II. USING THE LEONTIEF MATRIX TO ESTIMATE THE IMPACT OF INVESTMENTS UPON THE GLOBAL OUTPUT1

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Abstract

The study presents in the first chapter the applied methodology and the data used for the empirical research. The economic activities were grouped into 10 sectors by aggregating the extended input-output tables for Romania (with 105 branches). The chosen reference year is 2007 - the last year for which such statistical recordings were available.

The second chapter examines some of the Romanian economy's structural features revealed by the matrices A and (I-A)-1, insisting on the driving effects of interdependencies (direct and indirect) generated by cross-sector productive flows.

The third chapter focuses on the impact of gross fixed capital formation (GFCF) upon the output. On the one hand, the implications of changes in volume are estimated (for example, data on 2007 are recalculated for a variation of +/-5% in the GFCF). On the other hand, the influence of the sectoral structure of the indicator in question is quantified with the help of three different macroeconomic simulations. Further possible developments of the current investigation are discussed at the end of the paper.

Keywords: input-output analysis, multipliers, macroeconomic simulations

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1. Introduction

1. The research is based on the input-output tables for Romania with 105 branches from the last available year, namely 2007 (National Institute of Statistics, 2010).
In order to make the estimates more relevant in terms of macroeconomic analysis and prediction, the 105 branches were aggregated into 10 sectors deemed representative for the current configuration of the Romanian economy (Dobrescu, 2009). Table 1 explains the correspondence of the initial classification (105 branches) with the new more aggregated classification (10 sectors).

Table 1

Aggregated Sectoral Structure

Sector Code	Sector Name	Branch codes (of the classification based on 105 branches) included in the respective sector
1	Agriculture, forestry, hunting and fishing	1…6
2	Mining and quarrying	7,9,11…17
3	Production and distribution of electric and thermal power	79…82
4	Food, beverages and tobacco	18…27
5	Textiles, leather, pulp and paper, furniture	28...33, 77
6	Machinery and equipment, transport means, other metal products	60…65, 67…76
7	Other manufacturing industries	34,35,8,36,38…59, 78
8	Constructions	83
9	Transports, post and telecommunications	87…91, 93…95
10	Trade, business and public services	84…86, 96…105

By this operation, two computational advantages were obtained. First, the problem of branches with negative gross value added or with conventional statistical role has been circumvented. On the other hand, the matrix analysis itself has become very easy.

2. Methodologically, the standard scheme was used, which we recall briefly here. Thus, on the basis of the cross-sectoral exchange table, the technical coefficients aij (matrix A) were determined:

$$a_{ij} = x_{ij} / X_j \quad (1)$$

where: xij represents the part of sector i's output used in sector j, and Xj the total output of sector j.

Introducing the final use (demand), denoted by Yi, interdependencies within the economy are quantified through

$$X_i = \Sigma a_{ij} X_j + Y_i \quad (2)$$

which, in matrix representation, is written as

$$X=XA+Y \quad (2a)$$

and

$$X=(I - A)^{-1}Y \quad (2b)$$

2. Some structural features of the Romanian economy revealed by matrices A and $(I\text{-}A)^{-1}$

1. The technical coefficients matrix (A) for 2007, in previous year prices, is shown in Table 2.

Table 2

Technical Coefficients Matrix (A)

Sector Code	1	2	3	4	5	6	7	8	9	10
1	0.433	0.002	0.000	0.209	0.058	0.000	0.001	0.001	0.000	0.009
2	0.000	0.302	0.239	0.001	0.000	0.009	0.087	0.007	0.000	0.006
3	0.017	0.087	0.208	0.044	0.034	0.036	0.073	0.015	0.034	0.034
4	0.042	0.001	0.003	0.358	0.012	0.003	0.007	0.003	0.007	0.095
5	0.012	0.006	0.002	0.035	0.242	0.017	0.025	0.038	0.008	0.063
6	0.053	0.075	0.023	0.024	0.034	0.242	0.046	0.047	0.086	0.059
7	0.124	0.091	0.270	0.066	0.098	0.189	0.410	0.159	0.077	0.131
8	0.006	0.006	0.004	0.004	0.005	0.003	0.003	0.153	0.006	0.016
9	0.010	0.031	0.007	0.025	0.025	0.025	0.023	0.006	0.077	0.039
10	0.033	0.069	0.041	0.064	0.091	0.068	0.055	0.124	0.152	0.199

Source: Authors' own computations.

One should note that the inputs come not only from the national output, but also from imports by the provider sector.

Dynamic analyses showed that in Romania the coefficients aij were characterized by high volatility. This comes from the overlap of several key processes of transition:

- structural adjustment of the economy;
- technological change;
- changes, sometimes dramatic, in the relative prices.

Therefore, the data for the last input-output tables (2007) were used, assuming that they capture closer the current features of the economy.

2. Table 2 shows horizontally that for each sector the largest coefficients are related to their own inputs (values on the main diagonal). Naturally, significant inputs to the sector's output occur, for example;

- from sector 4 to sector 1;
- from sectors 3 and 7 to sector 2;
- from sectors 2 and 7 to sector 3;
- from sector 10 to sectors 4 and 5;
- from sector 9 to sector 6;

- from sectors 6 and 8 to sector 7;
- from sectors 5, 8 and 9 to sector 10.

Generally, however, the output conditioning of each sector on its inputs is high, which suggests that the direct cross-sector interdependencies are still low.

Future extension of the analysis to tables with 105 branches will highlight more accurately if this finding reflects the actual technological characteristics of the Romanian economy or it was induced (and in what proportion) by the aggregation operation (by using the classification based on 10 sectors).

3. Since the cross-sector exchange table is built in prices, the coefficients aij can sum up vertically (sc) and horizontally (sr); they are shown in the following table:

Table 3

Total of a_{ij} Coefficients by Vertical (s_c) and Horizontal (s_r) Summing

Sector Code	s_c	s_r	Sector Code	s_c	s_r
1	0.730	0.713	6	0.592	0.689
2	0.670	0.651	7	0.730	1.615
3	0.797	0.582	8	0.553	0.206
4	0.830	0.531	9	0.447	0.268
5	0.599	0.448	10	0.651	0.896

3.1. Vertical totals (s_c) estimate the share of intermediate consumption in the sector output.

The difference from the unit approximates the share of gross value added in output.

One may note that, without exception, the s_c values are below unit, thus confirming that in the aggregation used the gross value added is positive in all sectors. There are some notable differences regarding the size of the sc itself.

The best placed in terms of value added are (in order):

- Transport, post and telecommunications, largely because of the last subdivision, but also due to budgetary subsidies for the first component;
- Constructions, which in 2007 - last year for which the input-output tables were calculated – were under the impulse of the housing boom before the crisis;
- Equipment, machinery, transport equipment, other metal products, branches generally characterized by a high degree of processing of raw materials;
- Textiles, leather, pulp and paper, furniture, which are strong labour-intensive industries;
- Trade, business and public services, because of the trade markup rates and tariffs in the private sector, and the public sector output estimates based costs, where wages were prevailing;
- Mining and quarrying industry, due mainly to natural gas extraction.

The second part of the ranking includes:

- Food, beverages and tobacco, due to higher material costs specific to these industries and to the enormous accounting and tax evasion in this area;
- Production and distribution of electric and thermal power, still dominated by administered prices;

- Agriculture, forestry, hunting and fishing, being in a poor technological state and, as a consequence, with low production efficiency;
- Other manufacturing industries, a sector that brings together most of the raw processing industries.

3.2. Results of horizontal summing (s_r) approximate (by the difference from the unit) the relative contribution of domestic output to cover the intermediate consumption of the economy. The highest deficit in this regard is in sector 7 (Other manufacturing industries), indicating a high dependence on imports of industries that use materials and semi finished products. At the opposite end stands sector 8 (Constructions), mainly known as serving important segments of final demand (residential and productive investment, infrastructure works, etc.).

4. The analytical valences given by the inverse matrix $(I–A)^{-1}$ are known as numerical expression of interdependencies among the sectors of the economy - not only direct (indicated by the matrix A), but also indirect.

Table 4

Inverse Coefficients Matrix $(I–A)^{-1}$

Sector Code	1	2	3	4	5	6	7	8	9	10
1	1.837	0.035	0.035	0.628	0.173	0.031	0.041	0.040	0.035	0.121
2	0.135	1.592	0.612	0.155	0.114	0.155	0.347	0.125	0.092	0.142
3	0.147	0.254	1.441	0.209	0.144	0.158	0.257	0.117	0.119	0.162
4	0.161	0.050	0.056	1.652	0.080	0.049	0.064	0.059	0.062	0.225
5	0.082	0.060	0.067	0.138	1.368	0.075	0.099	0.109	0.056	0.154
6	0.218	0.235	0.195	0.194	0.149	1.412	0.205	0.160	0.192	0.197
7	0.634	0.531	0.908	0.602	0.472	0.635	2.033	0.547	0.366	0.564
8	0.023	0.021	0.020	0.024	0.018	0.015	0.018	1.192	0.018	0.033
9	0.063	0.089	0.073	0.095	0.073	0.073	0.085	0.048	1.118	0.096
10	0.195	0.239	0.236	0.278	0.249	0.215	0.238	0.281	0.283	1.389

Source: Authors' own computations.

4.1. Expressed in prices, the coefficients on the vertical – in the case of matrix $(I–A)^{-1}$ - could also be summed up (s_c). How could one interpret these totals? In fact, they approximate the output of all industrial branches induced by one unit of final demand addressed to the sector relative to the vertical in question (hence, in matrix $(I–A)^{-1}$, the coefficients on the main diagonal have over unit values). Vertical sums of matrix $(I–A)^{-1}$ are also called output multipliers (Miller and Blair, 2009, p.246).

4.1a. The s_r values resulted from matrix A do not involve the second quadrant of the input-output table. To make these values somewhat comparable with the corresponding vector S_c, we use the relation $s_r^*=s_r+1$. The two sets of values (S_c and s_r^*) are shown in Table 5.

The difference between S_c and s_r^* is attributable to the way the effect of productive interdependencies within the economy is expressed. While s_r^* is limited to the direct ones, S_c adds to them the indirect interdependencies (mediated by the links between the related branches).

Table 5

Coefficients S_c and s_r*

Sector Code	s_c	s_r*
1	3.495	1.713
2	3.106	1.651
3	3.643	1.582
4	3.975	1.531
5	2.84	1.448
6	2.818	1.689
7	3.387	2.615
8	2.678	1.206
9	2.341	1.268
10	3.083	1.896

4.1b. Thus an index of output indirect drive determined as the ratio $\omega = S_c / s_r^*$, is shown in graph ω (Figure1.).

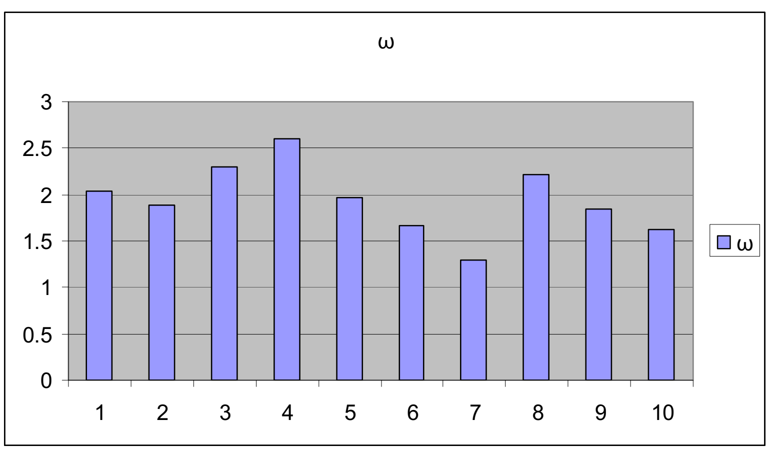
4.2. In the case of matrix $(I\text{-}A)^{-1}$, the sums of coefficients on horizontals (s_r), estimate the output required by the respective sector to ensure a unit increase in the final demand in all sectors of the economy.

Of course, these values would not be comparable if they refer to matrices of different sizes. This aggregation effect can be mitigated by dividing them by the number of sectors, thus obtaining values associated with a variation by a unit of the final demand in the economy.

Their use for explanatory purposes still requires caution because they are based on the less realistic assumption of uniform modification in final demand in each of the sectors.

Figure 1

Graph ω



Sector

3. Involving input-output techniques to estimate the effects of changes in gross fixed capital formation

Matrix $(I–A)^{-1}$ allows for a wide variety of analytical and predictive simulations. Among them, the identification of the effect that the change in final demand can have on the overall output (and, implicitly, on employment) is of particular interest, especially in periods of recession, when the macroeconomic policies should favor the recovery of production. We shall illustrate the assertion by the example of investment, whose key role in social development is universally recognized.

1. A first question concerns the influence of this indicator expressed in terms of volume over a given sectoral structure.

1.1. To answer the question "how the output changes in response to a change in GFCF", it is necessary to define a few preliminary calculation assumptions.

1.1a. The first hypothesis concerns the simulated range of the respective variation. It was natural to consider both the possibility of an upward dynamics and that of a contraction. In this respect, the current application has chosen the "plus-minus 5%" range, which is usually enough to identify relevant implications on the macroeconomic level, especially regarding the sense of evolution.

1.1b. "The chosen range is identical or differentiated by sectors?" This is another issue. Since the objective of the exercise aims to identify the influence of the change, the solution of equal percentage was preferred. In other words, the sectoral composition of GFCF is constant.

1.1c. As the simulation involves the entire input-output table, it was also necessary to specify the initial data set. Taking into account that the matrix (I–A)–1 is determined on the basis of information for 2007, the total gross fixed capital formation in that year was chosen as a reference.

1.2. The output change is denoted by +Y for the assumption of 5% growth in GFCF and by -Y if this indicator falls by 5%. The symbol V0 is also attached to these values, as they represent the sectoral statistical structure relative to 2007. The behavior of the Romanian economy during a boost in investment (positive or negative) is described in Table 6.

In the economy as a whole, therefore, an increase by 5% in gross fixed capital formation in the sectoral composition recorded in 2007 implied an extension by 0.815% of the global output, the effect being opposite in the case of a contraction of identical proportions.

One may note differences by branches. The most important variations were related to sectors 8 (Constructions) and 6 (Equipment, machinery, transportation equipment, other metal products). Reactivity was weaker in other sectors.

Table 6

Sectors' Output Reaction to a Variation by +/-5% in the Volume of Gross Fixed Capital Formation as according to the Structure of Variant V0 (in % as against the Statistical Level in 2007)

Sector Code	$^{+}$YV0	$^{-}$YV0
1	1.00019	0.99981
2	1.00000	1.00000
3	1.00000	1.00000
4	1.00000	1.00000
5	1.00345	0.99655
6	1.04669	0.95331
7	1.00000	1.00000
8	1.03768	0.96232
9	1.00000	1.00000
10	1.00052	0.99948
Total	1.00815	0.99185

2. The previous version (V0) was based on the use of sectoral structure of GFCF in 2007, as resulting from the statistical records. As expected, extensive processing of input-output tables (with 105 branches for the same year) indicated that the main suppliers of goods and investments were:

- Sector 6 (Equipment, machinery and transport equipment, other metal products) and
- Sector 8 (Constructions).

In 2007, other sectors seem to have a modest contribution to gross fixed capital formation:

- Sector 1 (Agriculture, forestry, hunting and fishing);
- Sector 5 (Textiles, leather, pulp and paper, furniture); and
- Sector 10 (Trade, public and business services).

2.1. To simulate the impact of the sectoral structure of GFCF upon the output, three somewhat different simulations were built.

2.1a. "The 2000-2007 average statistics simulation "(V1) uses the structure resulted from the aggregation of information across the above-mentioned range.

The next steps were followed when obtaining the estimates:

- GFCF deflators with fixed base were determined, using the annual price chain index for gross fixed capital formation;
- with these deflators the nominal values of GFCF were recalculated, which were then added for the entire period (2000-2007);
- the sizes obtained were converted into shares of the total economy.
  In this computation - in addition to the basic version (V0) – sector 7 also appears

with a small amount as a provider of GFCF. Numerical differences occur, of course, in the common sectors.

2.1b. "The 2004-2007average statistics simulation" (V2) was calculated in a similar manner, but for the 2004-2007 interval.

2.1c. Finally, "the projected simulation" (V3) involves the exogenous definition of the GFCF structure, based both on the experience of the last year, and also on the possible corrections envisaged by macroeconomic policies (stimulating the development of certain sectors, slowing down the dynamics of others, etc.).

For purely illustrative purposes, in this paper the average shares of 2000-2007 were amended as follows:

- sector 6 contribution was increased, having in view an enhancement of technological investment;
- sector 1 contribution was increased somewhat, to reflect the consistent revival of livestock breeding, tree-growing plantations, and forest farming;
- consequently the weights of other sectors, have been rounded and slightly decreased.

The rest of the application is similar to simulation V1.

2.1d. Table 7 shows the structures of simulations V1, V2 and V3 as compared to V0.

Table 7

Sector Shares (%) in Gross Fixed Capital Formation

Sector Code	Year 2007 (V0)	Average 2000-2007 (V1)	Average 2004-2007 (V2)	Illustrative Projection (V3)
1	0.15527	0.22894	0.285	1
2	0	0	0.000	0
3	0	0	0.000	0
4	0	0	0.000	0
5	2.16853	1.69317	1.750	1.7
6	46.14318	47.95672	47.914	50
7	0	0.00602	0.001	0.006
8	49.19515	46.53207	46.916	45
9	0	0	0.000	0
10	2.33788	3.58308	3.134	2.294
Total	100	100	100	100

Source: Authors' own computations.

2.2. Simply changing the structure of gross fixed capital formation changes the economic output (YV1, YV3 and YV2 simulations in Table 8), compared to statistics for 2007.

Table 8

Output Reaction to the Sectoral Change of GFCF (in % as against 2007)

Sector Code	YV1	YV2	YV3
1	0.998578	1.000828	1.020888
2	1	1	1
3	1	1	1
4	1	1	1
5	0.954002	0.967889	0.985092
6	0.480614	0.708608	1.078044
7	1.00004	1.000015	1
8	0.550726	0.722524	0.935734
9	1	1	1
10	0.996079	0.99838	0.999805
Total	0.9065	0.944836	0.99999

Source: Authors' own computations.

3. If the variation by+/-5% in the GFCF volume is applied to the three types of sectoral structure, the changes in output as compared to actual data on 2007, is as follows (Table 9):

Table 9

Output Reaction to Changes in the Sectoral Structure of GFCF in Terms of Volume Variation by +/-5%

Sector Code	$^{+}$YV1	$^{-}$YV1	$^{+}$YV2	$^{-}$YV2	$^{+}$YV3	$^{-}$YV3
1	0.99870	0.99846	1.00106	1.00059	1.01031	1.00896
2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5	0.95515	0.95285	0.96973	0.96604	1.00138	0.99468
6	0.50133	0.45990	0.74072	0.67649	1.08938	0.99194
7	1.00004	1.00004	1.00002	1.00001	1.00000	1.00000
8	0.56594	0.53551	0.74633	0.69872	1.00536	0.93308
9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
10	0.99676	0.99608	0.99930	0.99838	0.99760	0.99684
Total	0.91010	0.90315	0.95040	0.93961	1.00764	0.99139

Source: Authors' own computations.

One may note that changing the structure of sectoral gross fixed capital formation can significantly influence the dynamics of economic output.

Quantification – by using the input-output techniques – of the driving effects of direct and indirect interdependencies in the economy is, therefore, likely to put more clearly in evidence how important it is for Romania, even on short term, to boost and consistently promote investment programs.

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The authors intend to develop in several directions the research approach initiated by the current study.

In a first phase, they aim to extend the analysis to disaggregated input-output tables, with 34 and 105 branches, respectively. Future research will also focus on wider implementation of these techniques in the medium-term predictions.

Thus, more robust solutions will be sought for the forecasting of matrix A, which take into account in a realistic way the likely developments of technological progress and other factors that impact on the intermediate consumption of different branches (developing and refining the solutions advanced in Dobrescu, 2006).

More reliable methods to predict the final demand and its main components, including the gross fixed capital formation will also be tested.

To the extent that the sources of information will permit, we will proceed to the development of sectoral production functions, able to facilitate the operational approach of general equilibrium models.

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