsets of institutional facts that result from a collective assignment of functions which may be performed not only by virtue of their

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physical nature but by virtue of their social acceptance by the collectivity as having a status assigned to them.

For example, the noise made by a judge knocking at the end of a trial, without definite status of behaviour assigned to it, would only represent a natural noise. Conversely, taken inside the set of functions associated with the legal institution, the mentioned noise acquires a special meaning in terms of behaviour.

Thus, in order for functions to have action value that ensures the forming of an institutional structure , a number of reactions need to happen with ideas and functions (described below as mimes) that spread out among individuals in a collectivity, creating deontic powers such as: rights, duties, obligations, permissions, requirements. Not all them are only institutional but, institutions would not exist, without them, as stable dynamic structures<sup>1</sup>.

By creating institutional reality, human power is increased by its extended capacity for action. But, the possibility to fulfil desires within the institutional structures (like to get rich in an economical structure or to become a president in a political structure) is only based on the recognition, acknowledgement and acceptance of the deontic relationships.

The above statements require a basic intellectual cohesion that connects the members of a collectivity. This involves language as well as media means.

### **Mimes' dynamics**

The movement of ideas and principles among the members of collectivity creates a dynamics where socio-cultural niches are formed as described by K Popper<sup>2</sup>. To look deeper into the dynamic we will consider the mimes introduced by Dawkins (the virus like sentences in Hofstader's terminology) that describe the basic conceptual framework for such an analysis<sup>3</sup>.

There is a certain intercorellation of mimes inside a society that reacts and diffuses among individuals in that society.

Let's consider that the objective ontology mimes, A, are contributing to the generation of new mimes with social subjective ontology, X.

$$A \rightarrow X$$

The new mimes, X, are reacting with existing subjective ontology mimes to produce new function mimes, Y, along with the existing function mimes, D.

$$B + X \rightarrow Y + D$$

Further on we consider that the existence of new mimes of subjective ontology, X, in sufficient number, say  $2X$ , combined with the function mimes, Y, creates more subjective ontology mimes. Actually this process tells us that once people start believing in new ideas the associated functions are increasing this belief (e.g. one needs both a Bible and a church to have a religion).

<sup>1</sup> John R. Searle, "What is an institution", Journal of Institutional Economics, (2005).

<sup>2</sup> K. Popper, La logique de la decouverte scientifique, Payot, 1973

<sup>3</sup> I.I. Purica, "Creativity, intelligence and synergetic processes in the development of science", Scientometrics, Vol.13, Nos 1-2, (1988).



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We must mention that by new ideas we mean those subjective ontology principles that lay the basis for institutional structures (like new religion principles, new economic principles, new political principles, etc.).

Finally, the existing new mimes become traditional mimes, E, inside the created institutions.



Grouping the above reactions we have (also known as the Brusselator model):



We consider<sup>4</sup>, the concentration of the mimes as: a for A; b for B; n<sub>1</sub> for X; n<sub>2</sub> for Y.

Concentrations a and b will be fixed as representing the existing institutions' mimes, while n<sub>1</sub> and n<sub>2</sub> will be variables.

Thus the sources and sinks for X and Y are given below for each equation (based on typical chemical considerations<sup>5</sup>):

Reactions	Source X	Source Y	Sink X	Sink Y
A → X	a			
B + X → Y + D		b.n <sub>1</sub>	b.n <sub>1</sub>	
2X + Y → 3X	n <sub>1</sub> <sup>2</sup> .n <sub>2</sub>			n <sub>1</sub> <sup>2</sup> .n <sub>2</sub>
X → E			n <sub>1</sub>	
dn <sub>1</sub> /dt	a+n <sub>1</sub> <sup>2</sup> .n <sub>2</sub>		-b.n <sub>1</sub> -n <sub>1</sub>	
dn <sub>2</sub> /dt		b.n <sub>1</sub>		-n <sub>1</sub> <sup>2</sup> .n <sub>2</sub>

We will also consider that there is diffusion of mimes in a society among its members, x. The diffusion coefficients are respectively D<sub>1</sub> and D<sub>2</sub> for X and Y mimes.

The reaction diffusion process presented above is described by the equations given below:

$$\begin{aligned} dn_1/dt &= a - (b+1)n_1 + n_1^2 n_2 + D_1 d^2 n_1 / dx^2 \\ dn_2/dt &= bn_1 - n_1^2 n_2 + D_2 d^2 n_2 / dx^2 \end{aligned}$$

The boundary conditions considered for concentrations n<sub>1</sub>(x,t) and n<sub>2</sub>(x,t) may be of two kinds:

$$\begin{aligned} n_1(0,t) &= n_1(1,t) = a \\ n_2(0,t) &= n_2(1,t) = b/a \end{aligned}$$

or

$$n_j; j=1,2 \text{ remain finite for } x \rightarrow +/-\infty$$

The first set of conditions stems from the stationary state (dn<sub>j</sub>/dt=0 and D<sub>j</sub>=0) solutions of the equations which are:

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<sup>4</sup> H.Haken, Synergetics, Springer-Verlag, Berlin, 1977

<sup>5</sup> G.Nicolis, I.Prigogine, Self-Organization in Nonequilibrium Systems, Wiley, London, 1977



$$n_{10}=a \quad n_{20}=b/a$$

saying simply that the dynamics of new principles and functions starts from the existing ones; while the second set limits, to a finite value, the concentration level of new principles and functions over the diffusion space of the niche of persons exposed to these new principles and functions.

In order to check if new spatial or temporal structures show up, we do a stability analysis on the differential equations.

To this purpose we put  $n_1=n_{10}+q_1$ ;  $n_2=n_{20}+q_2$  and linearize the equations with respect to  $q_1$  and  $q_2$ .

We have:

$$\begin{aligned} dq_1/dt &= (b-1)q_1 + a^2q_2 + D_1 d^2 q_1 / dx^2 \\ dq_2/dt &= -bq_1 - a^2q_2 + D_2 d^2 q_2 / dx^2 \end{aligned}$$

The boundary conditions become:

$$q_1(0,t)=q_1(1,t)=q_2(0,t)=q_2(1,t)=0$$

or

$$q_j \text{ finite for } x \rightarrow +/-\infty$$

Putting  $q=(q_1 \ q_2)$  the equations become:

$$q=Lq$$

where the matrix L is defined as:

$$L = \begin{vmatrix} D_1 d^2 / dx^2 + b - 1 & a^2 \\ -b & D_2 d^2 / dx^2 - a^2 \end{vmatrix}$$

In order to satisfy the boundary conditions we put

$$q(x,t)=q_0 \exp(\lambda_i t) \sin(l\pi x)$$

with  $l=1,2,\dots$

Inserting the above expression for  $q(x,t)$  into the equation for  $q'$  yields a set of homogeneous linear algebraic equations for  $q_0$ . Non-vanishing solutions are only possible if the determinant is zero:

$$\begin{vmatrix} -D_1 + b - 1 - \lambda & a^2 \\ -b & -D_2 - a^2 - \lambda \end{vmatrix} = 0$$

where:  $\lambda=\lambda_l$  and  $D'_j=D_j l^2 \pi^2$ ,  $j=1,2$ .

To have a null determinant  $\lambda$  must obey the characteristic equation:

$$\lambda^2 - \alpha \lambda + \beta = 0$$

where:

$$\begin{aligned} \alpha &= (-D'_1 + b - 1 - D'_2 - a^2) \\ \beta &= (-D'_1 + b - 1)(-D'_2 - a^2) + ba^2 \end{aligned}$$

Instability occurs if  $\text{Re}(\lambda)>0$ . A dynamics of interest for our analysis is the one having a constant,  $a$  (the concentration of objective ontology mimes is rarely changing nowadays; although moments like 'e pur si muove' may be interesting to look at).

The following will consider the change of  $b$ , subjective ontology mimes, to reach a critical level  $b_c$  when the solution becomes unstable.

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So:  $\lambda = \alpha/2 \pm 1/2\sqrt{\alpha^2 - 4\beta}$

We first consider  $\lambda$  is real; which requires  $\alpha^2 - 4\beta > 0$  and  $\lambda > 0$  requires  $\alpha + \sqrt{\alpha^2 - 4\beta} > 0$

If  $\lambda$  is complex then  $\alpha^2 - 4\beta < 0$  and we need for instability  $\alpha > 0$ .

After transforming the inequalities above back to expressions in  $a$ ,  $b$ ,  $D'_1$ ,  $D'_2$  we obtain:

1) soft-mode instability:  $\lambda$  real;  $\lambda \geq 0$ ,

$$(D'_1 + 1)(D'_2 + a^2)/D'_2 < b$$

(which results from  $\alpha^2 - 4\beta < 0$  above)

2) hard-mode instability:  $\lambda$  complex;  $\operatorname{Re}(\lambda) \geq 0$ ,

$$D'_1 + D'_2 + 1 + a^2 < b < D'_1 - D'_2 + 1 + a^2 + 2a\sqrt{1 + D'_1 - D'_2}$$

The left inequality comes from  $\alpha < 0$  above while the right one from  $\alpha^2 - 4\beta < 0$ .

Considering the inequalities for  $b$ , the instability occurs for such a wave number for which the smallest  $b$  fulfils the inequalities for the first time.

As results from the analysis, a complex  $\lambda$  is associated with a hard mode excitation while,  $\lambda$  real is associated with a soft mode one.

Since soft mode instability occurs for  $k \neq 0$  and real  $\lambda$ , a partially inhomogeneous static pattern arises.

We follow Haken<sup>6</sup> in applying an adiabatic elimination of the stable modes to evidence the soft mode instability resulting in a bifurcation, after a critical value.

So, we put:

$$q(x,t) = \zeta_u q_{0,u} \sqrt{2} \sin l_c \pi x + \sum_j \zeta_{sj} q_{0,sj} \sqrt{2} \sin l \pi x$$

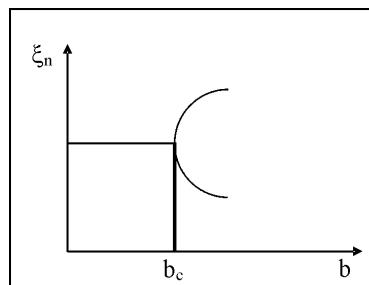
where, the index  $u$  refers to 'unstable', the sum over  $j$  contains the stable modes which are eliminated adiabatically leaving us in the soft mode case, for  $l$  even, with:

$$\zeta'_u = c_1(b - b_c)\zeta_u - c_3\zeta_u^3$$

describing the behaviour of the order parameter  $\zeta_u$ .

The coefficients  $c_1$  and  $c_3$  are functions of  $a$ ,  $b_c$ ,  $D'_1$ ,  $D'_2$  and  $l_c$  where  $l_c$  is the critical value of  $l$  for which instability occurs first. A plot of  $\zeta_u$  as a function of  $b$  is given in Figure 1.

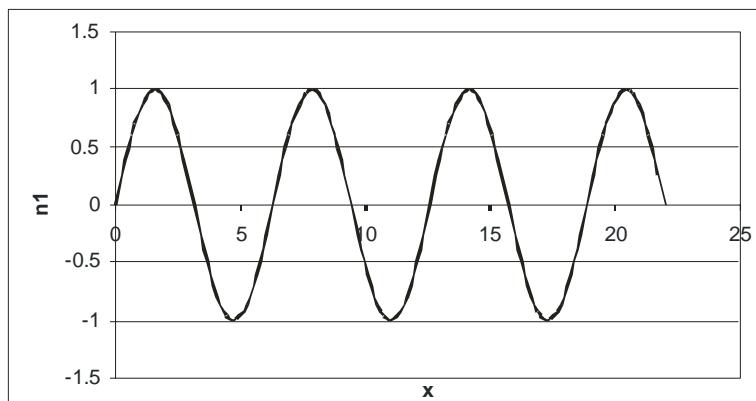
**Figure 1**



<sup>6</sup> H. Haken, Synergetics, Springer Verlag, Berlin, 1977

At  $b=b_c$  a point of bifurcation occurs and a spatially periodic structure is established figure 2.

**Figure 2**



If  $I$  is odd then the equation for  $\zeta_u$  reads:

$$\zeta'_u = c_1(b-b_c)\zeta_u + c_2\zeta_u^2 - c_3\zeta_u^3$$

where  $c_1$  and  $c_3$  are the same as above while  $c_2$  depends also on  $a$ ,  $b_c$ ,  $D_1$ ,  $D_2$  and  $I_c$ .

Hard and soft regimes are occurring here that we will not continue to describe them here, but, refer the reader to the abundant mathematical literature in the field.

### Conclusions

The point we wanted to make is largely explained by the analysis above. We found out, by applying a reaction diffusion model to describe the dynamics of mimes in a society that institutions are occurring as space time structures having a Benard-Taylor type of stability far from equilibrium<sup>7</sup>. The process of institutional structuring comes as a bifurcation, occurring over a critical value of the existing concentration of subjective ontology mimes that allows the setting up of a spatial-temporal structure of the new mimes concentration that describes the acceptance of principles and functions of an institution.

The approach that we take is opening a way to describe not only the space structures (in the sense of socio-cultural institutions), but, also their time dynamics. The conditions of change in political structures, in administrative ones, or, even the onset of new religious sects, that are generally referred to as revolutions and as changes, may be accommodated within this approach.

It is worth noting that the on-setting of dynamically stable structures of mimes concentration in societies occurs when the critical values are crossed, leading to

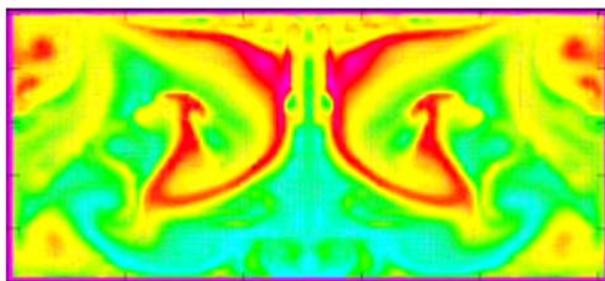
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<sup>7</sup> See B.M. Smirnov, Introduction to Plasma Physics, MIR Publishers, Moscow, 1977 and I. Purica, "Synergetic application of complex ordered processes", Proceedings of ENEA Workshops on Nonlinear Systems, Vol.3, Simulation of Nonlinear Systems in Physics, World Scientific Publishing Co., Singapore, 1991

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bifurcation. The dynamic stabilities far from thermal equilibrium that associate with the behaviour described by these processes may lead to a better understanding of the institutions' dynamics in the framework of socio-economic evolution inside the socio-cultural niches of human societies. See an illustration in Figure 3 that represents a simulation of this type of behaviour for a typical physical system.

**Figure 3**  
**Illustrative Benard-Taylor dynamic**



Finally, we state that the present days physical models, if applied to socio-economic evolution, show that Heraclit's 'panta rei' may represent patterns of dynamic institutional structures that change from one into another with the time constants of human history.

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